ELECTRICITY AND MAGNETISM

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REFERENCE.

(1) UNIVERSITY PHYSICS; 6th EDITION, 1981, BY; Francis W. Sears, Mark W. Zemansky and Hugh D. Young.
(3) Physics; 5th Printing, 1980, By; Marcelo Alonso and Edward J. Finn.
Chapter One: Electric Interaction
1.1- Electric Charge

Since there are two kinds of electrification, there are also two kinds of electric charge: positive and negative. A body exhibiting positive electrification has a positive electric charge, and one with negative electrification has a negative electric charge.

The net charge of a body is the algebraic sum of its positive and negative charges. A body having equal amounts of positive and negative charges (i.e., zero net charge) is called electrically neutral. On the other hand, a particle having a nonzero net charge is often called an ion. Since matter in bulk does not exhibit gross electrical forces, we may assume that it is composed of equal amounts of positive and negative charges. Bodies with like charges repel each other and bodies with unlike Charges attract each other.

1.2 Coulomb's Law:

The electrostatic interaction between two charged particles is proportional to their charges and to the inverse of the square of the distance between them, and its direction is along the line joining the two charges.

This may be expressed mathematically by

\[ F = \frac{k q_1 q_2}{r^2} \] (1.1)

\[ F = K e q_1 q_2 /r^2 \] (1.2)
Where $r$ is the distance between the two charge $q_1$ and $q_2$, $F$ is the force acting on either charge, and $K_e$ is a constant to be determined by our choice of units. For practical and computational reasons, it is more convenient to express $K_e$ in the form

$$K_e = \frac{1}{4\pi} \quad (1.3)$$

Where the new physical constant is called the vacuum permittivity. Accordingly, we shall normally write Eq.(1.2) in the form,

$$F = \frac{q_1q_2}{4\pi r^2} \quad (coulombs\ law) \quad (1.4)$$

When using eq.(1.4), we must include the charges $q_1$ and $q_2$ with their signs. A negative value of $F$ corresponds to attraction and a positive value corresponds to repulsion.

1.3 Unit Of Charge

Matter is made up of neutral atoms. A neutral atom consists of $Z$ negatively charged electrons surrounding a positively charged nucleus containing $Z$ positively charged protons and $A - Z$ neutrons. $Z$ is the atomic of the nucleus.

Using MKS system;

$$K_e = 10^{-7} \text{ c}^2$$

Where $c = 2.9979 \times 10^8 \text{ m/s}$ is the velocity of light in vacuum.

For practical purposes, we may say that $K_e = 9 \times 10^9 \text{ N.m}^2/\text{c}^2$.

Then $$F = (10^{-7}c^2) q_1 q_2/ r^2$$

Or $$F = 9 \times 10^9 q_1 q_2/r^2 \quad (1.5)$$

Coulomb; Is that charge which, when it is placed one meter from an equal charge in vacuum, repels it with a force of $10^{-7}c^2$ or $9 \times 10^9$ Newton's
1.4 Electric Field:
In any region where an electric charge experiences a force we say there is an electric field.
An electric field \( E \) exists at a point in space if there is a force \( F \) of electric origin on a test charge \( q \) at rest at point.
The electric field is defined as:

\[
E = \frac{F}{q} \quad \text{or} \quad F = qE
\]  

(1.6)
The intensity of electric field at a point is equal to the force per unit charge placed at that point. The electric field intensity \( E \) is expressed in Newton/Coulomb (N/C) or, using the fundamental units, \((\text{m} \cdot \text{kg} / \text{s}^2 \cdot \text{c}^{-1})\).

Electric field →\( E \)
Positive charge \(+\) →\( F=qE \)
Negative charge \(−\) →\( F=qE \)
Fig.(1.6) Direction of the force produced by an electric field on appositive and a negative charge.
If \( q \) is positive then \( F \) has the same direction of \( E \). If \( q \) is negative then \( F \) has opposite direction of \( E \).(fig.1.6).
An electric field may be represented by lines of force, which are the lines that, at each point, are tangent to the direction of the electric field at that point.
If we apply an electric field to a region where positive and negative particles or ions are present, the field will tend to move the positive and negative charged bodies in opposite directions, result charge separation, an effect called polarization.

Motion of an electric charge in a uniform electric field.

Uniform electric field is that field \( E \) has the same intensity and direction everywhere, a uniform electric field is represented by parallel lines of force. The best way of producing a uniform
electric field is by charging, with equal and opposite charges, two parallel plates.

The equation of motion of electric charge in E is given by equations;

\[ ma = F \quad \text{and} \quad F = qE \]

\[ ma = qE \quad \text{or} \quad a = \frac{qE}{m} \quad (1.7) \]

where, \( m \) is the mass of the charged particle, and \( a \) is its acceleration, \( q \) is the charge of the charged particle.

The acceleration of a body in an electric field depends therefore on the ratio \( q/m \). Since this ratio is in general different for different charged particles or ions, their acceleration is an electric field are also different.

The fig.(1.8) is illustrate the charged particles which passing through an electric field.

Fig.(1.8) Deflection of a positive charge by a uniform electric field.
We assume that the initial velocity \( V_0 \) of the particle when it enters the field is perpendicular to the direction of the electric field.

From the equation of motion in mechanics, we have;

\[
x = V_0 t \quad \text{and} \quad y = \frac{1}{2} at^2 \quad (1.8)
\]

Eliminating the time \( t \), by using eq.(1.7) we obtain

\[
y = \frac{1}{2} \left( \frac{q}{m} \right) \left( \frac{E}{V_0^2} \right) x^2 \quad (1.9)
\]

thus verifying that it is a parabola. We obtain the deflection \( a \) by calculating the slope \( dy/dx \) of the path at \( x=a \). The result is

\[
tana = (dy/dx)_{x=a} = \frac{qEa}{mV_0^2} = \frac{d}{L}
\]

If we place a screen \( S \) at distance \( L \), the particle with given \( q/m \) and velocity \( V_0 \) will reach a point \( c \) on the screen. Noting that \( tana \) is also approximately equal to \( d/L \) because the vertical displacement BD is small compared with \( d \) if \( L \) is large, we have

\[
\frac{qEa}{mV_0^2} = \frac{d}{L} \quad (1.10)
\]

By measuring \( d, L, a \) and \( E \), we may obtain the velocity \( V_0 \) (or the kinetic energy) if we know the ratio \( q/m \); or, conversely, we may obtain \( q/m \) if we knew \( V_0 \). Therefore when a stream of particles, all having the same ratio \( q/m \), passes through the electric field, they are deflected according to their velocities or energies. A device such as the one illustrated in fig.(1.8) may be used as an energy analyzer, which separates identical charged particles moving with different energies. For example, \( \beta \)-rays are electrons emitted by some radioactive materials; if we place a beta emitter at \( o \), all the electrons will concentrate at the same spot on the screen if they have the same energy. But if they are emitted with different energies, they will spread over a region of the screen. It is this second
situation that is found experimentally, a result of great impotence from the point of view of nuclear structure.

1.5 Electric Field Of A point Charge

Let us write the equation of Coulomb's law in the form

\[ F = q_2 \left( \frac{q_1}{4 \pi r^2} \right) \]

This gives the force produced by the charge \( q_1 \) on the charge \( q_2 \) placed a distance \( r \) from \( q_1 \). We may also say, eq.(1.6), that the electric field \( E \) at the point where \( q_2 \) is placed is such that

\[ F = q_2 \]

Therefore, by comparing both expressions for \( F \), we conclude that the electric field at distance \( r \) from a point charge \( q_1 \) is

\[ E = \frac{q_1}{4 \pi r^2} \]  \hspace{1cm} (1.11)

Or in vector form,

\[ \mathbf{E} = \frac{q_1}{4 \pi r^2} \mathbf{\hat{r}} \]  \hspace{1cm} (1.12)

Where \( \mathbf{\hat{r}} \) is the unit vector in the radial direction, away from the charge \( q_1 \), since \( F \) is along this direction. Expression (1.12), is valid for both positive and negative charges, with the direction of \( \mathbf{E} \) relative to \( \mathbf{\hat{r}} \) given by the sign of \( q_1 \). Thus \( \mathbf{E} \) is directed away from a positive charge and toward a negative charge.

Figure(1.9a) indicates the electric field at points near a positive charge, and fig.(1.9b) indicates the electric field near a negative charge. The lines of force of the electric field of a positive and of a negative charge are shown in fig.(1.10). They are straight lines passing through the charge.

When several charges are present, as in fig.(1.11) the resultant electric field is the vector sum of the electric fields produced by each charge.
That is,

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots + \mathbf{E}_n \]  

(1.13)

\[ \mathbf{E} = \sum_i q_i \mathbf{\hat{r}}_r / 4\pi \ r_i^2 \]  

(1.14)

If the charge distribution is continuous:

\[ \mathbf{E} = \sum_i q_i \mathbf{\hat{r}}_r / 4\pi \ r_i^2 \]  

\[ \mathbf{E} = 1/4\pi \ dq / r^2 \ \mathbf{\hat{r}}_r \]  

(1.5)

Where \( \mathbf{\hat{r}}_r \) points from the charge increment \( dq \) to the field point.

1.6 Electric Potential

A charged particle placed in an electric field has potential energy of its interaction with the field. The electric potential at a point is defined as the potential energy per unit charge placed at the point. Designating the electric potential energy of a charge \( q \) by \( U \), we have

\[ V = U / q \quad \text{or} \quad U = qV \]  

(1.16)

The electric potential is measured in Joules/Coulomb or J/C, a unit called volt, abbreviated \( V \) (\( V = m^2 \text{kg./s}^2 \text{c} \)).

If a charge \( q \) moves from one point \( p_1 \) to another point \( p_2 \) along any path, the work done by the electric field is

\[ W = U_{p1} - U_{p2} = q (V_1 - V_2) \] ,
Which gives the difference in potential between points $p_1$ and $p_2$ as,

$$V_1 - V_2 = \frac{W}{q}$$

(1.17)

Thus we may define the electric potential difference between two points as the work done by the electric field in moving the unit charge from one point to the other. For example, there is a potential difference of one volt between two points if the electric field does a work of one joule in moving a charge of one coulomb from one point to the other.

The electric field is the negative of the gradient of the electric potential; that is,

$$\hat{E} = -\text{grad } V$$

Accordingly, the component $E_x$ of the electric field along the direction corresponding to a displacement $dx$ is given by

$$E_x = -\frac{dv}{dx}$$

(1.18)

The negative sign shows that the electric field points in the direction in which the electric potential decreases.

Equation (1.18) indicates that the electric field can also be expressed in volts/meter, a unit which is equivalent to Newton/coulomb given before. This can be seen in the following way:

$$\text{Volt/meter} = \text{joule/coulomb-meter} = \text{Newton-meter/coulomb-meter} = \text{Newton/coulomb}$$

By common usage, the term volt/meter, abbreviated $v/m$, is preferred to $\text{N/C}$.

Equation (1.18) is used to find the electric potential $V$ when the field $\hat{E}$ is known, and conversely. We shall illustrate the method two simple cases.

(a) Uniform Electric Field,

Placing the $x$-axis parallel to the uniform electric field fig.(1.19), we may write,
\[ E = -\frac{dv}{dx} \]

**Fig.(1.19) uniform electric field**

since \( E \) is constant, and we assume \( v=0 \) at \( x=0 \) by integration, we have

\[
dv = -Edx = -E dx
\]

\[ V = -Ex \quad (1.19) \]

This is a very useful relation which has been represented graphically in Fig.(1.20). we may note that, because of the negative sign in Eq.(1.19), the electric field points in the direction in which the electric potential decreases. when we consider two points \( x_1 \) and \( x_2 \), Eq.(1.19) gives

\[ V_1 = -Ex_1 \text{ and } V_2 = -Ex_2. \] subtracting, we have \( V_2-V_1 = -E (x_2-x_1) \) or calling \( d=x_2-x_1 \), we obtain

\[ E = \frac{V_2-V_1}{d} \quad \text{or} \quad E = \frac{V_1-V_2}{d} \]

If the potential difference \( V_1 - V_2 \) is positive, the field points in the direction from \( x_1 \) to \( x_2 \), and if it is negative, the field points in the opposite direction.

**(b) Electric Potential of a point charge**
To obtain the electric potential due to a point charge, we use Eq. (1.18) with x replaced by the distance \( r \), since the electric field it produces is along the radius that is, 
\[
E = - \frac{dv}{dr}
\]

Remembering Eq. (1.11) \( E = \frac{q}{4\pi r^2} \)

We may write, \( \frac{q}{4\pi r^2} = - \frac{dv}{dr} \)

Integrating, and assuming \( V = 0 \) for \( r = \infty \), we obtain

\[
\int_{0}^{r} dv = - \frac{q}{4\pi} \frac{dr}{r^2}
\]

The result of these integration is

\[
V = \frac{q}{4\pi} r
\]  

(1.21)

The electric potential \( V \) is positive or negative depending on the sign of the charge \( q \).

If we have several charges \( q_1, q_2, q_3, \ldots \), the electric potential at a point \( p \) (Fig. 1.11) is the scalar sum of their individual potentials, that is, 

\[
V = \frac{q_1}{4\pi r_1} + \frac{q_2}{4\pi r_2} + \frac{q_3}{4\pi r_3} + \ldots = \frac{1}{4\pi} \sum_i q_i/r_i
\]  

(1.22)

If we place a charge \( q_2 \) at a distance \( r \) from a charge \( q_1 \), the potential energy of the system is \( U_p = q_2 V \), or

\[
U_p = \frac{q_1 q_2}{4\pi r}
\]  

(1.23)

and the potential energy of a system of charge is

\[
U_p = \sum_{all \ pairs} q_1 q_2 /4\pi r
\]

Surfaces having the same electric potential at all points - that is, \( V = \text{constant} \) - are called equipotential surfaces. At each point of an equipotential surface, the direction of the electric field is
Perpendicular to the surface, that is, the line of force are orthogonal to the equipotential surfaces.

1.6 Energy Relations In An Electric Field

The total energy of a charged particle or ion of mass \( m \) and charge \( q \) moving in an electric field is,

\[
U = U_k - U_p = \frac{1}{2}mv^2 + qV
\]  

(1.25)

When the ions move from position \( p_1 \) (where the electric potential is \( V_1 \)) to position \( p_2 \) (where the potential is \( V_2 \)), Eq. (1.25), combined with the principle of conservation of energy gives,

\[
\frac{1}{2}mv_1^2 + qV_1 = \frac{1}{2}mv_2^2 + qV_2,
\]

which we may write as

\[
\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = q(V_1 - V_2)
\]  

(1.26)

Note from Eq.(1.26) that a positively charged particle \((q > 0)\) gains kinetic energy when moving from a larger to a smaller potential \((V_1 > V_2)\), while a negatively charged particle \((q < 0)\), to gain energy, has to move from a lower to a higher potential \((V_1 < V_2)\).

If we choose the zero of electric potential at \( p_2 \) \((V_2 = 0)\) and arrange our experiment so that at \( p_1 \) the ions have zero velocity \((v_1 = 0)\), eq. (1.26) becomes (dropping the subscripts),

\[
\frac{1}{2}mv^2 = qV
\]  

(1.27)

an expression that gives the kinetic energy acquired by a charged particle when it moves.