**p – n junction**

The forward I-V characteristics of pn junction diodes are generally represented by

\[ I = I_o \left[ \exp \left( \frac{eV}{\eta kT} \right) - 1 \right] \]

where \( \eta \) is an ideality factor between 1 and 2 depending on the semiconductor material. Figure 1 shows the pn junction forward I-V characteristics of three different semiconductor diodes: Ge with \( \eta = 1 \), Si with \( \eta = 2 \) for \( V < 0.6 \) V and GaAs with \( \eta = 2 \). For semiconductor materials with \( E_g > \sim 1 \) eV, generally, \( \eta = 2 \) and the diode current is typically due to recombination of injected holes and electrons in the depletion region. The external current supplies the carriers lost by recombination and the I-V characteristics are given by the diode equation with \( \eta = 2 \).

In pn junctions made from semiconductors with an energy bandgap \( E_g \) > \( \sim 1 \) eV, typically the recombination current dominates the pn junction current as shown in Figure 1 for the case of GaAs and Si at lower diode currents. The observed current through the pn junction supplies the carriers that are lost by recombination in the space charge layer (SCL). The dependence of the current density \( J \) on the applied voltage is closely represented by

\[ J = J_o \exp \left( \frac{eV}{2kT} \right); \quad V >> kT/e \]

where \( J_o \) is a constant (temperature dependent) and the other symbols have their usual meanings. This equation cannot be derived in a simple fashion but it is nonetheless possible to argue a semiquantitative basis for its functional form.

When a forward bias is applied across a pn junction, the built-in potential \( V_o \) is reduced to \( V_o - V \), and minority carriers become injected across the junction. To calculate the recombination current, we need to calculate the rate of recombination of injected minority carriers across the junction from A to B. Consider the flow of one electron around the external circuit (driven by the external EMF) as depicted in Figure 2. When the electron enters the external circuit from the p-side it is equivalent to a hole entering the p-side. This electron actually leaves the valence band of the semiconductor to enter the external circuit. The electron in the external circuit, after going around the circuit, enters the n-side so that it supplies the n-side with an electron. Although there is one electron flowing around the external circuit, there is a hole flowing from the p to n-side and an electron flowing from the n to p-side. One externally flowing electron is therefore equivalent to one hole and one electron recombining in the depletion region. To find the total diode current, we only need to calculate the rate of recombination for electrons in AM and that for holes in BM.
Schematic sketch of typical $I-V$ characteristics of Ge, Si and GaAs $pn$ junctions as $\log(I)$ vs $V$. The slope indicates $e/(\eta kT)$ where $\eta$ is the ideality factor in the diode equation $I = I_0 \exp(eV/\eta kT)$; $V > kT/e$.

**Figure 1**

|pn| Junction $I-V$ Characteristics

The current density in a forward biased $pn$ junction is generally described by the Shockley equation,

$$J = \left( \frac{eD_n}{L_h N_d} + \frac{eD_p}{L_e N_a} \right) \eta_i \left[ \exp \left( \frac{eV}{kT} \right) - 1 \right]$$

where $e$ is the electronic charge, $k$ is Boltzmann’s constant, $T$ is temperature (K), $V$ is the voltage across the $pn$ junction, $\eta_i$ is the intrinsic concentration, $D$ is the diffusion coefficient, $L$ is the diffusion length and $N_a$ and $N_d$ are the acceptor and donor doping concentrations respectively. The subscripts $e$ and $h$ refer to
electrons and holes, respectively, as minority carriers; that is, holes in the $n$-side and electrons in the $p$-side. If $\tau$ is the charge carrier lifetime (recombination time) then $L = \sqrt{D\tau}$. The Shockley expression neglects the current component that is due to recombination in the depletion region, that is in the space charge layer (SCL). The electron and hole concentrations across the device are depicted (in an exaggerated way) in Figure 1. The application of a forward bias leads to the injection of minority carriers into the neutral regions of the diode. The minority carrier concentrations (e.g. holes and electron concentrations) at the space charge layer (SCL) boundaries in the neutral regions ($n$- and $p$-regions respectively) are represented as $p_n(0)$ and $n_p(0)$. In a long diode the minority carrier concentration profile falls exponentially towards the electrode, which means that there is a concentration gradient and hence diffusion. The minority carriers therefore diffuse towards the bulk giving rise to a diode current. These arguments lead to the Shockley equation stated in Equation (1) for a $p^n$ junction long diode.

Forward biased $pn$ junction and the injection of minority carriers. Carrier concentration profiles across the device under forward bias. Note: SCL = space charge layer and $W$ = width of the SCL with forward bias. Other symbols have their usual meanings.

Figure 1
The reverse current density component due to thermal generation of electron-hole pairs (EHPs) within the depletion region, as depicted in Figure 2, is given by

\[
J_{\text{gen}} = \frac{eWn_i}{\tau_g}
\]

where \( W \) is the width of the depletion region and \( \tau_g \) is the mean thermal generation time. Thermal generation of EHPs in the depletion region occurs through generation-recombination centers and depends on carrier concentrations, crystal defects and impurities. \( \tau_g \) in Equation (2) represents a mean thermal generation time calculated by integrating the rate of thermal generation across the depletion region.

There is also a contribution to the reverse current arising from the thermal generation of minority carriers in the neutral regions within a diffusion length to the SCL, their diffusion to the SCL, and subsequent drift through the SCL (Figure 2). This is essentially the Shockley model with a negative voltage, that is Equation (1) with a reverse bias. The battery replenishes the minority carriers that are lost in this way from the neutral regions. Stated differently, there is a reverse current due to the diffusion of minority carriers in neutral regions towards the SCL.

The width of the depletion region with a reverse bias \( V = -V_r \) is given by

\[
W = \left[ \frac{2\pi(N_a + N_d)(V_o + V_r)}{eN_aN_d} \right]^{1/2}
\]

where \( V_o = (kT/\epsilon)\ln((N_aN_d)/n_i^2) \) is the built-in voltage, \( \epsilon = \epsilon_0\varepsilon_r \) is the permittivity of the semiconductor material. Equation (3) assumes an abrupt pn junction.
Reverse biased $pn$ junction. Minority carrier concentration profiles and the origin of the reverse current. Note: EHP = electron-hole pair, SCL = space charge layer, $E = $ electric field, $W =$ width of the SCL with reverse bias, $V_r =$ reverse bias. Other symbols have their usual meanings. Subscript $o$ refers to "no-bias" condition.

**Figure 2**

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**Problem: The $pn$ junction**

Consider a long Si diode made of an abrupt $p^n$ junction which has $10^{12}$ donors $\text{cm}^{-3}$ on the $n$-side and $10^{18}$ acceptors on the $p$-side. The dependence of the hole and electron drift mobility on the dopant concentration is shown in Figure 3. The minority carrier recombination times are $\tau_h = 490$ ns for holes in the $n$-side and $\tau_e = 23.8$ ns for electrons in the $p^+$-side. The cross sectional area is 0.1 mm$^2$. Assume a long diode. The thermal generation time $\tau_g$ in the depletion region is $\sim 1$ ms. Assume that the reverse current is dominated by the thermal generation rate in the depletion region.

a) Calculate the forward current at 27 °C when the voltage across the diode is 0.6 V.

b) Estimate the forward current at 57 °C when the voltage across the diode is still 0.6 V.

c) Calculate the voltage across the diode at 57 °C if the forward current in a above at 27 °C is kept constant.
d What is the reverse current at 27 °C when the diode voltage is −5 V?

e Estimate the reverse current at 57 °C when the diode voltage is −5 V.

*Note:* Assume that the forward current is determined by the Shockley equation (minority carrier diffusion).

The variation of the drift mobility with dopant concentration in Si for electrons and holes.

**Figure 3**

**Solution**

**a** Consider room temperature, $T = T_i = 300$ K. $kT/e = kT_i/e = 0.2585$.

The general expression for the diffusion length is $L = \sqrt{D\tau}$ where $D$ is the diffusion coefficient and $\tau$ is the carrier lifetime. $D$ is related to the mobility of carriers, $\mu$, via the Einstein relationship, $D/\mu = kT/e$. We therefore need to know $\mu$ to calculate $D$ and hence $L$. Electrons diffuse in the $p$-region and holes in the $n$-region so that we need $\mu_e$ in the presence of $N_a$ acceptors and $\mu_h$ in the presence of $N_d$ donors.

From the drift mobility, $\mu$ vs. dopant concentration for silicon graph we have the following:

$N_a = 10^{18}$ cm$^{-3}$, $\mu_a \approx 250$ cm$^2$ V$^{-1}$ s$^{-1}$

$N_d = 10^{11}$ cm$^{-3}$, $\mu_h \approx 450$ cm$^2$ V$^{-1}$ s$^{-1}$

Thus,

$D_e = kT\mu_e/e \approx (0.02585 \text{ V})(250 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 6.46 \text{ cm}^2 \text{ s}^{-1}$

and

$D_h = kT\mu_h/e \approx (0.02585 \text{ V})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 11.63 \text{ cm}^2 \text{ s}^{-1}$

Diffusion lengths are

$L_e = \sqrt{D\tau_e} = \sqrt{(6.46 \text{ cm}^2 \text{ s}^{-1})(2.38 \times 10^{-5} \text{ s})} = 3.93 \times 10^{-4} \text{ cm}, \text{ or } 3.93 \mu\text{m}$

and

$L_h = \sqrt{D\tau_h} = \sqrt{(11.63 \text{ cm}^2 \text{ s}^{-1})(490 \times 10^{-9} \text{ s})} = 2.39 \times 10^{-3} \text{ cm}, \text{ or } 23.9 \mu\text{m}$

The built-in potential is
\[ V_o = \left( kT/e \right) \ln\left( N_d N_a / n_i^2 \right) = \left( 0.02585 \text{ V} \right) \ln\left( \left( 10^{18} \text{ cm}^{-3} \times 10^{15} \text{ cm}^{-3} \right) / \left( 1.45 \times 10^{10} \text{ cm}^{-3} \right)^2 \right) \]
\[ V_o = 0.755 \text{ V} \]

To calculate the forward current when \( V = 0.6 \text{ V} \), we need to evaluate both the diffusion and recombination components of the current. It is likely that the diffusion component will exceed the recombination component at this forward bias (this can be easily verified). Assuming that the forward current is due to minority carrier diffusion in neutral regions,

\[ I = I_{so} \{ \exp(eV/kT) - 1 \} = I_{so} \exp(eV/kT) \text{ for } V \gg kT/e \approx 26 \text{ mV} \]

where,

\[ I_{so} = AJ_{so} = A \eta_{i} [ (D_i/L_N) + (D_e/L_N) ] \approx A \eta_{i} D_i / L_N \]

as \( N_a \gg N_d \). In other words, the current is mainly due to the diffusion of holes in the n-region. Thus,

\[ I_{so} = \frac{0.1 \times 10^{-2} \text{ cm}^{2} \times (1.6 \times 10^{-19} \text{ C}) \times (1.45 \times 10^{10} \text{ cm}^{-3})^2 \times (11.66 \text{ cm}^{2} \text{ s}^{-1})}{23.9 \times 10^{-3} \text{ cm} \times (1 \times 10^{13} \text{ cm}^{-3})} \]

\[ I_{so} = 1.64 \times 10^{-13} \text{ A or } 0.164 \text{ pA} \]

The forward current is then

\[ I = I_{so} \exp(eV/kT) = (1.64 \times 10^{-13} \text{ A}) \exp\left( \frac{0.6 \text{ V}}{0.02585 \text{ V}} \right) \]
\[ I = 0.00197 \text{ A or } 1.97 \text{ mA} \]

b Consider \( T = T_2 = 57 \text{ K }+ 273 \text{ K } = 330 \text{ K } , kT/e = 0.02844 \). First, find the new \( n_i \) from \( n_i = \left( N_d N_a \right)^{1/2} \exp\left( -E_g / 2kT \right) \). Thus,

\[ \frac{n_i(T_2)}{n_i(T_1)} = \left( \frac{T_2^{3/2} \exp\left( \frac{-E_g}{2kT_2} \right)}{T_1^{3/2} \exp\left( \frac{-E_g}{2kT_1} \right)} \right) \]

substituting, \( T_1 = 300 \text{ K } , T_2 = 330 \text{ K } , n_i(300 \text{ K }) = 1.45 \times 10^{10} \text{ cm}^{-3} , E_g = 1.1 \text{ eV} \), we find \( n_i(330 \text{ K } ) = 1.16 \times 10^{11} \text{ cm}^{-3} \). Assuming that the temperature dependence of \( n_i \) dominates those of other terms in \( I_{so} \), the new \( I'_{so} \) is,

\[ I'_{so} = I_{so} \left( \frac{n_i(T_2)}{n_i(T_1)} \right)^2 \]

i.e.

\[ I'_{so} = (1.64 \times 10^{-13} \text{ A}) \left[ \frac{1.16 \times 10^{11} \text{ cm}^{-3}}{1.45 \times 10^{10} \text{ cm}^{-3}} \right]^2 = 1.05 \times 10^{-11} \text{ A} \]

The forward current is then
\[ I' = I'_0 \exp(eV / kT_2) = (1.05 \times 10^{-11} \text{ A}) \exp[(0.6 \text{ V})/(0.02844)] = 0.0152 \text{ A} \]

Suppose that the current is kept constant through the \textit{pn} junction from 27 °C to 57 °C. that is, \( I' = I = 0.00197 \text{ A} \). Suppose that the voltage across the \textit{pn} junction is now \( V' \). Then,

\[ I'_0 \exp(eV' / kT_2) = I = 0.00197 \text{ A} \]

Thus,

\[ V' = (kT_2 / e) \ln(I'/I'_0) = (0.02844 \text{ V}) \ln[(0.00197 \text{ A})/(1.05 \times 10^{-11} \text{ A})] = 0.542 \text{ V} \]

Notice that the voltage across the \textit{pn} junction decreases with temperature when the current through it is kept constant.

\[ \text{d} \]

When a reverse bias of \( V_r \) is applied, the potential difference across the depletion region becomes \( V_o + V_r \), and the width \( W \) of the depletion region is

\[ W = \left[ \frac{2e(N_a + N_d)(V_o + V_r)}{eN_a N_d} \right]^{1/2} = \left[ \frac{2e(V_o + V_r)}{eN_d} \right]^{1/2} \]

\[ W = \left[ \frac{2(1.9)(8.85 \times 10^{-12} \text{ F m}^{-2})(0.755 \text{ V} + 5 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{15} \text{ cm}^{-3} \times 10^{6} \text{ m}^{-3} / \text{ cm}^{-3})} \right]^{1/2} \]

\( i.e. \)

\[ W = 2.75 \times 10^{-6} \text{ m or 2.75 \mu m} \]

The thermal generation current \( I_{gen} \) with \( V_r = 5 \text{ V} \) is,

\[ I_{gen} = \frac{eAWn_i}{\tau_g} \]

\[ I_{gen} = \frac{(1.6 \times 10^{-15} \text{ C})(0.001 \text{ cm}^3)(2.75 \times 10^{-4} \text{ cm})(1.45 \times 10^{10} \text{ cm}^{-3})}{(1 \times 10^{-3} \text{ s})} \]

\[ I_{gen} = 6.39 \times 10^{-13} \text{ A or 0.639 \ pA} \]

The total reverse current is due to thermal generation in the depletion region and diffusion in the neutral regions.

\[ I_{re} = I_{gen} + I_{sp} = 0.639 \text{ pA} + 0.164 \text{ pA} = 0.80 \text{ pA} \]

\( \text{e} \)

Estimation of the reverse current at 57 °C is difficult because we need to know the temperature dependence of \( \tau_g \). If \( \tau_g \) were to remain very roughly the same then \( I_{gen} \propto n_i \) and the new thermal generation current would be,

\[ I'_{gen} = I_{gen} \cdot \frac{n_i(330 \text{ K})}{n_i(300 \text{ K})} = (0.639 \text{ pA}) \frac{(1.16 \times 10^{11} \text{ cm}^{-3})}{(1.45 \times 10^{10} \text{ cm}^{-3})} = 5.10 \text{ pA} \]

However, the reverse saturation current \( I_{ss} \propto n_i^2 \) which leads to \( I'_{ss} = 10.5 \text{ pA} \) at 57 °C as calculated above. It is clear that the diffusion component is now greater than the thermal generation component. The total reverse current at 57 °C.
\[ I'_{re} = I'_{ken} + I'_{so} = 5.1 \text{ pA} + 10.5 \text{ pA} = 15.6 \text{ pA} \]

Fig. 6.15: Reverse \( I-V \) characteristics of a \( pn \) junction.
Fig. 5.28: A semiconductor slab of length $L$, width $W$ and depth $D$ is illuminated with light of wavelength $\lambda$. $I_{ph}$ is the steady state photocurrent.