Weekly Hours: Theoretical: 2 UNITS: 5
Tutorial: 1
Experimental: 1

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ART. 2/2 FORCE

Replace the two forces by a single equivalent force R and find the angle θ between R and the x-axis. Solve both geometrically and by using unit vectors i and j.

**Geometric**

**Graphical**: construct parallelogram & measure R & θ.

**Trigonometric**: Law of cosines

\[ R^2 = 4^2 + 6^2 - 2(4)(6)\cos 80° \]
\[ R = 6.61 \text{ kN} \]

\[ R^2 = (6.61)^2 + 6^2 - 2(6.61)(6)\cos(θ - 30°) \]
\[ θ - 30° = \cos^{-1} 0.8029 = 36.6°, \quad θ = 66.6° \]

**Vector algebra**

\[ R_x = 6 \cos 30° - 4 \cos 50° = 2.63 \text{ kN} \]
\[ R_y = 6 \sin 30° + 4 \sin 50° = 6.06 \text{ kN} \]
\[ R = 2.63i + 6.06j \text{ kN}, \quad θ = \tan^{-1} \frac{6.06}{2.63} = 66.6° \]
ART. 2/3 RECTANGULAR COMPONENTS (2-D)

Force $F$ in rectangular components is given by $F = -40\mathbf{i} + 60\mathbf{j}$ N.
Determine the non-rectangular components of $F$ in the $y$- and $h$-directions.

**Graphical solution:**
Construct $F$ then form parallelogram. Measure $F_y$ and $F_h$.

**Trigonometric solution:**
\[
\alpha = \tan^{-1} \frac{40}{60} = 33.7^\circ, \quad F = \sqrt{(40)^2 + (60)^2} = 72.1 \text{ N}
\]
Law of sines \[
\frac{72.1}{\sin 30^\circ} = \frac{F_h}{\sin 33.7^\circ}, \quad F_h = 80.0 \text{ N}
\]
\[
\beta = 180 - 30 - 33.7 = 116.3^\circ
\]
\[
\frac{F_y}{\sin 116.3^\circ} = \frac{72.1}{\sin 30^\circ}, \quad F_y = 129.3 \text{ N}
\]
ART. 2/4 MOMENT (2-D)

Calculate the moment of the 400-N force about point O in five different ways.

From the geometry
\[ a + 0.04 = 0.120 \tan 60^\circ \]
\[ a = 0.168 \text{ m} \]
\[ 0.120 - b = 0.040 \tan 30^\circ \]
\[ b = 0.0969 \text{ m} \]
\[ d = b \cos 30^\circ \]
\[ = 0.0839 \text{ m} \]

(I) \( M_0 = F_d = 400(0.0839) = 33.6 \text{ N.m} \)

(II) \( M_0 = 400(0.12 \sin 60^\circ - 0.04 \cos 60^\circ) = 33.6 \text{ N.m} \)

(III) \( M_0 = F_x b = 400 \sin 60^\circ (0.0969) = 33.6 \text{ N.m} \)

(IV) \( M_0 = F_y a = 400 \cos 60^\circ (0.168) = 33.6 \text{ N.m} \)

(V) \( M_0 = i x F = (0.04i + 0.12j) \times 400(i \sin 60^\circ + j \cos 60^\circ) \)
\[ = 8k - 4.16k = -33.6k \text{ N.m} \]
ART. 2/5 COUPLÉ (2-D)

Replace the force and couple acting on the wrench by a single equivalent force \( F \) applied at \( D \). Determine \( b \).

Replace 60-N.m couple by an equivalent couple consisting of two 300-N forces a distance \( d \) apart placed to cancel the given force. Thus, resultant is \( F = 300 \text{ N} \) located at \( D \) where

\[
M_A = Fd; \quad 60 = 300d, \quad d = 0.2 \text{ m}
\]

\[
b = 0.2 / \cos 20^\circ = 0.213 \text{ m} \quad \text{or} \quad b = 21.3 \text{ mm}
\]
ART. 2/16 RESULTANTS (2-D)

Represent the resultant of the three forces and one couple by an equivalent force $R$ at $A$ and a couple $M$. Find $M$ and the magnitude and direction of $R$.

\[ R_x = \sum F_x = 2 \sin 30^\circ + 1.5 \]
\[ = 2.5 \text{ kN} \]

\[ R_y = \sum F_y = 2 \cos 30^\circ - 4 \]
\[ = -2.27 \text{ kN} \]

\[ R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.5^2 + 2.27^2} = 3.38 \text{ kN} \]

\[ M = \sum M_A = 1.5 + 4(2) - 2 \cos 30^\circ (2 + 1.5) - 1.5(1.5) \]
\[ = 1.188 \text{ kN} \cdot \text{m CW} \]

\[ \theta = \tan^{-1} \frac{2.27}{2.5} \]
\[ = 42.2^\circ \]
**ART. 217 RECTANGULAR COMPONENTS (3-D)**

For $a = 3\, \text{m}$, $b = 6\, \text{m}$, $c = 2\, \text{m}$, $F = 10\, \text{kN}$, determine the magnitudes of the components of $F$ along $AC$ and $AD$ and the projection of $F$ along $DC$.

Choose $x$-$y$-$z$ axes.

- $AB = \sqrt{3^2 + 6^2 + 2^2} = 7\, \text{m}$
- $AC = \sqrt{2^2 + 6^2} = 2\sqrt{10}\, \text{m}$
- $AD = \sqrt{2^2 + 3^2} = \sqrt{13}\, \text{m}$
- $DC = \sqrt{3^2 + 6^2} = 3\sqrt{5}\, \text{m}$

- $F_{AC} = F \cos \alpha = 10 \frac{2\sqrt{10}}{7} = 9.04\, \text{kN}$
- $F_{AD} = F \cos \beta = 10 \frac{\sqrt{13}}{7} = 5.15\, \text{kN}$

Let $\mathbf{n} = \text{unit vector along } DC = \frac{3}{3\sqrt{5}} \mathbf{i} + \frac{6}{3\sqrt{5}} \mathbf{j}$

- $F = 10 \left( \frac{-3}{7} \mathbf{i} + \frac{6}{7} \mathbf{j} + \frac{2}{7} \mathbf{k} \right) \, \text{kN}$
- $F_{DC} = F \cdot \mathbf{n} = \frac{10}{7} (-3 \mathbf{i} + 6 \mathbf{j} + 2 \mathbf{k}) \cdot \frac{1}{\sqrt{5}} (\mathbf{i} + 2 \mathbf{j})$
  - $= \frac{10}{7\sqrt{5}} (-3 + 12) = 5.75\, \text{kN}$
Determine the moment of the 500-N force \( F \) about the \( x \)-axis.

**Scalar solution**

\[
|F_x| = 500 \sin 30^\circ \cos 60^\circ = 125 \text{ N} \\
|F_y| = 500 \cos 30^\circ = 433 \text{ N} \\
|F_z| = 500 \sin 30^\circ \sin 60^\circ = 217 \text{ N}
\]

\[
M_x = -433 (0.3) + 217 (0.2) = -86.6 \text{ N} \cdot \text{m}
\]

**Vector solution**

\[
\mathbf{r}_A = 0.3 \mathbf{i} + 0.2 \mathbf{j} - 0.3 \mathbf{k} \text{ m}
\]

\[
\mathbf{M}_A = \mathbf{r}_A \times \mathbf{F} = \frac{0.3 \times (-433) - 0.2 \times 500 - 0.3 \times 125}{0.3 \times 0.2 - 0.2 \times (-0.3)} = -86.6 \text{ N} \cdot \text{m}
\]
Replace the two forces and couple by a wrench. Find the moment \( M \) of the wrench and the coordinates of point \( P \) in the y-z plane through which the force of the wrench passes.

\[
R = \Sigma F = 200\hat{i} + 150\hat{j} \text{ N}
\]

Assume positive wrench so direction cosines of \( M \) are those of \( R \) or 0.8, 0.6, 0.

\[
\Sigma M_p = 200(0.3-y)\hat{j} - 200(0.3-y)\hat{k} + 150\hat{i} + 150(0.2)\hat{k} - 30\hat{i}
= (-30 + 150z)\hat{i} + (60 - 200z)\hat{j} + (-30 + 200y)\hat{k} \text{ N•m}
\]

Equate direction cosines of \( \Sigma M_p \) & \( \Sigma F \) & get

\[
\begin{align*}
(-30 + 150z)/M &= 0.8 \\
(60 - 200z)/M &= 0.6 \\
(-30 + 200y)/M &= 0
\end{align*}
\]

Solve & get \( y = 0.15 \text{ m} \) or \( y = 150 \text{ mm} \)

\( z = 0.264 \text{ m} \) or \( z = 264 \text{ mm} \)

\( M = (-30 + 150(0.264))/0.8 = 12 \text{ N•m} \), \( M = 12(0.8\hat{i} + 0.6\hat{j}) \text{ N•m} \)
Determine the pull \( P \) on the rope exerted by the man to hold the crate in the position shown. Also find the tension \( T \) in the upper rope.

**Solution (I) x-y axes**

\[ \Sigma F_x = 0 : \quad 0.866P - \frac{1.5}{5}T = 0 \]
\[ \Sigma F_y = 0 : \quad -0.5P - 200(9.81) + 0.954T = 0 \]

Solve simultaneously and get

\[ P = 871 \text{ N}, \quad T = 2513 \text{ N} \]

**Solution (II) x'-y' axes**

\[ \Sigma F_{x'} = 0 : \quad P \cos(30^\circ + 17.45^\circ) - 200(9.81) \frac{1.5}{5} = 0 \]

\[ P = 871 \text{ N} \]

\[ \Sigma F_{y'} = 0 : \quad T - 871 \sin(30^\circ + 17.45^\circ) - 200(9.81) \cos 17.45^\circ = 0 \]

\[ T = 2513 \text{ N} \]
The uniform 40-kg bar with small end rollers is supported by the horizontal and vertical surfaces and by wire AC. Calculate the tension $T$ in the wire and the forces at $A$ and $B$. Solve by using two moment equations and one force equation.

$$W = mg = 40(9.81) = 392 \text{ N}$$

$$\theta = \tan^{-1} \frac{1.5}{1} = 33.7^\circ$$

$$\sum M_A = 0: \quad 2B - 392 \left(\frac{1.5}{2}\right) = 0$$

$$B = 147.2 \text{ N}$$

$$\sum M_E = 0: \quad (T \cos 33.7^\circ)2 - 392 \left(\frac{1.5}{2}\right) = 0$$

$$T = 176.9 \text{ N}$$

$$\sum F_y = 0: \quad A + 176.9 \sin 33.7^\circ - 392 = 0$$

$$A = 294 \text{ N}$$
Member OBC and sheave C have a mass of 500 kg with mass center at G. Calculate the magnitude of the force supported by the pin at O. Collar A provides horizontal support only.

Replace force by force and couple at C.

\[ \Sigma M_o = 0: \]
\[ 500(9.81)(1.5) - 2A + 3000(4.5 \cos 30^\circ + 3 \sin 30^\circ) + 1500 = 0 \]
\[ A = 12524 \text{ N or } A = 12.52 \text{ kN} \]

\[ \Sigma F_x = 0: \]
\[ 12.52 - 3 \sin 30^\circ - 0_x = 0, \quad 0_x = 11.02 \text{ kN} \]

\[ \Sigma F_y = 0: \]
\[ 0_y - 500(9.81) - 3 \cos 30^\circ = 0, \quad 0_y = 7.50 \text{ kN} \]

\[ O = \sqrt{(11.02)^2 + (7.50)^2} = 13.34 \text{ kN} \]
A high-voltage power line is suspended as shown. Tension in the line at the insulators is 3 kN. Calculate the tension $T$ in link $AD$ and the compression $C$ in links $AB$ and $AC$.

\[ \Sigma F_x = 0: \quad P - 2(3) \sin 15^\circ = 0 \]
\[ P = 1.553 \text{ kN} \]

\[ AC = AD = \sqrt{2^2 + (1.5)^2} = 2.5 \text{ m} \]

\[ \Sigma F_y = 0: \quad T \sin \theta - 1.553 = 0 \]
\[ T = \frac{1.553}{1.5/2.5}, \quad T = 2.59 \text{ kN} \]

\[ \Sigma F_y = 0: \quad 2C \cos \beta - 2.59 \cos \theta = 0 \]
\[ C = \frac{2.59(2/2.5)}{2(2/2.5)}, \quad C = 1.29 \text{ kN} \]
Connections at A, B, C, D are ball & socket joints. Neglect weight of members. Find compression P in legs BD & CD and magnitude of force at A.

\[ 300 (9.81) (4/5) = 2354 \text{ N} \]
\[ 300 (9.81) (3/5) = 1766 \text{ N} \]
\[ W = 300 (9.81) = 2943 \text{ N} \]

\[ \theta = \tan^{-1} \frac{2.4}{1.2} = 63.4^\circ \]

\[ 2P \sin \theta = D \]
\[ P = \frac{D}{2 \sin \theta} = \frac{\sqrt{5} D}{4} \]

\[ \Sigma F_y = 0: A_y - 2354 = 0, \quad A_y = 2354 \text{ N} \]
\[ \Sigma M_G = 0: 1.8A_z + 2354 (4.8) + 2943 (1.8) + 1766 (1.8) = 0, \]
\[ \Sigma F_z = 0: 1570 + 4P/\sqrt{5} - 2943 - 1766 = 0, \quad P = 1755 \text{ N} \]

\[ A = \sqrt{(2354)^2 + (1570)^2} = 2830 \text{ N} \]

[A\text{\_z} = 1570 \text{ N}]
ART. 4/3 METHOD OF JOINTS

Determine the forces in members FG, EG, and GD for the simple truss.

By inspection of joint F, FG = EF = 0

Joint E

\[ \theta = \tan^{-1} \frac{3}{4}, \quad \sin \theta = \frac{3}{5}, \quad \cos \theta = \frac{4}{5} \]

\[ \beta = \tan^{-1} \frac{3}{16} = 10.62^\circ \]

4 kN \[ \Sigma F_x = 0: \quad EG \left(\frac{4}{5}\right) - ED \cos 10.62^\circ = 0 \]

\[ \Sigma F_y = 0: \quad EG \left(\frac{3}{5}\right) + ED \sin 10.62^\circ - 4 = 0 \]

Solve to obtain \( EG = 5.33 \text{ KN T} \)

Joint G

\[ \Sigma F_y = 0: \quad GD - 5.33 \left(\frac{3}{5}\right) = 0 \]

\( GD = 3.20 \text{ KN C} \)
Determine the forces in members DI, DE, and EI for the simple truss.

\[ \sum F_x = 0: \text{DI} \cos 45^\circ - 18 \cos 45^\circ = 0 \]

\[ \text{DI} = 18 \text{ kN} \text{ C} \]

\[ \sum F_y = 0: \text{DE} (3 \cos 45^\circ) - 18 (3) = 0 \]

\[ \text{DE} = 25.5 \text{ kN C} \]

\[ \sum F_z = 0: \text{EI} = 0 \]
Determine the forces in members AD, BD, CD, & ED of the space truss loaded and supported as shown. Verify the adequacy of internal stability.

No. of members \( m = 12 \); No. of joints \( j = 6 \) \((m + 6 = 18) = (3j = 18)\) so members are adequate in number and comprise rigid tetrahedrons.

Joint B: \( \Sigma F_z = 0 \) gives \( \frac{3}{5} F_{BD} - \frac{P}{\sqrt{2}} = 0 \), \( F_{BD} = \frac{5P}{3\sqrt{2}} \) (17)

All unknown forces taken (+) tension

Joint D:

\[
\begin{align*}
-F_{BD} &= F_{BD} \left(-\frac{4}{5}i - \frac{3}{5}k\right) = \frac{P}{3\sqrt{2}} \left(-4j - 3k\right) \\
-F_{CD} &= F_{CD} \left(-k\right) \\
F_{AD} &= F_{AD} \left(-\frac{i}{\sqrt{2}} - \frac{k}{\sqrt{2}}\right) = \frac{F_{AD}}{\sqrt{2}} \left(-i - k\right) \\
F_{ED} &= F_{ED} \left(\frac{j}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \frac{F_{ED}}{\sqrt{2}} \left(i + j\right)
\end{align*}
\]

\( \Sigma F = 0 \) yields

\[
\begin{align*}
\{i\text{-terms:}\} &- F_{AD}/\sqrt{2} - F_{ED}/\sqrt{2} = 0 \\
\{j\text{-terms:}\} &-4P/(3\sqrt{2}) + F_{ED}/\sqrt{2} = 0 \\
\{k\text{-terms:}\} &-P/\sqrt{2} - F_{CD} - F_{AD}/\sqrt{2} = 0
\end{align*}
\]

Solve & get

\[
F_{AD} = -\frac{4P}{3} \quad (c) \quad F_{CD} = \frac{P}{3\sqrt{2}} \quad (7) \quad F_{ED} = \frac{4P}{3} \quad (T)
\]
Determine the total force (shear) supported by the pin at B for the loaded frame.

From FBD of entire frame
$$\Sigma M_A = 0:\]
C_x \left(0.5\sqrt{2}\right) - 50(9.81)\left(\frac{1}{\sqrt{2}} + 0.15\right) = 0
C_x = 595 \text{ N}
$$
$$\Sigma F_x = 0: A_x - 595 = 0, A_x = 595 \text{ N}
$$

From FBD of member BC
$$\Sigma M_B = 0:\]
595\left(0.5\right) - 50(9.81)(0.15) - C_y \left(0.5\sqrt{2}\right) = 0
C_y = 386 \text{ N}
$$
$$\Sigma F_x = 0: B_x + 50(9.81)/\sqrt{2} - 595 = 0, B_x = 248 \text{ N}
$$
$$\Sigma F_y = 0: 386 + 50(9.81)/\sqrt{2} - B_y = 0, B_y = 733 \text{ N}
$$
Total force (shear) $B = \sqrt{(248)^2 + (733)^2} = 774 \text{ N}$
Determine the magnitude of the force supported by the pin at C.

**Entire frame**

\[ \Sigma M_A = 0: \quad 0.4E - 0.6(600) = 0 \]
\[ E = 900 \text{ N} \]

\[ \Sigma F_x = 0: \quad A_x = 600 \text{ N} \]

\[ \Sigma F_y = 0: \quad A_y = 900 \text{ N} \]

**Link CD** \( \Sigma M_D = 0: \)

\[ C_x(0.2) - 600(0.4) = 0, \quad C_x = 1200 \text{ N} \]

**Link ABC** \( \Sigma M_B = 0: \)

\[ C_y(0.2) - 600(0.2) - 1200(0.2) = 0 \]
\[ C_y = 1800 \text{ N} \]
\[ C = \sqrt{(1200)^2 + (1800)^2} = 2160 \text{ N} \]
Determine the x-coordinate of the centroid of the shaded area.

\[ dA = (x_2 - x_1) dy \]
\[ = (\sqrt{by} - y/2) dy \]
\[ A = \int_0^b (\sqrt{by} - y/2) dy \]
\[ = \left[ \frac{2}{3} \sqrt{b} y^{3/2} - \frac{y^2}{4} \right]_0^b \]
\[ = \frac{5}{12} b^2 \]

\[ \bar{x} = \frac{\int x_c dA}{A} = \frac{5 b^3/24}{5 b^2/12} = \frac{b}{2} \]
Determine the z-coordinate of the mass center of the solid obtained by revolving the quarter-circular area about the z-axis.

Differential element is a washer of radii $r$ and $a$ and thickness $dz$ with volume

$$dV = \pi (r^2 - a^2)dz = \pi (a^2 - z^2 + 2a\sqrt{a^2 - z^2})dz$$

$$\int z dV = \int_0^a \pi (a^2 - z^3 + 2az\sqrt{a^2-z^2})dz$$

$$= \pi \left[ \frac{a^2z^2}{2} - \frac{z^4}{4} + \frac{2a}{3} \sqrt{(a^2-z^2)^3} \right]_0^a = \frac{11}{12} \pi a^4$$

$$\int dV = \int_0^a \pi (a^2 - z^2 + 2a\sqrt{a^2-z^2})dz$$

$$= \pi \left[ \frac{a^2z^3}{3} + a(z\sqrt{a^2-z^2} + a^2\sin^{-1}\frac{z}{a}) \right]_0^a = \pi a^3 \left( \frac{2}{3} + \frac{\pi}{2} \right)$$

$$\bar{z} = \frac{\int z dV}{\int dV} = \frac{(11/12)\pi a^4}{\pi a^3 (2/3 + \pi/2)}$$

$$\bar{z} = \frac{11a}{2 (4 + 3\pi)} = 0.410a$$
The semicircular and straight bars are made from stock with a mass of 7.5 kg per meter of length and are welded to the triangular plate made from material with a mass of 100 kg per square meter of area. Calculate the coordinates of the mass center of the assembly.

<table>
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<th>Part</th>
<th>m</th>
<th>( \bar{x} )</th>
<th>( \bar{y} )</th>
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<td>2181</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{6426}{32.86} = 195.6 \text{ mm}
\]

\[
\bar{y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{2181}{32.86} = 66.4 \text{ mm}
\]
Uniform 60-kg bar AB is subjected to force P. Smooth guides at B. At A, $\mu_s = 0.8$.

(a) If $P = 400 \text{ N}$, find friction force at A.

(b) Find $P$ required to cause slippage at A.

\[
W = mg = 60(9.81) = 589 \text{ N}
\]

(a) $P = 400 \text{ N}$. Assume equil.

\[
\Sigma F_y = 0: \quad N_1 - 589 = 0, \quad N_1 = 589 \text{ N}
\]

\[
\Sigma M_{C} = 0: \quad 400 \frac{L}{2} \sin 60^\circ + 589 \frac{L}{2} \cos 60^\circ - F \frac{L}{2} \sin 60^\circ = 0
\]

\[
F = 370 \text{ N} < \left[ \mu_s N_1 = 0.8(589) = 471 \text{ N} \right]
\]

so assumption is valid

(b) $F = \mu_s N_1 = 471 \text{ N}$

\[
\Sigma M_C = 0: \quad P \frac{L}{2} \sin 60^\circ + 589 \frac{L}{2} \cos 60^\circ - 471 (\frac{L}{2} \sin 60^\circ) = 0
\]

\[
P = 602 \text{ N}
\]
The hubs of the uniform 50-kg wheel rest on inclined rails. If support at A is removed, determine the friction force acting on the wheel if $\mu_s = 0.50$, $\mu_k = 0.40$. What would happen if $\mu_s = 0.30$ & $\mu_k = 0.25$?

First, assume equilibrium.

$\Sigma M_A = 0$:

$T(500 \cos 30^\circ) - 50(9.81)(250 \sin 30^\circ) = 0$

$T = 85.6 \text{ N}$

$\Sigma F_y = 0$:

$N - 50(9.81) \cos 30^\circ - 85.6 \sin 30^\circ = 0$

$N = 468 \text{ N}$

$\Sigma F_x = 0$:

$-F - 85.6 \cos 30^\circ + 50(9.81) \sin 30^\circ = 0$, $F = 171 \text{ N}$

Since ($F_{\text{needed}} = 171 \text{ N} < (F_{\text{max}} = \mu_s N = 0.50(468) = 234 \text{ N})$, equilibrium assumption is valid & $F = 171 \text{ N}$

If $\mu_s = 0.30$, $F_{\text{max}} = 0.30(468) = 140.4 \text{ N} < 171 \text{ N}$

so wheel will slip. But $F \neq 0.25(468) \text{ N}$ since $N \neq 468 \text{ N}$ under accelerating conditions.
Each screw of the jack has a mean diameter of 21 mm and a lead of 8 mm, one a right-hand and the other a left-hand thread. For $\theta = 30^\circ$ determine (a) the torque $M$ required to raise the load $P = 7.5$ kN and (b) the torque $M'$ required to lower the load. The coefficient of friction is $\mu = 0.20$.

For equilibrium

$W = 2C \cos 30^\circ$, $P = 2C \sin 30^\circ$

so $W = P \csc 30^\circ = 7.5 \sqrt{3} = 12.99$ kN

Friction angle $\phi = \tan^{-1} 0.20 = 11.31^\circ$

Helix angle $\alpha = \tan^{-1} \left( \frac{L}{2\pi r} \right) = \tan^{-1} \left( \frac{8}{2\pi(21/2)} \right) = 6.91^\circ$

$M = 2Wr \tan (\phi + \alpha) = 2(12.99)(21/2) \tan (11.31^\circ + 6.91^\circ)$

(a) $M = 89.8$ kN-mm or $M = 89.8$ N-m

$M' = 2Wr \tan (\phi - \alpha) = 2(12.99)(21/2) \tan (11.31^\circ - 6.91^\circ)$

(b) $M' = 21.0$ kN-mm or $M' = 21.0$ N-m
The coefficient of kinetic friction between the 30-mm-diameter pin and the pulley is 0.25. Calculate the tension \( T \) required to (a) raise the load and (b) lower the load at a constant speed. Neglect the mass of the pulley.

\[
\phi = \tan^{-1} 0.25 = 14.04^\circ
\]

\[
\tau = r \sin \phi = 0.015 \sin 14.04^\circ = 0.00364 \text{ m}
\]

\[
L = 500(9.81) = 4905 \text{ N}
\]

(a) To raise load: \( \Sigma M_B = 0 \)

\[
0.25T - 4905(0.125 + 0.00364) = 0
\]

\[
T = 2524 \text{ N or } T = 2.52 \text{ kN}
\]

(b) To lower load: \( \Sigma M_B = 0 \)

\[
0.25T - 4905(0.125 - 0.00364) = 0
\]

\[
T = 2381 \text{ N or } T = 2.38 \text{ kN}
\]
Calculate the force $P$ on the handle of the differential band brake that will prevent the flywheel from turning on its shaft to which the torque $M = 150 \text{ N}\cdot\text{m}$ is applied. The coefficient of friction between the band and the flywheel is $\mu = 0.40$.

**Diagram**

1. $T_1$: Force on the band
2. $T_2$: Force on the flywheel
3. Moment arm $r = 150 \text{ mm}$
4. $\alpha = 30^\circ$

**Equations**

1. Band: $T_2 = T_1 e^{\mu \beta}$
   \[ T_2 = T_1 e^{0.40 \frac{7\pi}{6}} = 4.33 T_1 \quad \cdots (1) \]

2. Flywheel: $\Sigma M = 0$; $150 + (T_1 - T_2)(0.150) = 0$
   \[ T_2 - T_1 = 1000 \text{ N} \quad \cdots (2) \]

3. Handle: $\Sigma M = 0$; $0.150 T_2 - (T_1 \sin 30^\circ)(0.075) - 0.450 P = 0$

Solve (1) & (2) to get $T_1 = 300 \text{ N}$, $T_2 = 1300 \text{ N}$

Solve for $P$ and get $P = 408 \text{ N}$
Calculate the moment of inertia of the shaded area about the x- and y-axes. Also find the radius of gyration $k_x$.

For rectangular area about its base $I = \frac{1}{3}bh^3$ so
\[
dI_x = \frac{1}{3}y^3\,dx = \frac{1}{3}(90x/4)^{3/2}\,dx
\]
\[
I_x = \frac{9}{8}(10)^{3/2}\int_0^{40} x^{3/2}\,dx
\]
\[
= \frac{9}{8}(10)^{3/2}\left[ \frac{2}{5}x^{5/2} \right]_0^{40} = \frac{9(31)}{80} \frac{40^5}{5} = 13.95(10)^4 \text{ mm}^4
\]

Area $A = \int y\,dx = \frac{3}{2} \sqrt{10} \int_0^{40} x^{1/2}\,dx = \frac{3}{2} \sqrt{10} \frac{2}{3} x^{3/2} |_0^{40} = 700 \text{ mm}^2$

$k_x = \sqrt{I_x/A} = \sqrt{13.95(10)^4/700} = 14.12 \text{ mm}$

$I_y = \int x^2\,dA = \int_0^{40} y\,dx = \frac{3}{2} \sqrt{10} \int_0^{40} x^{5/2}\,dx
\]
\[
= \frac{3}{2} \sqrt{10} \left[ \frac{2}{7}x^{7/2} \right]_0^{40} = 54.43 (10)^4 \text{ mm}^4$
Calculate the moment of inertia of the rectangular area about the x-axis and find the polar moment of inertia about point O.

For rectangular area recall

\[ I = \frac{1}{12}bh^3 - \frac{h^2}{12}b \]

\[ I_x = \bar{I}_x + Ad_x^2 \]
\[ = \frac{1}{12} (90)(60)^3 + (90)(60)(30+30)^2 = 21.06 \times 10^6 \text{ mm}^4 \]

\[ I_y = \bar{I}_y + Ad_y^2 \]
\[ = \frac{1}{12} (60)(90)^3 + (90)(60)(45)^2 = 14.58 \times 10^6 \text{ mm}^4 \]

\[ I = I_x + I_y = 21.06 \times 10^6 + 14.58 \times 10^6 = 35.64 \times 10^6 \text{ mm}^4 \]
Compute the moment of inertia about the $x$-axis and the polar radius of gyration about $O$ for the area shown.

For quarter circular area $A = \frac{\pi}{4} (40)^2 = 1257 \text{ mm}^2$

$I_x = I_y = \frac{1}{4} \left( \frac{\pi}{4} r^4 \right) = \frac{\pi}{16} (40)^4 = 503 \times 10^3 \text{ mm}^4$

$I_z = I_x + I_y = 2 \times (503) \times 10^3 = 1005 \times 10^3 \text{ mm}^4$

For square area $A = -20(20) = -400 \text{ mm}^2$

$I_x = I_y = -\frac{1}{3} bh^3 = -\frac{1}{3} (20)(20)^3 = -53.3 \times 10^3 \text{ mm}^4$

$I_z = I_x + I_y = -2 \times (53.3) \times 10^3 = -106.7 \times 10^3 \text{ mm}^4$

For net area $I_x = (503 - 53.3) \times 10^3 = 449 \times 10^3 \text{ mm}^4$

$k_z = k_0 = \sqrt{I_z/A} = \sqrt{\frac{1005 - 106.7}{1257 - 0.4}} = 32.4 \text{ mm}$