to have a temperature gradient

\[ T = T_0 + By \]

\[ B = +0.00651 \text{ C/m} \]

\[ \rho = \frac{P}{RT} = \frac{P}{R(T_0 + By)} \]

\[ dP = \frac{P}{R(T_0 + By)} \cdot \rho \, dy \]

\[ \frac{dP}{d\rho} = R(T_0 + By) \]

\[ \rho = \frac{P}{RT_0} \]

\[ R = \frac{L}{\kappa} \]

\[ \kappa = \frac{C}{\varepsilon} \]
Units and Scales of pressure measurement.

Pressure may be expressed with respect to any arbitrary datum. The usual datum are absolute zero and local atmospheric pressure.

\[
\text{absolute pressure} = \text{pressure gage} + \text{barometric reading}
\]

\[
P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}.
\]
1. Bourdon gage

Typical devices used for measuring gage pressure.

2. Barometer

Devices used to measure the local atmospheric pressure.
Manometers

It is used to determine the difference in pressure.

\( P_A + \gamma h = \gamma h \) for \( P_A \neq \gamma h \)

\( P_A + \gamma h_1 - \gamma h_2 = 0 \)

\( P_A - \gamma h_1 - \gamma h_2 + \gamma h_3 = P_B \)

\( P_A + \gamma h_1 - \gamma h_2 - \gamma h_3 = P_B \)
Micromanometer

It is used to measure very small differences in pressure precisely.

\[ P_c + \gamma_1 (h_1 + \Delta y) + \gamma_2 (h_2 + R - \Delta y) - \gamma_2 (h_1 + \Delta y) - \gamma_1 (h_1 - \Delta y) = P_D \]

Where

* \( R \) = manometer reading
* \( \alpha \) = expansion coefficient
* \( A \) = area of cross-section

\[ P_c - P_D = R \left\{ \gamma_2 - \gamma_1 \left(1 - \frac{\alpha}{A}\right) - \gamma_1 \frac{R}{A} \right\} \]
Inclined Manometer

\[ P_1 - S \gamma_w (h+\Delta h) = P_2 \]
\[ P_1 - P_2 = S \gamma_w (h+\Delta h) \]  \( \text{--- (1)} \)

\[ h = R \sin \alpha \quad \text{and} \quad \Delta h = R \Delta \alpha \]  \( \text{--- (2)} \)

- \( P_1 - P_2 = S \gamma_w (R \sin \alpha + \frac{g}{A} R) \)
- \( P_1 - P_2 = S \gamma_w (\sin \alpha + \frac{g}{A}) R \)  \( \text{--- (3)} \)

\[ |P_1 - P_2| = CR \]

When \( C = S \gamma_w (\sin \alpha + \frac{g}{A}) \)
Force on plane area

- The distributed forces resulting from the action of fluid on finite area may be conveniently replaced by a resultant force.

1. Horizontal Surface

The magnitude of the force acting on one side of the surface is

\[ F = \int P \, dA = P \int dA = PA \]

To find the action of the resultant force, take an element and take the moment of the distribution force about any axis say xy plane.

The moment about y-axis

\[ PA \bar{x} = \int x \, P \, dA \]  

since \( P \) is constant

\[ \bar{x} = A \int x \, dA \]

\( \bar{x} \) is the distance from the y-axis to the centroid of the area.
also \( \bar{y} = \frac{1}{A} \int y \, dy \, da \)

2. Inclined surface

\[
\bar{y} = \frac{1}{A} \int_{h \sin \theta}^{h} y \, dy \, da
\]

For an element with area

\( SA \) as a strip with thickness

\( dy \) with long edges horizontal

the magnitude of \( SF \) acting on it is

\[
SF = P \delta A = \gamma h SA = \gamma y \sin \theta \, SA
\]

Since all elemental forces are parallel

\[
F = F_2 \, Sp \, da = \gamma \sin \theta \, Sy \, da
\]

From appendix (A) \( Sy \, da = \bar{y} A \)
\[ F y \sin \theta A = y \Delta A = \frac{P_o A}{F} \]

Pressure at the centroid x area

Center of Pressure

\[ y_p = \frac{1}{F} \int x p dA \]

To find the pressure center: take a moment about the axis, i.e.

\[ x_p F = \int x p dA, \quad y_p F = \int y p dA \]

\[ x_p = \frac{1}{F} \int x p dA, \quad y_p = \frac{1}{F} \int y p dA \]

From Fig.

\[ x_p = \frac{1}{\frac{4}{A}} \int x y y \sin \theta dA \]

\[ = \frac{1}{\frac{4}{A}} \int x y dA \]

From Appendix A

\[ \int x y dA = I_{xy} \]

\[ x_p = \frac{I_{xy}}{\frac{4}{A}} \]

also \( I_{xy} = I_{yx} + \frac{\bar{x} I_A}{A} \)

\[ x_p = \frac{\bar{x} I_A + I_{xy}}{\frac{4}{A}} \]
\[ y_p = \frac{1}{SA} \int y^2 \, dA \]

and

\[ \int y^2 \, dA = I_x \quad (\text{Appendix A}) \]

\[ 2I_x = I_g + \frac{I_a}{SA} \]

\[ y_p = \bar{y} + \frac{I_a}{SA} \]

**Force Components on Curved Surface:**

1. **Horizontal Component**

   The horizontal component is equal to the pressure force exerted on a projection of the curved surface. The vertical plane of projection is normal to the direction of the component.

\[ SF_x = P \cdot SA \cdot \cos \theta \]

\[ F_x = \int P \cdot \cos \theta \, dA \]

\[ \cos \theta \cdot SA \]

\[ F_x = P_A \]
2. **Vertical Component**

It is equal to the weight of liquid vertically above the curved surface and extending up to the free surface:

\[ F_v = \int P \cos \theta \, da \]

\[ F_v = \int \gamma \, dV \]

\[ \bar{x} = \frac{1}{V} \int x \, dV \]

---

- \( \bar{x} \): The distance from \( O \) to the line of action
- \( dV \): The volume of the prism of height \( h \)

and base \( \cos \theta SA \) or the volume of liquid vertically above the area element.
Buoyant Force

It is the resultant force exerted on a body by a static fluid in which it is submerged or floating. It is always acts vertically upward.

\[ F_B = \gamma V \]

\[ \gamma \text{: specific weight of the liquid } \text{N/m}^3 \]

\[ V \text{: volume of fluid displaced} \]

\[ \bar{x} = \frac{1}{V} \int x \, dV \]

\[ F_B = F_v - F_v \]

The buoyant force acts through the centroid of the displaced volume of fluid.

\[ F \leftrightarrow \text{centroid of } ACDEFA \]
In solving a static problem involving submerged or floating objects, weighing an odd-shaped object suspended in two different fluids yields sufficient data. To determine its weight, volume, unit gravity force, and relative density as shown in Fig.

\[ F_1 = \frac{\rho_1 V_1}{\rho_2} \]
\[ W = \text{gravity force} \]
\[ V_2 = \text{volume of liquid displaced} \]

\[ F_1 + \gamma_1 V = W \]
\[ F_2 + \gamma_2 V = W \]
\[ V = \frac{F_1 - F_2}{\gamma_2 - \gamma_1} \]
\[ W = \frac{F_1 \gamma_2 - F_2 \gamma_1}{\gamma_2 - \gamma_1} \]

The equilibrium equation are written

Hydrometer: It is uses the principle of buoyant force to determine the relative density of liquids.
\[ V_o = \text{Volume displaced} \]
\[ \gamma = \text{specific weight of desired water} \]
\[ s = \text{relative density of liquid to be determined} \]

**Hydrometer**

The equation for the hydrometer is:

\[ \gamma V_o = W \]

To find the specific density of the liquid, we have:

\[ s = \frac{W}{(V_o - \Delta V) \gamma} \]

And

\[ \Delta V = a \Delta h \]

Substituting \( \Delta h \) in 2:

\[ \gamma V_o = (V_o - a \Delta h) \gamma - s \]

\[ V_o = s V_o + a s \Delta h \]

\[ \Delta h = \frac{V_o}{a} \left( \frac{s - 1}{s} \right) \]
Ex. 2.12

\[ W = 1.5 \text{ N} \]
\[ F = 1 \text{ N} \]
\[ Y = 2 \]
\[ YV = 2 \]

\[ W = F + YV \]
\[ 1.5 = 1.1 + 9806 \text{ V} \]
\[ 0.4 = 9806 \text{ V} \]
\[ V = 0.000408 \text{ m}^2 \]
\[ V = 408 \text{ cm}^3 \]

\[ W = 5 \cdot YV \]
\[ 1.5 = 5 \cdot V \]
\[ V = 0.3175 \]
\[ 9806 \cdot 0.000408 \]

Stability of floating and submerged bodies

A body floating in a static liquid has vertical stability.

Stable, Unstable, Neutral.

A body has linear stability when a small linear displacement in any direction sets up restoring force tending to return it to its original position.
Determination of Rotational Stability of floating objects:

Any floating object with center of gravity below its center of buoyancy (center of displaced volume) floats in stable equilibrium.

Stable  
Unstable

When a body submerged or floating in a liquid as shown below

$B'$ the center of buoyant force acts upward and $G$ the center of gravity of the body
The intersection of buoyant force and the centerline is called the metacenter, designated \( M \).

\[ M \text{ above } G \quad \text{the body is stable} \\
M \text{ at } G \quad 4 \quad 4 \quad = \text{neutral} \\
M \text{ below } G \quad 4 \quad 4 \quad = \text{unstable} \]

The distance \( MG \) is called the metacentric height.

Then the restoring coupling is \( m = W \cdot MG \sin \theta \)

In which \( \theta \) is the angular displacement and \( W \) the weight of the body.

Ex: As shown in fig.(x), above a block 6m wide and 20 m long has a gross mass of 200 Mg. Its center of gravity is 30 cm above the water surface. Find the metacentric height and restoring couple when \( d \gamma = 30 \) cm.

The depth of submersion in water is

\[ F_B = W \]

\[ \gamma \cdot V = 200,000 \times 9.81 \]

\[ 9810 \times 6 \times 20 \times h = 200,000 \times 9.81 \]

\[ h = 1.667 \text{m} \]
The centroid in the tipped position is located with moment about $AC$ and $CD$

$$\bar{x} = \frac{1.367 \times 6.3 + 0.6 \times 6 \times \frac{1}{2} \times 2}{1.667 \times 6} = 2.82 \text{ m}$$

$$\bar{y} = \frac{1.367 \times 6 \times \frac{1.367}{2} + 0.6 \times 6 \times \frac{1}{2} (0.2 + 1.367)}{1.667 \times 6} = 0.842$$

By similar triangle $AEO$ and $B'B'M$

$$\frac{\Delta y}{\delta} = \frac{B'B}{MB}$$

$$\Delta y = 0.3, \quad \frac{b}{2} = 3 \text{ m}, \quad B'P = 3 - 2.82 = 0.18 \text{ m}$$

$$MB = 1.8 \text{ m}$$

$$G_i = 1.967 \text{ from the bottom (CD)}$$

$$CB = 1.967 - 0.842 = 1.125 \text{ m}$$

$$MG = MB - CB = 1.8 - 1.125 = 0.675 \text{ m}$$

...the body is stable where $MG$ is positive.

The restoring coupling $W$ $MG$ $\sin \delta$

$$= 200000 \times 9.806 \times 0.675 \times \frac{0.3}{\sqrt{3^2 + 0.3^2}}$$

$$= 131 \text{ kN} \cdot \text{m}$$
Nonprismatic Cross-section body

For a floating object of variable cross-section, such as a ship:

\[ \Delta F_B \cdot S = W \cdot r \]

where \( W \) is the weight of the body and \( r \) is the distance shift.

If we take an element \( SA \) on horizontal section through the body at liquid surface:

The volume \( \Theta \cdot SA \)

Force \( \Delta V = \Theta \cdot \Theta \cdot SA \)

moment about \( O = \Theta \cdot x^2 \cdot \Theta \cdot SA \) for small \( \Theta \)

\[ \Delta F_B \cdot S = \Theta \cdot \Theta \cdot \int x^2 \, dA = \Theta \cdot I \]

where \( \int x^2 \, dA = I \) (moment of Inertia)
\[ \gamma \theta I = wr = \gamma V r \]

Where \( V \) is the total volume of liquid displaced since \( \theta \) is very small

\[ MB \sin \theta = MB \theta = r \quad \text{or} \quad MB = \frac{r}{\theta} = \frac{I}{V} \]

The metacentric height is then

\[ MC = MB + CB \]

or \[ MC = \frac{I}{V} + CB \]

The minus sign is used if \( C \) is above \( B \) and the plus sign when \( C \) is below \( B \).

Ex. A body displacing 1 Cg has the horizontal cross-section at the waterline shown in Fig. Its center of buoyancy is 2 m below the water surface and its center of gravity is 0.5 m above the water surface. Determine the metacentric height for rolling about \( y-y \) axis and pitching about \( x-x \) axis.

Mass displaced = 1 Cg = 1,000,000 kg
Center of buoyancy = 2 m below water surface

\text{Gravity} = 0.5 \text{ m}

\text{C.B} = 2 - 0.5 = 1.5 \text{ m}

as for rolling about y-y axis

\begin{align*}
V &= \text{mass displaced} = \frac{1000000}{1000} = 1000 \text{ m}^3 \\
&= \frac{1}{12} x 24 x 20^3 + 4 \times \frac{1}{12} x 6 x 5^3 \\
&= 2250 \text{ m}^4
\end{align*}

\begin{align*}
I_{yy} &= \frac{bh^3}{12} + 4 \left( \frac{1}{12} bh^3 \right) = \frac{1}{12} x 24 x 20^3 + 4 \times \frac{1}{12} x 6 x 5^3 \\
&= 2250 \text{ m}^4
\end{align*}

\begin{align*}
I_{xx} &= \frac{1}{12} x 10 x 24^3 + 2 \times \frac{1}{36} x 10 x 6^3 \\
&= 23400 \text{ m}^4
\end{align*}

For rolling \quad M_G = \frac{I}{V} - \text{C.B} = \frac{2250}{1000} - 1.5 = 0.75 \text{ m}

For pitching \quad M_G = \frac{23400}{1000} - 1.5 = 21.9 \text{ m}

...the body is stable.
Relative Equilibrium

Fluid masses in relative equilibrium for steady flow motion in motion no shear stress will occur if there is no relative motion between adjacent layer of the fluid.

1. Uniform linear acceleration:
   a. horizontal acceleration:

   \[ \sum F = ma \]

   \[ P_1 dA - P_2 dA_z = \frac{\Delta}{g} \int dA \alpha \]

   or

   \[ \frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{\Delta x}{g} dA \wedge \alpha = \frac{h_1 - h_2}{g} \]

   \[ \tan \theta = \frac{\Delta x}{g} \]

   From fig. the left side is the slope.
b. Vertical acceleration

\[ \Sigma F_y = m a_y \]

\[ = P \cdot dA - y \cdot h \cdot da = \frac{y \cdot h}{g} \cdot da \cdot a_y \]

\[ = P = \frac{y \cdot h \left(1 + \frac{a_y}{g}\right)} \text{ upward} \]

\[ P = \frac{y \cdot h \left(1 - \frac{a_y}{g}\right)} \text{ downward} \]

The general equation for a tank moved in two direction x & y.

\( A_x = \) The acceleration in x-dir.

\( a_y = \) \( a \) in y-dir.

\( P_0 = \) The initial pressure and equal to atmospheric pressure when the tank is opened.

\[ P = P_0 - \frac{y}{g} \frac{A_x}{x} - \frac{y}{g} \left(1 + \frac{a_y}{g}\right) y \]

and

\[ \tan \theta = -\frac{A_x}{a_y + g} \]
2. Uniform Rotational Vortex Flow

Consider liquid rotating about the central axes with angular velocity \( \omega \) rad/sec.

The slope of water caused by normal acceleration \( (A_n) \) and the gravitational acceleration \( (g) \):

\[
\text{slope } = \frac{\text{dh}}{\text{dr}} = \frac{A_n}{g}
\]

\[
\Rightarrow \text{dh} = \frac{A_n}{g} \text{ dr}
\]

Since \( A_n = \omega^2 r \)

\[
\Rightarrow \text{dh} = \frac{\omega^2 r^2}{g} \text{ dr}
\]

or \( h = \frac{\omega^2 r^2}{2g} + c \) at \( r = 0 \) \( h = 0 \)

\[ h = \frac{\omega^2 r^2}{2g} \quad \text{(1)} \]

\[
P = P_0 + \gamma \frac{\omega^2 r^2}{2g} - \gamma y
\]

\[ P = P_0 + \gamma \frac{\omega^2 r^2}{2g} - \gamma y \quad \text{(2)} \]