Composition and Resolution of Forces

Composition of Forces is the process of replacing a force system by its resultant. The resultant of the two concurrent forces \( P \) and \( Q \), can be determined by means of the parallelogram law.

\[ R = \sqrt{P^2 + Q^2 - 2 \cdot P \cdot Q \cdot \cos \theta} \]

The value of \( \theta \) can be determined as follows:

\[ \sin \alpha = \frac{Q}{R} \]

In obtaining rectangular components, the parallelogram reduces to a rectangle. For example:

\[ R = \sqrt{P^2 + Q^2} \]

\[ \sin \alpha = \frac{Q}{R} \]
Resolution 8. Is the processes of replacing a force by its components.

1) Resolving a force along rectangular components:

\[ F_x = 0 \alpha = F \times \cos \theta \]
\[ F_x = F \times \frac{4}{5} \rightarrow \text{through} \ O \]
\[ F_y = 0B = F \times \sin \theta \]
\[ F_y = F \times \frac{3}{5} \uparrow \text{through} \ O \]

For a force in space (3-dimensional) the resolution is as following:

* The angles \( \theta_x, \theta_y, \theta_z \) are the angles between the resultant force and the positive coordinate axes. The cosine of these angles can be used in the resolution process.
Angles are called direction cosines. If the angles is greater than 90°, the cosine is negative.

3) Resolving a force along non-rectangular components:
   Several methods are available.
   1) Graphically: by drawing the parallelogram to any convenient scale.
   2) Algebraically: by using law of sines.

\[ \frac{OH}{\sin B} = \frac{F}{\sin(180° - x - B)} \]

3) Resolve the force into rectangular components and equal each of the rectangular components of F to the sum of the rectangular components of OA and OB.

Ex) Resolve the 100 N force along OA and OB?

Method 1 is:

\[ \frac{D}{\cos 30°} = \frac{100}{\cos 40°} \]

\[ P = \frac{100 \times \sin 40°}{\cos 40°} \]
method 2 8-

\[ P = 100 \times \frac{0.6427}{0.9397} = 68.4 \text{ N through 0} \]

\[ \frac{Q}{\sin 30} = \frac{100}{\sin 110} \Rightarrow Q = 100 \times \frac{0.5}{0.9397} = 53.2 \text{ N } \theta \text{ through 0} \]

\[ F_x = 100 \times \cos 30 = 86.6 \text{ N} \]

\[ F_y = 100 \times \sin 30 = 50 \text{ N} \]

\[ Q_x = Q \times \cos 70 \]

\[ Q_y = Q \times \sin 70 \]

\[ F_x = Q_x + P \]

\[ F_y = Q_y + 0 \]

\[ 50 = Q_y \Rightarrow 50 = Q \times \sin 70 \Rightarrow Q = 53.2 \text{ N } \theta \text{ through 0} \]

\[ 86.6 = 53.2 \times \cos 70 + P \Rightarrow 0 = 68.4 \text{ N } \theta \text{ through 0} \]
**Moment of a Force**

The moment of a force is a measure of its tendency to turn or rotate a body about the moment axis.

In Fig. 2-1, the magnitude of the moment of the horizontal force $F$ about (with respect to) the vertical line $AB$ is the product $(Fd)$, where $d$ is the perpendicular distance from point $A$ (the intersection of the line $AB$ and the horizontal plane) to the force, and $F$ is often referred to as the moment of the force with respect to point $A$.

If the force does not lie in a plane perpendicular to the moment axis, it may be resolved into two components, one being parallel to the moment axis and the other lying in a plane perpendicular to the axis.

To determine the moment of the force $F$ in Fig. 2-1 with respect to the line $BC$, the force is resolved into components $F_1$ and $F_2$. The moment of $F_1$ with respect to $BC$ is (Zero) since $F_1$ is parallel to $BC$. The component $F_2$ is in a plane which is perpendicular to $BC$, and its moment about $BC$ is $(F_2d_2)$. 

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*Fig. 2-1*
To illustrate that a force parallel to an axis has no moment with respect to the axis, the shaft and pulley shown in Fig 2-2, the forces $T_1$ and $T_2$ tend to rotate the pulley on the shaft (ab), whereas the force $R_2$, which is parallel to the shaft, tends to translate the pulley along the shaft (ab), which means the moment of $R$ with respect to ab is zero.

**Note:** The moment of a force acts either clockwise or counterclockwise about a particular moment axis; thus it is a vector quantity with a definite sense of rotation about a definite axis.

The sense of rotation is indicated by placing the arrowhead on the vector pointing in the direction a right-hand screw would advance if turned in the direction of the sense of rotation of moment.

The moment of a force $F$ with respect to a point $A$ is defined as a vector with magnitude equal to the product of the perpendicular distance from $A$ to $F$ times the magnitude of $F$, with direction perpendicular to plane containing $A$ and $F$. (Fig 2-3)
Note: the moment \( MA \) of Fig. 2-3 can be resolved into rectangular components \( M_{AB} \), \( M_{AC} \), and \( M_{AD} \). These three rectangular components of the moment \( MA \) are actually the moments of the force \( F \) about the lines \( AB \), \( AC \), and \( AD \) respectively.

\[
M_{AB} = F \times d
\]

\[
M_{AC} = F \times d
\]

\[
M_{AD} = F \times d
\]

**Principle of Moments of Forces**

The principle of moments as applied to a force system states that the moment of the resultant of the force system with respect to any axis is equal to the algebraic sum of the moments of the forces of the system with respect to the same axis.

"The application of this principle to a pair of Concurrent force is known as "Varignon's Theorem". Varignon's theorem may be demonstrated as follows: In Fig. 2-4,

\[
\text{mom. of } R = \text{mom. of } P + \text{mom. of } Q
\]

\[
R_r = Q_q + P_p
\]
EX) Determine the moment of the force \( F \) in Fig. with respect to the vertical line \( ab \).

Solution:

1) \[ CD = \sqrt{5^2 + 4^2 + 7^2} = 9.487 \text{ cm} \]

The scale of \( F \) is

\[ \frac{400}{9.487} = 42.2 \text{ N/cm} \]

\( F_x = 42.2 \times 7 = 295.4 \text{ N} \)
\( F_y = 42.2 \times 4 = 168.8 \text{ N} \)
\( F_z = 42.2 \times 5 = 211 \text{ N} \)

2) \[ M_{ab} = F_z \times 6 - F_x \times 4 \]

\[ = 211 \times 6 - 295.4 \times 4 \]

\[ = 84.4 \text{ N cm} \]

\[ M_{ab} = 84.4 \text{ N cm} \]