Note: Answer (6) questions only, 10 marks for each.

Q1) Prove by induction (for n = 1) : \(1 + 4 + 7 + \ldots + (3n - 2) = 1/2 n (3n - 1)\)

Q2) for the following relations on the set \(A = \{1, 2, 3\}\):
\(R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}\), & \(Q = A \times A = \) the universal relation
Determine if \(R \) & \(Q\) is:
(a) reflexive; (b) symmetric; (c) transitive; (d) antisymmetric.

Q3) Sketch the graph of the g: \(R \rightarrow R\) where \(g(x) = x^3 + 5\)
Is \(g(x)\) : 1) Function?
2) One-to-one?
3) Onto? (Mention the reasons)

Q4) Find minimum spanning tree and its weight for the following graph using Kruskal algorithm

Q5) In a survey of 100 students produced the following:
- 32 study mathematics
- 20 study physics
- 45 study biology
- 15 study mathematics & biology
- 7 study mathematics & physics
- 10 study physics & biology
- 30 do not study any of the three subjects
(a) Find the number of students studying all three subjects?
(b) Fill in the correct number of students in each of the eight regions of the Venn diagram.

Q6) Consider the algebraic expression: \((6 - (2 + (6 / 3)))^* ((5 - 2) + 1)\)
1- Rewrite the expression into prefix polish notation form
2- Draw the corresponding ORT

Q7) design a finite state machine that recognizes the sequence pattern "ab" in the input string \(x \in A^*\).
Where the input \(A = \{a, b\}\), and the output \(Z = \{0, 1\}\).
Trace the input string \(x \in A^*\) : \(x = a\ a\ a\ b\ b\ a\ b\ b\ a\ b\ b\)