Not: Choose only 4 questions (15 mark for each)

1. Let, \( n = 16 \)
   1. Find all the residue classes of \( n \)
   2. Find all the prime residue classes of \( n \)
   3. Find all the non-negative residue classes of \( n \)
   4. Find all the odd residue classes of \( n \)
   5. Find a system

b) Solve the bellow
   1. \( [7]_{12} + [8]_{12} = \)
   2. \( [7]_{5} \times [8]_{5} = \)
   3. \( [7]_{11} + [8]_{11} = \)

2. Find all the solutions to
   \( X \equiv 1 \mod 2 \)
   \( X \equiv 2 \mod 3 \)
   \( X \equiv 3 \mod 5 \)
   \( X \equiv 4 \mod 11 \)

3. a) Define prime number and show if the bellow integers are prime or not
   97, 47, -43, -415

b) Find the remainder of
   1. \( 23 \mod 7 \)
   2. \( -4 \mod 5 \)
   3. \( 121 \mod 0 \)
   4. \( -333 \mod -10331 \)

4. Expand the rational numbers \( \frac{239}{51} \) as simple continued fractions.

5. Let \( a=13, b=2222, \) find
   a) \( \text{gcd}(a,b), \text{gcd}(-a,b), \text{gcd}(a,-b), \text{gcd}(-a,b), \text{gcd}(b,a) \)
   b) \( \text{lcm}(a,b), \text{lcm}(-a,b), \text{lcm}(a,-b), \text{lcm}(-a,b), \text{lcm}(b,a) \)
1. (a) 
5) \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}

1. (b) 
1) [3]_{12}
2) [4]_{5}
3) [1]_{11}

2. (a) 
\[ M = \prod_{i=1}^{m} m_i = 2 \times 3 \times 5 \times 11 = 330 \]

\[ M_i = \frac{m}{m_i} \quad \rightarrow \quad M_1 = \frac{330}{2} = 165 \]
\[ M_2 = \frac{330}{3} = 110 \]
\[ M_3 = \frac{330}{5} = 66 \]
\[ M_4 = \frac{330}{11} = 30 \]

\[ \begin{array}{c|c|c}
\hline
M_1 & S_1 & \equiv 1 \mod m_1 \\
\hline
& S_1 = 165^{-1} \mod 2 & 1 \\
\rightarrow & S_1 = 1 \\
\hline
\end{array} \quad \begin{array}{c|c|c}
\hline
M_3 & S_3 & \equiv 1 \mod m_3 \\
\hline
& S_3 = 66^{-1} \mod 5 & 1 \\
\rightarrow & S_3 = 1 \\
\hline
\end{array} \]

\[ \begin{array}{c|c|c}
\hline
M_2 & S_2 & \equiv 1 \mod m_2 \\
\hline
& S_2 = 110^{-1} \mod 3 & 2 \\
\rightarrow & S_2 = 2 \\
\hline
\end{array} \quad \begin{array}{c|c|c}
\hline
M_4 & S_4 & \equiv 1 \mod m_4 \\
\hline
& S_4 = 30^{-1} \mod 11 & 7 \\
\rightarrow & S_4 = 7 \\
\hline
\end{array} \]

\[ X = 1 \cdot 165 \cdot 1 + 2 \cdot 110 \cdot 2 + 3 \cdot 66 \cdot 1 + 4 \cdot 30.7 \mod 330 \]
\[ = 1643 \mod 330 \]

3. (a) 
Prime number: an integer \( p > 1 \) is a prime if it has no positive divisor other than 1 and itself.

- 97
  \[ \sqrt{97} < \sqrt{100} = 10 \]
  Prime less than 10 are \( \{2, 3, 5, 7\} \) and none of these divides 97 and so 97 is a prime

- 47
  \[ \sqrt{47} < \sqrt{49} = 7 \]
  Prime less than 10 are \( \{2, 3, 5\} \) and none of these divides 47 and so 47 is a prime

- 43
  By definition -43 is not prime
-415
By definition -415 is not prime

3. (b)
1) since \(23=7 \cdot 3 + 2 \mod 7 = 2\)
2) since \(-4=5(-1) + 12 \mod 5 = 1\)
3) not possible since \(b=0\)
4) not possible by the definition of remainder “for \(b>0\) define \(a\) and \(b=r\), where \(r\) is the remainder \(a\) is divided by \(b\)

4.
\[
239=51 \times 4 + 35
\]
\[
51=35 \times 1 + 16
\]
\[
35=16 \times 2 + 3
\]
\[
16=3 \times 5 + 1
\]
\[
3=3 \times 1 + 0
\]
\[
\frac{239}{51} = \{4, 1, 2, 5, 3\}
\]

5. (a) : \(a=13, b=2222\)
Since 13, and 2222 relatively prime then \(\gcd(13,2222)=1\)
\(\gcd(13,2222) = \gcd(-13,222) = \gcd(13,-222) = \gcd(-13,-222) = 1\)

5. (b): \(a=13, b=222\)
Since 13, and 2222 relatively prime then \(\gcd(13,2222)=1\)
\[
\text{lcm}(13,2222) = \frac{13 \times 2222}{\gcd(13,2222)} = 28886
\]
\[
\text{lcm}(13,222) = \text{lcm}(-13,222) = \text{lcm}(13,-222) = \text{lcm}(-13,-222) = 28886
\]