5-BJT Transistor Modeling
The key to the small-signal approach is the use of ac equivalent circuits or models. There are two methods regarding the equivalent circuit to be substituted for the transistor, the **hybrid parameters and the re model**. A model is the combination of circuit elements, properly chosen, that best approximates the actual behavior of BJT under specific operating conditions. In summary the ac equivalent circuit of BJT amplifier is obtained by:

1- **Setting all dc sources to zero-potential equivalent and replacing them by a short circuit connection to ground.**

![Transistor circuit under examination](image1)

2- **Replacing all capacitors short circuit equivalent.**

3- **Removing all element bypassed by the short circuit equivalents introduced by steps 1 & 2.**

4- **Redrawing the circuit in a more convenient and logical forms** (Fig-5).

![The network of Fig-5](image2)

5- **Use the hybrid or re equivalent circuit of the BJT to complete the equivalent circuit of the amplifier.**

6- **Finally, the following important parameters are determined for the amplifier:**
   1- Input impedance \(Z_i\)
   2- Output impedance \(Z_o\)
   3- Voltage gain \(A_v\)
4-Current gain $A_i$
5-phase relationship ($\theta$)

**The $r_e$ Transistor Model**
The $r_e$ model employs a diode and controlled current source to duplicate the behavior of a transistor. A current-controlled current source is one where the parameters of the current source are controlled by a current elsewhere in the network, in general, BJT transistor amplifiers are referred to as current-controlled device.

**Common-Base Configuration (CB)**

The ac resistance of a diode can be determined by the equation $r_{ac} = 26 \text{ mV} / I_D$.

Same equation can be used to find the ac resistance of the diode of Fig 5-4(a) if we simply substitute the emitter current as follows:

$$r_e \approx \frac{26 \text{ mV}}{I_E}$$

$e$ of $r_e$ was chosen to emphasize that it is the dc level of emitter current that determines the ac level of the resistance of the diode of Fig 5-4(b). Substituting the resulting value of $r_e$ in Fig 5-4(b) will result in the very useful model of Fig 5-5.

For the CB, $Z_i$ range from a few ohms to a maximum of about 50 Ω

If we set the signal to zero ($V_\text{i}=0$) then $I_e = 0\text{ A}$ and $I_c = \alpha I_e = \alpha (0\text{ A}) = 0\text{ A}$, resulting in an open-circuit equivalence at the output terminals.

For the CB configuration, values of $Z_o$ are in MΩ range for CB the input impedance is relatively small and the output impedance quite high.
Example 3: CB configuration with $I_E = 4\text{mA}$, $\alpha = 0.98$, and an ac of $2\text{mV}$ applied between the base and emitter. Determine the $Z_i$. Calculate $A_v$ if a load of $0.56\text{k}\Omega$ is connected to the output terminals, find the $Z_o$, and $A_i$.

**Solution:**

- $Z_i = \frac{26 \text{mV}}{4 \text{mA}} = 6.5 \Omega$
- $I_c = \frac{V_i}{Z_i} = \frac{2 \text{mV}}{6.5 \Omega} = 0.30769 \text{mA}$
- $V_o = I_c R_L = \alpha I_c R_L = (0.98)(0.30769 \text{mA})(0.56 \text{k}\Omega)$
  = 168.86 mV
- $A_v = \frac{V_o}{V_i} = \frac{168.86 \text{mV}}{2 \text{mV}} = 84.43$
- $A_i = \frac{\alpha R_L}{r_c} = \frac{(0.98)(0.56 \text{k}\Omega)}{6.5 \Omega} = 84.43$
- $Z_o \approx \infty \Omega$
- $\frac{I_o}{I_i} = -\alpha = -0.98$
Common-Emitter Configuration (CE)

For the CE configuration the emitter is common between input and output ports of amplifier. Substituting the $r_e$ equivalent cct for npn transistor will result in fig 5-8, $I_B$ is the input current while $I_C$ is the output current.

\[
I_e = \beta I_B \\
I_e = I_c + I_b = \beta I_B + I_b \\
I_c = (\beta + 1)I_b \\
I_e = \beta I_B
\]

![Fig 5-8(a) CE BJT (b) approximate model](image)

\[Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}\]

\[V_i = V_{be} = I_e r_e \approx \beta I_B r_e\]

![Fig 5-9 Z_i using the approximate model](image)

\[Z_i = \frac{V_{be}}{I_B} = \frac{\beta I_B r_e}{I_B} = \beta r_e_{CE}\]

![Fig 5-10 r_e on input impedance](image)

For the CE configuration $Z_i$ defined by $\beta r_e$ rang from a few hundred ohms to the $\text{k}\Omega$ range, with maximums of about 6-7 $\text{k}\Omega$. 

\[Z_i = r_e_{CE}\]
Example 4: $\beta=120$ and $I_E=3.2\,mA$ for CE configuration with $r_o=\infty\Omega$, determine

(a) $Z_v$,
(b) $A_v$ if a load of 2 k\Omega is applied.
(c) $A_i$ with the 2 k\Omega load.

$$r_o = \frac{26 \, mV}{3.2 \, mV} = 8.125 \, \Omega$$

and $Z_i = \beta r_o = (120)(8.125 \, \Omega) = 975 \, \Omega$

$$A_v = \frac{R_L}{r_o} = \frac{2 \, k\Omega}{8.125 \, \Omega} = -246.15$$

$$A_i = \frac{I_o}{I_i} = \beta = 120$$

**Common-Collector Configuration (CC)**

For the CC configuration the model of CE configuration is normally applied.
The Hybrid (h-parameter) Equivalent Model

The r<sub>e</sub> model for the transistor is sensitive to the dc level of operation of the amplifier. For the hybrid equivalent model the parameters are defined at an operating point that may or may not reflect the actual operating conditions of the amplifier.

For the basic three-terminal electronic device there are two ports (pairs of terminals) of interest. The set at the left will represent the input terminals, and the set at the right, the output terminals.

![Fig5-14 Two port system](image)

For each set of terminals, there are two variables of interest

\[
\begin{align*}
V_i &= h_{11}I_i + h_{12}V_o \\
I_o &= h_{21}I_i + h_{22}V_o
\end{align*}
\]

The parameters relating the four variables are called h-parameters from the word hybrid (V & I).

Set V<sub>o</sub>=0 (short circuit the output terminals) and solve Eq[5-8a] for h<sub>11</sub>

\[
h_{11} = \left. \frac{V_i}{I_i} \right|_{V_o=0} \quad \text{ohms}
\]

= \( h_i(\Omega) \) short circuit input impedance parameter

Set I<sub>i</sub> equal to zero by opening the input, the following will result for h<sub>12</sub>

\[
h_{12} = \left. \frac{V_i}{V_o} \right|_{I_i=0} \quad \text{unitless}
\]

= \( h_r \) open circuit reverse transfer voltage ratio

Eq[5-8b] V<sub>o</sub>=0 by shorting the output terminals, will result for h<sub>21</sub>

\[
h_{21} = \left. \frac{I_o}{I_i} \right|_{V_o=0} \quad \text{unitless}
\]

= \( h_f \) short cct forward transfer current ratio

Again opening the input leads by set I<sub>i</sub> =0 and solving for h<sub>22</sub>

\[
h_{22} = \left. \frac{I_o}{V_o} \right|_{I_i=0} \quad \text{siemens}
\]

= \( h_o(S) \) open-circuit output admittance

Since each term of Eq[5-8a] has the unit volt, let us apply KVL in reverse to find a circuit that fits the equation as in shown Fig5-15.
Since each term of Eq[5-16b] has the units of \textbf{current}, let us now apply \textbf{KCL in reverse} to obtain the circuit of Fig5-16.

The complete \textbf{ac equivalent circuit} for the basic three-terminal linear device is indicated in Fig5-17 with a new set of subscripts for the \textit{h}-parameters.

The circuit of Fig5-17 is applicable to any linear three-terminal electronic device or system with no internal independent sources.

\textbf{Common Emitter Configuration (CE)}

The hybrid equivalent network for the CE configuration is shown Fig5-18, Note that:

\begin{align*}
I_i &= I_b \\
I_o &= I_c \\
V_i &= V_{be} \\
V_o &= V_{ce}
\end{align*}
Common Base Configuration (CB)

The hybrid equivalent network for the CB configuration is shown Fig 5-19, Note that:

\[ I_i = I_e \]
\[ I_o = I_c \]
\[ V_i = V_{eb} \]
\[ V_o = V_{cb} \]

Fig 5-19 CB configuration (a) graphical symbol (b) hybrid

The networks of Figs 5-18 & 5-19 are applicable for pnp or npn transistors. There are three different sets of h-parameters (Table 5-1).

<table>
<thead>
<tr>
<th>BJT configuration</th>
<th>h-parameters sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Common-Emitter</td>
<td>( h_{ie}, h_{re}, h_{ir}, h_{oe} )</td>
</tr>
<tr>
<td>2 Common-Collector</td>
<td>( h_{ic}, h_{ro}, h_{re}, h_{oe} )</td>
</tr>
<tr>
<td>3 Common-Base</td>
<td>( h_{ib}, h_{ro}, h_{re}, h_{oe} )</td>
</tr>
</tbody>
</table>

Table 5-2 lists typical parameter values in each of the three transistor configurations

<table>
<thead>
<tr>
<th>h-parameters</th>
<th>CE</th>
<th>CC</th>
<th>CB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{re} )</td>
<td>1 kΩ</td>
<td>1 kΩ</td>
<td>20 kΩ</td>
</tr>
<tr>
<td>( h_{re} )</td>
<td>2.5 × 10^4</td>
<td>1</td>
<td>3.0 × 10^4</td>
</tr>
<tr>
<td>( h_{re} )</td>
<td>50</td>
<td>-50</td>
<td>-0.98</td>
</tr>
<tr>
<td>( h_{re} )</td>
<td>25</td>
<td>25</td>
<td>0.5</td>
</tr>
<tr>
<td>( 1/h_{oe} )</td>
<td>40 kΩ</td>
<td>40 kΩ</td>
<td>2 MΩ</td>
</tr>
</tbody>
</table>

Approximate CE & CB hybrid equivalent circuit

Since \( h_{re} \) & \( h_{rb} \) are normally small quantity, their removal is approximated by \( h_{re} \approx 0 \) and \( h_{re}V_o = 0 \), resulting in a short-circuit equivalent for the feedback element as shown in Fig 5-20. The resistance determined by \( 1/h_{oe} \) & \( 1/h_{ob} \) are large enough to be ignored, in comparison to a parallel load, which can be replaced by an open circuit equivalent for the CE and CB models.

Fig 5-20 Effect of removing \( h_{re} \) & \( h_{oe} \)

The resulting equivalent circuit (Fig 5-21) is quite similar to the general structure of the CB & CE

Fig 5-21 approximate hybrid model
Hybrid versus $r_e$ model CE configuration

Hybrid versus $r_e$ model CB configuration

Fig 5-22 Hybrid versus $r_e$ model (a) CE configuration (b) CB configuration

Note that the minus sign in Eq [5-12] account for the fact that the current source of the standard hybrid equivalent circuit is pointing down rather than in the actual direction as shown in the $r_e$ model of fig 5-22b.

Example 5:

Given $I_E = 2.5$ mA, $h_{ie} = 140$, $h_{re} = 20 \mu$S (mho), and $h_{ib} = 0.5 \mu$S, determine
(a) The common-emitter hybrid equivalent circuit.
(b) The common-base $r_e$ model.

Solution:

(a) $r_e = \frac{26 \text{ mV}}{2.5 \text{ mA}} = \frac{26 \text{ mV}}{2.5 \text{ mA}} - 10.4 \Omega$

(b) $r_e = \frac{1}{h_{re}} = \frac{1}{20 \mu\text{S}} = 50 \text{k}\Omega$

Fig 5-23 CE hybrid cct for Ex 5:
SUMMARY

1- Amplification in the ac domain cannot be obtained without the application of dc biasing level.

2- For most applications the BJT amplifier can be considered linear, permitting the use of the superposition theorem to separate the dc and ac analyses and designs.

3- A model is the combination of circuit elements, carefully chosen, that best approximates the behavior of a BJT for a particular set of operating conditions.

4- When introducing the ac model for a BJT:
   a. all dc sources are set to zero and replaced by a short circuit connection to ground.
   b. all capacitors are replaced by a short-circuit equivalent.
   c. all elements in parallel with an introduced short-circuit equivalent should be removed from the network.
   d. the network should be redrawn as often as possible.

5- The input impedance of an ac network cannot be measured with an ohm-meter.

6- The output impedance of an amplifier is measured with the applied signal set to zero. It cannot be measured with an ohmmeter.

7- For all transistor amplifiers, the no-load gain is always greater than the loaded gain.

8- The gain from source to load is always reduced by the internal resistance of the source.

9- The current gain of an amplifier is very sensitive to the input impedance of the amplifier and the applied load.

10- The re model for a transistor is very sensitive to the dc biasing network of the amplifier.

11- An output impedance for the re model can be included only if obtained from a data sheet or from a graphical measurement from the characteristic curves.

12- For the common-base configuration, the input impedance is generally quite small and the output impedance quite large. In addition, the voltage gain can be quite large, but the current gain is always very close to 1.

13- For the common-emitter configuration, the input impedance generally is approximately a few kilohms, and the output impedance is relatively large. In addition, the common-emitter configuration can have a relatively high voltage and current gain.
14- The parameters of a hybrid equivalent model for a transistor are provided for a particular set of dc operating conditions. However, four parameters are provided rather than the two that normally appear for the re model. For some applications the reverse transfer voltage ratio and the typical output impedance normally found in the re model can be quite important.

**Equation**

\[
\begin{align*}
  r_e &= \frac{26 \text{ mV}}{I_E} \\
  Z_i &= r_e \\
  A_v &\approx \frac{R_L}{r_e} \\
  A_i &\approx -1 \\
  Z_i &\approx \beta r_e \\
  A_v &= \frac{R_L}{r_e} \\
  A_i &= \beta \\
  h_{ie} &= \beta r_e \\
  h_{ij} &= \beta \alpha \\
  h_{ih} &= r_e \\
  h_{ij} &= -\alpha \approx -1
\end{align*}
\]
6-BJT small signal Analysis

1-Common-Emitter Fixed-Bias Configuration (CE)

Input signal $V_i$ is applied to the base of the transistor, recognized that the input current $I_i$ is not the base current but the source current, while the output current $I_o$ is the collector current. The small-signal ac analysis begins by:

1- Removing the dc effects of $V_{cc}$ and replacing the dc blocking capacitors $C_1$ and $C_2$ by short-circuit equivalents, resulting in the network of Fig6-2.

Note in Fig6-2 that the common ground of the dc supply and the emitter resistor permits the relocation of $R_B$ and $R_C$ in parallel with the input and output sections of the transistor.

![Fig 6-1CE fixed bias configuration](image1)

![Fig 6-2 removal of effects of $V_{CC}$, $C_1$, $C_2$](image2)

2-Substituting the approximate $r_e$ small-signal equivalent circuit for the transistor of Fig6-2 will result in the network of Fig6-3 performing following results.

![Fig6-3 $r_e$ model to the network](image3)

3- The next step is to determine $\beta$, $r_e$, and $r_o$

The magnitude of $\beta$ obtained from a specification sheet
The magnitude of $r_e$ determine from a dc analysis of the system
The magnitude of $r_o$ obtained from a specification sheet

$Z_i$ from fig6-3

$$Z_i = R_B \| \beta r_e$$ [6-1]

If $R_B \gg 10 \beta r_e$ then

$$Z_i \equiv \beta r_e \quad \text{ohms}$$ [6-2]

$Z_o$ Determined when $V_i=0$, $I_i=I_o=0$, resulting in an open-circuit equivalence for the current source, the result is shown in fig 6-4.

![Fig6-4 Determining $Z_o$ for the network](image4)
If $r_o \geq 10 R_C$

the approximation $R_C||r_o = R_C$ is frequently applied and

$$Z_o = R_C$$  \hspace{1cm} \text{ohms} \hspace{1cm} [6-3]$$

$A_v$, the resistor $r_o$ and $R_C$ are in parallel, and

$$V_o = -\beta I_b(R_C||r_o)$$

But

$$I_b = \frac{V_i}{\beta r_e}$$

So that

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right)(R_C||r_o)$$

And

$$A_v = \frac{V_o}{V_i} = \frac{(R_C||r_o)}{r_o}$$ \hspace{1cm} [6-4]$$

If $r_o \geq 10 R_C$ \hspace{1cm} [6-5]$$

$$A_v = \frac{R_C}{r_o}$$ \hspace{1cm} r_o \geq 10 R_C$$ \hspace{1cm} [6-6]$$

$A_i$

$$I_o = \frac{(r_o)(\beta I_b)}{r_o + R_C} \quad \text{and} \quad \frac{I_b}{I_o} = \frac{r_o \beta}{r_o + R_C}$$

With

$$I_b = \frac{(R_B)(I_i)}{R_B + \beta r_e} \quad \text{or} \quad \frac{I_B}{I_i} = \frac{R_B}{R_B + \beta r_e}$$

$$A_i = \frac{I_o}{I_i} = \left(\frac{I_o}{I_b}\right)\left(\frac{I_b}{I_i}\right) = \left(\frac{r_o \beta}{r_o + R_C}\right)\left(\frac{R_B}{R_B + \beta r_e}\right)$$

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$ \hspace{1cm} [6-7]$$

If $r_o \geq 10 R_C$ and $R_B \geq 10 \beta r_e$ \hspace{1cm} [6-8]$$

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o)(R_B)} \quad A_i \equiv \beta$$ \hspace{1cm} r_o \geq 10 R_C, R_B \geq 10 \beta r_e$$

For checking

$$A_i = -A \frac{Z_o}{R_c}$$ \hspace{1cm} [6-9]$$

**Phase Relationship:** the negative sign in the resulting equation for the $A_v$ reveals that a $180^\circ$ phase shift occurs between the input and output signals, as shown in fig6-5
Fig 6-5 180º phase shift between input & output

The simplicity of moving from one model to other by: \( h_{fe} = \beta \) and \( h_{ie} = \beta r_e \)

**Example 1:** For the network of Fig 6-6

- Determine \( r_e \).
- Find \( Z_i \) (with \( r_o = \infty \) Ω).
- Calculate \( Z_o \) (with \( r_o = \infty \) Ω).
- Determine \( A_v \) (with \( r_o = \infty \) Ω).
- Find \( A_i \) (with \( r_o = \infty \) Ω).
- Repeat parts (c) through (e) including \( r_o = 50 \) kΩ

**Solution:**

(a) DC analysis:

\[
I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 \text{ V} - 0.7 \text{ V}}{470 \text{ kΩ}} = 24.04 \mu\text{A}
\]

\[
I_E = (\beta + 1)I_B = (101)(24.04 \mu\text{A}) = 2.428 \text{ mA}
\]

\[
r_e = \frac{-26 \text{ mV}}{2.428 \text{ mA}} = 10.71 \Omega
\]

(b) \( \beta r_e = (100)(10.71 \Omega) = 1.071 \Omega \)

\[
Z_i = R_B || \beta r_e = 470 \text{ kΩ} || 1.071 \text{ kΩ} = 1.069 \text{ kΩ}
\]

(c) \( Z_o = \frac{R_C}{3 \text{ kΩ}} = 3 \text{ kΩ} \)

(d) \( A_v = \frac{-R_C}{r_e} = \frac{-3 \text{ kΩ}}{10.71 \Omega} = -280.11 \)

(e) Since \( R_B \geq 10\beta r_e (470 \text{ kΩ} > 10.71 \text{ kΩ}) \)

\[
A_v = \beta = 100
\]

\[
Z_o = r_o || R_C = 50 \text{ kΩ} || 3 \text{ kΩ} = 2.83 \text{ kΩ} \text{ vs. } 3 \text{ kΩ}
\]

\[
A_v = \frac{-r_o || R_C}{r_e} = \frac{2.83 \text{ kΩ}}{10.71 \Omega} = -264.24 \text{ vs. } -280.11
\]

\[
A_i = \frac{\beta R_{by} r_o}{(r_o + R_C)(R_B + \beta r_e)} = \frac{(100)(470 \text{ kΩ})(50 \text{ kΩ})}{(50 \text{ kΩ} + 3 \text{ kΩ})(470 \text{ kΩ} + 1.071 \text{ kΩ})} = 94.13 \text{ vs. } 100
\]

As a check

\[
A_i = -A_v \frac{Z_i}{R_C} = \frac{(-264.24)(1.069 \text{ kΩ})}{3 \text{ kΩ}} = 94.16
\]
2-Voltage Divider Bias (bypassed CE configuration)

Substituting the approximate $r_o$ equivalent circuit will result the network of Fig7-8.

$Z_i$

\[
Z_i = R' || \beta r_e
\]  \[6-10\]

$Z_o$ with $V_i = 0V$ resulting in $I_b = 0 \mu A$ and $\beta I_b = 0 \mu A$

\[
Z_o = R_C || r_o
\]  \[6-11\]

If $r_o \geq 10 R_C$

\[
Z_o \equiv R_C \quad r_o \geq 10 R_C
\]  \[6-12\]

$A_v$ since $R_C$ and $r_o$ are in parallel

\[
V_o = -(\beta I_b)(R_C || r_o)
\]

\[
I_b = \frac{V_i}{\beta r_e}
\]

so that

\[
V_o = -\beta \left( \frac{V_i}{\beta r_e} \right) \left( R_C || r_o \right)
\]

and

\[
A_v = \frac{V_o}{V_i} = \frac{-R_C || r_o}{r_o}
\]  \[6-14\]

For $r_o \geq 10R_C$

\[
A_v = \frac{V_o}{V_i} = \frac{R_C}{r_o}
\]  \[6-15\]
\[ R' = R_1 \| R_2 = R_B, \]

For \( r_o \geq 10 R_C \)

\[ A_i = \frac{I_o}{I_i} = \frac{\beta R'r_o}{r_o(R' + \beta r_o)} \quad [6-16] \]

\[ A_v = \frac{I_o}{I_i} = \frac{\beta R'}{R' + \beta r_o} \quad [6-17] \]

For checking

\[ A_v = -A_i \frac{Z_v}{R_c} \quad [6-18] \]

\[ A_v = -A_i \frac{Z_v}{R_c} \quad [6-19] \]

**Phase Relationship**: the negative sign in the resulting equation for the \( A_v \) reveals that a 180° phase shift occurs between the output \( V_o \) and input \( V_i \).

**Example 2**:

(a) Determine \( r_o \).
(b) Find \( Z_v \).
(c) Calculate \( Z_v \) (with \( r_o = \infty \Omega \)).
(d) Determine \( A_i \) (with \( r_o = \infty \Omega \)).
(e) Find \( A_v \) (with \( r_o = \infty \Omega \)).

(f) Find the parameters of parts (b) through (e) if \( r_o = 1/h_{oe} = 50 \text{k}\Omega \) and compare.

**Solution**:

**The approximate approach**:

\[ V_E = \frac{R_2}{R_1 + R_2} \quad V_{CE} = \frac{(8.2 \text{k}\Omega)(22 \text{V})}{56 \text{k}\Omega + 8.2 \text{k}\Omega} = 2.81 \text{V} \]

\[ V_C = V_E - V_{CE} = 2.81 \text{V} - 0.7 \text{V} = 2.11 \text{V} \]

\[ i_E = \frac{V_E}{R_E} = \frac{2.11 \text{V}}{1.5 \text{k}\Omega} = 1.41 \text{mA} \]

\[ r_e = \frac{26 \text{mV}}{1.41 \text{mA}} = 18.44 \text{k}\Omega \]

(b) \( R' = R_1 \| R_2 = (56 \text{k}\Omega)\| (8.2 \text{k}\Omega) = 7.15 \text{k}\Omega \)

(c) \( Z_v = R' = 7.15 \text{k}\Omega \)

(d) \( A_v = \frac{R_C}{r_e} = \frac{6.8 \text{k}\Omega}{18.44 \text{k}\Omega} = -368.76 \)
Since $R_e$ is often much greater than $r_e$, $\beta$ is normally much greater than 1, the approximate equation is the following:

$$Z_e = \frac{Z_e'}{1 + \beta R_e}$$

Fig 6-12, the input impedance of an un bypassed CE configuration looking into the network to the right of $R_e$ is

$$Z = \frac{V_i}{I_i} = \frac{Z_e'}{1 + \beta R_e}$$

or

$$Z = \frac{V_i}{I_i} = \frac{Z_e'}{1 + \beta R_e}$$

The condition $R_e \geq 10 \beta R_e$ is not satisfied. Therefore $3$-CE Emitter-Bias Configuration a. Un bypassed

Un bypassed configurations appear in Fig 6-10. Substituting the approximate $r_e$ equivalent model will result in Fig 6-11.

The condition $r_o \geq 10 R_c$ is not satisfied. Therefore

$$A = \frac{V_o}{V_i} = \frac{r_o}{1 + \beta R_e}$$

$$A = \frac{V_o}{V_i} = \frac{r_o}{1 + \beta R_e}$$

$$A = \frac{V_o}{V_i} = \frac{r_o}{1 + \beta R_e}$$

$$A = \frac{V_o}{V_i} = \frac{r_o}{1 + \beta R_e}$$
\[ Z_i = R_B \| Z_i \]  
\[ Z_o = R_C \]

\( Z_o \) with \( V_i \) set to zero, \( I_b = 0 \) and \( \beta I_b \) can be replaced by an open-circuit

\[ A_v \]

\[ I_b = \frac{V_i}{Z_b} \]
\[ V_o = -I_o R_C = -\beta I_b R_C \]
\[ = -\beta \left( \frac{V_i}{Z_o} \right) R_C \]

\[ A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_o} \]

\( Z_b = \beta (r_o + R_E) \) gives

\[ A_v = \frac{V_o}{V_i} \approx -\frac{R_C}{r_o + R_C} \]

For the approximation \( Z_b = \beta R_E \)

\[ A_i \]

\[ I_b = \frac{R_e I_i}{R_B + Z_o} \]
\[ I_b = \frac{R_B}{R_B + Z_o} \]

In addition,

\[ I_o = \beta I_b \]
\[ I_o = \frac{I_o}{I_b} = \beta \]

so that

\[ A_i = \frac{I_o}{I_i} = \frac{I_o}{I_b} \]
\[ = \beta \frac{R_B}{R_B + Z_o} \]

and

\[ A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{R_B + Z_o} \]

or

\[ A_i = -A_v \frac{Z_i}{R_C} \]

**Phase Relationship:** the negative sign in the resulting equation for the \( A_v \) reveals that a 180° phase shift occurs between the output \( V_o \) and input \( V_i \).
Effect of \( r_o \)

\[ Z_i = \beta r_e + \frac{(\beta + 1) + R_e/r_o}{1 + (R_C + R_E)/r_o} R_E \]  

Since the ratio \( R_C / r_o \) is always **much less** than \((\beta + 1)\)

\[ Z_e = \beta r_e + \frac{(\beta + 1)R_E}{1 - (R_C + R_E)/r_o} \]

**For** \( r_o \geq 10(R_C + R_E) \)

\[ Z_o = \beta r_e + (\beta + 1)R_E \]

Since \( \beta + 1 \approx \beta \) the following equation is an **excellent one for most application**:

\[ Z_o = \frac{R_C}{r_o} \quad (r_o \geq 10(R_C + R_e)) \]  

**However,** \( r_o \gg r_e \) and

\[ Z_o \approx R_C \left| r_o \right| \left[ 1 + \frac{\beta + 1}{1 + \beta r_e / R_E} \right] \]

This can be written as

\[ 1/\beta < 1 \text{ and } r_e/R_E < 1 \text{ a sum usually less than one.} \text{ The result is a multiplying factor for } r_o \text{ greater than one.} \]

\[ Z_o = R_C \left| 51 r_o \right| \quad \text{Which is certainly simply } R_C. \text{ Therefore} \]

\[ Z_o = R_C \]  

**Any level of } r_o \]

\[ A_v = \frac{V_o}{V_i} = \frac{-\beta R_e}{Z_o} \left[ 1 + \frac{r_e + R_e}{r_o} \right] \]

\[ \frac{r_e}{r_o} \ll 1 \]

\[ A_v = \frac{V_o}{V_i} = \frac{-\beta R_e}{Z_o} \left[ 1 + \frac{R_e}{r_o} \right] \]

\[ r_o \geq 10R_C \]  

\[ A_v = \frac{V_o}{V_i} = \frac{-\beta R_e}{Z_o} \left[ 1 + \frac{R_e}{r_o} \right] \]

\[ r_o \geq 10R_C \]  

\[ 106 \]
b- Bypassed CE configuration

If $R_E$ of fig 6-7 is bypassed by an emitter capacitor $C_E$ the complete $r_e$ equivalent model can be substituted resulting the same equivalent network as for fig 6-8.

Example 3: For the network of fig 6-13, without $C_E$ (un bypassed) determine:

![Fig 6-13 Example 3](image)

(a) $r_e$
(b) $Z_v$
(c) $Z_o$
(d) $A_v$
(e) $A_i$

Solution:

(a) DC: $I_e = \frac{V_{CC} - V_{BE}}{R_E + (\beta + 1)R_E} = \frac{20 \text{ V} - 0.7 \text{ V}}{470 \text{ k}\Omega + (121)0.56 \text{ k}\Omega}$

$$I_e = (\beta + 1)I_e = (121)(35.89 \mu\text{A}) = 4.34 \text{ mA}$$

and $r_e = \frac{126 \text{ mV}}{4.34 \text{ mA}} = 5.99 \Omega$

(b) Testing the condition $r_e \geq 10(R_C + R_B)$

$40 \text{ k}\Omega \geq 10(2.2 \text{ k}\Omega + 0.56 \text{ k}\Omega)$

$40 \text{ k}\Omega \geq 10(2.76 \text{ k}\Omega) = 27.6 \text{ k}\Omega$ (satisfied)

Therefore

$$Z_o = \beta(r_e + R_B) = 120(5.99 \Omega + 560 \Omega)$$

$$Z_o = 6792 \text{ k}\Omega$$

and $Z_v = R_v || Z_b = 470 \text{ k}\Omega || 6792 \text{ k}\Omega$

$$Z_v = 59.34 \text{ k}\Omega$$

$c) Z_v = R_C = 2.2 \text{ k}\Omega$

$r_o \geq 10R_C$ is satisfied. Therefore

$$A_v = \frac{V_v}{V_i} = \frac{\beta R_C}{Z_o} = \frac{(120)(2.2 \text{ k}\Omega)}{6792 \text{ k}\Omega} = -3.89$$

Compared to -3.93 using $A_v = -R_C/R_E$

$$A_i = \frac{Z_v}{R_C} = -(-3.89)\left(\frac{59.34 \text{ k}\Omega}{2.2 \text{ k}\Omega}\right) = 104.92$$

Compared to 104.85 using $A_i = \beta R_B / (R_B + Z_b)$
Example 4: Repeat the analysis of Example: 3 with $C_E$ in place

Solution:

(a) The dc analysis is the same, and $r_e = 5.99 \, \Omega$.
(b) $R_E$ is “shorted out” by $C_E$ for the ac analysis. Therefore,

$$Z_i = R_E \| Z_b = R_R \| \beta R_c = 470 \, k\Omega \| (120)(5.99 \, \Omega)$$

$$= 470 \, k\Omega \| 718.8 \, \Omega \approx 717.70 \, \Omega$$

(c) $Z_o = R_C = 2.2 \, k\Omega$

(d) $A_v = \frac{R_C}{r_e}$

$$= \frac{2.2 \, k\Omega}{5.99 \, \Omega} = -367.28 \quad \text{(a significant increase)}$$

(e) $A_i = \frac{\beta R_R}{R_R + Z_b} = \frac{(120)(470 \, k\Omega)}{470 \, k\Omega + 718.8 \, \Omega}$

$$= 119.82$$

Example 5: For the network of Fig6-14, determine (using appropriate approximation)

\begin{center}
\includegraphics[width=0.5\textwidth]{fig6-14}
\end{center}

Solution:

\begin{align*}
\beta R_R & > 10R_2 \\
(210)(0.68 \, k\Omega) & > 10(10 \, k\Omega) \\
142.8 \, k\Omega & > 100 \, k\Omega \quad \text{(satisfied)}
\end{align*}

\begin{align*}
V_B &= \frac{R_2}{R_1 + R_2} V_{CC} = \frac{10 \, k\Omega}{90 \, k\Omega + 10 \, k\Omega} (16 \, V) = 1.6 \, V \\
V_E &= V_B - V_{BE} = 1.6 \, V - 0.7 \, V = 0.9 \, V \\
I_E &= V_E \frac{R_E}{R_E + 0.68 \, k\Omega} = 0.9 \, V \frac{0.68 \, k\Omega}{1.324 \, mA} = 1.324 \, mA \\
r_e &= \frac{26 \, mV}{1.324 \, mA} = 19.64 \, \Omega
\end{align*}

(b) The ac equivalent circuit is provided in Fig6-15. The resulting configuration is now different from Fig6-11 only by the fact that now $R_b = R' = R_C \| R_2 = 9 \, k\Omega$

\begin{center}
\includegraphics[width=0.5\textwidth]{fig6-15}
\end{center}

Testing condition of $r_o \geq 10(R_C + R_E)$ and $r_o \geq 10R_C$ are both satisfied. Using appropriate approximations yields
Repeat Example 5: with $C_E$ in place

The dc analysis is the same, and $r_e = 19.64 \, \Omega$.

$Z_p = \beta r_e = (210)(19.64 \, \Omega) = 4.12 \, k\Omega$

$Z_i = R_B|Z_p = 9 \, k\Omega|4.12 \, k\Omega$

$= 2.83 \, k\Omega$

$Z_o = R_C = 2.2 \, k\Omega$

(d) $A_v = \frac{R_C}{r_e} = \frac{2.2 \, k\Omega}{19.64\Omega} = -112.02$ (a significant increase)

(e) $A_i = -A_v \frac{Z_i}{R_i} = -(-112.02) \left(\frac{2.83 \, k\Omega}{2.2 \, k\Omega}\right)$

$= 144.1$

4-Emitter-Follower Configuration

Since the output is taken from the emitter terminal of the transistor as shown in Fig6-16, the network is emitter-follower. The output voltage is always slightly less than the input signal due to the drop from base to emitter, but the approximation $A_v \approx 1$. The fact that $V_o$ “follows” the magnitude of $V_i$ with an in-phase relationship for the emitter-follower. The emitter-follower configuration used for impedance-matching purposes, it presents high impedance at the input and low impedance at the output.

![Fig 6-16 Emitter-follower configuration](image)

![Fig6-17 re model for network of fig 6-16](image)
Construct the network defined by the equation above; the configuration of fig 6-18 will result

With $V_i$ set to zero

Since $R_E$ is typically much greater than $r_e$, the following approximation is

\[ A_v \approx \frac{r_e}{r_e + R_E} \]

Since $R_E$ is usually much greater than $r_e$, $(R_E + r_e) \approx R_E$ and
Phase Relationship: the resulting equation for the $A_v$ reveals that the output $V_o$ and input $V_i$ are in phase for the emitter-follower configuration.

Effect of $r_0$

$Z_i$

$R_o \geq 10R_E$ is satisfied

$Z_i = \beta r_e + (\beta + 1)R_E$

$Z_i = \beta (r_e + R_E)$ \hspace{1cm} $r_0 \geq 10R_E$

$Z_o$

$Z_o = r_o \| R_E \| r_e$

$Z_o \approx R_E \| r_e$ \hspace{1cm} Any $r_o$

$A_v$

$R_o \geq 10R_E$ is satisfied and we use the approximation $\beta + 1 \approx \beta$

$A_v = \frac{\beta R_E}{Z_o}$

$Z_o = \beta (r_e + R_E)$

$A_v = \frac{\beta R_E}{\beta (r_e + R_E)}$
Example 7: For the emitter-follower network of Fig 6-19 determine:

Repeat part (b) through (e) with \( r_o = 25\,\text{k}\Omega \) and compare result

Solution:

(a) \( I_e = \frac{V_{CC} - V_{BE}}{R_b + (\beta + 1)R_e} \)
\[ = \frac{12 \,\text{V} - 0.7 \,\text{V}}{220 \,\text{k}\Omega + (101)3.3 \,\text{k}\Omega} = 20.42 \,\mu\text{A} \]
\( I_e = (\beta + 1)I_b \)
\[ = (101)(20.42 \,\mu\text{A}) = 2.062 \,\text{mA} \]
\( r_e = \frac{26 \,\text{mV}}{I_e} = \frac{26 \,\text{mV}}{2.062 \,\text{mA}} = 12.61 \,\Omega \)

(b) \( Z_v = \beta r_e + (\beta + 1)R_e \)
\[ = (100)(12.61 \,\Omega) + (101)(3.3 \,\text{k}\Omega) \]
\[ = 1.261 \,\text{k}\Omega + 333.3 \,\text{k}\Omega \]
\[ = 334.56 \,\text{k}\Omega \approx \beta R_e \]
\( Z_v = R_b \| Z_o = 220 \,\text{k}\Omega \| 334.56 \,\text{k}\Omega \]
\[ = 132.72 \,\text{k}\Omega \]

(c) \( Z_v = R_b \| r_e = 3.3 \,\text{k}\Omega \| 12.61 \,\Omega \]
\[ = 12.56 \,\Omega \approx r_e \]

(d) \( A_v = \frac{V_o}{V_i} = \frac{R_b}{R_b + r_e} = \frac{3.3 \,\text{k}\Omega}{3.3 \,\text{k}\Omega + 12.61 \,\Omega} \]
\[ = 0.996 \approx 1 \]

(e) \( A_v = -\frac{\beta R_o}{R_b + Z_v} = -\frac{(100)(220 \,\text{k}\Omega)}{220 \,\text{k}\Omega + 334.56 \,\text{k}\Omega} = -39.67 \]

versus
\( A_v = -A \frac{Z_v}{R_e} = -(0.996) \left( \frac{132.72 \,\text{k}\Omega}{3.3 \,\text{k}\Omega} \right) = -40.06 \)
Checking the condition $r_o \approx 10R_E$, we have

\[ 25 \text{ k}\Omega \approx 10(3.3 \text{ k}\Omega) = 33 \text{ k}\Omega \]

which is not satisfied. Therefore,

\[
Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \frac{R_E}{r_o}} = \frac{(100)(12.61 \text{ } \Omega)}{1 + \frac{3.3 \text{ k}\Omega}{25 \text{ k}\Omega}} + \frac{(100 + 1)(3.3 \text{ k}\Omega)}{1 + \frac{3.3 \text{ k}\Omega}{25 \text{ k}\Omega}}
\]

\[ = 1.261 \text{ k}\Omega + 294.43 \text{ k}\Omega = 295.7 \text{ k}\Omega \]

\[ Z_i = \frac{R_B}{Z_b} = 220 \text{ k}\Omega \parallel 295.7 \text{ k}\Omega = 126.15 \text{ k}\Omega \text{ vs. } 132.72 \text{ k}\Omega \quad \text{as obtained earlier} \]

\[ Z_o = \frac{R_E}{r_o} = 12.56 \text{ } \Omega \quad \text{as obtained earlier} \]

\[ A_v = \frac{(\beta + 1)R_E}{Z_b} = \frac{(100 + 1)(3.3 \text{ k}\Omega)/295.7 \text{ k}\Omega}{1 + \frac{3.3 \text{ k}\Omega}{25 \text{ k}\Omega}} = 0.996 = 1 \]

Matching the earlier result.

Therefore, a good approximation for the actual results can be obtained by simply ignoring the effects of $r_o$ for this configuration.

The network of fig6-20 equations changed only by replacing $R_B$ by $R' = R_1 \parallel R_2$.

Fig6-21 will also provide the input/output characteristics of an emitter-follower but includes a collector resistor $R_C$.

In this case $R_B$ is again replaced by the parallel combination of $R_1$ and $R_2$. The input impedance $Z_i$ and output impedance $Z_o$ are unaffected by $R_C$ since it is not reflected into the base or emitter equivalent. In fact, the only effect of $R_C$ will be to determine the Q-point of operation.

![Fig6-20E-follower with voltage divider](image1.png)  ![Fig6-21 E-follower with $R_C$](image2.png)