CHAPTER

TWO

RECTIFYING CIRCUITS

A rectifier circuit is one which links an a.c. supply to a d.c. load, that is, it converts an alternating voltage supply to a direct voltage. The direct voltage so obtained is not normally level, as from a battery, but contains an alternating ripple component superimposed on the mean (d.c.) level.

The various circuit connections described, although all giving a d.c. output, differ in regard to the a.c. ripple in the output, the mean voltage level, efficiency, and their loading effects on the a.c. supply system.

2-1 CIRCUIT NOMENCLATURE

Rectifying circuits divide broadly into two groups, namely, the half-wave and full-wave connections.

The half-wave circuits are those having a rectifying device in each line of the a.c. supply, all cathodes of the varying devices being connected to a common connection to feed the d.c. load, the return from the load being to the a.c. supply neutral. The expression *half-wave* describes the fact that the current in each a.c. supply line is unidirectional. An alternative to the description *half-wave* is to use the expression *single-way* in describing these circuits.

The full-wave circuits are those which are in effect two half-wave circuits in series, one feeding into the load, the other returning load current directly to the a.c. lines, eliminating the need to employ the a.c. supply neutral. The expression *full-wave* is used because the current in each a.c. supply line, although not necessarily symmetrical, is in fact alternating. The full-wave circuits are more commonly called *bridge circuits,* but alternatively are also known as *double-way circuits.*

The control characteristics of the various circuits may be placed broadly into one of three categories: namely, uncontrolled, fully-controlled, and half-controlled.

The uncontrolled rectifier circuits contain only diodes, giving a d.c. load voltage fixed in magnitude relative to the a.c. supply voltage magnitude.

In the fully-controlled circuits all the rectifying elements are thyristors (or power transistors). In these circuits, by suitable control of the phase angle at which the thyristors are turned on, it is possible to control the mean (d.c.) value of, and to reverse, the d.c. load voltage. The fully-controlled circuit is often described as a *bidirectional converter,* as it permits power flow in either direction between supply and load.

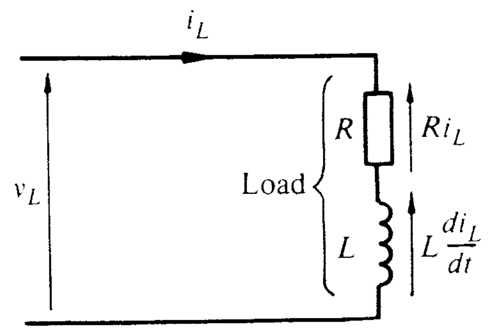


Figure 2-3 Load equivalent circuit.

The majority of d.c. loads (such as d.c. motors) respond to the mean (d.c.) value of the voltage, hence the r.m.s. value of the output voltage is generally of little interest. However, the a.c. ripple content in the direct voltage waveform, that is, the instantaneous variation of the load voltage relative to the mean value, is often a source of unwanted losses and is part of the characteristic of the circuit.

To select a suitable diode for the circuit, both the diode current and voltage waveforms must be studied. The diode voltage v_D shows a peak reverse voltage value of $V_{\max}$.

Almost all d.c. loads contain some inductance; the waveforms shown in Fig. 2-2*c* are for an inductive load having the equivalent circuit shown in Fig. 2-3. Current flow will commence directly the supply voltage goes positive, but the presence of the inductance will delay the current change, the current still flowing at the end of the half cycle, the diode remains on, and the load sees the negative supply voltage until the current drops to zero.

Reference to Fig. 2-3 shows that the instantaneous load voltage

$$v_L = Ri_L + L\, di_L/dt \tag{2-3}$$

which enables the waveshape of the load current i_L to be determined. A guide to the required current rating of the diode would be given by determining the r.m.s. value of its current waveform.

The mean voltage for Fig. 2-2*c* is given by

$$V_{\text{mean}} = (1/2\pi)\int_{\theta=0}^{\theta=\phi} V_{\max} \sin\theta\, d\theta \tag{2-4}$$

and is lower than the case of no inductance.

The single-phase half-wave circuit can be controlled by the use of a thyristor as shown in Fig. 2-4*a*. The thyristor will only conduct when its voltage v_T is positive and it has received a gate firing pulse i_g. Figures 2-4*b* and *c* show the conduction of the thyristor delayed by an angle α beyond that position where a diode would naturally conduct (or commutate); in this case, the firing delay angle α is expressed relative to the supply voltage zero.

Without the commutating diode, the waveforms would be similar to those of Fig. 2-2 with the exception of a delayed start. The waveforms of Fig. 2-4 do, how-

The half-controlled rectifier circuits contain a mixture of thyristors and diodes which prevent a reversal of the load voltage, but do allow adjustment of the direct (mean) voltage level. The half-controlled and uncontrolled (diode only) circuits are often described as *unidirectional converters,* as they permit power flow only from the a.c. supply into the d.c. load.

Pulse-number is a manner of describing the output characteristic of a given circuit, and defines the repetition rate in the direct voltage waveform over one cycle of the a.c. supply. For example, a six-pulse circuit has in its output a ripple of repetition rate six times the input frequency, that is, the fundamental ripple frequency is 300 Hz given a 50 Hz supply.

2-2 COMMUTATING DIODE

Many circuits, particularly those which are half- or uncontrolled, include a diode across the load as shown in Fig. 2-1. This diode is variously described as a free-wheeling, flywheel, or by-pass diode, but is best described as a commutating diode, as its function is to commutate or transfer load current away from the rectifier whenever the load voltage goes into a reverse state.

The commutating diode serves one or both of two functions; one is to prevent reversal of load voltage (except for the small diode volt-drop) and the other to transfer the load current away from the main rectifier, thereby allowing all of its thyristors to regain their blocking state.

2-3 SINGLE-PHASE HALF-WAVE (OR SINGLE-WAY)

Although the uncontrolled single-phase half-wave connection shown in Fig. 2-2*a* is very simple, the waveforms of Figs. 2-2*b* and *c* illustrate fundamentals which will constantly recur in the more complex circuits.

The assumption is made that the magnitude of the supply voltage is such as to make the diode volt-drop negligible when conducting. The waveforms are developed on the assumption that the diode will conduct like a closed switch when its anode voltage is positive with respect to its cathode, and cease to conduct when its current falls to zero, at which time it acts like an open switch. The turn-on and turn-off times of the diode, being only a few microseconds, may be taken as instantaneous times in relation to the half cycle time for a 50 Hz supply.

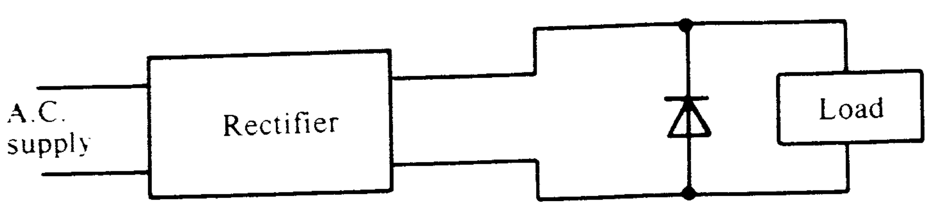


Figure 2-1 Position of commutating diode.

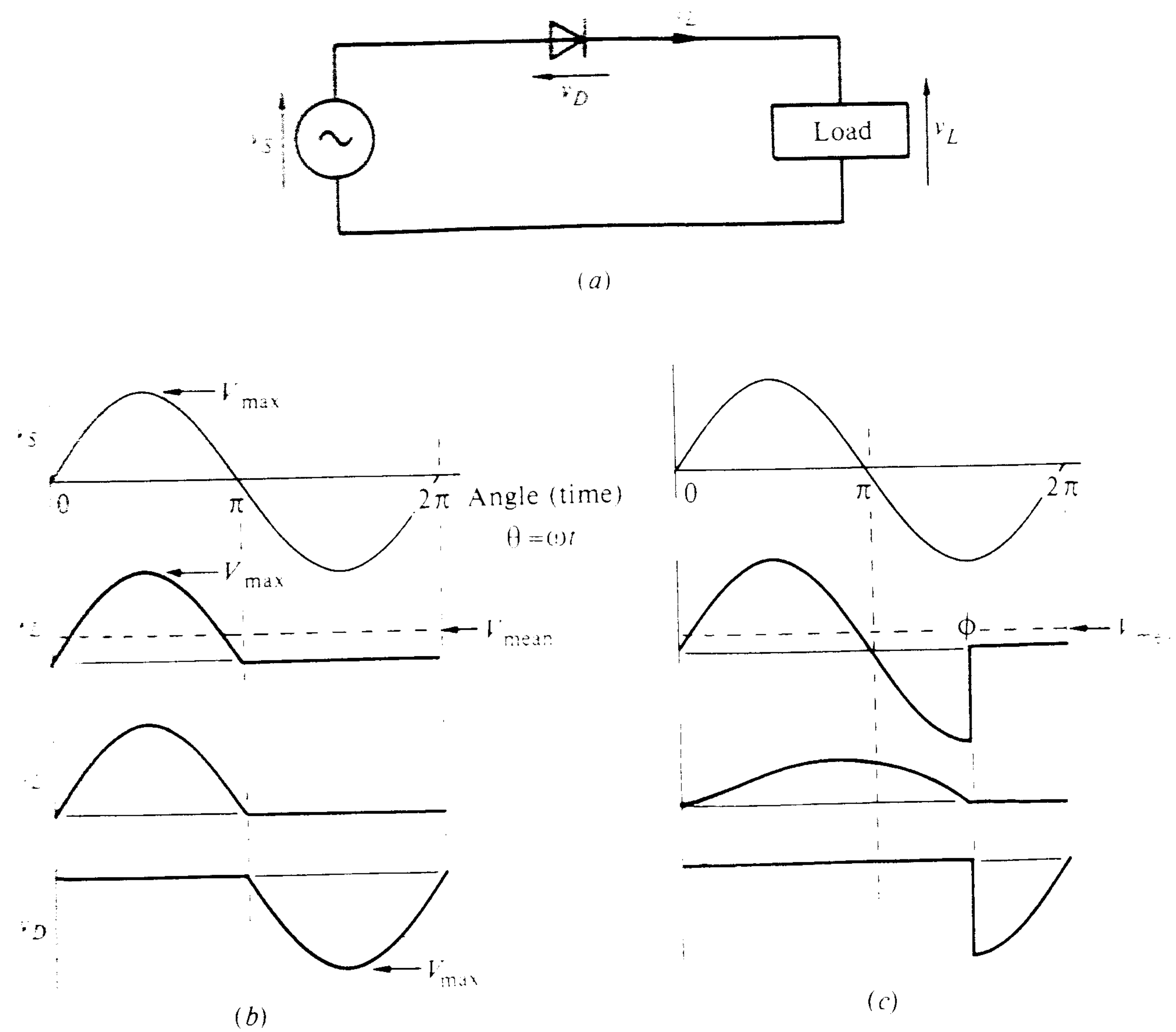


Figure 2-2 Single-phase half-wave circuit. (*a*) Connection. (*b*) Waveforms when the load is pure resistance. (*c*) Waveforms when the load contains some inductance.

The waveforms shown in Fig. 2-2*b* are for a load of pure resistance, the supply voltage v_S being sinusoidal of peak value $V_{\mathbf{max}}$. Immediately prior to v_S going positive, there is no load current, hence no load voltage, the negative supply voltage appearing across the diode. As v_S goes positive, the diode voltage changes to anode positive relative to the cathode, current flow then being possible. Neglecting the small diode volt-drop, the load current $i_L = v_S/R$ until the current falls to zero at the end of the positive half cycle. As the diode prevents reverse current, the entire supply negative voltage appears across the diode.

The load voltage waveform in Fig. 2-2*b* has the mean value of a half sinewave, namely,

$$V_{\mathbf{max}}/\pi \tag{2-1}$$

which could be calculated from

$$V_{\mathbf{mean}} = (1/2\pi)\int_{\theta=0}^{\theta=\pi} V_{\mathbf{max}} \sin\theta \, d\theta \tag{2-2}$$

where $\theta = \omega t$ is any angle on the waveform.

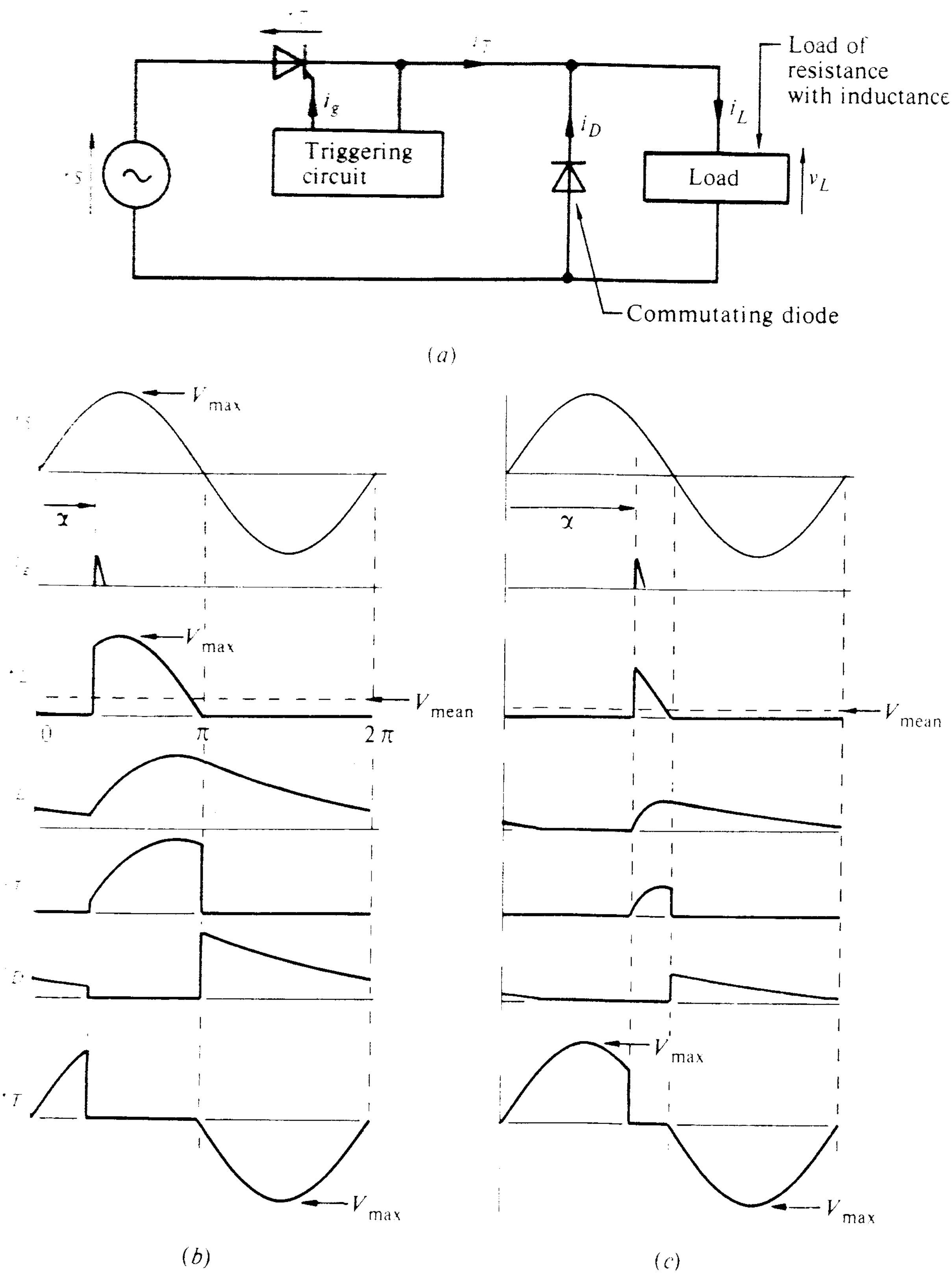


Figure 2-4 Single-phase half-wave controlled circuit with commutating diode. (*a*) Connection. (*b*) Small firing delay angle, and continuous current. (*c*) Large firing delay angle, and discontinuous current.

ever, assume the presence of a commutating diode which prevents the load voltage reversing beyond the diode volt-drop value, resulting in the waveforms shown.

During the thyristor on-period, the current waveform is dictated by Eq. (2-3), but once the voltage reverses, v_L is effectively zero and the load current follows an exponential decay. If the current level decays below the diode holding level, then the load current is discontinuous as shown in Fig. 2-4c. Figure 2-4b shows a continuous load-current condition, where the decaying load current is still flowing when the thyristor is fired in the next cycle. Analysis of the load-voltage waveform gives a mean value of

$$V_{\text{mean}} = (1/2\pi) \int_{\alpha}^{\pi} V_{\text{max}} \sin\theta \, d\theta = \frac{V_{\text{max}}}{2\pi} (1 + \cos\alpha) \tag{2-5}$$

Inspection of the waveforms shows clearly that the greater the firing delay angle α, the lower is the mean load voltage, Eq. (2-5) confirming that it falls to zero when $\alpha = 180°$.

The thyristor voltage v_T waveform shows a positive voltage during the delay period, and also that both the peak forward and peak reverse voltages are equal to V_{max} of the supply.

Inspection of the waveforms in Fig. 2-4 clearly shows the two roles of the commutating diode, one to prevent negative load voltage and the other to allow the thyristor to regain its blocking state at the voltage zero by transferring (or commutating) the load current away from the thyristor.

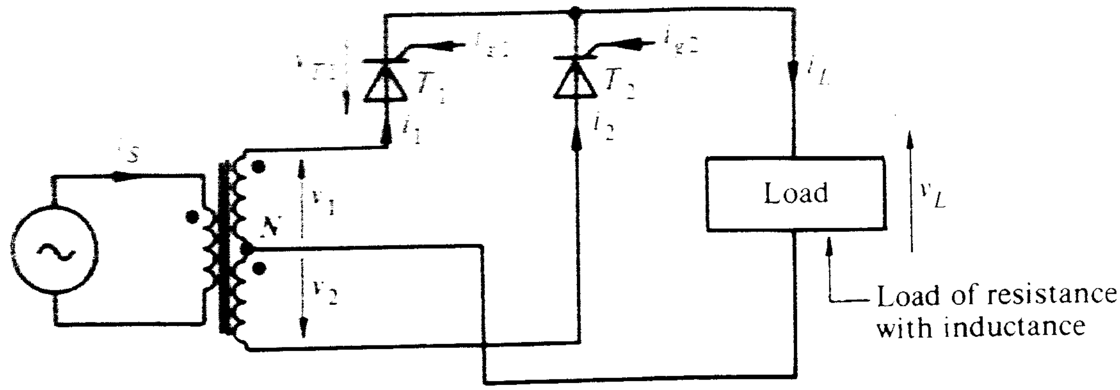
2-4 BI-PHASE HALF-WAVE (OR SINGLE-WAY)

The bi-phase connection of Fig. 2-5a provides two voltages v_1 and v_2 in anti-phase relative to the mid-point neutral N. In this half-wave connection, the load is fed via a thyristor in each supply line, the current being returned to the supply neutral N.

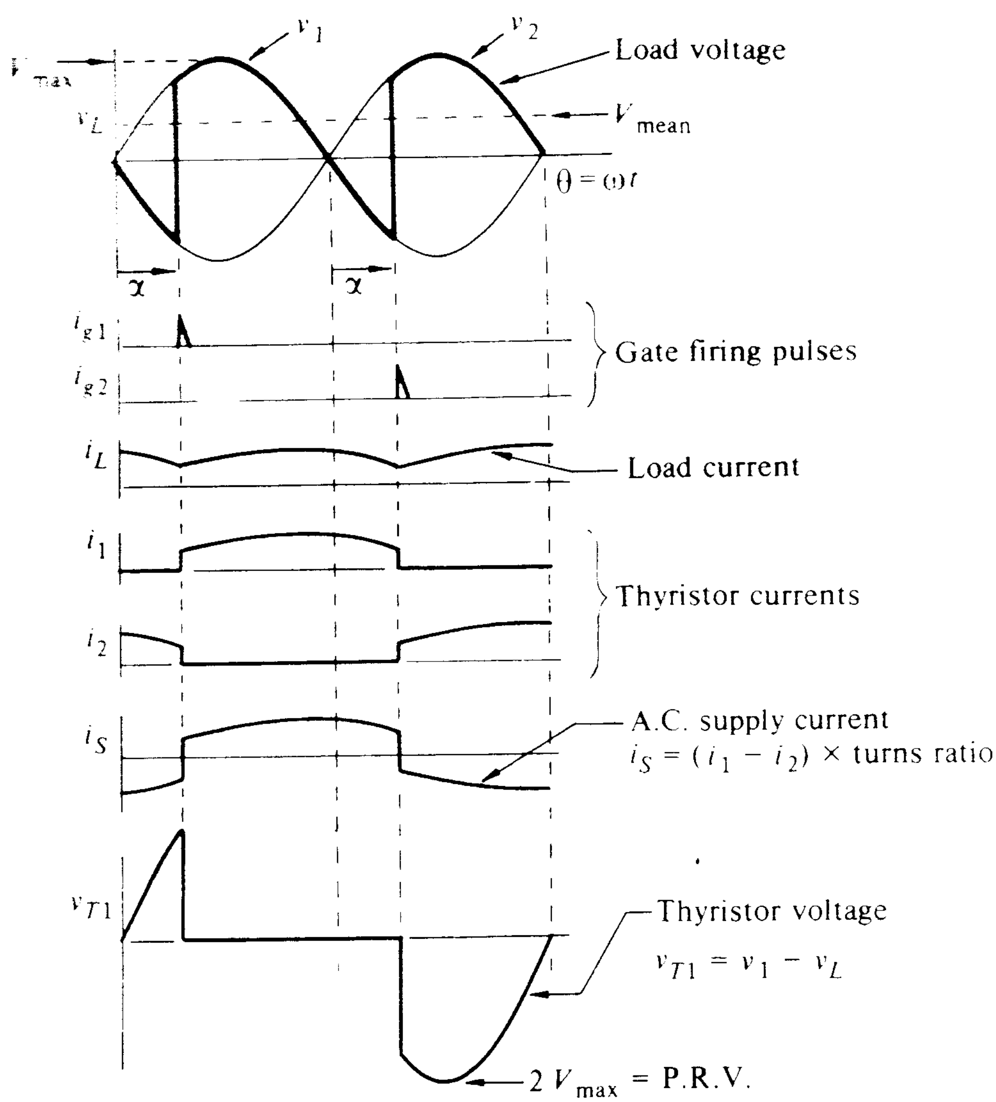
In any simple half-wave connection, only one rectifying device (thyristor or diode) will conduct at any given time, in the diode case this being that one connected to the phase having the highest voltage at that instant. In the controlled circuit, a given thyristor can be fired during any time that its anode voltage is positive relative to the cathode.

With reference to Fig. 2-5, thyristor T_1 can be fired into the on-state at any time after v_1 goes positive. The firing circuits are omitted from the connection diagram to avoid unnecessary confusion to the basic circuit, but can be assumed to produce a firing pulse into the respective thyristor gates as shown in the waveforms. Each pulse is shown delayed by a phase angle α relative to the instant where diodes would conduct, that is, if the thyristors were replaced by diodes, α would be zero.

Once thyristor T_1 is turned on, current builds up in the inductive load, maintaining thyristor T_1 in the on-state into the period when v_1 goes negative. However, once v_1 goes negative, v_2 becomes positive, and the firing of thyristor T_2 immediately turns on thyristor T_2 which takes up the load current, placing a reverse voltage on thyristor T_1, its current being commutated (transferred) to thyristor T_2. The thyristor voltage v_T waveform in Fig. 2-5b shows that it can be fired into conduction at any time when v_T is positive. The peak reverse (and forward) voltage that appears across the thyristor is $2V_{\text{max}}$, that is, the maximum value of the com-



(a)



(b)

Figure 2-5 Bi-phase half-wave circuit. (*a*) Connection. (*b*) Waveforms.

plete transformer secondary voltage. Figure 2-6 illustrates this fact more clearly in that when thyristor T_2 is on and effectively a short-circuit, the entire transformer voltage appears across the off-state thyristor T_1.

Inspection of the load voltage waveform in Fig. 2-5*b* reveals that it has a mean

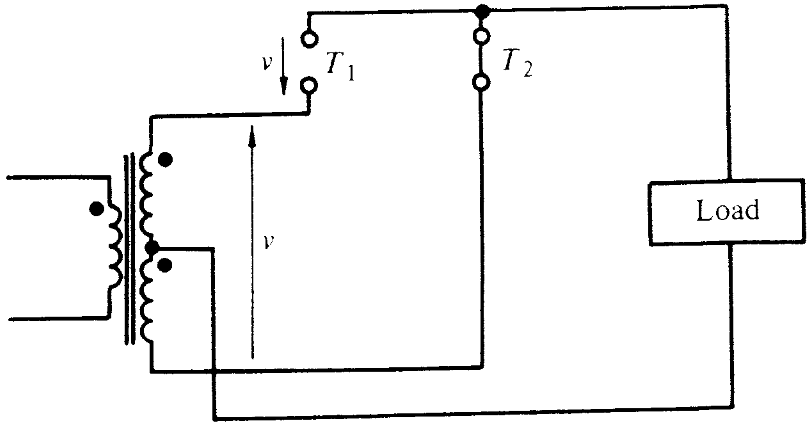


Figure 2-6 Illustrating instantaneous circuit condition.

value of

$$V_{\text{mean}} = (1/\pi)\int_{\alpha}^{\pi+\alpha} V_{\max} \sin\theta \, d\theta = \frac{2V_{\max}}{\pi} \cos\alpha \qquad (2\text{-}6)$$

In practice, the load voltage will be reduced by the volt-drop across one thyristor, as at all times there is one thyristor in series with the supply to the load. Also, this equation assumes high enough load inductance to ensure continuous load current. The highest value of the mean voltage will be when the firing delay angle α is zero, that is, the diode case. When the firing delay angle is 90°, the load voltage will contain equal positive and negative areas, giving zero output voltage, a value confirmed by Eq. (2-6) which shows the mean voltage follows a cosine variation with firing delay angle. As the load voltage waveform repeats itself twice in the time of one supply cycle, the output has a two-pulse characteristic.

The current waveforms in Fig. 2-5*b* show a continuous load current, but the ripple will increase as the mean voltage is reduced until, given a light load inductance, the load current will become discontinuous, the load voltage then having zero periods. The thyristor currents are of half-cycle duration and tend to be square-shaped for continuous load current. The a.c. supply current can be seen to be non-sinusoidal and delayed relative to the voltage.

2-5 SINGLE-PHASE BRIDGE (OR DOUBLE-WAY)

The bridge (full-wave or double-way) connection can be arranged to be either uncontrolled, fully-controlled, or half-controlled configurations, and this section will describe each connection in turn.

2-5-1 Uncontrolled

The single-phase bridge circuit connection is shown in Fig. 2-7*a* in its simplest diagrammatic layout. This layout, whilst almost self-explanatory and widely used in electronic circuit layouts, does not at a glance demonstrate that it is two half-wave

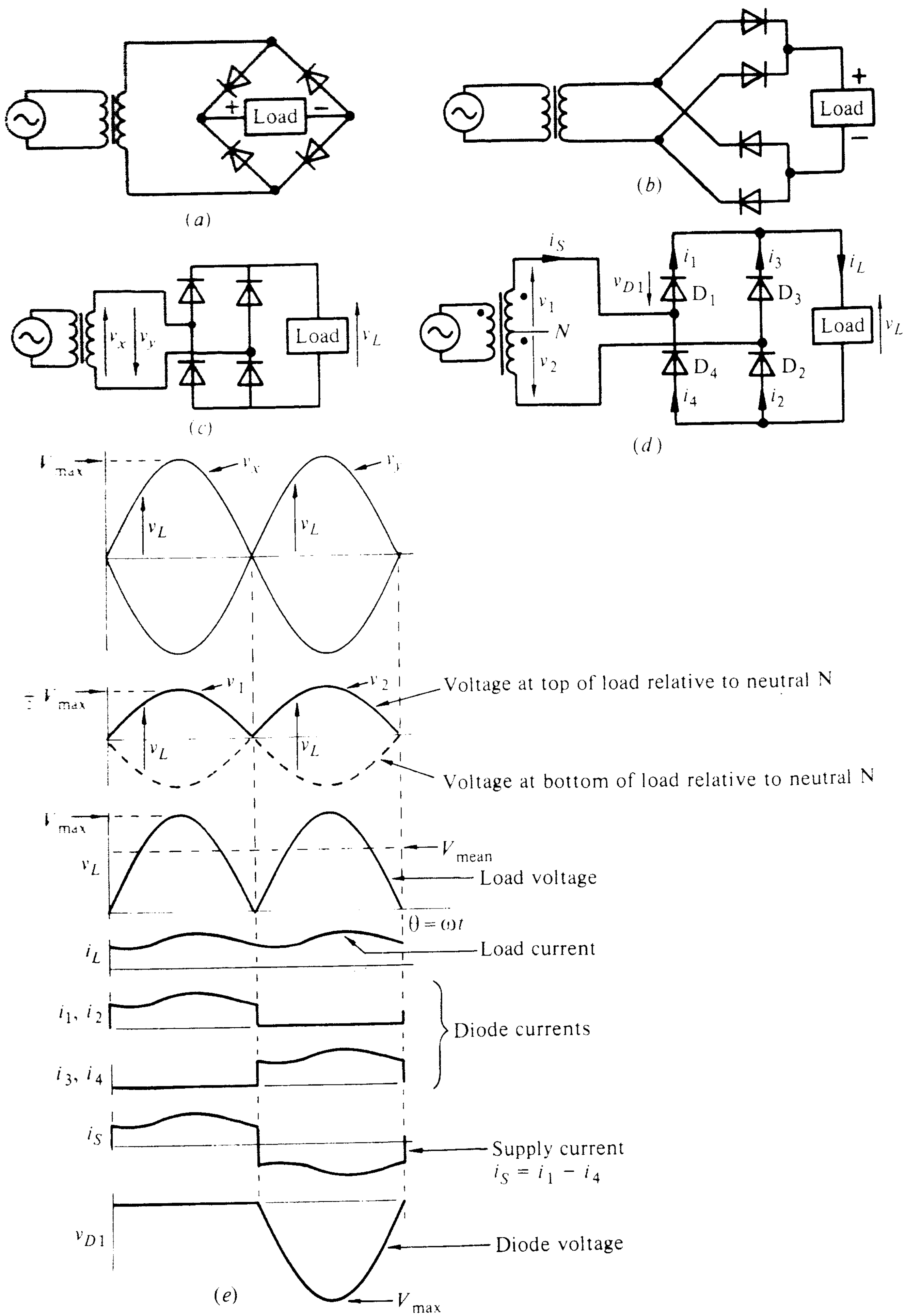


Figure 2-7 Single-phase bridge circuit. (*a*) – (*d*) Connection drawn to differing diagram lay-

circuits in series, nor is it possible to draw a similar layout for the three-phase circuits. The same circuit drawn to a different diagrammatic layout as in Fig. 2-7*b* shows clearly the concept of two half-wave circuits in series making the full-wave connection, two diodes with common cathodes feeding into the load, two diodes with common anodes returning the load current to the other supply line. However, the layout of Fig. 2-7*b* is rather cumbersome, and for power applications the layout of Fig. 2-7*c* is used.

In constructing the voltage waveforms, some circuit reference must be used, and in this respect one can construct the supply waveforms with reference to a mid-point neutral N as shown in Fig. 2-7*d*, thus enabling a comparison to be made to the half-wave circuit of Fig. 2-5. As only a simple two-winding transformer is required, the mid-point is neither required nor available in practice, and in this respect it is useful to look at Fig. 2-7*c* where the supply is given two labels v_x and v_y shown in the waveforms of Fig. 2-7*e*.

The load voltage shown in Fig. 2-7*e* can be constructed either by taking the waveforms of v_x and v_y when each is positive, or by constructing the voltages on each side of the load relative to the neutral N, the difference between them being the load voltage v_L. The use of the neutral N does demonstrate that the load voltage is the addition of two half-wave circuit voltages in series, making a full-wave connection. The diode voltage v_{D1} has a peak reverse value of the maximum value of the supply voltage, this being only half the value in the half-wave connection of Fig. 2-5 for the same load voltage; however, two diodes are always conducting at any given instant, giving a double volt-drop.

The diode and supply current waveforms shown in Fig. 2-7*e* are identical in shape to the half-wave connection of Fig. 2-5. The output characteristic is two-pulse, hence as regards the load response and supply requirements the bridge connection is similar to the bi-phase half-wave circuit.

2-5-2 Fully-controlled

The fully controlled circuit shown in Fig. 2-8 has thyristors in place of the diodes of Fig. 2-7. Conduction does not take place until the thyristors are fired and, in order for current to flow, thyristors T_1 and T_2 must be fired together, as must thyristors T_3 and T_4 in the next half cycle. To ensure simultaneous firing, both thyristors T_1 and T_2 are fired from the same firing circuit as shown in Fig. 2-9, the output being via a pulse transformer as the cathodes of the respective thyristors are at differing voltages in the main circuit.

The load voltage is the same as that described for the bi-phase half-wave connection with a mean value as for Eq. (2-6) of

$$V_{\text{mean}} = \frac{2V_{\text{max}}}{\pi} \cos \alpha \tag{2-7}$$

less in this case by two thyristor volt-drops. This equation will not apply if the load current is not continuous.

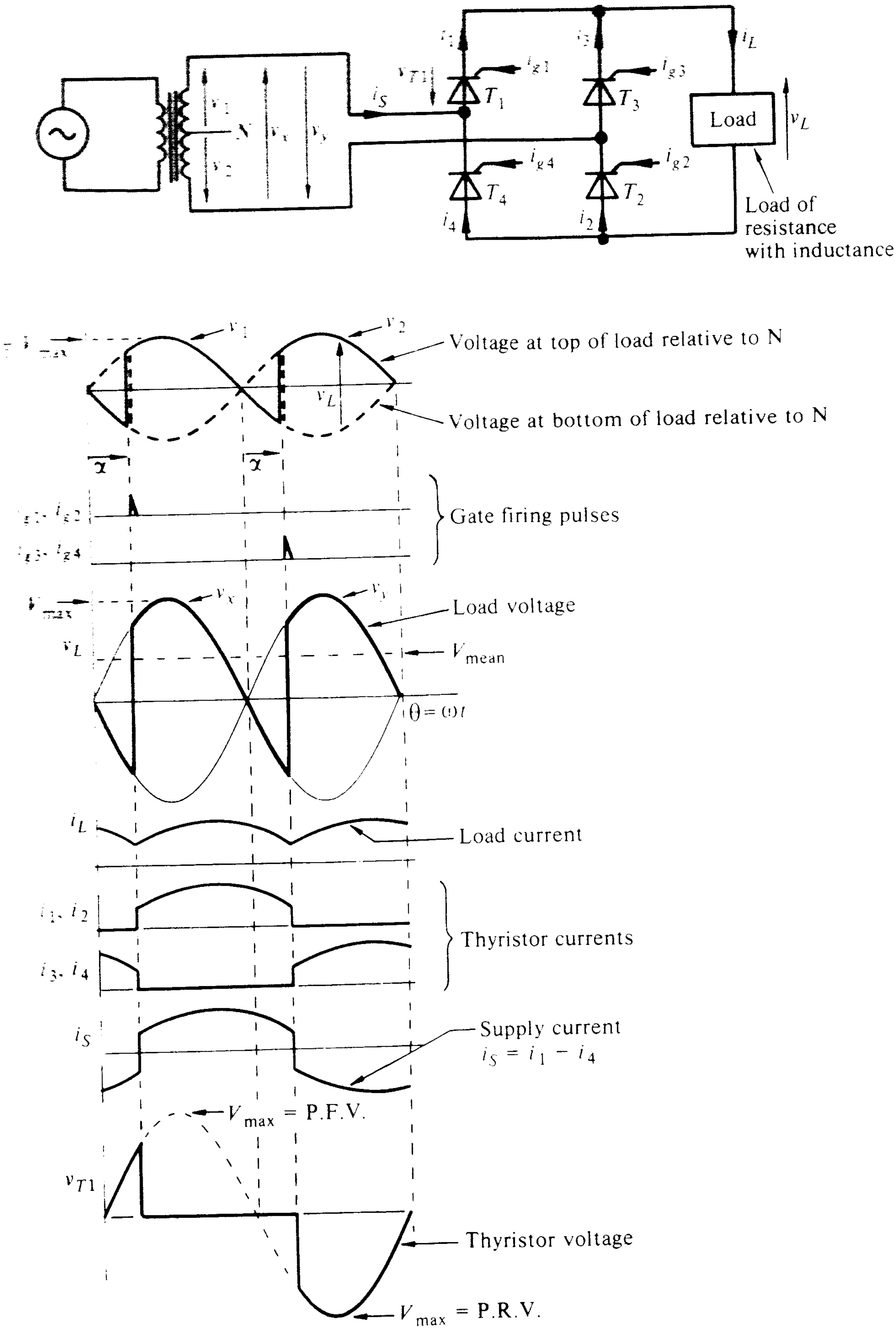


Figure 2-8 Fully-controlled single-phase bridge. (*a*) Connection. (*b*) Waveforms.

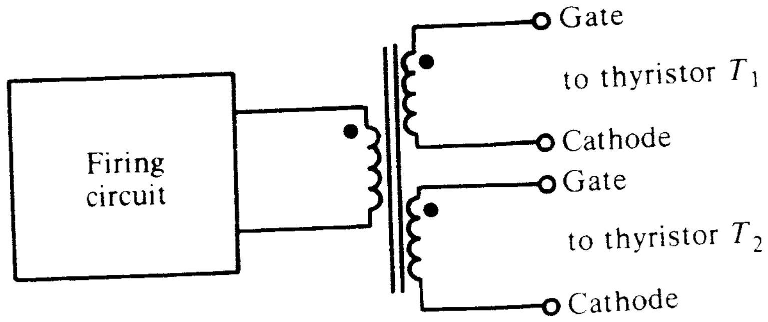


Figure 2-9 Firing circuit output connections.

2-5-3 Half-controlled

It is possible to control the mean d.c. load voltage by using only two thyristors and two diodes, as shown in the half-controlled connection of Fig. 2-10*a*. Drawing the circuit to a different diagrammatic layout, as shown in Fig. 2-11, shows clearly that the full-wave connection is the addition of two half-wave circuits, the input current to the load being via the thyristors, whilst the diodes provide the return path. The approach to determining the load-voltage waveform in Fig. 2-10*b* is to follow previous reasoning by plotting the voltage at each end of the load relative to the supply neutral N. The thyristors commutate when fired, the diodes commutating at the supply voltage zeros. The load voltage so constructed never goes negative and follows a shape as if the load were pure resistance, reaching a zero mean value when the firing delay angle α is 180°.

The presence of the commutating diode obviously prevents a negative load voltage, but this would in any case be the situation, even without the commutating diode. After the supply voltage zero and before (say) thyristor T_3 is fired, thyristor T_1 would continue conducting, but the return load-current path would have been commutated from diode D_2 to diode D_4, hence a free-wheeling path for the load current would be provided via T_1 and D_4, resulting in zero supply current. The commutating diode will provide a preferential parallel path for this free-wheeling load current compared to the series combination of a thyristor and diode, and hence enable the thyristor to turn off and regain its blocking state.

The mean value of the load voltage will be given by

$$V_{\text{mean}} = (1/\pi)\int_{\alpha}^{\pi} V_{\max}\sin\theta\, d\theta = \frac{V_{\max}}{\pi}(1+\cos\alpha) \tag{2-8}$$

different by the various thyristor and diode volt-drops.

The current duration in the thyristors and main diodes is less than 180° by the firing delay angle α, leading to an a.c. supply current which has zero periods. The commutating diode conducts the decaying load current during the zero voltage periods.

Compared to the fully-controlled circuit, the half-controlled circuit is cheaper, but the a.c. supply current is more distorted due to its zero periods. The half-controlled connection cannot be used in the inversion mode described in Sec. 3-3, only the fully-controlled connection allows a reversal of the mean direct voltage.

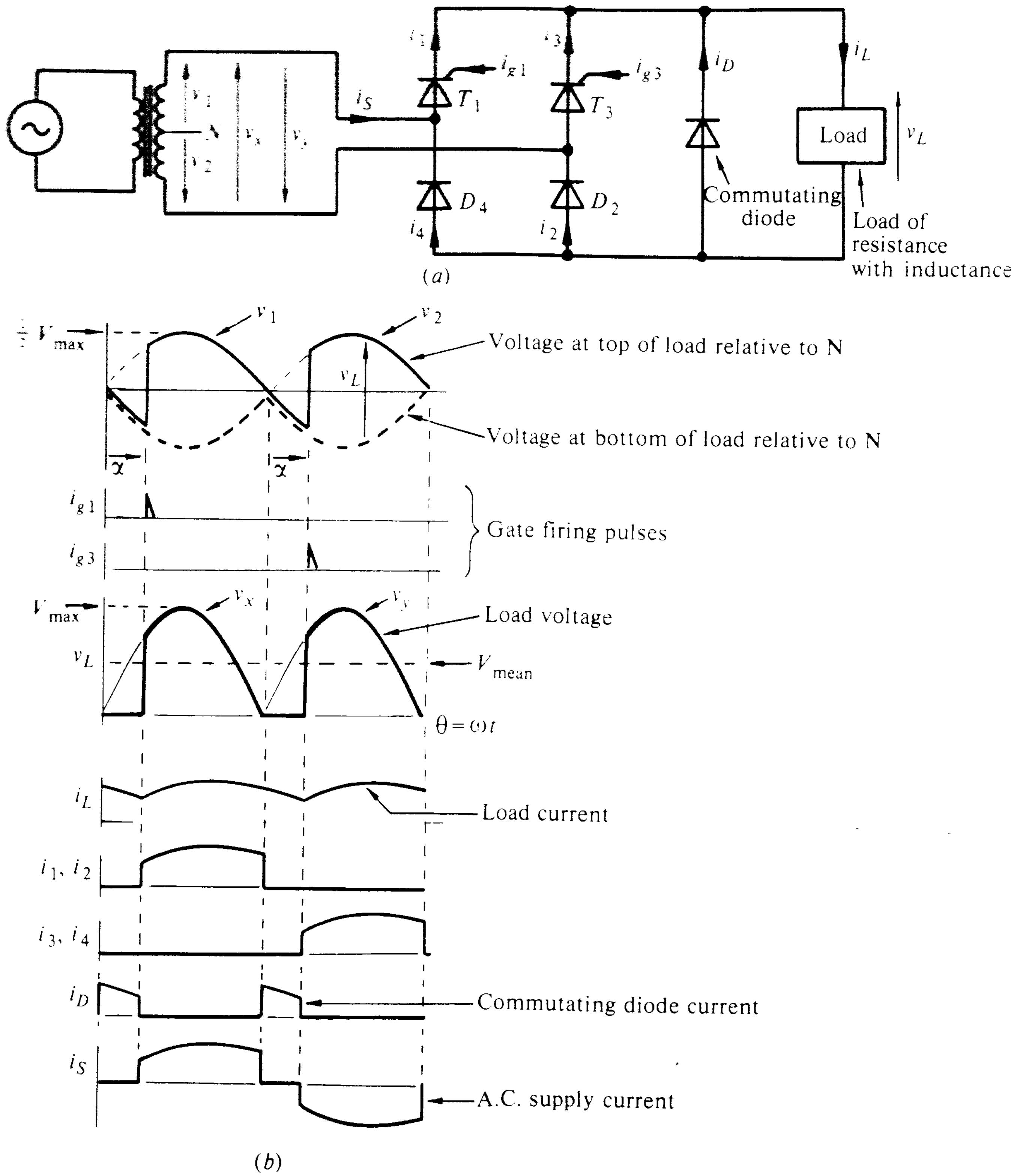


Figure 2-10 Half-controlled single-phase bridge. (*a*) Connection. (*b*) Waveforms.

2-6 THREE-PHASE HALF-WAVE (OR SINGLE-WAY)

The three-phase half-wave connection is the basic element in most of the polyphase rectifier circuits, although its use in its own right is limited, due in part to it requiring a supply transformer having an interconnected-star (zig-zag) secondary. However, to ease the explanation, the supply will initially be assumed as a simple star-connected winding.

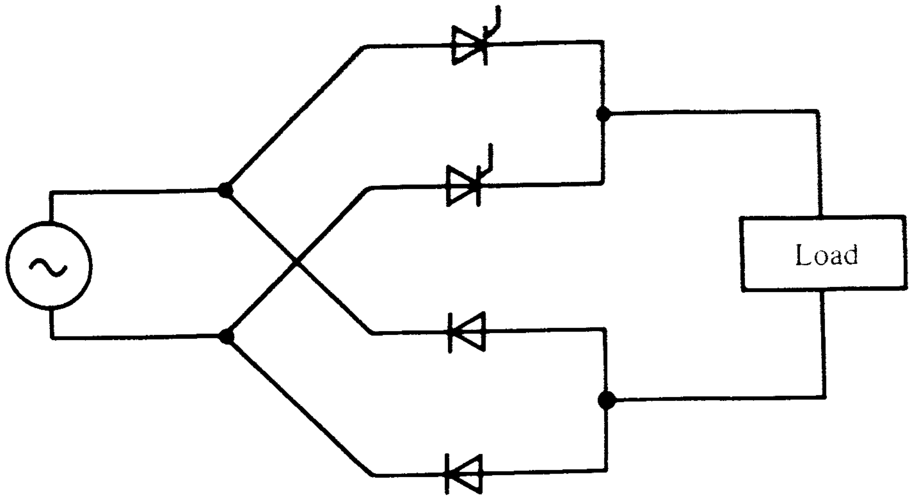


Figure 2-11 Half-controlled full-wave (bridge) connection.

With the polyphase connections, the time intervals between the repetitions in the d.c. load waveforms are shorter than for single-phase connections, and also in practice they will be supplying larger loads having heavier inductance. The net result is for the ripple content of the load current to be less, and it is reasonable to assume the current to be continuous and level. Hence, in developing the current waveforms for the polyphase circuits, it will be assumed that the load current is continuous and level, that is, it has negligible ripple.

The connection of the three-phase half-wave circuit is shown in Fig. 2-12*a*, each supply phase being connected to the load via a diode and, as in all half-wave connections, the load current being returned to the supply neutral.

The circuit functions in a manner such that only one diode is conducting at any given instant, that one which is connected to the phase having the highest instantaneous value. This results in the load voltage v_L having the waveform shown in Fig. 2-12*b*, which is the top of the successive phase voltages. While v_1 is the most positive phase, diode D_1 conducts but, directly v_2 becomes more positive than v_1, the load current commutates (transfers) from diode D_1 to diode D_2. Confirmation of the instant of commutation can be seen by examining the diode voltage waveform v_D, which goes negative directly v_1 has an instantaneous value below v_2, hence diode D_1 turns off.

The instantaneous d.c. load voltage varies between the maximum value of the phase voltage and half this value, and it also repeats itself three times per cycle, thus having a three-pulse characteristic. Comparison of the load voltage in Fig. 2-12*b* to the two-pulse load voltage of Fig. 2-5 or 2.7 shows the three-pulse connection has a much smaller ripple.

The mean value of the load voltage is given by

$$V_{\text{mean}} = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_{\text{max}} \sin\theta \, d\theta = \frac{3\sqrt{3}}{2\pi} V_{\text{max}} \qquad (2\text{-}9)$$

less by the single diode volt-drop.

Assuming level d.c. load current I_L, the diode currents shown in Fig. 2-12*b*

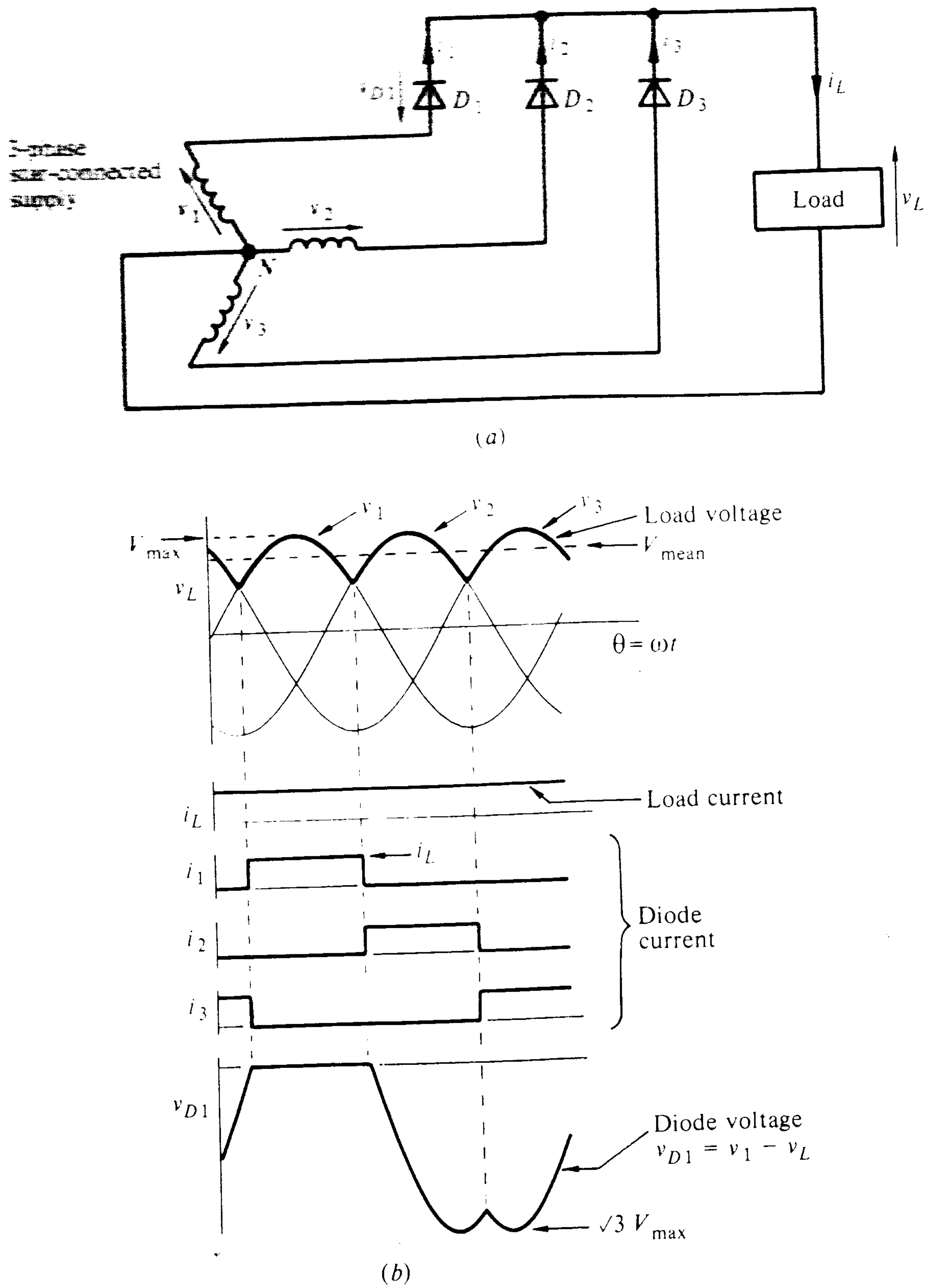


Figure 2-12 Three-phase half-wave circuit using diodes. (*a*) Connection. (*b*) Waveforms.

are each blocks one-third of a cycle in duration. Using the r.m.s. value of the diode current for the required rating purposes for each diode,

$$I_{rms} = I_L/\sqrt{3} \tag{2-10}$$

an expression which can be calculated by using calculus, or more simply by taking the square root of the mean of the sum of the (current)^2 over three equal intervals

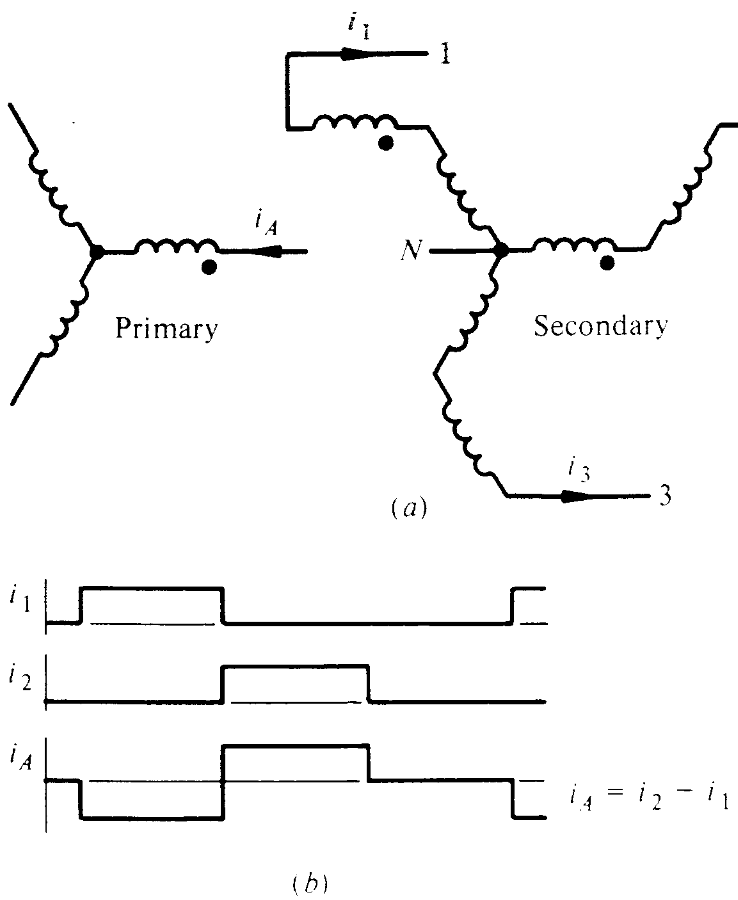


Figure 2-13 Interconnected-star connection of secondary. (*a*) Transformer circuit. (*b*) Current waveform.

in the cycle, that is,

$$I_{\text{rms}} = \left(\frac{I_L^2 + 0^2 + 0^2}{3}\right)^{1/2} = I_L/\sqrt{3}$$

Examination of the diode voltage waveform shows the peak reverse voltage to be $\sqrt{3}V_{\text{max}}$, which is the maximum value of the voltage between any two phases, that is, the maximum value of the line voltage.

It was stated earlier that the simple star connection of the supply was not appropriate, the reason being that the unidirectional current in each phase will lead to possible d.c. magnetization of the transformer core. To avoid this problem the interconnected-star (sometimes called zig-zag) winding shown in Fig. 2-13*a* is used as the secondary of the supply transformer. The current which is reflected into the primary is now a.c. as shown in Fig. 2-13*b*, being as much positive as negative, hence avoiding any d.c. component in the core m.m.f.

When the diodes of Fig. 2-12*a* are replaced by thyristors as shown in Fig. 2.14*a*, the circuit becomes fully controllable, with the mean load voltage being adjustable by control of the firing delay angle α. Again the firing circuits are not shown, but it may be assumed that each thyristor has a firing circuit connected to its gate and cathode, producing a firing pulse relative in position to its own phase voltage. A master control will ensure that the three gate pulses are displaced by 120° relative to each other, giving the same firing delay angle to each thyristor.

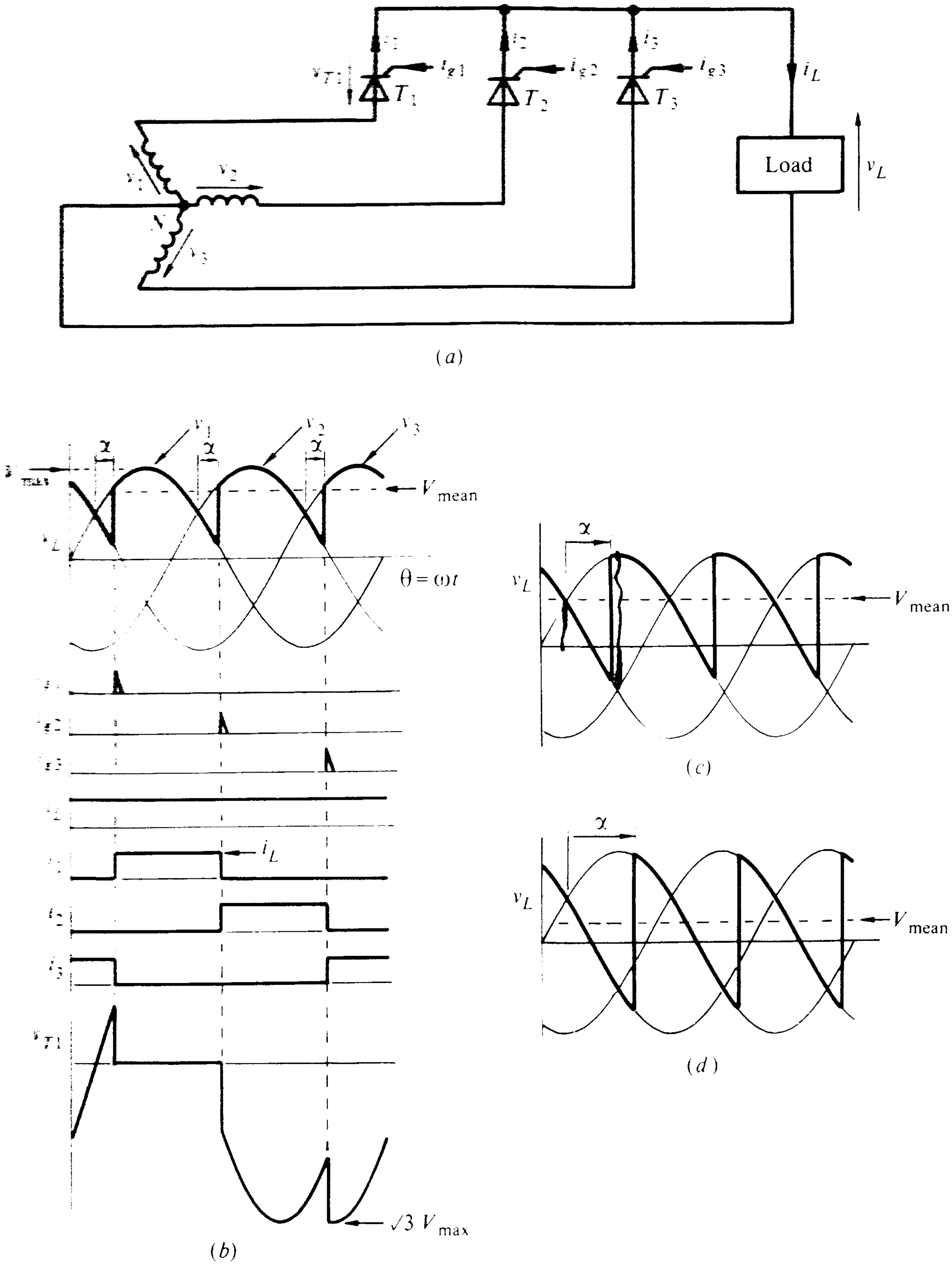


Figure 2-14 Three-phase half-wave controlled circuit using thyristors. (*a*) Connection. (*b*) Waveforms with samll firing delay angle. (*c*) and (*d*) Load voltage waveform with large firing delay angle.

The firing delay angle α is defined such that it is zero when the output mean voltage is a maximum, that is, the diode case. Hence, the firing delay angle α shown in Fig. 2-14 is defined relative to the instant when the supply phase voltages cross and diodes would commutate naturally, not the supply voltage zero.

Reference to Fig. 2-14b shows that the thyristors will not take up conduction until turned on by the gate pulse, thereby allowing the previous phase voltage to continue at the load, so giving an overall lower mean load voltage. The ripple content of the load voltage is increased, but it still has the three-pulse characteristic. The current waveform shapes have not changed to, but are delayed by, the angle α relative to the diode case. The thyristor voltage v_T shows that the thyristor anode voltage is positive relative to the cathode after the position of zero firing delay angle.

The load voltage waveforms of Fig. 2-14c and d show the effect of a larger delay angle, the voltage having instantaneous negative periods after the firing delay angle $\alpha = 30°$. The mean load voltage is given by

$$V_{\text{mean}} = \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_{\text{max}} \sin\theta \, d\theta = \frac{3\sqrt{3}}{2\pi} V_{\text{max}} \cos\alpha \tag{2-11}$$

less by the single thyristor volt-drop.

As in the two-pulse Eq. (2-6), the mean voltage is proportional to the cosine of the firing delay angle α, being zero at 90° delay. The assumption of level continuous load current will be less valid as the mean voltage approaches zero, due to the greatly increased ripple content of the load voltage.

2-7 SIX-PHASE HALF-WAVE (OR SINGLE-WAY)

The connections, with waveforms, of the six-phase half-wave circuit using a simple star supply are shown in Fig. 2-15. The theory of the connection is an extension of the three-phase half-wave circuit, each thyristor conducting for one sixth cycle.

The load-voltage waveform is the top of the six-phase voltages for the diode case, delayed by the firing delay angle α as shown in Fig. 2-15b when thyristors are used. The load-voltage waveform is of six-pulse characteristic, having a small ripple in the diode case at a frequency of six times the supply frequency. Including firing delay, the mean value of the load voltage is

$$V_{\text{mean}} = \frac{1}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_{\text{max}} \sin\theta \, d\theta = \frac{3}{\pi} V_{\text{max}} \cos\alpha \tag{2-12}$$

The thyristor voltage waveform of Fig. 2-15b shows the peak reverse (and forward) voltage to be twice the maximum value of the phase voltage. The diode is inefficiently used as it conducts for only one-sixth of the cycle, giving an r.m.s. value of

$$I_{\text{rms}} = I_L/\sqrt{6} \tag{2-13}$$

for a level load current I_L.

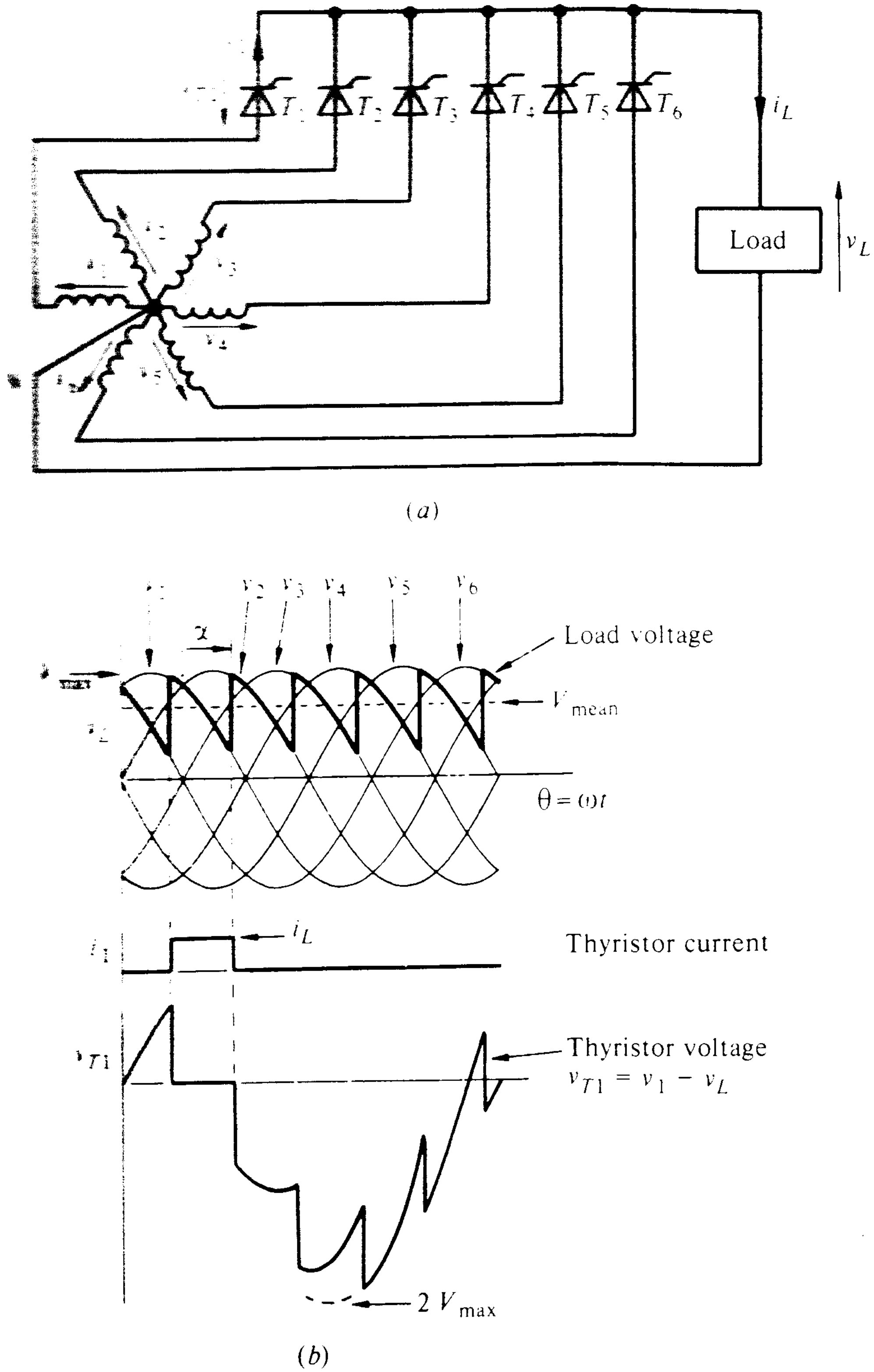


Figure 2-15 Simple six-phase half-wave circuit. (a) Connection. (b) Waveforms.

The simple star connection of Fig. 2-15*a* is not used in practice as the currents reflected into the primary winding have a large third-harmonic component. To eliminate the third-harmonic component, the fork connection shown in Fig. 2-16 can be used, but more frequently the double-star connection shown in Fig. 2-17 is used.

The double-star connection is essentially two independent three-phase half-wave circuits operating in parallel to give a six-pulse output. Fig. 2-17*a* shows the

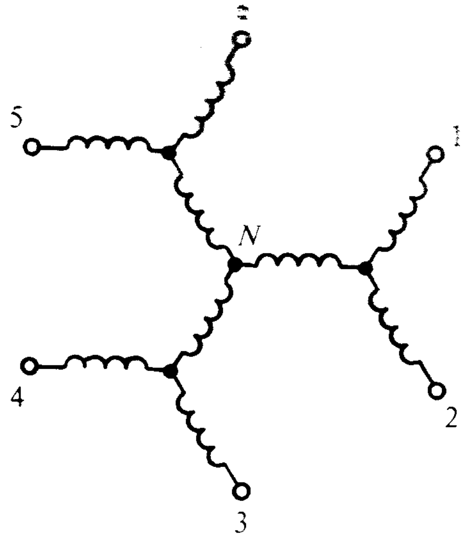


Figure 2-16 The six-phase fork connection.

two star groups supplied at 180° to each other, that is, if the two star points were rigidly connected, we would have a simple six-phase system. The two star points are linked via an interphase transformer, which is best considered as a reactor rather than a transformer. The load current is returned to the centre of the reactor.

Reference to the load-voltage waveform in Fig. 2-17*b* shows the two three-pulse waveforms of each star group relative to its own star point. The reactor allows each star group to conduct at the same time by taking up the voltage difference between the two star points, the load voltage then being midway between the two three-pulse groups. The load voltage has a six-pulse characteristic with a maximum instantaneous value of $(\sqrt{3}/2)V_{max}$ occurring where the phase voltages cross.

For the diode case shown in Fig. 2-17*b*, the load voltage can be calculated by finding the mean value of either three-pulse group, or directly from the actual six-pulse load-voltage waveform, giving

$$V_{mean} = \frac{3\sqrt{3}}{2\pi} V_{max} \tag{2-14}$$

less by one diode volt-drop as the two groups are in parallel.

Because the two groups act independently, each diode conducts for one third of each cycle, hence at any instant one diode in each group is conducting, each carrying one half of the load current. The current waveforms in Fig. 2-17*b* are developed to show that in a delta-connected primary a stepped current waveform is drawn from the three-phase a.c. supply. Compared to the simple six-pulse connection of Fig. 2-15, the current utilization of the diode and the input a.c. waveform are both much superior.

The reactor voltage v_R waveform shown in Fig. 2-17*b* is the difference between the two star groups, having an approximately triangular shape with a maximum value of one half that of the phases, and is at a frequency of three times that of the supply. In order for a voltage to be developed across the reactor, there must be a changing magnetic flux, which can only be developed with a magnetizing current.

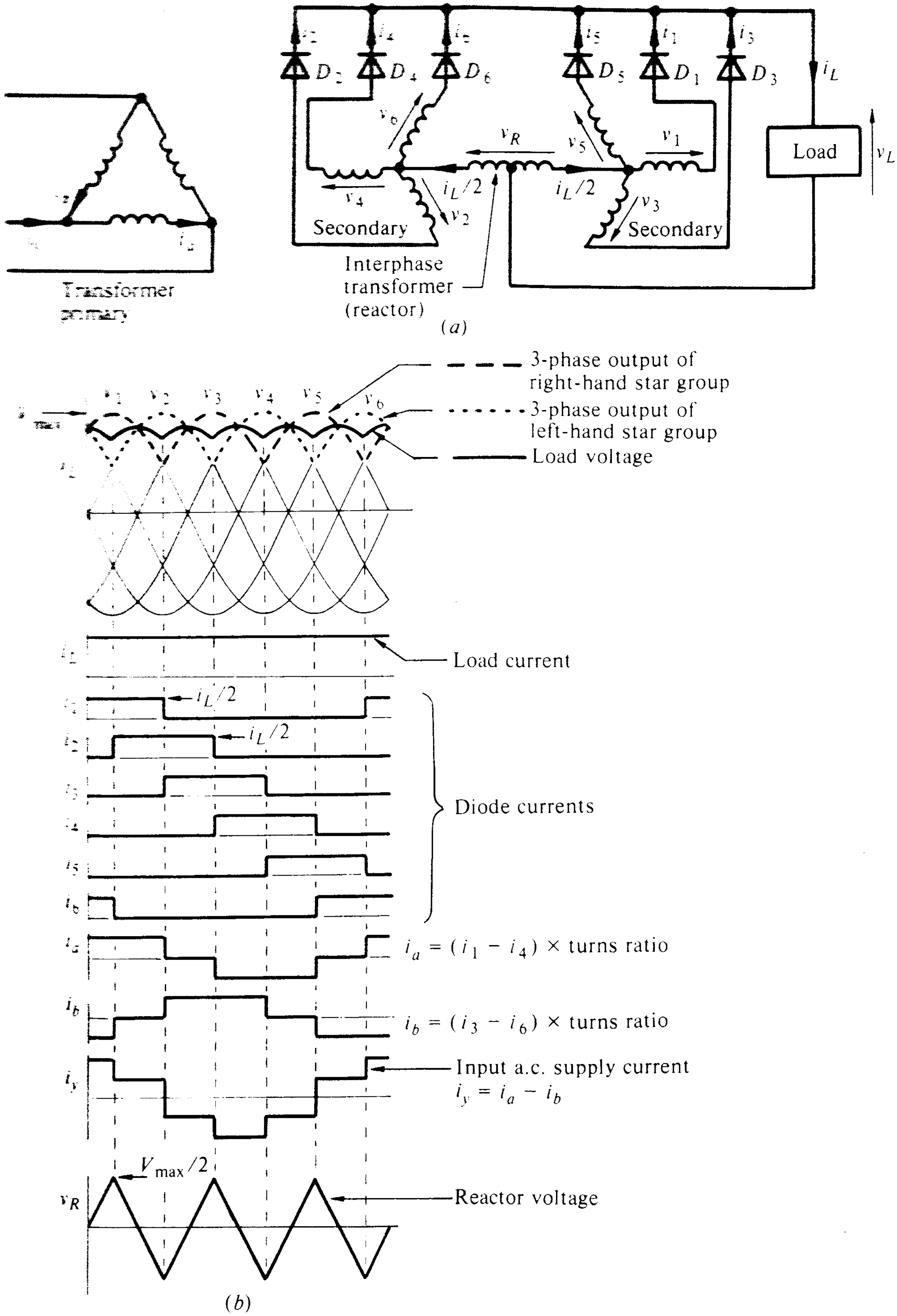


Figure 2-17 Double-star six-phase half-wave circuit. (*a*) Connection. (*b*) Waveforms.

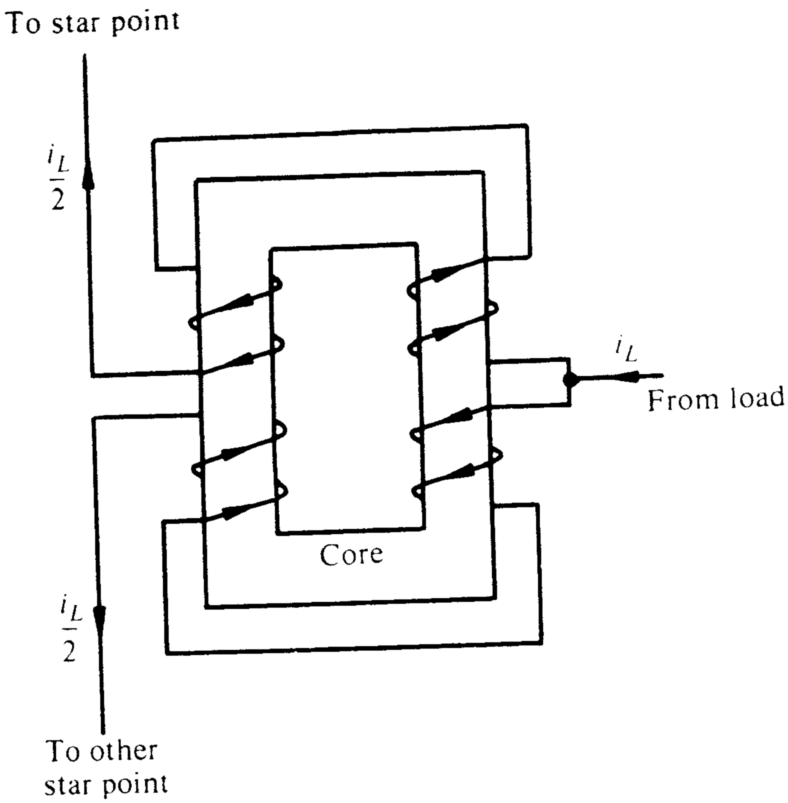


Figure 2-18 Construction of interphase transformer (reactor).

The path for the magnetizing current must be through the diodes, which is only possible when there is load current flowing – the magnetizing-current path being through the reverse diode as a component slightly reducing the diode forward current. The level of the load current must exceed the level of the magnetizing current for it to exist. If the load is disconnected, no magnetizing current can flow; hence no voltage can be developed across the reactor, so the star points are electrically common, and the circuit acts as a simple six-phase half-wave connection. To guarantee correct functioning of the circuit under all load conditions, a small permanent load in excess of the magnetizing current must be connected across the rectifier.

A typical construction of the interphase transformer (reactor) is shown in Fig. 2-18, which shows a two-legged core with two closely coupled windings on each leg. The close coupling of the windings will ensure an m.m.f. balance as in a transformer, forcing the load current to divide equally between the windings. The magnetizing current flowing from one star point to the other star point will act in the same direction in all coils to satisfy the flux requirement. As in the normal transformer, the magnetizing current represents the slight unbalance between the total current in the two windings on the same leg.

Each diode will have a peak reverse voltage requirement of $2V_{\max}$, as they will be required to withstand this voltage if the interphase transformer fails to excite, the circuit then acting as the simple six-phase half-wave connection shown in Fig. 2-15.

Replacing the diodes shown in Fig. 2-17*a* by thyristors converts the double-star connection into a fully-controlled circuit. The load-voltage waveform with a

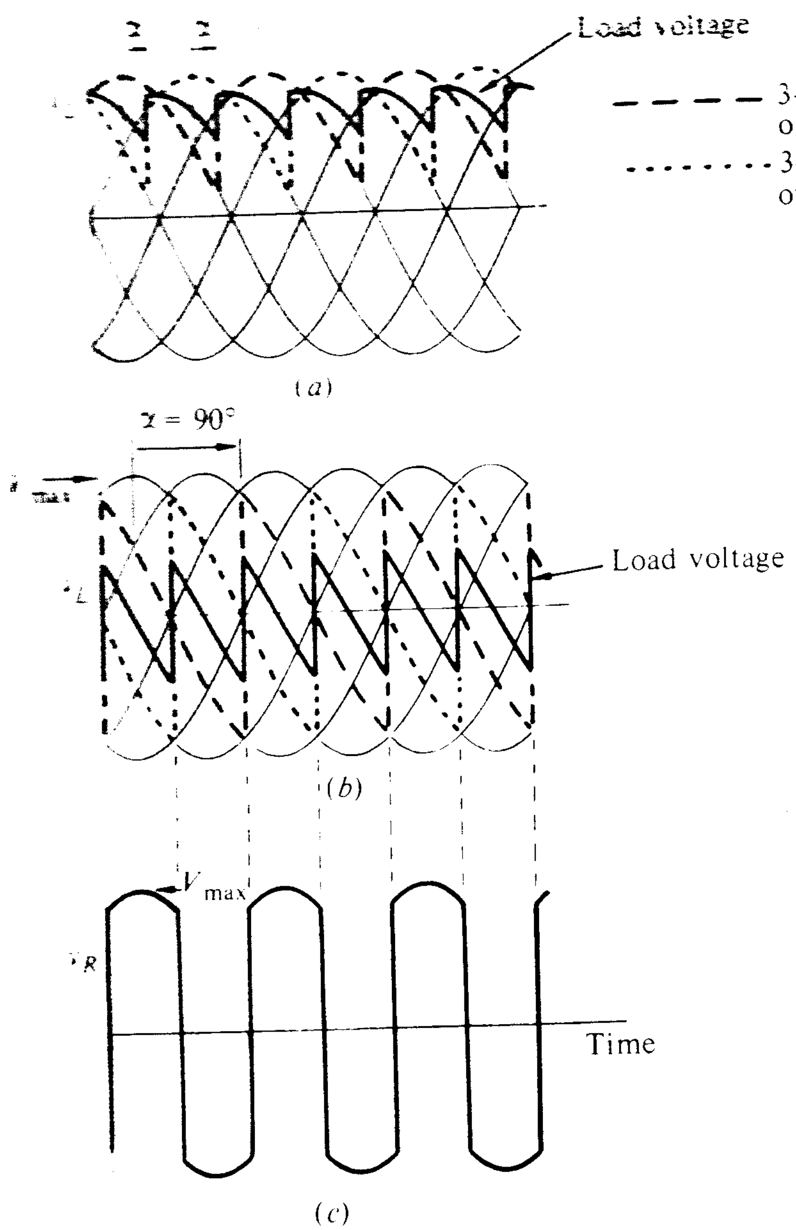


Figure 2-19 Voltage waveforms with controlled double-star connection. (*a*) Load voltage with small firing delay angle α. (*b*) Load voltage waveform with $\alpha = 90°$, V_{mean} = zero. (*c*) Interphase transformer (reactor) voltage at $\alpha = 90°$.

small firing delay angle α is shown in Fig. 2-19*a*, developed in the same manner as the diode case with the six-pulse load-voltage waveform midway between the two three-pulse groups. The mean voltage is, as in the earlier circuits, proportional to $\cos \alpha$ when the load current is continuous.

When the firing delay angle is 90°, the mean voltage is zero, and the load-voltage waveform is as shown in Fig. 2-19*b*. At this condition of zero mean load voltage, the interphase transformer (reactor) voltage is approximately rectangular as shown in Fig. 2-19*c*. The flux change in the reactor is proportional to the area of the voltage-time curve (from $v = d\phi/dt$ giving $\delta\phi = \int v\,dt$). Comparison of the rectangular waveform of Fig. 2-19*c* to the triangular waveform in the diode case shows an area three times larger; hence, with a flux change three times greater, the interphase transformer will be physically three times larger in the fully-controlled circuit as compared to the diode circuit.

2-8 THREE-PHASE BRIDGE (OR DOUBLE-WAY)

The three-phase bridge connection is most readily seen as a full-wave or double-way connection by reference to the circuit layout shown in Fig. 2-20. The load is fed via a three-phase half-wave connection, the return current path being via another half-wave connection to one of the three supply lines, no neutral being required. However, the circuit connection layout is more usually drawn as shown in Fig. 2-21*a*.

The derivation of the load-voltage waveform for the all-diode connection of Fig. 2-21 can be made in two ways. Firstly, one can consider the load voltage to be the addition of the two three-phase half-wave voltages, relative to the supply neutral N, appearing at the positive and negative sides of the load respectively. As the voltage waveforms of Fig. 2-21*b* show, the resultant load voltage is six-pulse in characteristic, having as its maximum instantaneous value that of the line voltage. An alternative approach to deriving the load-voltage waveform is to consider that the two diodes which are conducting are those connected to the two lines with the highest voltage between them at that instant. This means that when v_a is the most positive phase diode D_1 conducts, and during this period first v_b is the most negative with diode D_6 conducting, until v_c becomes more negative when the current in diode D_6 commutates to diode D_2. The load voltage follows in turn six sinusoidal voltages during one cycle, these being $v_a - v_b$, $v_a - v_c$, $v_b - v_c$, $v_b - v_a$, $v_c - v_a$, $v_c - v_b$, all having the maximum value of the line voltage, that is, $\sqrt{3}$ times the phase voltage. Although the supply is shown as star-connected in Fig. 2-21, a delta connection can equally well be used.

The mean value of the load voltage can either be calculated from the sum of the two three-pulse waveforms which, using Eq. (2-9), gives

$$V_{\text{mean}} = 2 \times \frac{3\sqrt{3}}{2\pi} V_{\text{ph(max)}} = \frac{3}{\pi} V_{\text{line(max)}} \qquad (2\text{-}15)$$

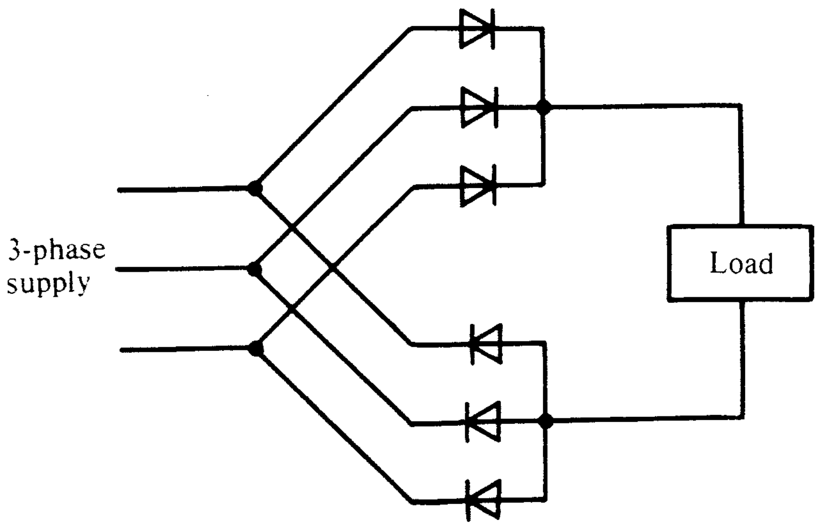


Figure 2-20 Three-phase full-wave circuit.

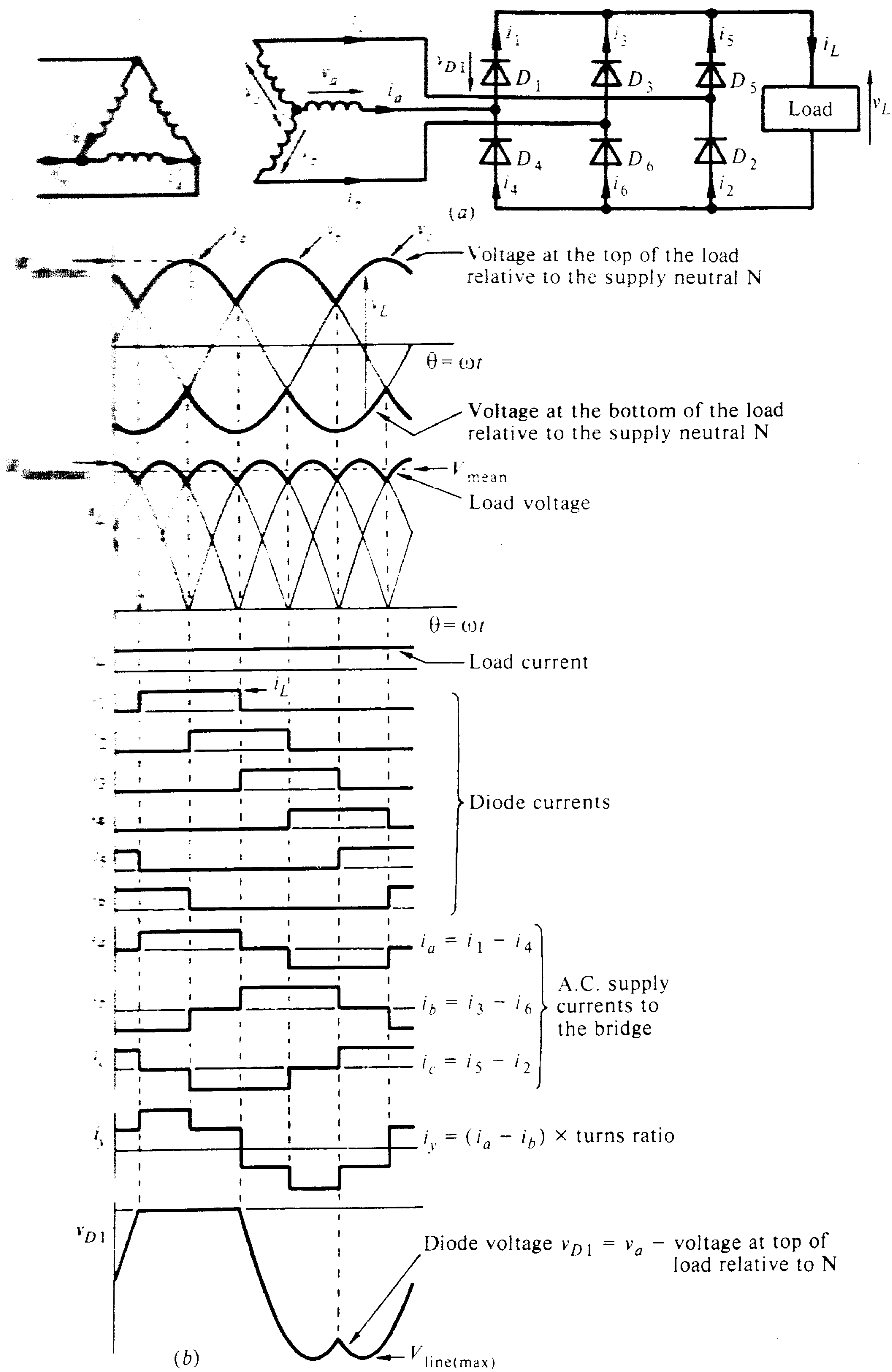


Figure 2-21 Three-phase bridge circuit. (a) Connection. (b) Waveforms.

or it can be calculated directly from the six-pulse load-voltage waveform which from Eq. (2-12) yields the same as Eq. (2-15) above. As two diodes are in series with the load, the mean value is reduced by two diode volt-drops.

The diode current waveforms shown in Fig. 2-21*b* reveal that each diode conducts the full-load current for one third of a cycle, the order of commutation determining the numbering of the diodes in the circuit. The diode voltage v_{D1} waveform can be determined as the difference between the phase voltage v_a and the voltage at the top of the load relative to the supply neutral N. The peak reverse voltage appearing across the diode is the maximum value of the line voltage.

Figure 2-21*b* shows the a.c. supply current to be symmetrical, but of a quasi-square shape. However, the current waveforms are closer to a sinusoidal shape than those in the single-phase bridge connection.

The three-phase bridge can be made into a fully-controlled connection by making all six rectifying elements thyristors, as shown in Fig. 2-22*a*. As in previous circuits, the mean load voltage is now controllable by delaying the commutation of the thyristors by the firing delay angle α.

With a small firing delay angle as shown in Fig. 2-22*b*, the waveshapes can be readily understood by reference to earlier circuits. The two three-pulse waveforms add to give the six-pulse load-voltage waveform. The current-waveform shapes are similar to the diode case, except they are delayed by the angle α.

A problem does arise with the bridge circuit that was not present in the earlier circuits, and that is the one of starting. When connected to the a.c. supply, firing gate pulses will be delivered to the thyristors in the correct sequence but, if only a single firing gate pulse is used, no current will flow, as the other thyristor in the current path will be in the off-state. Hence, in order to start the circuit functioning, two thyristors must be fired at the same time in order to commence current flow. With reference to Fig. 2-22*b* (say), the supply is connected when v_a is at its peak value, the next firing pulse will be to thyristor T_2. However, thyristor T_2 will not conduct unless at the same time thyristor T_1 is pulsed, as reference to the waveforms shows these are the two thyristors conducting at that instant. Hence, for starting purposes, the firing circuit must produce a firing pulse 60° after its first pulse. Once the circuit is running normally, the second pulse will have no effect, as the thyristor will already be in the on-state.

The starting pulse can be fed to the thyristor by each firing circuit having two isolated outputs, one to its own thyristor and the other to the previous thyristor. Alternatively, the firing circuits can be electronically linked so that, when each firing circuit initiates a pulse to its own thyristor, it also does likewise to the previous firing circuit.

When the firing delay is large, with the load voltage having negative periods, it is difficult to visualize the load-voltage waveform from the two three-pulse pictures; hence, as shown in Fig. 2-22*c*, the six line voltages $v_a - v_b$, $v_a - v_c$, $v_b - v_c$, $v_b - v_a$, $v_c - v_a$, $v_c - v_b$ give a direct picture of the load-voltage waveform and clearly show that zero mean voltage is reached when the firing delay angle is 90°.

The value of the mean load voltage is given by

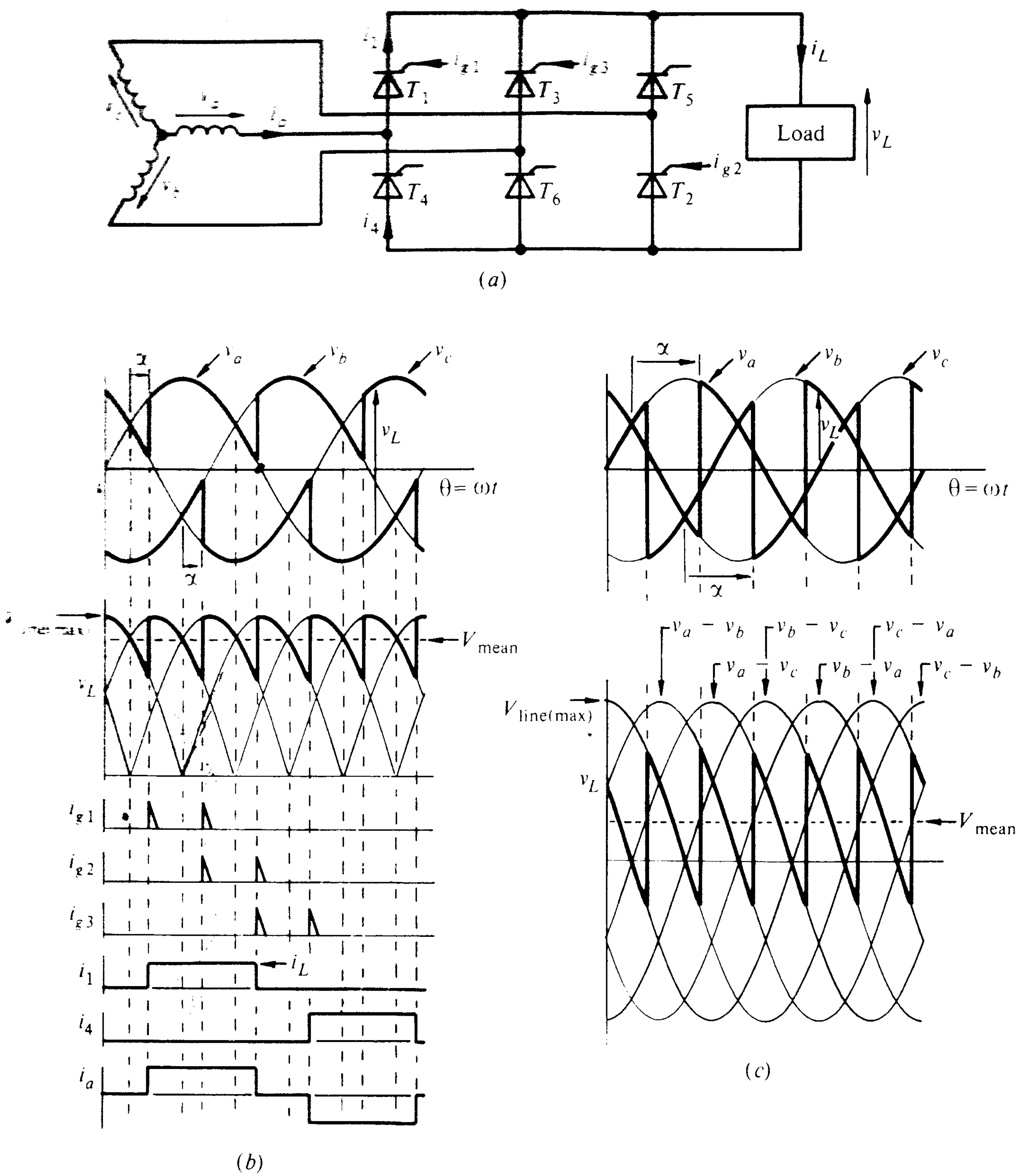


Figure 2-22 Fully-controlled three-phase bridge. (*a*) Connection. (*b*) Waveforms with small firing delay angle. (*c*) Voltage waveforms with large firing delay angle.

$$V_{\text{mean}} = \frac{3}{\pi} V_{\text{line(max)}} \cos \alpha \tag{2-16}$$

less by the two thyristor volt-drops.

Reference to Fig. 2-20 would indicate quite correctly that control of the load voltage is possible if three thyristors were used in the half-wave connection feeding

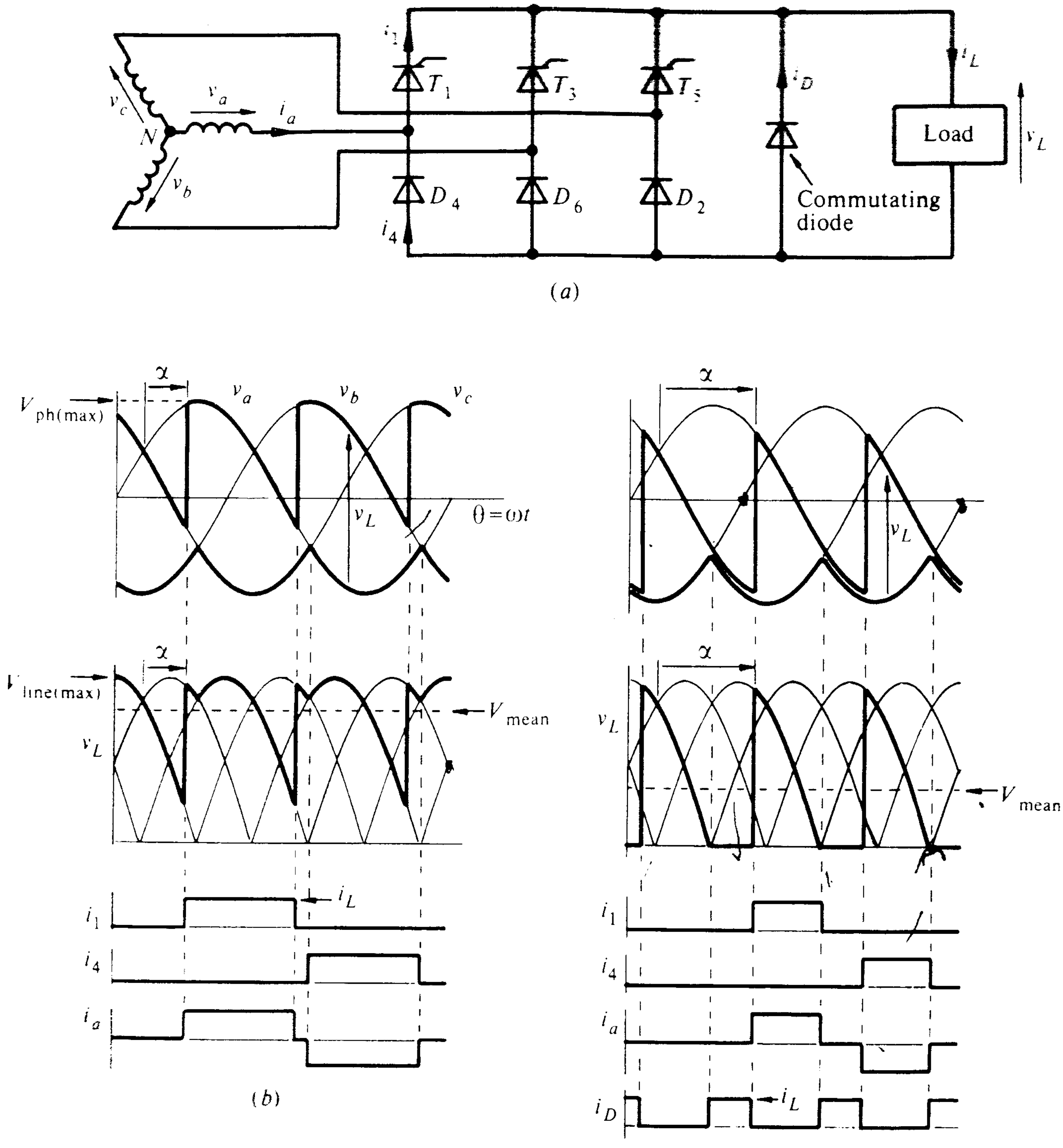


Figure 2-23 Half-controlled three-phase bridge. (*a*) Connection. (*b*) Waveforms with small firing delay angle. (*c*) Waveforms with larger firing delay angle.

the load, and diodes only used to return the current to the supply. Such a connection is shown in Fig. 2-23*a*, with the addition of a commutating diode whose function is similar to that in the single-phase half-controlled bridge.

The action of the circuit is most clearly explained by the two three-pulse voltage waveforms shown in Fig. 2-23*b*, where the upper waveform shows a small firing delay, whereas the lower waveform is that of the diode case. The addition of these two waveforms gives the load voltage v_L, with only three notches of voltage removed per cycle, not the six notches of the fully-controlled circuit. The waveform is now three-pulse, having a higher harmonic ripple component than the fully-controlled connection.

The current waveforms of Fig. 2-23b show that the thyristor current i_1 is delayed, but i_4 remains in phase with its voltage, resulting in an unsymmetrical a.c. supply-line current which will contain even harmonics.

Figure 2-23c shows a firing delay angle above 90°, making the upper waveform more negative than positive [to get to (c) from (b) think of the vertical line at the firing position moving to the right]. The load voltage now has periods of zero voltage, the commutating diode taking the freewheeling load current in preference to the series arm (of a thyristor plus diode) in a like manner to that described for the single-phase half-controlled bridge.

Inspection of the load-voltage waveforms in Fig. 2-23 shows that zero mean load voltage is reached when the firing delay angle α reaches 180°. The mean voltage can be considered as the addition of the two half-wave three-pulse voltages giving from Eqs. (2-9) and (2-11)

$$V_{\text{mean}} = \frac{3\sqrt{3}}{2\pi} V_{\text{ph(max)}} (1 + \cos\alpha) = \frac{3}{2\pi} V_{\text{line(max)}} (1 + \cos\alpha) \tag{2-17}$$

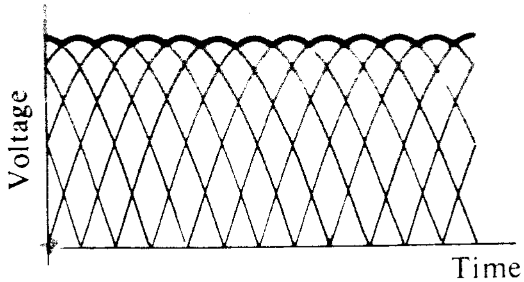
Compared to the fully-controlled circuit, the half-controlled circuit is cheaper, has no starting problems, but has a higher harmonic content in its load-voltage and supply-current waveforms.

2-9 TWELVE-PULSE CIRCUITS

Figure 2-24 illustrates the twelve-pulse voltage waveform for an uncontrolled, that is, diode, connection, where it is clearly close to a smooth direct voltage. The associated current shown is typical of the waveshape of the current drawn from a three-phase a.c. supply, this being closer to sinusoidal form than in the lower-pulse circuits. It is possible to conclude that the higher the pulse number of a rectifier, the closer it comes to the ideal of giving a level direct voltage and drawing a sinusoidal current from the a.c. supply.

Three of the most common connections which give a twelve-pulse characteristic are shown in Fig. 2-25. The half-wave connection of Fig. 2-25a is an extension of the double-star circuit described in Sec. 2-7. Here four star groups are displaced to give twelve phases 30° apart, linked via interphase transformers (reactors) to the load. Four diodes conduct simultaneously with only one diode volt-drop reducing the mean load voltage.

The full-wave connections involve linking two three-phase bridges as shown in Figs. 2-25b and c. The a.c. supply is from a transformer having two secondaries: one star-connected, the other delta-connected. In this manner the three-phase voltages supplying the two bridges are displaced by a phase angle of 30°, hence the two six-pulse outputs are symmetrically displaced to give an overall twelve-pulse output.



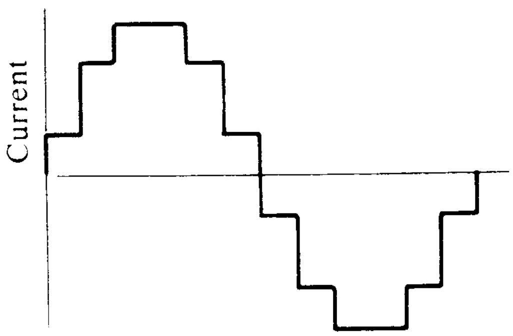


Figure 2-24 Twelve-phase waveforms. (*a*) Voltage of uncontrolled circuit. (*b*) Typical a.c. input current.

The series connection of Fig. 2-25*b* is for loads demanding a high voltage as the individual bridge outputs add, but the diode ratings relate to the individual bridge. The series connection also provides access to a centre point for earthing purposes. The two bridges may be joined in parallel as shown in Fig. 2-25*c*.

Higher pulse-number circuits may be built using the basic three-phase building blocks as for the twelve-pulse connections.

The circuits of Fig. 2-25 may be fully controlled by using thyristors, or partly controlled using a combination of thyristors and diodes.

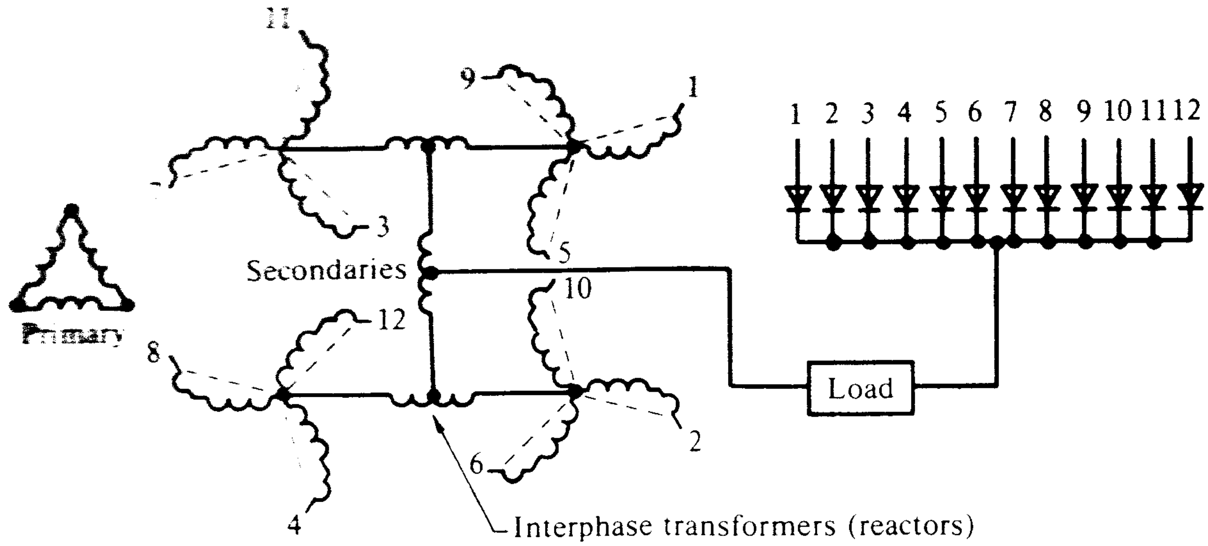
2-10 TRANSFORMER RATING

It is evident from the rectifier circuits described that the supply transformers carry currents which are non-sinusoidal, and the secondary is sometimes a connection of windings from different legs of the transformer core. The rating (or size) of the transformer must take these factors into account.

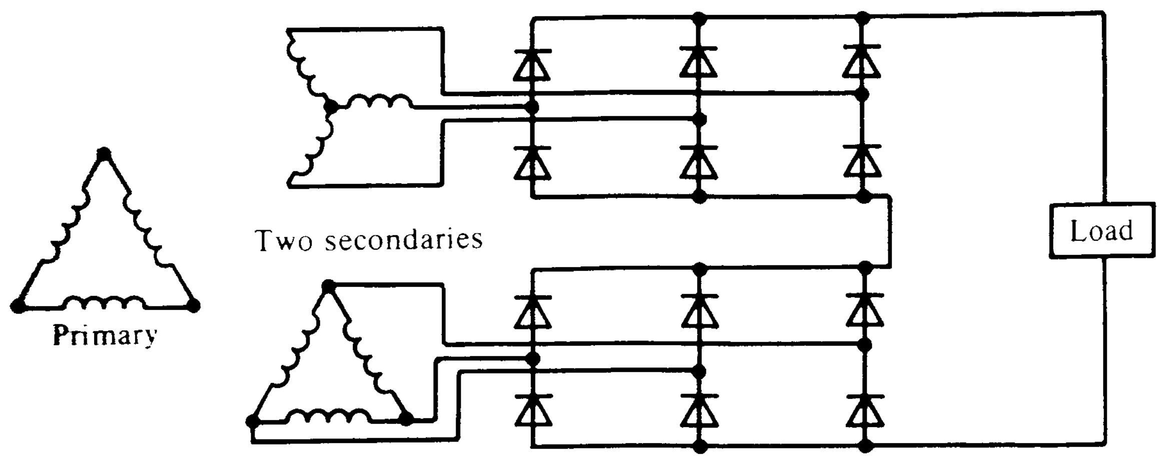
The winding rating of a transformer is the sum of the products of the number of windings, times their r.m.s. voltage, times the winding r.m.s. current.

The primary rating may differ from the secondary, particularly in the half-wave circuits due to the better current waveform and the absence of phases composed of windings linked from different legs. The fork connection of Fig. 2-16 demonstrates a connection where the secondary winding has a higher rating than the primary.

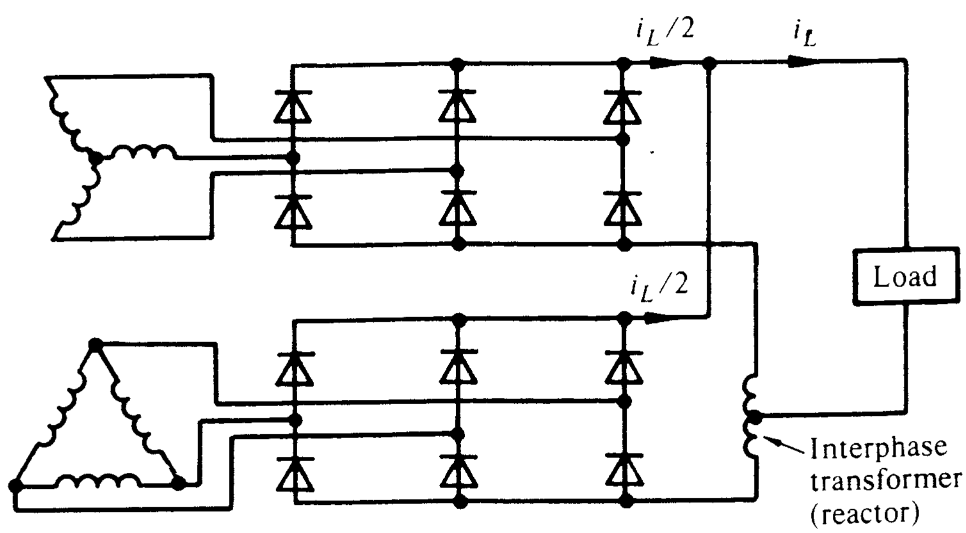
In those transformers where there are two or more secondary windings linked



(*a*)



(*b*)



(*c*)

Figure 2-25 Typical twelve-pulse connections. (*a*) Half-wave (single-way). (*b*) Bridge, series connection. (*c*) Bridge, parallel connection.

to a single primary winding, such as in the bi-phase, interconnected-star, or double-star secondaries, the winding design must ensure that the mean distance between windings is the same. The secondaries are sectionalized and interlinked to give the same space and hence the same leakage flux between the primary and each secondary winding. Each secondary winding must extend for the same length as the primary, so that there is an m.m.f. balance, otherwise there would be excessive mechanical stresses.

2-11 SUMMARY

In this chapter several rectifying circuits have been described, so that in a given application one can be aware of the comparisons between the various circuits in order to make the right choice.

A low-voltage load, say 20 V, will impose no severe voltage stresses on the diode (or thyristor) voltage ratings but, at this low voltage, the difference between the one-diode volt-drop of the half-wave circuit and the double volt-drop of the full-wave bridge circuits is significant, suggesting a lower loss with the half-wave circuits.

A high-voltage load, say 2 kV, will indicate that the choice should be a bridge circuit, as the diode (or thyristor) voltage ratings would be excessive in a half-wave circuit. The double volt-drop in the bridge circuit would be insignificant with high-voltage loads.

In the medium-voltage range, the more complex transformer design would possibly rule out the half-wave circuits on cost considerations.

Single-phase circuits are limited to lower power applications, say 15 kW, because there is a limit to the distorted current which can be drawn from the supply, in addition to the usual reasons for using the three-phase supply for heavier loads.

Where applications require a reversal of the mean load voltage, the fully-controlled connection must be used. The half-controlled connections are cheaper where no load-voltage reversal is required, but the greater distortion in the voltage and current waveforms leads to technical limitations to their use.

In Chapter 7 it is shown that, in order to reduce the harmonic content of the waveforms, it is necessary to adopt the higher pulse-number connections. Restrictions imposed by the electricity supply authorities on the harmonic current which can be drawn by a load enforces the use of a higher pulse-number circuit to supply heavy loads. Where an application requires a very smooth direct voltage, the use of a high pulse-number connection may be the most economical solution.

2-12 WORKED EXAMPLES

Example 2-1

A circuit is connected as shown in Fig. 2-2 to a 240 V 50 Hz supply. Neglecting the diode volt-drop, determine the current waveform, the mean load voltage, and the mean load current for a load of (i) a pure resistor of 10 Ω, (ii) an inductance of 0.1 H in series with a 10 Ω resistor.

SOLUTION The supply voltage quoted is an r.m.s. value relating to a sinewave, hence referring to Fig. 2-2b,.

$$V_{\max} = 240\sqrt{2} = 339.4 \text{ V}$$

(i) For a load of pure resistance R, the load current will be a half sinewave of maximum value

$$I_{\max} = V_{\max}/R = 339.4/10 = 33.94 \text{ A.}$$

From Eq. (2-2), $V_{\text{mean}} = 339.4/\pi = 108$ V

$$I_{\text{mean}} = V_{\text{mean}}/R = 108/10 = 10.8 \text{ A.}$$

(ii) The current waveform may be determined by considering the circuit to be a series combination of $R = 10$ Ω, $L = 0.1$ H, being switched to an a.c. supply at its voltage zero, current ceasing when it falls to zero.
The equation to determine the current i is

$$V_{\max} \sin \omega t = L\, di/dt + Ri, \text{ with } i = 0 \text{ at } t = 0,$$

$$339.4 \sin 2\pi 50 t = 0.1\, di/dt + 10i.$$

Using the Laplace transform method of solution where $\bar{i}(s)$ is the transform of $i(t)$, then

$$339.4 \times \frac{2\pi 50}{s^2 + (2\pi 50)^2} = 0.1(s\bar{i} - i_0) + 10\bar{i}, \text{ where } i_0 = 0,$$

$$\bar{i} = \frac{1\,066\,000}{[s^2 + (2\pi 50)^2]\,(s + 100)}$$

Transforming this equation yields

$$i = 9.81\, e^{-100t} + 10.29 \sin (2\pi 50 t - 1.262) \text{ A}$$

Alternatively the current expression may be determined by considering the current to be composed of the steady-state a.c. value plus a decaying d.c. transient of initial value such as to satisfy the initial condition of zero current.
The a.c. impedance is 10 Ω resistance in series with a reactance of $2\pi 50 \times 0.1 =$ 31.4 Ω giving an impedance of 32.97 Ω. The a.c. component of the current is therefore $339.4/32.97 = 10.29$ A peak, lagging the voltage by arctan $(31.4/10) =$ $72.3° = 1.262$ rad.
The a.c. component of the current is $10.29 \sin (2\pi 50t - 1.262)$ A, which has a value of -9.81 A at $t = 0$.
The time constant of the circuit is $0.1/10 = 1/100$ second, hence the d.c. component of the current is $9.81\, e^{-100t}$ A.

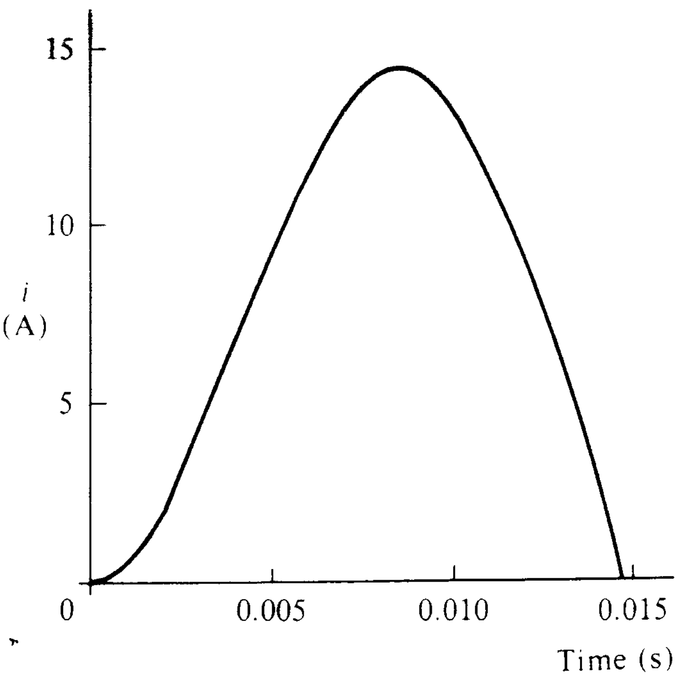


Figure 2-26

$iR + L\frac{di}{dt} = 0$

$\frac{di}{i} = -\frac{R}{L}dt$

$\ln i = -\frac{R}{L}t + k_1$

$i = Ae^{-\frac{R}{L}t}$

$i = \frac{V_m}{Z}\sin(wt-\theta) + Ae^{-\frac{R}{L}t}$

at $wt = \alpha$, $i = 0$

$-\frac{V_m \sin(\alpha-\theta)}{Z} = Ae^{-\frac{R\alpha}{Lw}}$

$A = -\frac{V_m \sin(\alpha-\theta)}{Z} \cdot e$

The total current equation is

$$i = 9.81\,e^{-100t} + 10.29 \sin(2\pi 50t - 1.262) \text{ A}$$

A sketch of the current waveform is shown in Fig. 2-26, current ceasing when it attempts to reverse.

The current will cease when $i = 0$ yielding a time $t = 0.01472$ s which gives an angular duration of current of 265°.

$2\times 180 \times 50$

t

265

Hence $$V_{\text{mean}} = \frac{1}{2\pi}\int_{0^\circ}^{265^\circ} 339.4 \sin\theta\, d\theta = 58.8 \text{ V}$$

The mean current can be calculated with the aid of calculus, but it is easier to find it by dividing the mean voltage by the d.c. impedance, that is, the resistance:

$$I_{\text{mean}} = 58.8/10 = 5.88 \text{ A.}$$

Example 2-2

If the diode of Fig. 2-2 is replaced by a thyristor, determine the mean load voltage and current if the load is 10 Ω in series with an inductor of 0.1 H, and the firing of the thyristor is delayed by 90°. The a.c. supply is 240 V, 50 Hz, and the thyristor volt-drop is to be neglected.

SOLUTION This problem is similar to that of Example 2-1(ii) except for the delayed start, the waveshapes being as shown in Fig. 2-27.

Assuming that time $t = 0$ at the instant of firing, then the steady-state a.c. component of current is $10.29 \sin(2\pi 50t - 1.262 + 1.571)$ A, which has a value of 3.12 A at $t = 0$.

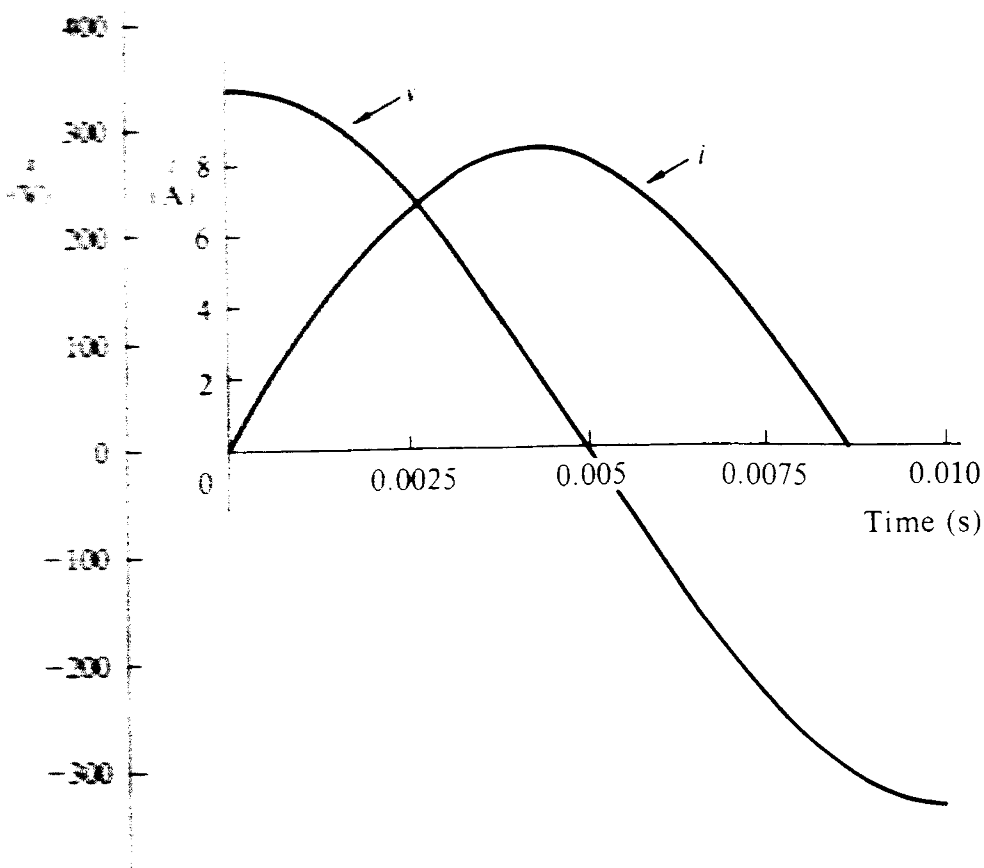


Figure 2-27

Hence the equation to the current is

$$i = 10.29 \sin(2\pi 50t + 0.309) - 3.12\, e^{-100t} \text{ A.}$$

The current will cease when $i = 0$ which occurs at $t = 0.0086$ s which is equivalent to 155°.

$$V_{\text{mean}} = \frac{1}{2\pi}\int_{90°}^{90°+155°} 339.4 \sin\theta \, d\theta = 22.8 \text{ V}$$

$$I_{\text{mean}} = 22.8/10 = 2.28 \text{ A}$$

Example 2-3

The single-phase half-wave circuit with a commutating diode as shown in Fig. 2-4 is used to supply a heavily inductive load of up to 15 A from a 240 V a.c. supply. Determine the mean load voltage for firing delay angles of 0°, 45°, 90°, 135°, and 180°, neglecting the thyristor and diode volt-drops. Specify the required rating of the thyristor and diode.

SOLUTION Using Eq. (2-5), the mean load voltage is

$$V_{\text{mean}} = \frac{240\sqrt{2}}{2\pi}(1 + \cos\alpha)$$

which yields the following values

α	0°	45°	90°	135°	180°
V_{mean}	108 V	92 V	54 V	16 V	0 V

Thyristor rating:
Peak forward (or reverse) voltage = V_{max}

$$\text{P.F.V.} = \text{P.R.V.} = 240\sqrt{2} = 340\text{ V}.$$

The thyristor will conduct for a maximum duration at $\alpha = 0°$ of one half cycle and, if one assumes level current, then using two equal time intervals the r.m.s. current rating can be calculated as

$$I_{rms} = \left(\frac{15^2 + 0^2}{2}\right)^{1/2} = 10.6\text{ A}$$

Diode rating:

$$\text{P.R.V.} = V_{max} = 340\text{ V}$$

As the firing delay approaches 180°, the diode will conduct for almost the whole cycle; hence the required current rating would be 15 A; however, in practice, some decay in this current would occur.

Example 2-4

Using the single-phase half-wave circuit of Fig. 2-4, a low-voltage load is supplied by a 20 V a.c. supply. Assuming continuous load current, calculate the mean load voltage when the firing delay angle is 60°, assuming forward volt-drops of 1.5 V and 0.7 V across the thyristor and diode respectively.

SOLUTION Using Eq. (2-5) and neglecting volt-drops,

$$V_{mean} = \frac{20\sqrt{2}}{2\pi}(1 + \cos 60°) = 6.752\text{ V}.$$

The thyristor will conduct for $(180 - 60)°$, giving an average volt-drop over the cycle of $\dfrac{120}{360} \times 1.5 = 0.5$ V.

The diode when conducting imposes a 0.7 V drop across the load; in this case, it averages over the cycle to $0.7 \times \dfrac{180 + 60}{360} = 0.467$ V. Therefore the mean load voltage is $6.752 - 0.5 - 0.467 = 5.78$ V.

It can be seen that at a low voltage the device volt-drops are not negligible.

Example 2-5

A load of 10 Ω resistance, 0.1 H inductance, is supplied via the circuit of Fig. 2-4 from a 70.7 V, 50 Hz a.c. supply. If the thyristor is fired at a delay angle of 90°,

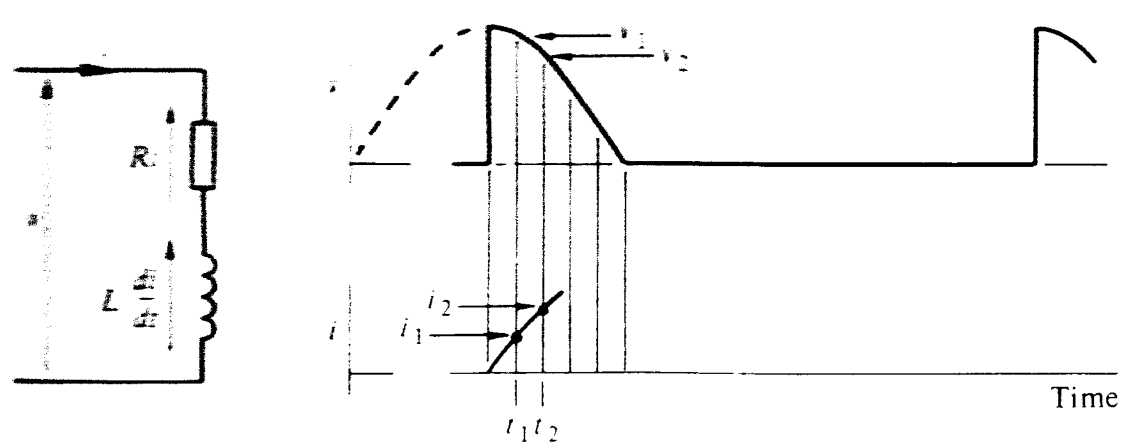


Figure 2-28

Determine by a graphical method the waveform of the current during the first two cycles of operation. Neglect device volt-drops.

SOLUTION Figure 2-28 relates to a graphical method where use is made of $v = Ri + L\,di/dt$.

Now taking any time (say) t_1, when the current is i_1, then $v_1 = Ri + L\delta i/\delta t$, that is, $\delta i = \dfrac{v_1 - Ri_1}{L}\delta t$, where the time interval $\delta t = t_2 - t_1$. Substitute the values from the graph (Fig. 2-28) for v_1, i_1 and δt into the equation, and δi is obtained. Plot i_2 as $i_1 + \delta i$, and the next point on the current graph is obtained. Proceed in a like manner to find the next current value i_3 at t_3 by using values at t_2.

At 50 Hz the cycle time is 20 ms so, if 20 intervals are selected, then $\delta t = 1$ ms.

With $R = 10$ and $L = 0.1$, $\delta i = 0.01v_n - 0.1i_n$.

The first calculation is when the load is switched to the voltage peak of 100 V (i.e. $70.7\sqrt{2}$) and no current, that is, $i_0 = 0$, giving

$$\delta i = (0.01 \times 100 \sin 90°) - (0.1 \times 0) = 1 \text{ A}$$

$$i_1 = 1 \text{ A}$$

The second calculation gives

$$\delta i = (0.01 \times 100 \sin 108°) - (0.1 \times 1) = 0.85 \text{ A}$$

$$i_2 = 1 + 0.85 = 1.85 \text{ A}$$

Continuing in a like manner, at the end of 5 ms (90°) the load voltage is zero, $i_5 = 2.83$ A.

For the next 15 ms, the commutating diode is conducting, with v in Fig. 2-28 being zero, hence, to find i_6,

$$\delta i = 0 - (0.1 \times 2.83) = -0.283 \text{ A}$$

giving $i_6 = 2.83 - 0.28 = 2.55$ A.

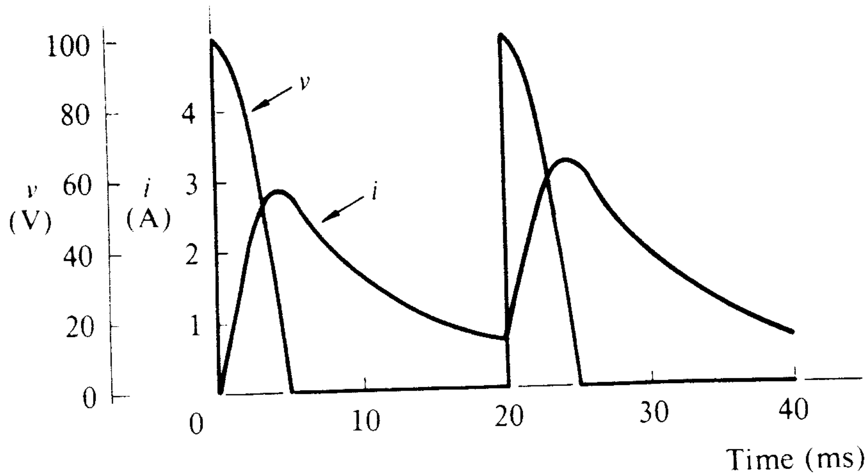


Figure 2-29

Continuing in a like manner $i_{20} = 0.59$ A when the thyristor is again fired, so to calculate i_{21},

$$\delta i = (0.01 \times 100 \sin 90°) - (0.1 \times 0.59) = 0.94 \text{ A}$$

$i_{21} = 0.59 + 0.94 = 1.53$ A until at the next voltage zero, $i_{25} = 3.17$ A and after a further 15 ms, $i_{40} = 0.65$ A.
The current is shown plotted in Fig. 2-29, from which it is possible to conclude that by the end of the third or fourth cycle steady-state conditions will have been reached. To improve the accuracy of the plot, shorter time intervals could be taken. An exact calculation could be made along the lines used in Example 2-1. In the steady state one would have to assume a current I_1 at the start of the commutating diode conduction period falling exponentially to I_2 when the thyristor is fired, then

$$I_2 = I_1 e^{-100t} \qquad (t = 15 \text{ ms})$$

During the thyristor on-period,

$$i = \frac{100}{32.97} \sin(\omega t - 1.262) + I_x e^{-100t},$$

I_x being the d.c. transient component found by substituting at $t = 0, i = I_2$ and at $t = 5$ ms, $i = I_1$, giving $I_1 = 3.09$ A and $I_2 = 0.69$ A.

Example 2-6

The bi-phase half-wave circuit shown in Fig. 2-5 is supplied at 120 V line to neutral. Determine the mean load voltage for firing delay angles of 0°, 30°, 60°, and 90°, assuming the load current to be continuous and level with a constant 1.5 V drop on each thyristor.
Determine the required thyristor ratings given that the load current is 15 A.

SOLUTION Using Eq. (2-6), $V_{\text{mean}} = \dfrac{2}{\pi} 120\sqrt{2} \cos\alpha - 1.5$, giving these values:

α	0°	30°	60°	90°
V_{mean}	106.5 V	92.1 V	52.5 V	0 V

Thyristor rating: from Fig. 2-5*b*, P.R.V. = P.F.V. = $2V_{max} = 2 \times 120\sqrt{2} = 340$ V; and for half cycle conduction, $I_{rms} = 15/\sqrt{2} = 10.6$ A.

Example 2-7

A single-phase diode bridge is supplied at 120 V. Determine the mean load voltage assuming each diode to have a volt-drop of 0.7 V.

SOLUTION Figure 2-7 relates to this circuit and, using Eq. (2-7) with $\alpha = 0$,

$$V_{mean} = \frac{2}{\pi} 120\sqrt{2} - (2 \times 0.7) = 106.6 \text{ V}.$$

Example 2-8

A single-phase fully-controlled bridge is supplied at 120 V. Determine the mean load voltage for firing delay angles of 0°, 45°, and 90°, assuming continuous load current. Allow a thyristor volt-drop of 1.5 V. Determine also the required peak voltage of each thyristor.

SOLUTION Figure 2-8 and Eq. (2-7) relate to this question.

$V_{mean} = \frac{2}{\pi} 120\sqrt{2} \cos\alpha - (2 \times 1.5)$, giving values:

α	0°	45°	90°
V_{mean}	105 V	73.4 V	0 V

The peak voltage across each thyristor = $V_{max} = 120\sqrt{2} = 170$ V.

Example 2-9

The half-controlled single-phase bridge circuit shown in Fig. 2-10 is supplied at 120 V. Neglecting volt-drops, determine the mean load voltage at firing delay angles of 0°, 60°, 90°, 135°, and 180°. If the load is highly inductive taking 25 A, determine the required device ratings.

SOLUTION Using Eq. (2-8), $V_{mean} = \frac{120\sqrt{2}}{\pi}(1 + \cos\alpha)$, giving these values:

α	0°	60°	90°	135°	180°
V_{mean}	108 V	81 V	54 V	16 V	0 V

Each thyristor and diode must withstand $V_{max} = 120\sqrt{2} = 170$ V.
The bridge components conduct for a maximum of one half-cycle, hence for level current $I_{rms} = 25/\sqrt{2} = 17.7$ A.
The commutating diode will conduct for almost the complete cycle when $\alpha \rightarrow 180°$,

therefore it must be rated to 25 A. In practice, the current would decay somewhat at low load voltages.

Example 2-10

Repeat the calculation for the mean voltage in Example 2-9 at $\alpha = 90°$, assuming thyristor and diode volt-drops of 1.5 V and 0.7 V respectively.

SOLUTION At $\alpha = 90°$ the bridge components conduct for half the time, hence over one cycle their volt-drops will reduce the mean voltage by $(1.5 + 0.7)/2 = 1.1$ V. The commutating diode imposes a 0.7 V negative voltage on the load for the other half of the time, averaging over the cycle to $0.7/2 = 0.35$ V.

$$V_{mean} = \frac{120\sqrt{2}}{\pi}(1 + \cos 90°) - 1.1 - 0.35 = 52.6 \text{ V.}$$

Example 2-11

A highly inductive d.c. load requires 12 A at 150 V from a 240 V single-phase a.c. supply. Give design details for this requirement using (i) bi-phase half-wave, and (ii) bridge connection. Assume each diode to have a volt-drop of 0.7 V. Make comparisons between the two designs.

SOLUTION (i) Referring to Fig. 2-5 and Eq. (2-6), using diodes,

$$V_{mean} = 150 = \frac{2}{\pi}V_{max} - 0.7$$

$$V_{max} = \frac{\pi}{2}(150 + 0.7) = 236.7 \text{ V}$$

Hence each section of the transformer secondary requires an r.m.s. voltage of $236.7/\sqrt{2} = 167.4$ V, and carries an r.m.s. current of $12/\sqrt{2} = 8.5$ A.

Transformer secondary rating $= 2 \times 167.4 \times 8.5 = 2.84$ kVA.

Transformer voltage ratio $= 240/167.4$.

Transformer primary current is square wave of r.m.s. value $12(167.4/240) = 8.4$ A.

P.R.V. for each diode $= 2V_{max} = 474$ V, with $I_{rms} = 12/\sqrt{2} = 8.5$ A.

(ii) Referring to Fig. 2-7,

$$V_{mean} = \frac{2}{\pi}V_{max} - (2 \times 0.7)$$

$$V_{max} = \frac{\pi}{2}(150 + 1.4) = 237.8 \text{ V}$$

Transformer secondary r.m.s. voltage $= 237.8/\sqrt{2} = 168.2$ V.

The secondary-current waveform is square wave 12 A and, as the current is level, its r.m.s. value $= 12$ A.

Transformer secondary rating $= 168.2 \times 12 = 2.02$ kVA.

Transformer voltage ratio = 240/168.2.
Transformer primary r.m.s. current = 12(168.2/240) = 8.4 A.
For each diode, P.R.V. = V_{max} = 238 V; $I_{rms} = 12/\sqrt{2} = 8.5$ A.
Comparing the two circuits, the bridge is superior on transformer size and diode voltage rating requirements.
The diode loss in (i) is 0.7/150 = 0.47% of the load power, compared to 1.4/150 = 0.93% in circuit (ii).

Example 2-12

Repeat Example 2-11 using a load voltage of 15 V.

SOLUTION Calculations now give:

	(i)	(ii)
transformer secondary voltage	17.4 V	18.2 V
transformer secondary rating	296 VA	219 VA
P.R.V. of diode	50 V	26 V
% loss to load power	4.7%	9.3%

The comparison now favours the bi-phase, half-wave circuit, as the P.R.V. requirement is inconsequential and the relative losses are important.

Example 2-13

An alternative connection for the half-controlled single-phase bridge is shown in Fig. 2-30. Assuming level load current, sketch the current waveshapes in the thyristor and diodes at a firing delay angle of 90°.

SOLUTION Figure 2-31 shows the waveforms. When thyristor T_1 is fired, it and diode D_2 will conduct. During the periods of zero load voltage, both diodes will conduct, carrying the freewheeling load current.

Example 2-14

Draw the load-voltage waveform which would occur with the half-controlled single-phase bridge circuit of Fig. 2-10 if there were no commutating diode, the firing delay angle were 150°, and one thyristor missed firing in one cycle. Assume continuous load current.

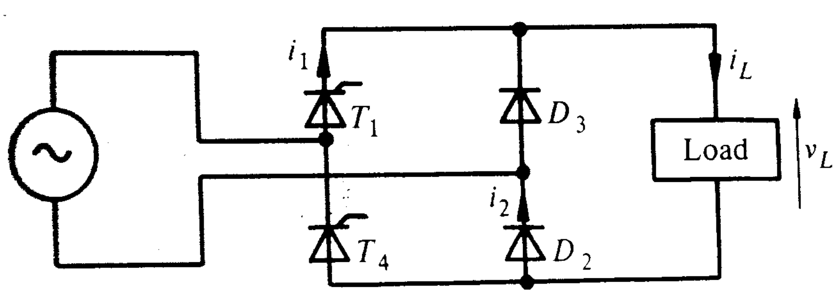
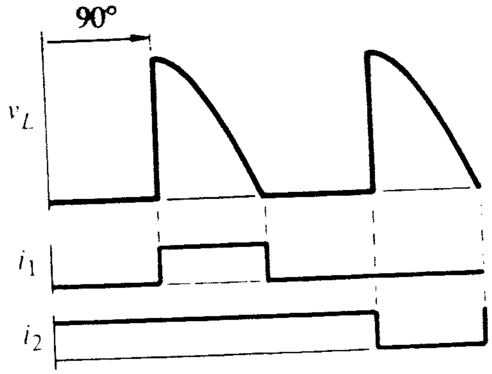


Figure 2-30



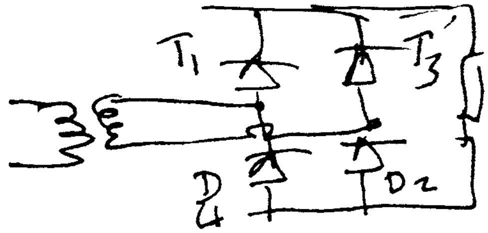


Figure 2-31

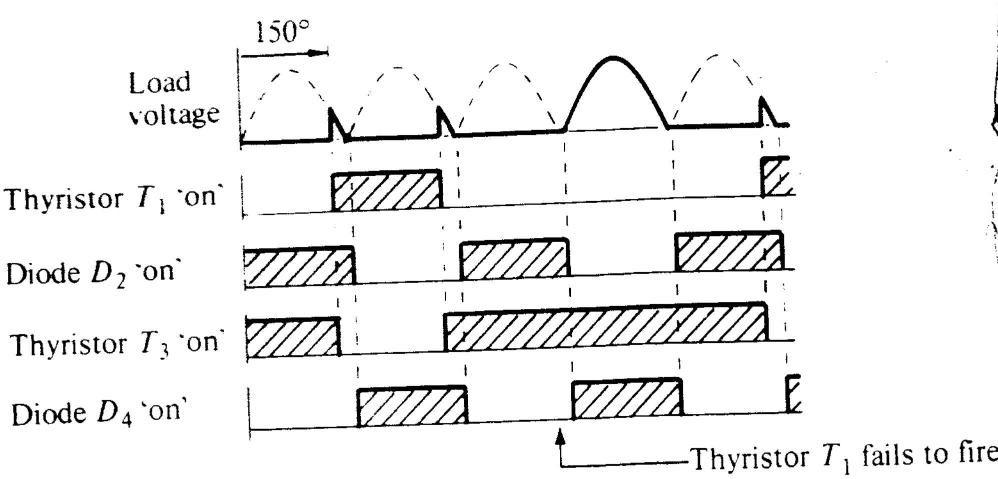
SOLUTION The normal operation without a commutating diode is shown in the first two half-cycles of Fig. 2-32 where, during the zero load-voltage period, the inductive load current freewheels via the series combination of a thyristor and diode.

However, if thyristor T_1 fails to fire, thyristor T_3 remains conducting to the start of the next half-cycle, when the supply voltage reverses and thyristor T_3, together with diode D_4, conducts a complete half-cycle into the load. The sudden burst of energy into the load could result in dangerous conditions, particularly with a motor load.
The presence of a commutating diode would prevent this danger, as thyristor T_3 would have been turned off at the end of the previous half-cycle. Misfiring would have merely resulted in the absence of one output voltage period.

Example 2-15

A three-phase half-wave rectifier as shown in Fig. 2-12 has a supply of 150 V/phase. Determine the mean load voltage and the required diode rating, assuming the load current is level at 25 A. Assume each diode has a volt-drop of 0.7 V.

SOLUTION From Eq. (2-9), $V_{\text{mean}} = \dfrac{3\sqrt{3}}{2\pi} 150\sqrt{2} - 0.7 = 174.7$ V.



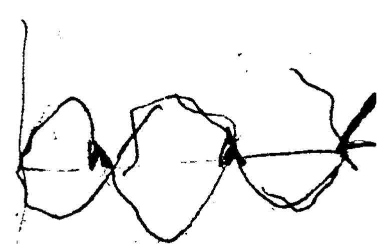


Figure 2-32

Diode rating from Fig. 2-12, and Eq. (2-10):

$$\text{P.R.V.} = \sqrt{3}V_{\max} = \sqrt{3} \times 150\sqrt{2} = 386 \text{ V}, \quad I_{\text{rms}} = \frac{25}{\sqrt{3}} = 14.4 \text{ A}.$$

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Example 2-16

A three-phase half-wave rectifier circuit is fed from an interconnected star transformer (shown in Fig. 2-13). If the load (highly inductive) is 200 V at 30 A, specify the transformer, given the a.c. supply to be 415 V and each diode to have a volt-drop of 0.7 V.

SOLUTION Using Eq. (2-9) to find the secondary voltage,

$$200 = \frac{3\sqrt{3}}{2\pi} V_{\max} - 0.7, \text{ giving}$$

$V_{\text{rms}} = 171.6$ V.

This voltage is the phasor addition of two windings equal in voltage magnitude displaced 60°; voltage of each section $= \dfrac{171.6}{2 \cos 30^\circ} = 99.1$ V.

For a star-connected primary, the winding voltage $= 415/\sqrt{3} = 239.6$ V.

Each secondary winding carries a block of 30 A for a one-third cycle giving $I_{\text{rms}} = 30/\sqrt{3} = 17.3$ A.

The 30 A is reflected into the primary to be $30(99.1/239.6) = 12.4$ A and, referring to Fig. 2-13*b*, the primary winding r.m.s. current value is

$$I_{\text{rms}} = \left(\frac{12.4^2 + 12.4^2 + 0^2}{3}\right)^{1/2} = 10.1 \text{ A}.$$

Secondary rating $= 6 \times 99.1 \times 17.3 = 10.3$ kVA.

Primary rating $= 3 \times 239.6 \times 10.1 = 7.3$ kVA.

Example 2-17

A three-phase half-wave controlled rectifier has a supply of 150 V/phase. Determine the mean load voltage for firing delay angles of 0°, 30°, 60°, and 90°, assuming a thyristor volt-drop of 1.5 V and continuous load current.

SOLUTION From Eq. (2-11), $V_{\text{mean}} = \dfrac{3\sqrt{3}}{2\pi} 150\sqrt{2} \cos\alpha - 1.5$, giving these values:

α	0°	30°	60°	90°
V_{mean}	173.9 V	150.4 V	86.2 V	0 V

Example 2-18

If the circuit of Fig. 2-14*a* for the three-phase half-wave controlled rectifier is modified by placing a commutating diode across the load, plot a curve of mean load

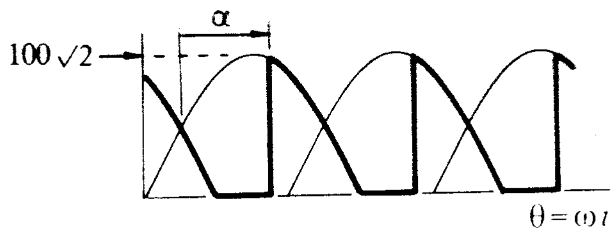

Figure 2-33

voltage against firing delay angle. Take the supply to be 100 V/phase and neglect device volt-drops.

SOLUTION The effect of the commutating diode is to prevent reversal of the load voltage, leading to the waveform shown in Fig. 2-33. From inspection of the waveform,

$$V_{\text{mean}} = \frac{1}{2\pi/3}\int_{30^\circ+\alpha}^{180^\circ} 100\sqrt{2}\sin\theta\, d\theta \qquad \text{when } \alpha \gtrless 30^\circ$$

but for firing delay angles below 30°, Eq. (2-11) applies.
The values of mean load voltages against firing delay angle are shown in the plot of Fig. 2-34, together with (for comparison) the fully controlled values with continuous current and no commutating diode.

Example 2-19

The simple six-phase half-wave connection using diodes supplies a d.c. load of 40 V, 50 A. Determine the required diode rating, and specify the transformer for (i) simple six-phase (Fig. 2-15), (ii) fork connection (Fig. 2-16). Assume level load current, a diode volt-drop of 0.7 V, and that the transformer primary is delta-connected, fed from a 415 V supply.

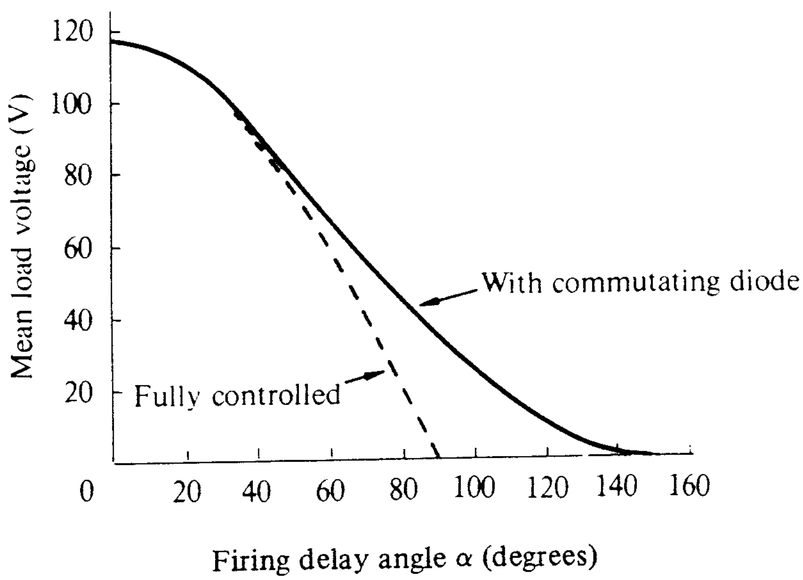

Figure 2-34

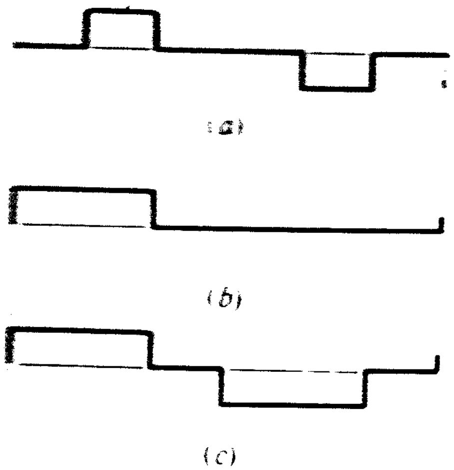

Figure 2-35 (*a*) Primary winding current, simple star secondary. (*b*) Inner winding current in fork secondary. (*c*) Primary winding current, fork secondary.

SOLUTION Using Eq. (2-12), $V_{\text{mean}} = \frac{3}{\pi} V_{\text{max}} - 0.7$, giving the r.m.s. voltage/phase as 30.14 V.
Referring to Fig. 2-15, and Eq. (2-13) for the diode rating, P.R.V. = $2 \times 30.14\sqrt{2} =$ 56 V. $I_{\text{rms}} = 50/\sqrt{6} = 20.4$ A.
To determine the transformer ratings, the current waveforms in the sections must be known. Figure 2-35 shows the various waveforms which can be derived from the outgoing phase currents.
(i) R.M.S. secondary current = $50/\sqrt{6} = 20.4$ A
Primary/secondary voltage = 415/30.14
Magnitude of primary current = 50(30.14/415) = 3.63 A
R.M.S. value of primary current = $3.63/\sqrt{3} = 2.1$ A
Secondary rating = $6 \times 30.14 \times 20.4 = 3.69$ kVA
Primary rating = $3 \times 415 \times 2.1 = 2.61$ kVA
(ii) R.M.S. voltage of each section $= \dfrac{30.14}{2 \cos 30^\circ} = 17.40$ V
R.M.S. current in outer winding section = 20.4 A
R.M.S. current in inner winding section = $50/\sqrt{3} = 28.9$ A
Magnitude of primary current = 50(17.4/415) = 2.1 A
R.M.S. value of primary current $= \left(\dfrac{2.1^2 + 2.1^2}{3}\right)^{1/2} = 1.71$ A
Secondary rating = $(6 \times 17.4 \times 20.4) + (3 \times 17.4 \times 28.9) = 3.64$ kVA
Primary rating = $3 \times 415 \times 1.71 = 2.13$ kVA

Example 2-20

Using the same data as for Example 2-19, determine diode, transformer and interphase transformer specifications using the double-star connection shown in Fig. 2-17.

SOLUTION Using Eq. (2-14), r.m.s. voltage/phase $= \dfrac{(40 + 0.7)2\pi}{3\sqrt{3}\sqrt{2}} = 34.8$ V. Re-

ferring to Fig. 2-17, each diode carries $50/2 = 25$ A for a one-third cycle. hence diode ratings are: P.R.V. $= 2 \times 34.8\sqrt{2} = 99$ V, $I_{rms} = 25/\sqrt{3} = 14.4$ A.
Transformer rating:
R.M.S. secondary current = 14.4 A
Primary/secondary current = 415/34.8
Magnitude of primary current = 25(34.8/415) = 2.1 A

R.M.S. value of primary current $= \left(\dfrac{2.1^2 + 2.1^2}{3}\right)^{1/2} = 1.71$ A

Secondary rating $= 6 \times 34.8 \times 14.4 = 3.01$ kVA
Primary rating $= 3 \times 415 \times 1.71 = 2.13$ kVA
Interphase transformer rating: The voltage rating is a function of the total flux change $\delta\phi$ which from $v = d\phi/dt$ gives $\delta\phi = \int v\,dt$, the area under the voltage-time curve.
As transformers are normally rated to sinewaves, comparison of the area under the curve of v_R (Fig. 2-17*b*) to a sinewave will yield an r.m.s value for rating (size) purposes; v_R is the first 30° of the voltage between two phase voltages 60° apart, that is, $v_R = 34.8\sqrt{2}\sin\omega t$.
Area under v_R curve for the first quarter cycle

$$= \int_0^{\pi/6} 34.8\sqrt{2}\sin\omega t\, d\omega t = 6.593 \text{ units}$$

Area under sine curve of r.m.s. value V

$$= \int_0^{\pi/6} \sqrt{2}V \sin 3\omega t\, d\omega t = 0.472V \text{ units},$$

$6.593 = 0.472V$, giving $V = 13.98$ volts.
The current rating $= (1/2) \times$ load current $= 50/2 = 25$ A.
Interphase transformer rating $= 13.98 \times 25 = 350$ VA.

Example 2-21

A double-star fully-controlled rectifier is supplied by a transformer having a secondary voltage of 200 V/phase. Determine the load voltage with firing delay angles of 0°, 30°, 45°, and 90°, assuming continuous load current. Determine the maximum mean load voltage if the interphase transformer failed to excite. Neglect thyristor volt-drops.
Determine the rating of the interphase transformer if the load current is 40 A, and what rating it would have if the circuit were uncontrolled.

SOLUTION Using Eq. (2-14) times cos α for the controlled case,
$V_{mean} = \dfrac{3\sqrt{3}}{2\pi} 200\sqrt{2}\cos\alpha$, giving these values:

α	0°	30°	45°	90°
V_{mean}	234 V	203 V	165 V	0 V

If the load current is insufficient to excite the interphase transformer, then the circuit operates as the simple six-phase circuit of Fig. 2-15. Maximum mean load voltage occurs at zero firing delay angle, giving

$$V_{\text{mean(max)}} = \frac{1}{2\pi/6}\int_{60°}^{120°} 200\sqrt{2}\sin\theta\, d\theta = 270\,\text{V}.$$

Following the reasoning developed in Example 2-20 for the equivalent r.m.s. voltage V across the interphase transformer, the area under the v_R curve of Fig. 2-19c for the first quarter cycle is

$$\int_{\pi/3}^{\pi/2} 200\sqrt{2}\sin\omega t\, d\omega t = 141.4 \text{ units}$$

Sinewave area = $0.472V$ units, hence $V = 141.4/0.472 = 300$ volts.
Rating = 300(40/2) = 6 kVA.
If the circuit were uncontrolled, the r.m.s. voltage is 80.4 V, giving the rating as 80.4(40/2) = 1.61 kVA.

Example 2-22

Derive a general expression for the mean load voltage of a p-pulse fully-controlled rectifier.

SOLUTION Figure 2-36 defines the general waveform where p is the pulse-number of the output. The angles are defined relative to the peak of the voltage waveform. From the waveform,

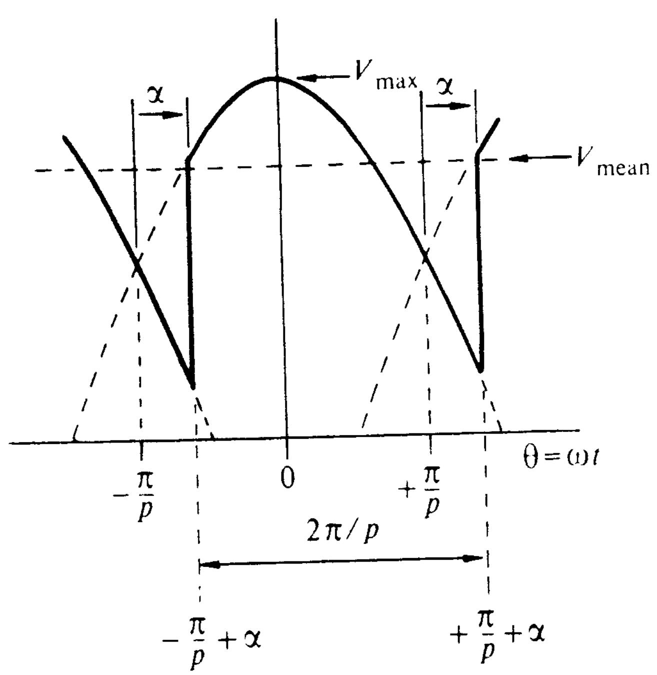

Figure 2-36

$$V_{\text{mean}} = \frac{1}{2\pi/p}\int_{-\frac{\pi}{p}+\alpha}^{+\frac{\pi}{p}+\alpha} V_{\max}\cos\theta\, d\theta$$

$$= \frac{pV_{\max}}{2\pi}\left[\sin\left(\frac{\pi}{p}+\alpha\right) - \sin\left(-\frac{\pi}{p}+\alpha\right)\right]$$

$$= \frac{pV_{\max}}{2\pi}\left[\sin\frac{\pi}{p}\cos\alpha + \cos\frac{\pi}{p}\sin\alpha - \sin\left(-\frac{\pi}{p}\right)\cos\alpha + \cos\left(-\frac{\pi}{p}\right)\sin\alpha\right]$$

$$= \frac{pV_{\max}}{\pi}\sin\frac{\pi}{p}\cos\alpha$$

Example 2-23

Determine the r.m.s. value of a level current I that flows for $1/p$ of each cycle.

SOLUTION Divide the cycle into p intervals, then the current in one interval is I, and zero in the other intervals.
The sum of the squares for each interval $= I^2$
Mean value of the sum of the squares $= I^2/p$

The r.m.s. value of the current $= \left(\dfrac{I^2}{p}\right)^{1/2} = \dfrac{I}{\sqrt{p}}$

Example 2-24

A three-phase bridge rectifier supplies a d.c. load of 300 V, 60 A from a 415 V, 3-phase, a.c. supply via a delta-star transformer. Determine the required diode and transformer specification. Assume a diode volt-drop of 0.7 V, and level load current.

SOLUTION Divide the cycle into p intervals; then the current in one interval is I.
$\dfrac{3}{\pi}V_{\text{line(max)}} - (2 \times 0.7)$, giving $V_{\text{line(max)}} = 315.6$ V, giving an r.m.s. phase voltage of $315.6/(\sqrt{3} \times \sqrt{2}) = 128.9$ V
Diode rating: P.R.V. $= V_{\text{line(max)}} = 315.6$ V; $I_{\text{rms}} = 60/\sqrt{3} = 34.6$ A

Secondary phase current r.m.s. value $= \left(\dfrac{60^2 + 60^2}{3}\right)^{1/2} = 49$ A

Transformer rating $= 3 \times 128.9 \times 49 = 18.9$ kVA
The primary and secondary ratings are the same, because each winding current has the same waveform.
Turns ratio primary/secondary $= 415/128.9$
Primary phase r.m.s. current $= 49(128.9/415) = 15.2$ A

Example 2-25

A fully-controlled three-phase rectifier bridge circuit is supplied by a line voltage of

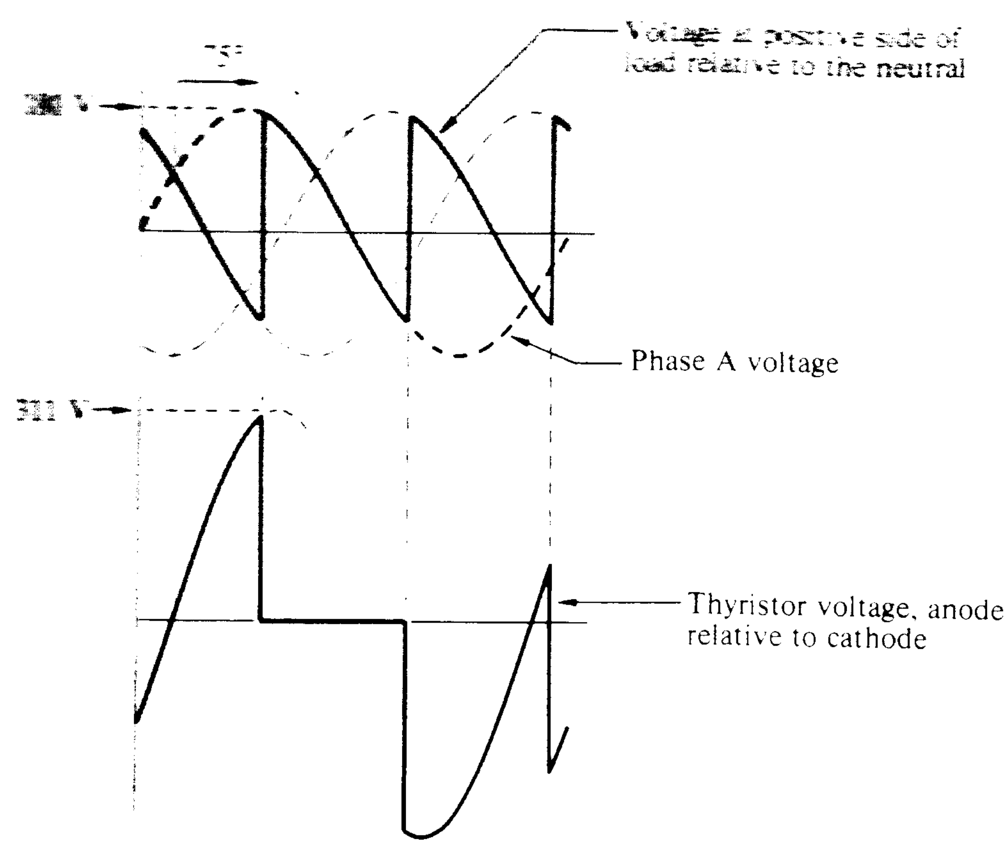

Figure 2-37

220 V. Assuming continuous load current and a thyristor volt-drop of 1.5 V, determine the mean load voltage at firing delay angles of 0°, 30°, 45°, 60°, and 90°. Plot the waveform of the thyristor voltage at a firing delay angle of 75°.

SOLUTION Using Eq. (2-16), $V_{\text{mean}} = \dfrac{3}{\pi} 220\sqrt{2} \cos\alpha - (2 \times 1.5)$, giving these values:

α	0°	30°	45°	60°	90°
V_{mean}	294 V	254 V	207 V	146 V	0 V

Referring to Fig. 2-22, the voltage across (say) thyristor T_1 is the difference between the phase A voltage and the voltage at the top of the load relative to the neutral. Figure 2-37 shows the thyristor voltage at a firing delay angle of 75°. It can be seen that the thyristor voltage is positive throughout the delayed firing period.

Example 2-26

If a commutating diode is placed across the load in the fully-controlled three-phase bridge circuit of Fig. 2-22, explain how the load-voltage waveform will be affected. Given the a.c. line voltage to be 200 V, plot values of mean load voltage against firing delay angle, neglecting any thyristor or diode volt-drops.
Sketch the a.c. line-current waveform at a firing delay angle of 90°.

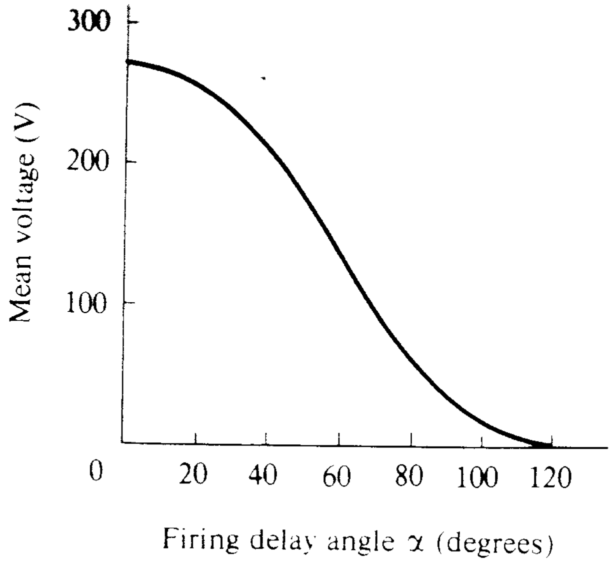

Figure 2-38

SOLUTION The effect of placing a diode across the load is to prevent voltage reversal, and so commutate the load current away from the thyristors.

Referring to the load-voltage waveforms in Fig. 2-22, up to $\alpha = 60°$ the voltage is always positive and Eq. (2-16) applies. Above $\alpha = 60°$ the load voltage loses its negative component, so the mean load voltage is given by

$$V_{\text{mean}} = \frac{1}{2\pi/6}\int_{60°+\alpha}^{180°} 200\sqrt{2}\sin\theta\, d\theta$$

being zero at $\alpha = 120°$.

The values of mean load voltage against firing delay angle are shown plotted in Fig. 2-38.

Assuming level load current, the presence of the diode modifies the waveforms at $\alpha = 90°$ to those shown in Fig. 2-39, drawn to the references as given in Fig. 2-22*a*. Note that the current waveforms while being further removed from a sinewave are still symmetrical. The commutating diode conducts during the zero load-voltage periods.

Example 2-27

A three-phase half-controlled rectifier bridge as shown in Fig. 2-23 is supplied at a line voltage of 415 V. Plot a curve relating mean load voltage to firing delay angle, and sketch the load-voltage waveform at firing delay angles of 0°, 30°, 60°, 90°, 120°, and 150°. Neglect thyristor and diode volt-drops.

SOLUTION Using Eq. (2-17),

$$V_{\text{mean}} = \frac{3}{2\pi}415\sqrt{2}(1 + \cos\alpha)$$

the values of which are shown plotted in Fig. 2-40.

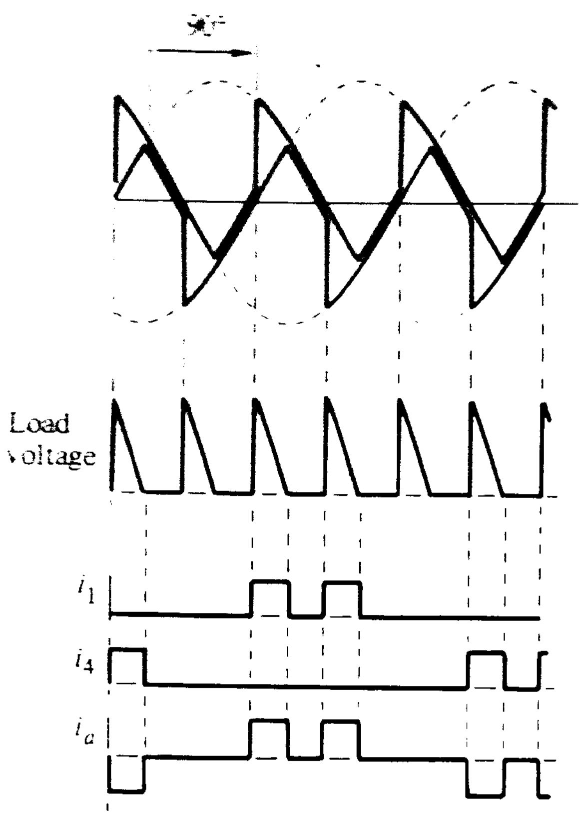

Figure 2-39

The waveforms of the load voltage are shown in Fig. 2-41. These waveforms may be most easily constructed by allowing the vertical line at commutation to move to the right as the firing delay increases.

Example 2-28

A d.c. load requires control of voltage from its maximum down to one quarter of that value. Using the half-controlled three-phase bridge, determine the current rating required for the commutating diode if the load current is level at 20 A.

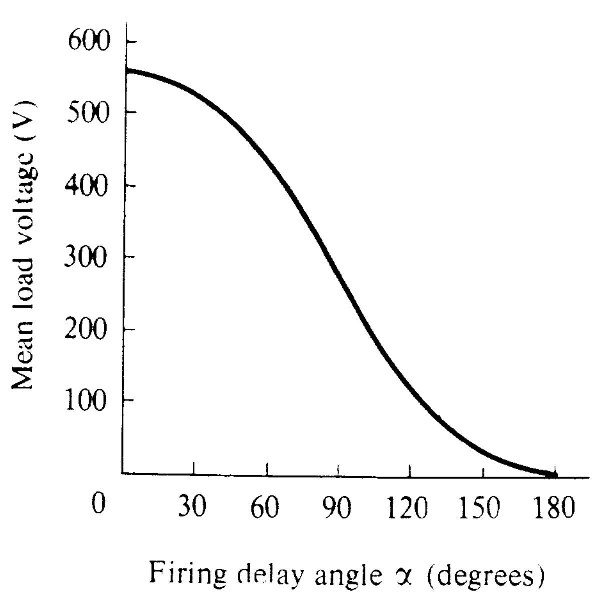

Figure 2-40

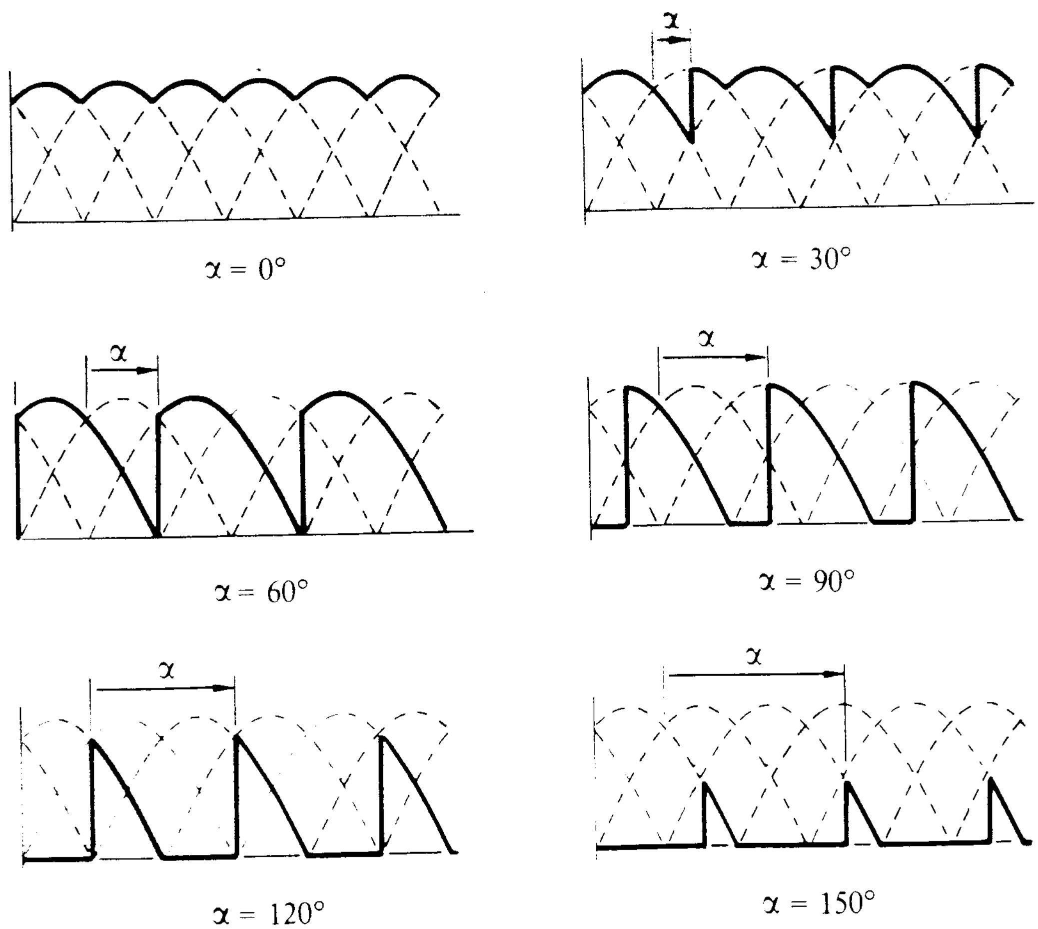

Figure 2-41

SOLUTION From Eq. (2-17), the mean load voltage is proportional to $(1 + \cos\alpha)$; therefore at one quarter voltage we have $\dfrac{1}{4} = \dfrac{1+\cos\alpha}{1+\cos 0}$, giving $\alpha = 120°$.

The sketch in Fig. 2-41 shows that at $\alpha = 120°$ the commutating diode will conduct for $60°$ every $120°$ (during the zero voltage periods); therefore r.m.s. current in diode $= \left(\dfrac{20^2 + 0^2}{2}\right)^{1/2} = 14.14$ A.

Example 2-29

Determine the percentage value of the peak-to-peak ripple voltage compared to the mean voltage for uncontrolled rectifiers having pulse numbers of 2, 3, 6, 12, and 24.

SOLUTION Taking the waveform to have a peak value of V, then for a p-pulse rectifier the minimum value of the waveform will be $V\cos\dfrac{\pi}{p}$ (see Example 2-22 with $\alpha = 0$).

Peak-to-peak value $= V - V\cos\dfrac{\pi}{p}$ and from Example 2-22, $V_{\text{mean}} = \dfrac{pV}{\pi}\sin\dfrac{\pi}{p}$

Hence relative percentage values are:

Pulse-number	2	3	6	12	24
% ripple	157.1	60.46	14.03	3.447	0.858

Example 2-30

A d.c. load of 2000 V, 300 A is to be supplied by a twelve-pulse bridge rectifier. Determine the required thyristor (or diode) ratings and transformer secondary voltages for both the series and parallel connections. Neglect thyristor or diode volt-drops.

SOLUTION The series connection is shown in Fig. 2-25*b*.
Mean voltage of each bridge = 2000/2 = 1000 V.
Each bridge has a six-pulse characteristic, hence Eq. (2-15) applies, giving a maximum line voltage of 1047 V.
Each thyristor (or diode) carries the total load current for one third cycle.
Thyristor (or diode) ratings are: P.R.V. = 1047 V, $I_{\text{rms}} = 300/\sqrt{3} = 173$ A.
The star secondary winding r.m.s. voltage = $1047/(\sqrt{3} \times \sqrt{2}) = 428$ V.
The delta secondary winding r.m.s. voltage = $1047/\sqrt{2} = 740$ V.
The parallel connection is shown in Fig. 2-25*c*. Compared to the series connection, voltages are doubled but currents halved. Hence thyristor (or diode) ratings are 2094 V, 87 A; winding voltages, star 855 V, delta 1481 V.

Example 2-31

Estimate the total thyristor losses compared to the load power for each of the twelve-pulse circuits shown in Fig. 2-25, when the mean voltage is 60 V, and thyristors are used having a volt-drop of 1.5 V.

SOLUTION Half-wave connection, one thyristor volt-drop, loss = 1.5/60 = 2.5%.
Bridge, parallel connection, two thyristor volt-drops, loss = (2 × 1.5)/60 = 5%.
Bridge, series connection, four thyristor volt-drops, loss = (4 × 1.5)/60 = 10%.

Example 2-32

Derive the current waveshape in the primary winding of the twelve-pulse bridge circuit.

SOLUTION With reference to Fig. 2-25*b*, the current in the star and delta secondary windings are as shown in Fig. 2-42. The shapes are as derived in Fig. 2-21, the delta connection converting the quasi-square shaped line current into the stepped waveform. Deriving directly the stepped waveform shape of the delta phase current is difficult but, by finding the difference between two stepped phase currents 120° apart, one easily arrives at the quasi-square wave for the line current.
The turns ratio for the delta is different from the star by $\sqrt{3}$, hence $I_{\text{primary}} = I + \sqrt{3}I$ giving the stepped wave shown in Fig. 2-42.

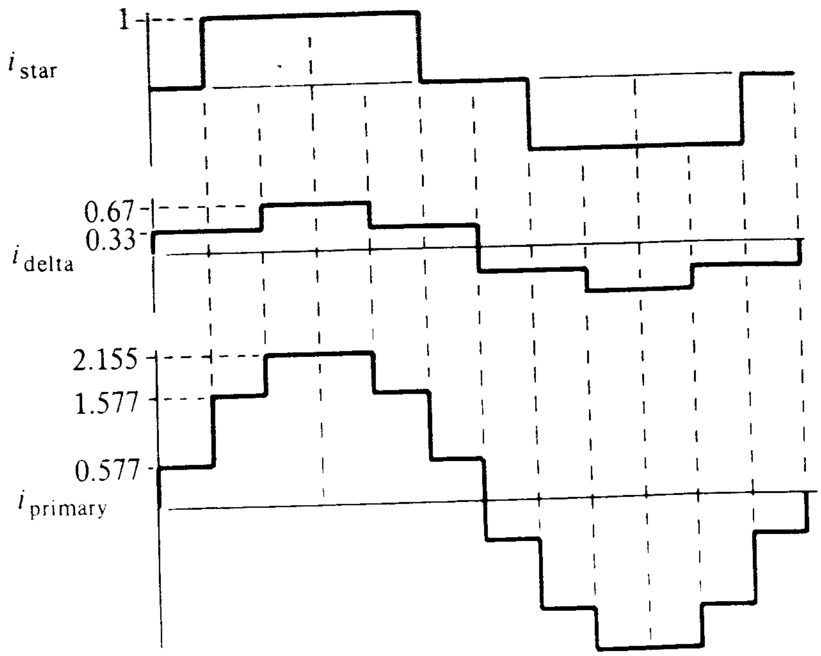

Figure 2-42

Example 2-33

A connection of three single-phase bridges fed from a three-phase supply as shown in Fig. 2-43 will give a six-pulse output. Sketch the waveform of the a.c. line current i_y, and determine the required diode and transformer specifications, if the load is

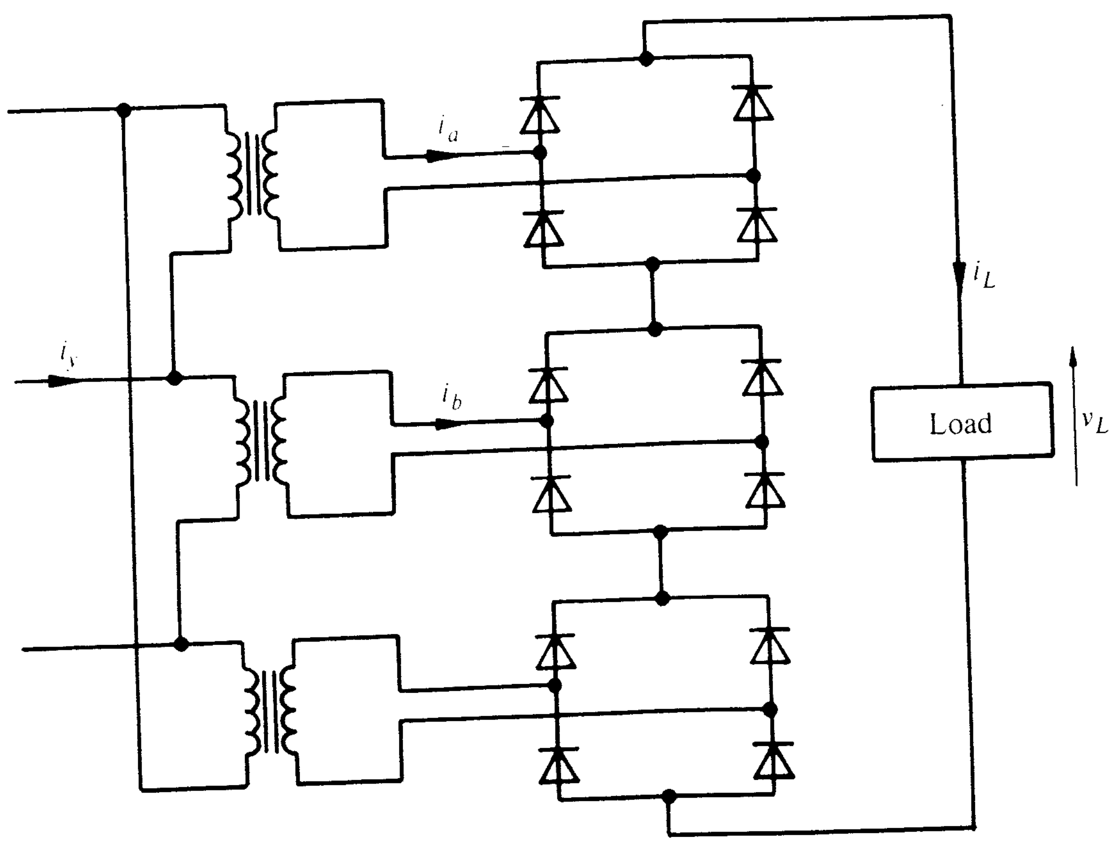

Figure 2-43 Three-phase high-voltage bridge connection.

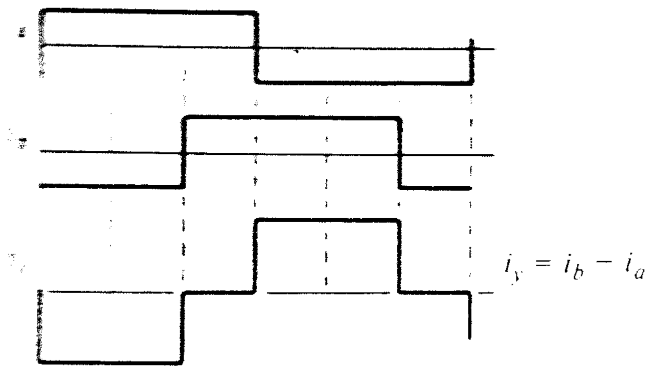

Figure 2-44

60 A at 300 V. Assume a diode volt-drop of 0.7 V and level load current. Compare this circuit to the normal three-phase bridge.

SOLUTION The output voltage is the addition of three two-pulse voltages displaced so as to give a six-pulse output. The mean voltage of each separate bridge is one third of the mean load voltage.

Let V = r.m.s. voltage of supply to each bridge, then

$$V_{mean} = \frac{300}{3} = \frac{2}{\pi} V\sqrt{2} - (2 \times 0.7)$$

for each two-pulse bridge, giving $V = 112.6$ volts.
Each diode conducts the complete load current for one half cycle as shown in Fig. 2-44. The figure also shows the input line current i_y.
Diode rating: P.R.V. $= \sqrt{2}V = 159$ volts, $I_{rms} = 60/\sqrt{2} = 42.4$ A.
Transformer rating $= 3 \times 112.6 \times 60 = 20.3$ kVA.
Comparison with the normal three-phase bridge can be examined by looking at Example 2-24, as the load specification is the same.
The P.R.V. is halved, but the current rating is higher for each diode by a factor of 1.22.
The total transformer rating (size) is greater. The input current waveform is the same, but a third-harmonic component does circulate in the transformer primary.
The volt-drop is higher, as at all times six diodes are conducting in series.
In conclusion, this circuit would only be used for high-voltage loads, because the peak reverse-voltage rating of each diode is less than for the normal bridge circuit.

CHAPTER

THREE

CONVERTER OPERATION

In Chapter 2, the basic characteristics of the common rectifying circuits were introduced, ignoring the effect of the a.c. supply impedance and concentrating only on the characteristics of the circuits as rectifiers. In this chapter, the analysis of those circuits will be widened to include the effect of the supply impedance, the power factor of the current drawn from the supply, and extend the study to reverse power flow.

The word *rectification* implies conversion of energy from an a.c. source to a d.c. load. In practice, under certain conditions, the power flow can be reversed, when the circuit is said to be *operating in the inverting mode*. As the circuit can be operated in either direction of power flow, the word *converter* better describes the circuit, the words *rectifier* and *inverter* being retained when the converter operates in those particular modes.

3-1 OVERLAP

In Chapter 2, the assumption was made that the transfer or commutation of the current from one diode (or thyristor) to the next took place instantaneously. In practice, inductance and resistance must be present in the supply source, and time is required for a current change to take place. The net result is that the current commutation is delayed, as it takes a finite time for the current to decay to zero in the outgoing diode (or thyristor), whilst the current will rise at the same rate in the incoming diode.

The inductive reactance of the a.c. supply is normally much greater than its resistance and, as it is the inductance which delays the current change, it is reasonable to neglect the supply resistance. The a.c. supply may be represented by its Thévenin equivalent circuit, each phase being a voltage source in series with its inductance. The major contributor to the supply impedance is the transformer leakage reactance.

To explain the phenomenon associated with the current transfer, the three-phase half-wave rectifier connection will be used, as once the explanation with this circuit has been understood, it can be readily transferred to the other connections. Figure 3-1a shows the three-phase supply to be three voltages, each in series with an inductance L. Reference to the waveforms in Fig. 3-1b shows that at commutation there is an angular period γ during which both the outgoing diode and incoming

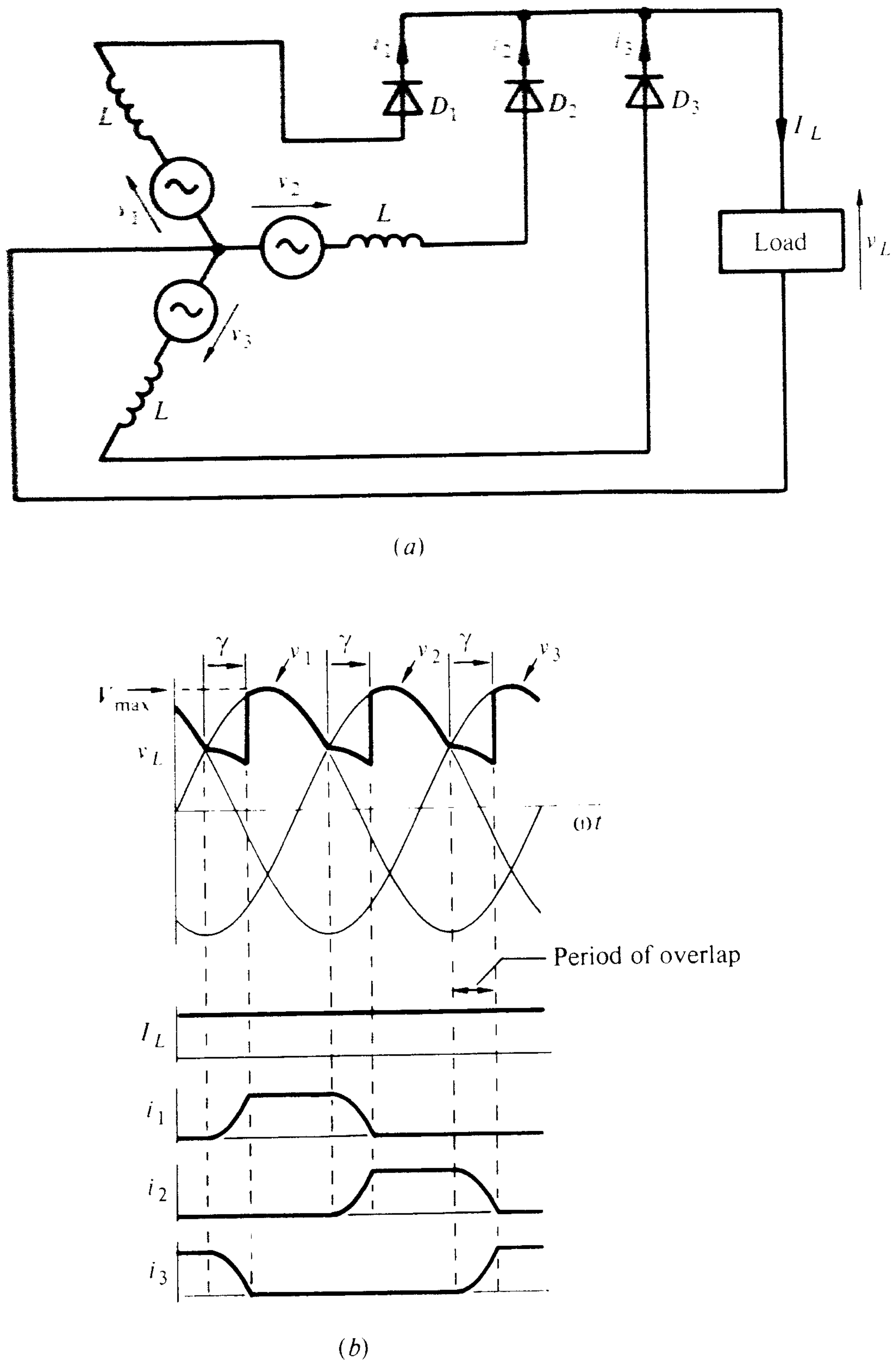

Figure 3-1 Overlap in the three-phase half-wave rectifier. (*a*) Circuit reference. (*b*) Waveforms.

diode are conducting. This period is known as the *overlap period*, and γ is defined as the *commutation angle* or alternatively the *angle of overlap*. During the overlap period, the load current is the addition of the two diode currents, the assumption being made that the load is inductive enough to give a sensibly level load current. The load voltage is the mean of the two conducting phases, the effect of overlap being to reduce the mean level.

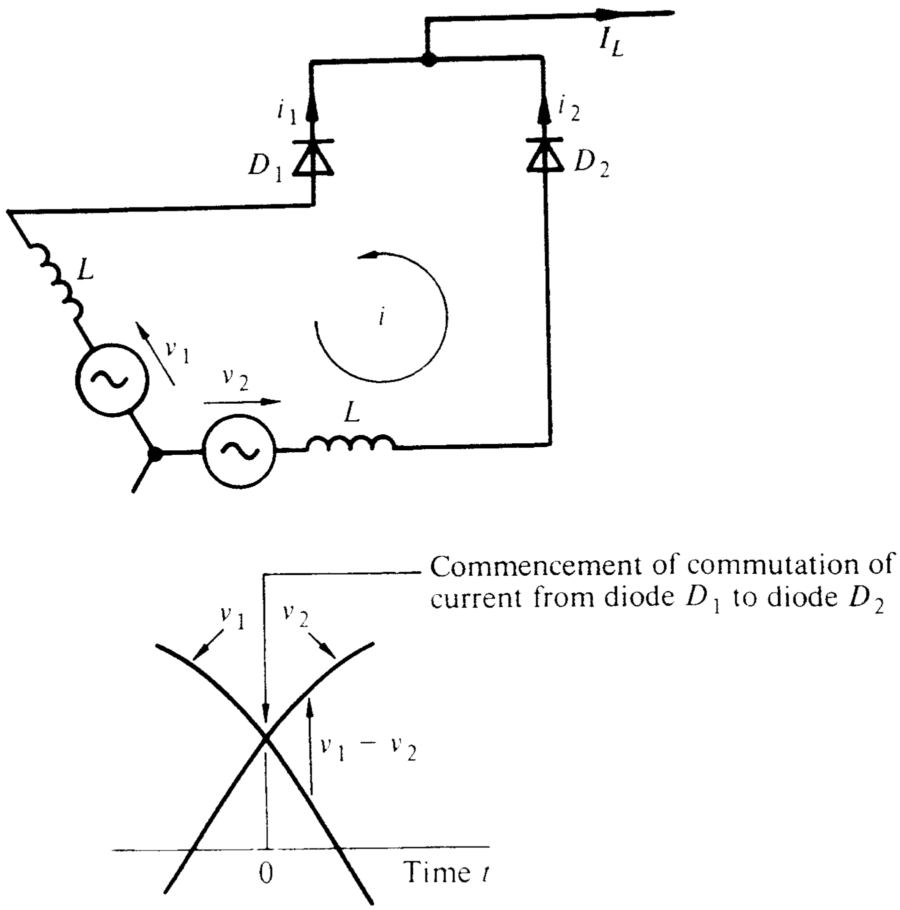

Figure 3-2 Conditions during the overlap period.

The overlap is complete when the current level in the incoming diode reaches the load-current value. To determine the factors on which the overlap depends, and to derive an expression for the diode current, a circulating current i can be considered to flow in the closed path formed by the two conducting diodes D_1 and D_2 as shown in Fig. 3-2. Ignoring the diode volt-drops,

$$v_2 - v_1 = L\,di/dt + L\,di/dt \tag{3-1}$$

The voltage $v_2 - v_1$ is the difference between the two phases, having a zero value at $t = 0$, the time at which commutation commences. The voltage difference between two phases is the *line voltage* having a maximum value $\sqrt{3}V_{\max}$ where $V_{\max}$ is of the phase voltage.

Using Eq. (3-1),

$$\sqrt{3}\,V_{\max} \sin \omega t = 2L \frac{di}{dt}$$

$$di = \frac{\sqrt{3}\,V_{\max}}{2L} \sin \omega t \, dt$$

Integrating both sides,

$$i = \frac{\sqrt{3}\,V_{\max}}{2L}\left(-\frac{\cos \omega t}{\omega}\right) + C$$

at $t = 0$, $i = 0$, $\therefore C = \dfrac{\sqrt{3}V_{max}}{2\omega L}$

$$\therefore i = \frac{\sqrt{3}V_{max}}{2\omega L}(1 - \cos \omega t) \tag{3-2}$$

The overlap is complete when $i = I_L$, at which instant $\omega t = \gamma$, the overlap angle. Also $\omega L = X$, the supply source reactance. Hence,

$$I_L = \frac{\sqrt{3}V_{max}}{2X}(1 - \cos \gamma) \tag{3-3}$$

or

$$\cos \gamma = 1 - \frac{2I_L X}{\sqrt{3}V_{max}} \tag{3-4}$$

From Eq. (3-2), the current change in the diodes during overlap is cosinusoidal, as illustrated in Fig. 3-1*b*.

It is worth noting that for commutation involving two phases of a three-phase group, conditions during overlap are as a line-line short-circuit fault. As the positive and negative phase sequence reactance values of a transformer are equal, then the commutating reactance value is the normal short-circuit reactance.

To determine the mean voltage of the waveform shown in Fig. 3-1*a*, one can use calculus to find the area under the two sections of the curve, one based on the sinewave shape after overlap is complete and the other during overlap. During overlap, the load voltage is the mean between two sinewaves, that is, the shape is sinusoidal, but if we consider the curve as a cosine wave, then the integration limits will be 0 to γ on a cosine wave of peak value $V_{max} \sin \frac{\pi}{6}$, giving

$$V_{mean} = \frac{1}{2\pi/3}\left[\int_{\frac{\pi}{6}+\gamma}^{\frac{5\pi}{6}} V_{max} \sin \theta \, d\theta + \int_0^{\gamma} V_{max} \sin \frac{\pi}{6} \cos \phi \, d\phi\right]$$

$$= \frac{3\sqrt{3}V_{max}}{4\pi}(1 + \cos \gamma) \tag{3-5}$$

If one neglects overlap, that is, let $\gamma = 0$, then Eq. (3-5) is identical to Eq. (2-9).

An alternative approach to analysing the effect of the supply inductance is to consider the relationship of the inductor voltage to its current as the current rises from zero to the load value (or collapses from the load value to zero).

$$L \, di/dt = v, \text{hence} \int L \, di = \int v \, dt$$

therefore

$$LI = \int v \, dt = \text{volt-seconds} \tag{3-6}$$

where I is the change in current and $\int v \, dt$ is the area under a curve representing the instantaneous voltage across the inductance during overlap. Therefore, if the mean value of the load voltage is found via terms in volt-seconds, we can subtract LI_L to take account of overlap.

For example, using the waveform in Fig. 3.1,

$$V_{\text{mean}} = \frac{1}{2\pi/3\omega}\left[\int_{t=\pi/6\omega}^{t=5\pi/6\omega} V_{\max}\sin\omega t\,dt - LI_L\right] = \frac{3\sqrt{3}V_{\max}}{2\pi} - \frac{3\omega}{2\pi}LI_L \tag{3-7}$$

From Eq. (3-3) we can substitute for I_L, giving

$$V_{\text{mean}} = \frac{3\sqrt{3}V_{\max}}{2\pi} - \frac{3\omega L\times\sqrt{3}V_{\max}}{2\pi\times 2X}(1-\cos\gamma) = \frac{3\sqrt{3}V_{\max}}{4\pi}(1+\cos\gamma)$$

an expression which is identical to Eq. (3-5).

In the controlled 3-pulse circuit, the overlap will lead to the waveform shown in Fig. 3-3 (circuit reference Fig. 3-1*a* using thyristors), where it can be seen that with a firing delay angle α a finite voltage is present from the start of commutation. Using Eq. (3-1), $v_2 - v_1 = \sqrt{3}V_{\max}\sin(\omega t+\alpha)$, where t is the time from the start of commutation, when $i = 0$.

Therefore $\qquad \sqrt{3}V_{\max}\sin(\omega t+\alpha) = 2L\,di/dt$

which yields
$$i = \frac{\sqrt{3}V_{\max}}{2\omega L}[\cos\alpha - \cos(\omega t+\alpha)] \tag{3-8}$$

overlap being complete when $i = I_L$ and $\omega t = \gamma$.

$$I_L = \frac{\sqrt{3}V_{\max}}{2\omega L}[\cos\alpha - \cos(\gamma+\alpha)] \tag{3-9}$$

Compared to the uncontrolled case ($\alpha = 0$), the overlap angle γ will be shorter and the current change during commutation will be towards a linear variation. The

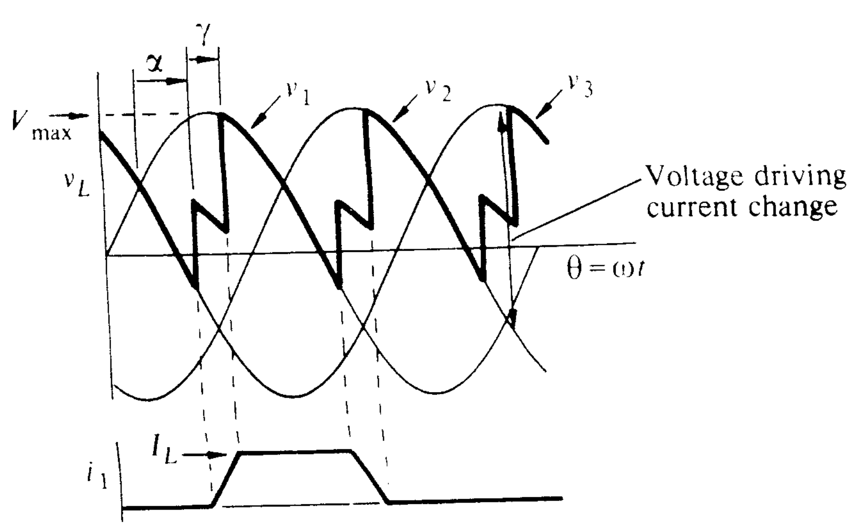

Figure 3-3 Overlap in a controlled 3-pulse rectifier.

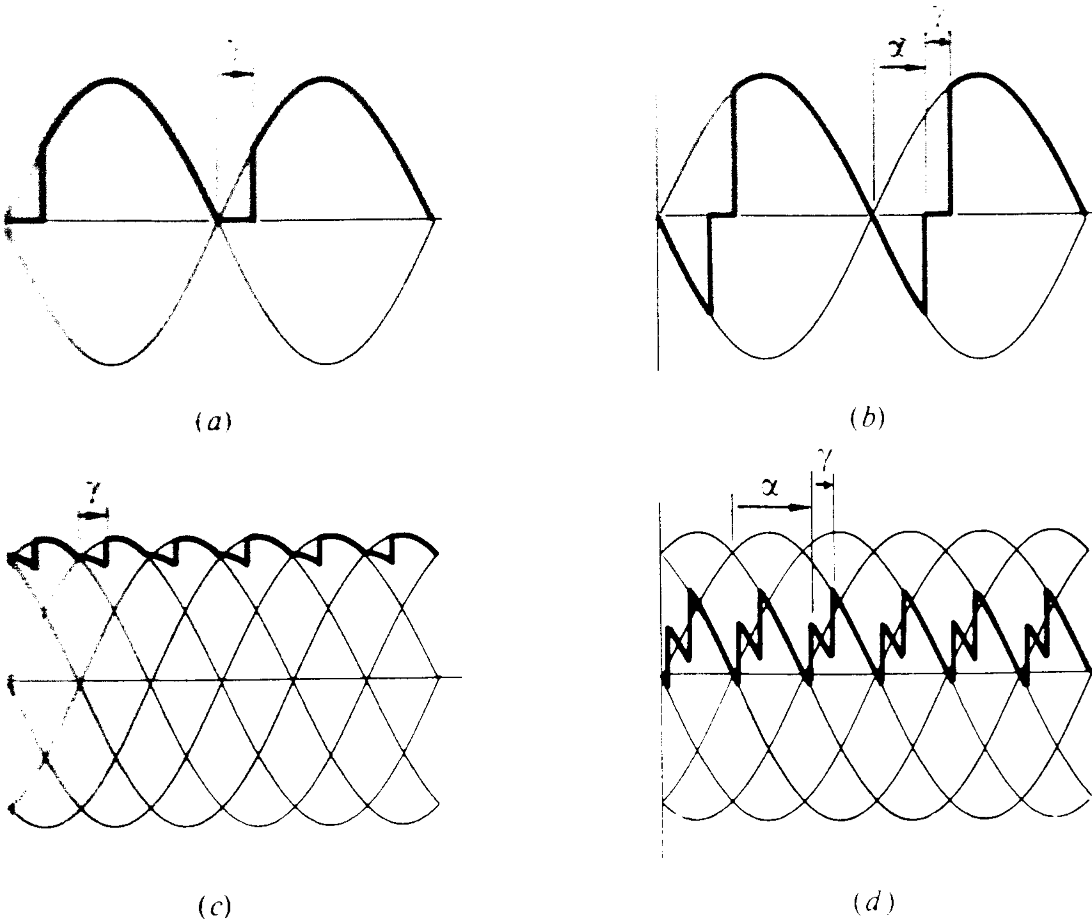

Figure 3-4 Typical load-voltage waveforms showing overlap. (*a*) 2-pulse uncontrolled. (*b*) 2-pulse controlled. (*c*) 6-pulse uncontrolled. (*d*) 6-pulse controlled.

mean load voltage is given by

$$V_{\text{mean}} = \frac{1}{2\pi/3}\left\{\int_{\frac{\pi}{6}+\alpha+\gamma}^{\frac{5\pi}{6}+\alpha} V_{\max}\sin\theta\, d\theta + \int_{\alpha}^{\alpha+\gamma} V_{\max}\sin\frac{\pi}{6}\cos\phi\, d\phi\right\}$$

$$= \frac{3\sqrt{3}\,V_{\max}}{4\pi}[\cos\alpha + \cos(\alpha+\gamma)] \qquad (3\text{-}10)$$

The effect of overlap is present in all rectifiers and Fig. 3-4 shows typical waveforms with pulse-numbers other than that discussed in detail above. The location of the waveform during overlap is at a position midway between the outgoing and incoming voltages. With the 2-pulse waveform, the load voltage is zero during the overlap period.

Circuits with a commutating diode across the load will experience the overlap effect in so far as time is required to transfer the load current away from the supply and into the diode. Typically, the condition can be represented at overlap by the circuit shown in Fig. 3-5. When the supply voltage v reverses, that is, to the direction shown in Fig. 3-5, then a circulating current i will be set up in the closed path formed with the diode. Commutation is complete when i equals the load current I_L. The load will not influence the commutating conditions if I_L is assumed to remain at a constant level.

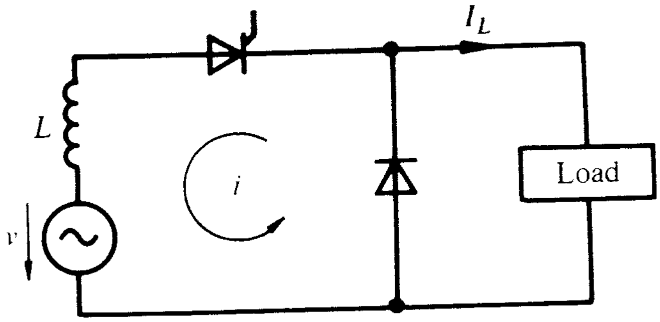

Figure 3-5 Circuit conditions at overlap when the load current is being transferred to the commutating diode.

Using similar reasoning to that developed in the derivation of Eq. (3-2), then from Fig. 3-5, and ignoring thyristor and diode volt-drops, $v = L\, di/dt$, where $v = V_{max} \sin \omega t$, and $i = 0$ at $t = 0$, starting from the instant when the load voltage attempts to reverse; hence we have the equations

$$i = \frac{V_{max}}{\omega L}(1 - \cos \omega t) \tag{3-11}$$

$$I_L = \frac{V_{max}}{\omega L}(1 - \cos \gamma) \tag{3-12}$$

The neglect of the device volt-drops could lead to considerable errors, particularly in the bridge circuits where two devices are concerned.

Following the commutation diode conduction period, the next thyristor is fired and the load current reverts back to the supply, giving another overlap period during which time the load voltage remains effectively zero. Figure 3-6 shows the circuit conditions which apply but, unlike those relating to Fig. 3-5, the voltage v this time will be above zero at the instant of thyristor firing, hence giving a shorter overlap period.

It is possible, given high enough supply reactance, for the overlap period to continue into the time when the next commutation is due to occur, say if, for example, overlap exceeds 60° in a 6-pulse connection. Conditions when this happens must be analysed with reference to the particular connection being used. In practice, it is rare for overlap to continue through to the next commutation, although it does occur, for example, when starting d.c. motors at low voltage.

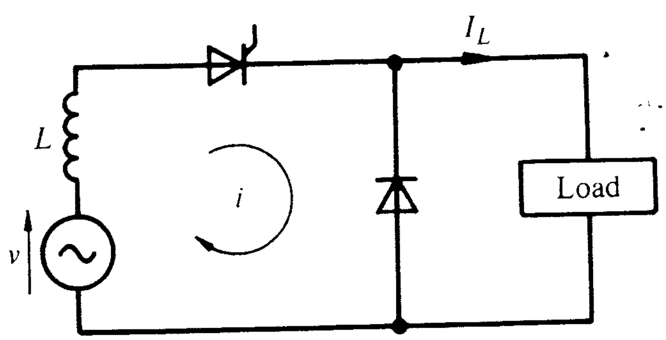

Figure 3-6 Circuit conditions at overlap when the load current is being transferred away from the commutating diode.

3-2 POWER FACTOR

The power factor of a load fed from an a.c. supply is defined as

$$\text{power factor} = \frac{\text{mean power}}{V_{\text{rms}} I_{\text{rms}}} \tag{3-13}$$

In the usual a.c. system where the current is sinusoidal, the power factor is the cosine of the angle between current and voltage. The rectifier circuit, however, draws non-sinusoidal current from the a.c. system, hence the power factor cannot be defined simply as the cosine of the displacement angle.

Inspection of the waveforms of the various controlled rectifiers in Chapter 2 shows that firing delay has the effect of delaying the supply current relative to its phase voltage. The current does contain harmonic components which result in its overall r.m.s. value being higher than the r.m.s. value of its fundamental component, therefore the power factor is less than that calculated from the cosine of its displacement angle.

Normally, the supply phase voltage can be taken as being sinusoidal, hence there will be no power associated with the harmonic current, which therefore results in

$$\text{power} = V_{1(\text{rms})} I_{1(\text{rms})} \cos \phi_1 \tag{3-14}$$

where the suffix 1 relates to the fundamental component of the current, ϕ_1 being the phase angle between the voltage and the fundamental component of the current.

For a sinusoidal voltage supply, substituting Eq. (3-14) into Eq. (3-13) yields

$$\text{power factor} = \frac{I_{1(\text{rms})}}{I_{\text{rms}}} \cos \phi_1 \tag{3-15}$$

where

$$\frac{I_{1(\text{rms})}}{I_{\text{rms}}} = \text{input distortion factor} \tag{3-16}$$

and

$$\cos \phi_1 = \text{input displacement factor.} \tag{3-17}$$

ϕ_1 will equal the firing delay angle α in the fully-controlled connections that have a continuous level load current.

The power factor will always be less than unity when there are harmonic components in the supply current, even when the current is in phase with the voltage, as in the diode case.

3-3 INVERSION

The 3-pulse converter has been chosen to demonstrate the phenomenon of inversion, although any fully-controlled converter could be used. Assuming continuous load current, consider the firing delay angle to be extended from a small value to almost 180° as shown in Fig. 3-7*b* to *f*. Up to a delay of 90° the converter is rectifying, but at 90° the voltage is as much negative as positive, resulting in a

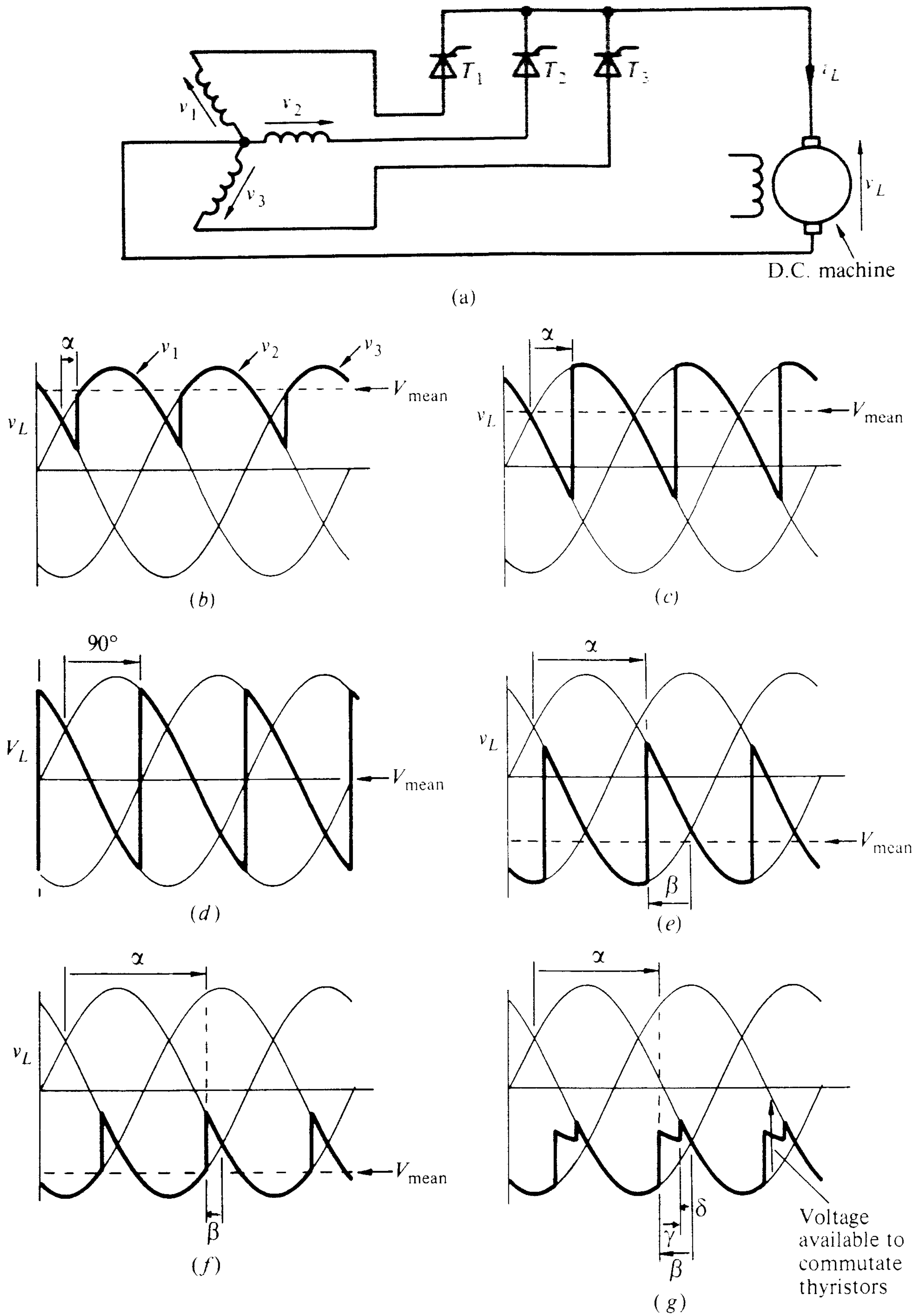

Figure 3-7 3-pulse waveform showing the effect on the load voltage when firing delay is extended towards 180°. (*a*) 3-pulse connection with a d.c. machine as the load. (*b*) Rectifying, small firing delay angle. (*c*) Rectifying, but instantaneous voltage partly negative. (*d*) Firing delay = 90°, V_{mean} is zero. (*e*) Inverting, V_{mean} is negative. (*f*) Inverting, approaching limit as $\beta \rightarrow 0$. (*g*) Inverting waveform including overlap effect.

zero mean value. Further delay beyond 90° will result in the waveform having an overall negative mean value, until, as α approaches 180°, the waveform shape is similar to that at $\alpha = 0$, except it is reversed. To construct these waveforms, let the vertical line at the instants of commutation move to the right as the firing delay is extended.

The circuit connection of Fig. 3-7*a* shows a d.c. machine as the load element, which acts as a motor while the converter is rectifying. However, once the load voltage v_L reverses, the d.c. machine acts as a generator, and the converter is now said to be operating in the inverting mode. The current direction cannot reverse, as it is constrained by the thyristor direction, hence, if the machine runs in the same direction of rotation, it can only generate by having its armature or field connections reversed.

The reversal of the direct voltage means that current is flowing in each phase when the phase voltage is negative, that is, power is being fed back into the a.c. system from the d.c. generator.

In order for the thyristors to commute, the converter must be connected to a large a.c. synchronous system, such as the public supply, so that the a.c. voltages are of a defined waveform. The energy fed back into the a.c. system will be absorbed by the many other loads on the system.

It is only possible to commutate current from (say) thyristor T_1 to thyristor T_2 while the instantaneous voltage of phase 2 is higher than phase 1 (that is, while v_2 is less negative than v_1). At $\alpha = 180°$, $v_2 = v_1$ and the relative voltage between the two phases after this reverses, making commutation impossible, hence $\alpha = 180°$ is the limit of operation. When in the inverting mode, it is more usual to designate the firing position as firing advance angle β as shown in Fig. 3-7*e* and *f*, the relationship between β and α being

$$\beta = 180° - \alpha \tag{3-18}$$

The limit of 180° and the relation between β and α apply whatever the pulse-number of the converter.

In deriving the waveforms of Fig. 3-7*b* to *f*, the effect of overlap has been ignored, so as to simplify the explanation. In Fig. 3-7*g* the overlap is shown, the effect being to delay the commutation, the waveform having a voltage midway between the incoming and outgoing voltages. If the commutation is not complete before the two commutating phases reach equal voltage values, then transfer of current is impossible as the load (generator) current will revert to the outgoing thyristor. Therefore, the overlap angle γ must be less than the angle of firing advance β. In practice, β can never be reduced to zero. In Fig. 3-7*g*, an angle

$$\delta = \beta - \gamma \tag{3-19}$$

is shown where δ represents as an angle the time available to the outgoing thyristor to regain its blocking state after commutation. δ is known as the *recovery* or *extinction angle* and would typically be required to be not less than 5°.

The firing circuits to the thyristors are designed so that, irrespective of any other control, a firing pulse is delivered to the thyristor early enough to ensure complete commutation. For example, at (say) $\beta = 20^\circ$, a firing (end-stop) pulse will always be delivered to the thyristor gate.

A more detailed study of converter operation in the inverting mode can be explained by reference to Fig. 3-8. Taking the generator as the source of power feeding into the a.c. system, it is convenient to reverse the voltage references as compared to the rectifiying mode. Compared to Fig. 3-7*a*, the d.c. machine shown in Fig. 3-8*a* has its armature connections reversed, so as to emphasize that for a given direction of rotation the voltage direction at the brushes is unchanged, whether the machine is generating or motoring, only the current flow in the brushes being reversed. Further, the references to the three-phase voltages are reversed to emphasize that the a.c. system is absorbing power, that is, the current is flowing into the positive (arrowhead) end of the voltage reference.

The generator voltage waveform in Fig. 3-8*b* now becomes the inverse of that shown in Fig. 3-7*g* because of the reversal of the frame of reference voltages. It can now be seen that the angle of firing advance β is an angle in advance of the commutation limit, a similar reasoning to the delay meaning in the rectifier case. Because the frame of reference has been reversed, it needs to be emphasized that inversion will take place only if the firing delay angle α is extended beyond 90°: one must not be misled from Fig. 3-8*b* into a belief that a movement of the firing pulse forward in time will give inversion.

The generator mean voltage can be calculated from the waveform in a similar manner to the derivation of Eqs. (3-5) and (3-10), hence giving

$$V_{\text{mean}} = \frac{1}{2\pi/3}\left[\int_{\frac{\pi}{6}-\beta+\gamma}^{\frac{5\pi}{6}-\beta} V_{\max}\sin\theta\,d\theta + \int_{-\beta}^{-\beta+\gamma} V_{\max}\sin\frac{\pi}{6}\cos\phi\,d\phi\right]$$

$$= \frac{3\sqrt{3}\,V_{\max}}{4\pi}\left[\cos\beta + \cos(\beta-\gamma)\right] \tag{3-20}$$

The generator voltage will be higher than V_{mean} by the thyristor volt-drop.

Assuming continuous-level direct current in the generator, the individual thyristor currents are as shown in Fig. 3-8*b*. It can be seen that these currents are in advance of their respective phase voltages, hence the power is being fed back into the a.c. system at a leading power factor.

The thyristor voltage waveform in Fig. 3-8*b* shows that the anode-[illegible] voltage is reversed for only the short time represented by δ, which gives [illegible] off time available to the thyristor to regain its blocking state. The [illegible] demonstrate that the anode voltage is positive with respect to the cathode [illegible] of its off-time.

3-4 REGULATION

The term *regulation* is used to describe the characteristic of equip[illegible]

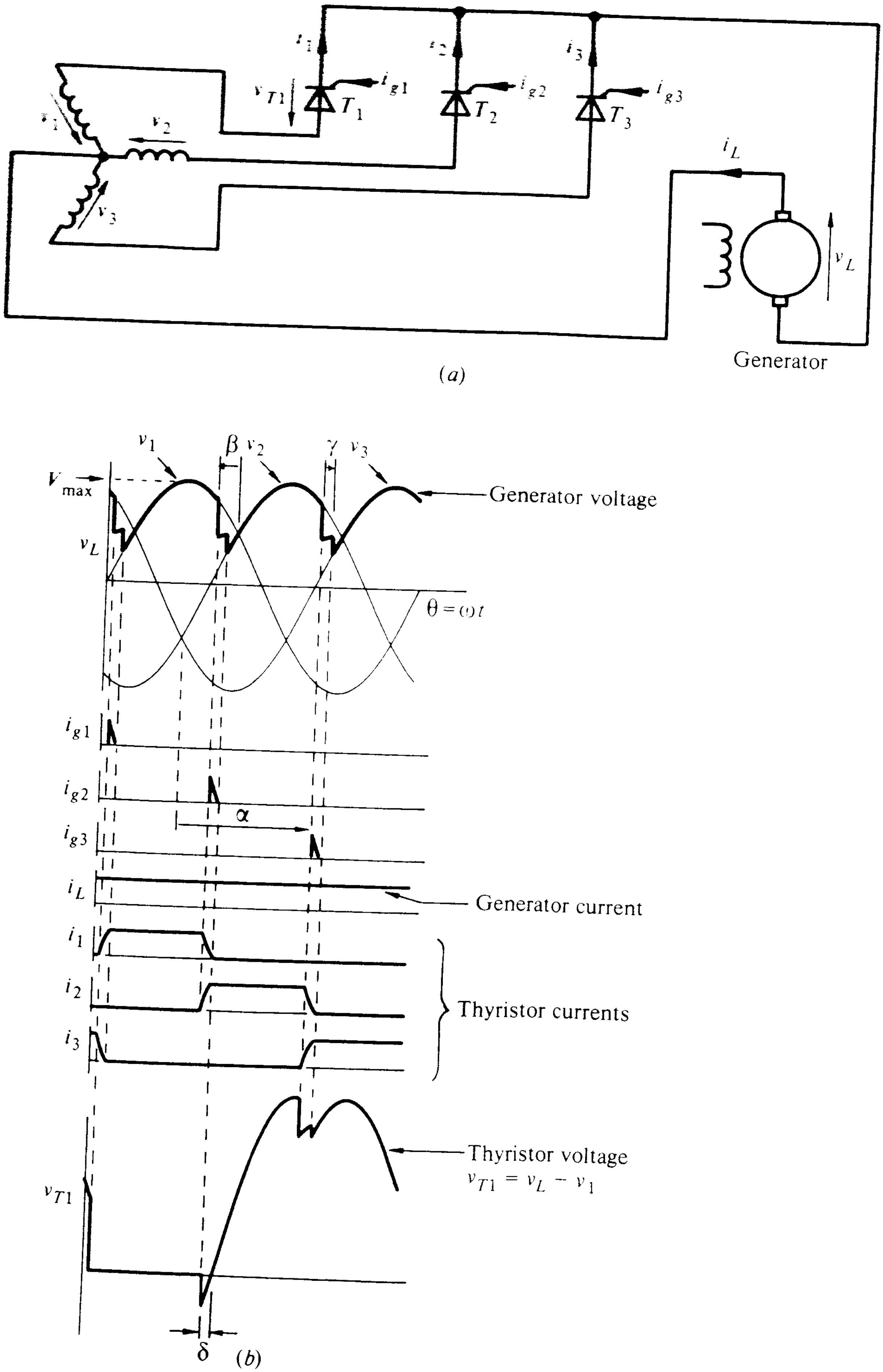

Figure 3-8 3-pulse inverter operation. (*a*) Connection and circuit reference. (*b*) Waveforms.

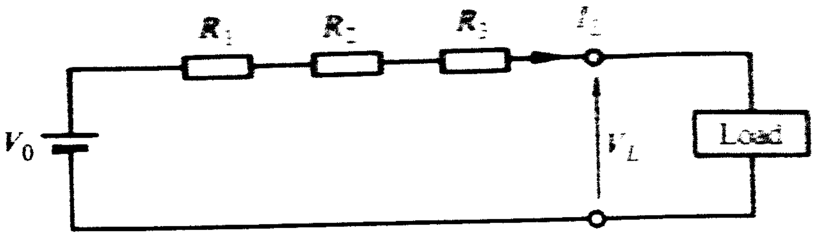

Figure 3-9 Equivalent circuit of a loaded rectifier.

loaded. In the case of the rectifier, regulation describes the drop in mean voltage with load relative to the no-load or open-circuit condition.

There are three main sources contributing to loss of output voltage:

1. The voltage drop across the diodes and/or thyristors.
2. The resistance of the a.c. supply source and conductors.
3. The a.c. supply source inductance.

These three voltage drops can be represented respectively by the three resistors R_1, R_2 and R_3 in the equivalent circuit of Fig. 3-9. The open-circuit voltage is given by V_0 and the actual load voltage by V_L. If the load current I_L is taken to be level, that is, a pure direct current, then any voltage drop can only be represented by resistors.

The voltage drop across the thyristors and diodes can in the first instance be taken as a constant value, or secondly can more accurately be represented by a smaller constant volt-drop (junction potential) plus a resistance value for the bulk of the silicon. In circuits containing a mixture of thyristors and diodes, the volt-drop and equivalent resistance attributed to this cause may depend on the degree of firing delay.

The resistance of the leads and a.c. source resistance can be considered constant in most cases. If (say) throughout a cycle the current is always flowing in two of the supply phases, then the resistance per phase can be doubled and added to the d.c. lead resistance to give the value for the equivalent circuit.

The voltage drop due to the a.c. supply source inductance is the overlap effect, and was calculated for the three-pulse uncontrolled circuit in Eq. (3-7). If the fully controlled case is considered, then the limits in the integral will be

$$\left(\frac{\pi}{6\omega}+\frac{\alpha}{\omega}\right) \text{ to } \left(\frac{5\pi}{6\omega}+\frac{\alpha}{\omega}\right),$$

giving

$$V_{\text{mean}} = \frac{3\sqrt{3}\,V_{\text{max}}}{2\pi}\cos\alpha - \frac{3\omega}{2\pi}LI_L \tag{3-21}$$

Examination of Eq. (3-21) shows that, independent of whether the rectifier is controlled or not, the load voltage is reduced by $(3\omega L/2\pi)I_L$ for the three-pulse output, hence this voltage can be represented in the equivalent circuit of Fig. 3-9 as a resistance of value $3\omega L/2\pi$. Unlike the other equivalent circuit resistances, this value does not represent any power loss, but merely represents the voltage drop due to overlap.

The voltage drop due to overlap is changed to a higher value if overlap continues into the period of the next commutation. Simple overlap is known as a mode 1 condition, whilst overlap involving three elements is known as a *mode* 2 condition. The paper by Jones, V. H. (see Bibliography) analyses in detail these conditions.

In the inverting mode, the voltage drop due to overlap can be determined for the 3-pulse connection in a like manner to the method used for deriving Eqs. (3-7) and (3-21), and with reference to Fig. 3-8b gives

$$V_{\text{mean}} = \frac{1}{2\pi/3\omega}\left[\int_{\frac{\pi}{6\omega}-\beta}^{\frac{5\pi}{6\omega}-\beta} V_{\max}\sin\omega t\,dt + LI_L\right]$$

$$= \frac{3\sqrt{3}V_{\max}}{2\pi}\cos\beta + \frac{3\omega L}{2\pi}I_L \qquad (3\text{-}22)$$

3-5 EQUATIONS FOR p-PULSE CONVERTER

A general expression for the mean load voltage of a p-pulse fully-controlled rectifier, including the effects of the overlap angle γ, can be determined by reference to Fig. 3-10, where the mean voltage is given by

$$V_{\text{mean}} = \frac{1}{2\pi/p}\left[\int_{-\frac{\pi}{p}+\alpha+\gamma}^{\frac{\pi}{p}+\alpha} V_{\max}\cos\theta\,d\theta + \int_{\alpha}^{\alpha+\gamma} V_{\max}\cos\frac{\pi}{p}\cos\phi\,d\phi\right]$$

$$= \frac{pV_{\max}}{2\pi}\left[\sin\left(\frac{\pi}{p}+\alpha\right) - \sin\left\{-\frac{\pi}{p}+(\alpha+\gamma)\right\}\right.$$

$$\left. + \cos\frac{\pi}{p}\sin(\alpha+\gamma) - \cos\frac{\pi}{p}\sin\alpha\right]$$

$$\therefore\; V_{\text{mean}} = \frac{pV_{\max}}{2\pi}\sin\frac{\pi}{p}[\cos\alpha + \cos(\alpha+\gamma)] \qquad (3\text{-}23)$$

less the device volt-drops.

By substituting $\alpha = \pi - \beta$ and calling the mean voltage positive then, for the inverting mode,

$$V_{\text{mean}} = \frac{pV_{\max}}{2\pi}\sin\frac{\pi}{p}[\cos\beta + \cos(\beta-\gamma)] \qquad (3\text{-}24)$$

The voltage drop due to the supply commutating reactance $X\,\Omega$/phase when a p-pulse rectifier supplies a load of current value I_L can be determined from

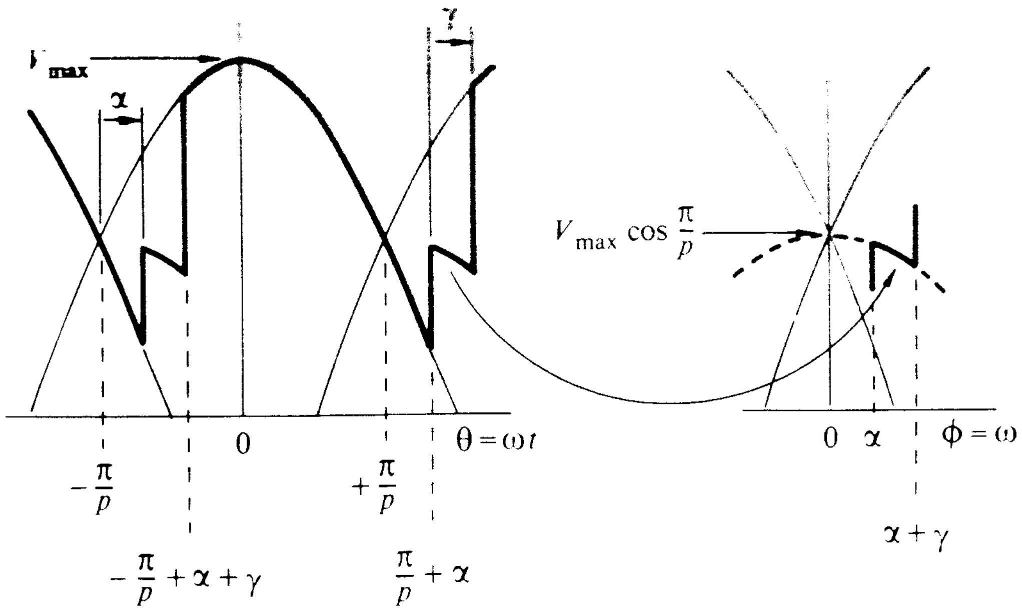

Figure 3-10 General waveform of a p-pulse rectifier.

Fig. 3-10. Taking the base as time, but allowing for the loss of area due to overlap as LI_L as shown in developing Eq. (3-7), then

$$V_{\text{mean}} = \frac{1}{2\pi/p\omega}\left[\int_{-\frac{\pi}{p\omega}+\frac{\alpha}{\omega}}^{\frac{\pi}{p\omega}+\frac{\alpha}{\omega}} V_{\max} \cos \omega t \, dt - LI_L\right]$$

$$= \frac{pV_{\max}}{\pi} \sin \frac{\pi}{p} \cos \alpha - \frac{pX}{2\pi} I_L \tag{3-25}$$

where $X = \omega L$.

This equation represents a voltage $V_0 \cos \alpha$, the mean open-circuit voltage, minus a voltage drop $(pX/2\pi)I_L$, giving an equivalent circuit as shown in Fig. 3-11, ignoring device and true resistance volt-drops. V_0 is the open-circuit voltage at zero firing delay angle.

The relationship between the overlap angle γ, load current I_L, supply voltage $V_{\max}$, and commutating reactance X for a p-pulse rectifier operating at any firing delay angle α, can be determined by equating Eqs. (3-23) and (3-25), yielding

$$XI_L = V_{\max} \sin \frac{\pi}{p} [\cos \alpha - \cos(\alpha + \gamma)] \tag{3-26}$$

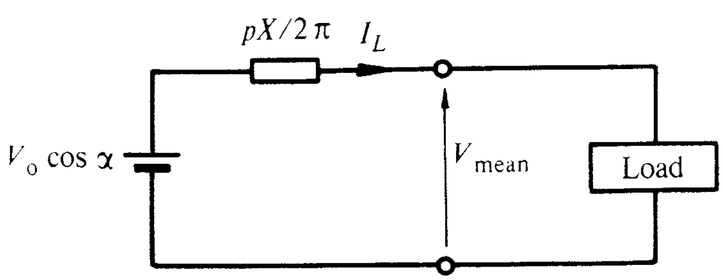

Figure 3-11 Equivalent circuit of a p-pulse fully-controlled rectifier, allowing for the commutating reactance.