3-NORTON’S THEOREM  Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig. (a)

1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.

$R_N$:  
3. Calculate $R_N$ by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin’s theorem will determine the proper value of $R_N$.

$I_N$:  
4. Calculate $I_N$ by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
Conclusion:

5. **Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.**

Note: The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation Fig. (b)

![Fig. (b) Converting between Thévenin and Norton equivalent circuits.](image)

**EXAMPLE (1)** Find the Norton equivalent circuit for the network in the shaded area of Fig. (1)

![Fig. (1) Network diagram](image)
Solution:
Steps 1 and 2 are shown in Fig. (1-a)

**Step 3** find \( R_N \) as shown in Fig. (1-b)

\[
R_N = R_1 \ || \ R_2 = 3 \ \Omega \ || \ 6 \ \Omega = \frac{(3 \ \Omega)(6 \ \Omega)}{3 \ \Omega + 6 \ \Omega} = \frac{18 \ \Omega}{9} = 2 \ \Omega
\]
**Step 4** find \( (I_N) \) as shown in Fig. (1-c), clearly indicating that the short-circuit connection between terminals (a) and (b) is in parallel with \( R_2 \) and eliminates its effect. \( I_N \) is therefore the same as through \( R_1 \), and the full battery voltage appears across \( R_1 \).

\[
V_2 = I_2 R_2 = (0)6 \ \Omega = 0 \ \text{V}
\]

Therefore,

\[
I_N = \frac{E}{R_1} = \frac{9 \ \text{V}}{3 \ \Omega} = 3 \ \text{A}
\]

**Step 5**: See Fig. (1-d). Substituting the Norton equivalent circuit for the network external to the resistor \( R_L \) of fig.(1)
A simple conversion indicates that the Thévenin circuits are, in fact, the same as Norton circuit (Fig. 1-e).

\[ I_N = 3 \text{ A} \]
\[ R_N = 2 \Omega \]
\[ E_{\text{Th}} = I_N R_N = (3 \text{ A})(2 \Omega) = 6 \text{ V} \]

Fig.(1-e)

Converting the Norton equivalent circuit of Fig. (1) To a Thévenin equivalent circuit.

**EXAMPLE 2** Find the Norton equivalent external to the 9Ω resistor in Fig. (2)
Solution:

Steps 1 and 2: See Fig. (2-a).

Step 3: find \( R_N \) See Fig. (2-b),

![Fig(2-b)](image)

\[
R_N = R_1 + R_2 = 5 \, \Omega + 4 \, \Omega = 9 \, \Omega
\]

Step 4: find \( I_N \) As shown in Fig. (2-c) the Norton current is the same as the current through the 4\( \Omega \) resistor. Applying the current divider rule,
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Figure 2(c) Determining $I_N$ for the network of Fig. (2)

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5 \, \Omega)(10 \, \text{A})}{5 \, \Omega + 4 \, \Omega} = \frac{50 \, \text{A}}{9} = 5.556 \, \text{A}$$

Step 5: See Fig. (2-d)

Substituting the Norton equivalent circuit for the network external to the resistor $R_L$ of Fig. (2).
**EXAMPLE (3)** (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of a-b in Fig. (3)

![Fig.(3)](image)

**Solution:**

Steps 1 and 2: See Fig. (3-a)

![Fig.(3-a)](image)
**Step 3** is shown in Fig. (3-b), and

\[ R_N = R_1 \parallel R_2 = \frac{4 \Omega \times 6 \Omega}{4 \Omega + 6 \Omega} = \frac{24 \Omega}{10} = 2.4 \Omega \]

**Step 4:** (Using superposition)

For the 7V battery (3-c),
Fig.(3-c) Determining the contribution to $I_N$ from the voltage source $E_1$.

$$I'_N = \frac{E_1}{R_1} = \frac{7 \text{ V}}{4 \Omega} = 1.75 \text{ A}$$

For the 8 A source (Fig. 3-d) we find that both $R_1$ and $R_2$ have been “short circuited” by the direct connection between $a$ and $b$, and

![Diagram showing short circuited components]

Fig.(3-d) Determining contribution to $I_N$ from the current source $I$.

$$I''_N = I = 8 \text{ A}$$

The result is

$$I_N = I''_N - I'_N = 8 \text{ A} - 1.75 \text{ A} = 6.25 \text{ A}$$
Step 5: See Fig. (3-e).

Substituting the Norton equivalent circuit for the network to the left of terminals (a-b) in Fig. (3).
4-MAXIMUM POWER TRANSFER THEOREM

The maximum power transfer theorem states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as “seen” by the load.

![Diagram](image)

Fig.(a)

Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

For the network of Fig. (a), maximum power will be delivered to the load when:

\[
R_L = R_{Th}
\]

For the network of Fig. (a):
For the Norton equivalent circuit of Fig. (b), maximum power will be delivered to the load when:

\[ R_L = R_N \]

Defining the conditions for maximum power to a load using the Norton equivalent circuit.

The dc operating efficiency of a system is defined by the ratio of the power delivered to the load to the power supplied by the source; that is,
The power delivered to $R_L$ under maximum power conditions ($R_L = R_{Th}$) is

$$\eta\% = \frac{R_L}{R_{Th} + R_L} \times 100\%$$

For the Norton circuit of Fig. (b),

$$I = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{2R_{Th}}$$

$$P_L = I^2R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} \text{ (watts, W)}$$

Example 4: A dc generator, battery, and laboratory supply are connected to a resistive load RL in Fig. 4(a), (b), and (c), respectively.
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Fig. (4)

a. For each, determine the value of $R_L$ for maximum power transfer to $R_I$.

b. Determine $R_L$ for 75% efficiency.
Solutions:

a. For the dc generator,

\[ R_L = R_{Th} = R_{int} = 2.5 \ \Omega \]

For the battery,

\[ R_L = R_{Th} = R_{int} = 0.5 \ \Omega \]

For the laboratory supply,

\[ R_L = R_{Th} = R_{int} = 40 \ \Omega \]

b. For the dc generator,

\[ \eta = \frac{P_o}{P_s} \quad (\eta \text{ in decimal form}) \]

\[ \eta = \frac{R_L}{R_{Th} + R_L} \]

\[ \eta(R_{Th} + R_L) = R_L \]

\[ \eta R_{Th} + \eta R_L = R_L \]

\[ R_L(1 - \eta) = \eta R_{Th} \]

and

\[ R_L = \frac{\eta R_{Th}}{1 - \eta} \]

\[ R_L = \frac{0.75(2.5 \ \Omega)}{1 - 0.75} = 7.5 \ \Omega \]

For the battery,

\[ R_L = \frac{0.75(0.5 \ \Omega)}{1 - 0.75} = 1.5 \ \Omega \]

For the laboratory supply,

\[ R_L = \frac{0.75(40 \ \Omega)}{1 - 0.75} = 120 \ \Omega \]
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Note:

😊 For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when

\[ R_L = R_{\text{int}} \quad \text{or} \quad R_L = R_S \]

EXAMPLE 5: Analysis of a transistor network resulted in the reduced configuration of Fig(5). Determine the \( R_L \) necessary to transfer maximum power to \( R_L \), and calculate the power of \( R_L \) under these conditions.

Solution:

\[ R_L = R_s = 40 \, \text{k\Omega} \]

\[ P_{\text{max}} = \frac{I_N^2 R_N}{4} = \frac{(10 \, \text{mA})^2 (40 \, \text{k\Omega})}{4} = 1 \, \text{W} \]

EXAMPLE 6: For the network of Fig. (6), determine the value of \( R \) for maximum power to \( R \), and calculate the power delivered under these conditions.
Determining $R_{Th}$ for the network external to the resistor $R$ of Fig(6).

**Solution:**

$$R_{Th} = R_3 + R_1 \parallel R_2 = 8 \Omega + \frac{(6 \Omega)(3 \Omega)}{6 \Omega + 3 \Omega} = 8 \Omega + 2 \Omega$$

$$R = R_{Th} = 10 \Omega$$
See Fig. (6-b) using voltage divider rule to find \( V_{R2} \):

Where \( E_{Th} = V_{R2} \)

\[
E_{Th} = \frac{R_2E}{R_2 + R_1} = \frac{(3 \Omega)(12 \text{ V})}{3 \Omega + 6 \Omega} = \frac{36 \text{ V}}{9} = 4 \text{ V}
\]

Then:

\[
P_{L_{\text{max}}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(4 \text{ V})^2}{4(10 \Omega)} = 0.4 \text{ W}
\]

**EXAMPLE 7** Find the value of \( R_L \) in Fig(7) for maximum power to \( R_L \), and determine the maximum power.
Solution:
See Fig. (7-a).

\[ R_{Th} = R_1 + R_2 + R_3 = 3 \, \Omega + 10 \, \Omega + 2 \, \Omega = 15 \, \Omega \]

and
\[ R_L = R_{Th} = 15 \, \Omega \]

Note Fig. (7-b), where
Fig(7-b) Determining $E_{Th}$ for the network external to the resistor $R_L$ of Fig. (7)

\[ V_1 = V_3 = 0 \text{ V} \]

and \[ V_2 = I_2 R_2 = I R_2 = (6 \text{ A})(10 \text{ Ω}) = 60 \text{ V} \]

Applying Kirchhoff’s voltage law,

\[ \sum V = -V_2 - E_1 + E_{Th} = 0 \]

and \[ E_{Th} = V_2 + E_1 = 60 \text{ V} + 68 \text{ V} = 128 \text{ V} \]

Thus,

\[ P_{L_{\text{max}}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128 \text{ V})^2}{4(15 \text{ Ω})} = 273.07 \text{ W} \]