Notice That: Answer Five Questions Only

Q1: Solve the following system of linear equations by Gauss elimination and matrix inverse methods.

\[ \begin{align*}
\begin{bmatrix}
 x + y + z &= 3 \\
 2x - y - 3z &= 1 \\
 x - 2y - z &= 2
\end{bmatrix}
\end{align*} \]

(20 marks)

Q2: A) Find the eigenvalues and eigenvectors of the matrix

\[ A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} \]

B) Evaluate \( \int_{0}^{1} \sqrt{\sin x + \cos x} \, dx \) using Simpson's one-third rule by dividing the interval into five equal parts.

(20 marks)

Q3: Solve the following differential equation using Laplace Transformation

\[ y'' - 3y' + 2y = 5e^x \quad y(0) = y'(0) = 1 \]

(20 marks)

Q4: Solve the following heat conduction equation with

\[ \frac{\partial^2 U}{\partial x^2} - \frac{1}{4} \frac{\partial U}{\partial t} \quad 0 < x < L, \quad t > 0 \]

Boundary conditions: \( U(0, t) = U(L, t) = 0 \)

Initial condition: \( U(x, 0) = x \)

(20 marks)

Q5: Find the least square approximation of the form \( y = \frac{a}{x} + b\sqrt{x} \) for the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

(20 marks)

Q6: From the following table, find the first and second derivatives at \( x = 1.2 \)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

(20 marks)

GOOD LUCK