Notice That: Answer Five Questions Only

Q1: - Solve the following differential equations

1) \( y'' - 3y' + 2y = 5 e^x \)
2) \( y' = (y - x) / (y + x) \)
3) \( y'' + xy = xy^2 \)
4) \( y''' - 2y'' - y' + 2y = 0 \)  

(20 marks)

Q2: - A) Find the local extreme values of the function:

\[ F(x,y) = x^2 + xy + 3x + 2y + 5 \]

B) Find the derivative of the function \( f(x,y,z) = xy + yz + zx \) at \((1,1,2)\) in the direction of \( A = 3i + 6j - 2k \)  

(20 marks)

Q3: - A) Find the equation of plane through \((1,1,-1)\), \((2,0,2)\) and \((0,-2,1)\).

B) Find the point of intersection of the lines:

\[ x = t, \quad y = -t + 2, \quad z = t + 1 \quad \& \quad x = 2s + 2, \quad y = s + 3, \quad z = 5s + 6 \]  

(20 marks)

Q4: - Evaluate the following multiple integrals

1) \[ \int_{-1}^{1} \int_{0}^{1-y} dz dy dx \]
2) \[ \int_{0}^{1} \int_{0}^{x} e^{xy} dx dy \]
3) \[ \int_{0}^{1} \int_{0}^{x^2+y^2} e^{-(x^2+y^2)} dy dx \]  

(20 marks)

Q5: - A) Find the average height of the paraboloid \( z = x^2 + y^2 \) over the square \( 0 \leq x \leq 2 \) and \( 0 \leq y \leq 2 \)  

B) Find the Taylor series generated by \( f(x) = \cos x \) at \( a = \pi/4 \)  

(20 marks)

Q6: - Test the convergence and divergence of the following series:

1) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} (\sqrt{n} + 1)} \] (use integral test)
2) \[ \sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}} \] (use limit comparison test)
3) \[ \sum_{n=1}^{\infty} \frac{(n+3)!}{3^n n! 3^n} \] (use ratio test)
4) \[ \sum_{n=1}^{\infty} (-1)^n \ln \left(1 + \frac{1}{n} \right) \] (use alternating series test)  

(20 marks)

GOOD LUCK !