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University of Technology
Materials Engineering Department
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Assistant Professor Dr. Akram Raheem Jabur
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References


Electrical resistivity
Electrical resistivity

Electrical resistivity $\rho$ (Greek: rho) is defined by,

$$\rho = \frac{E}{J}$$

where

- $\rho$ is the static resistivity (measured in ohm-metres, $\Omega$-m)
- $E$ is the magnitude of the electric field (measured in volts per metre, V/m),
- $J$ is the magnitude of the current density (measured in amperes per square metre, A/m²),

and $E$ and $J$ are both inside the conductor.

Many resistors and conductors have a uniform cross section with a uniform flow of electric current and are made of one material. (See the diagram) In this case,

the above definition of $\rho$ leads to:

$$\rho = \frac{RA}{\ell},$$

where

- $R$ is the electrical resistance of a uniform specimen of the material (measured in ohms, $\Omega$)
- $\ell$ is the length of the piece of material (measured in metres, m).
- $A$ is the cross-sectional area of the specimen (measured in square metres, m²).
A piece of resistive material with electrical contacts on both ends.

The reason resistivity has the dimension units of ohm-metres can be seen by transposing the definition to make resistance the subject:

\[ R = \frac{\rho \ell}{A} \]

The resistance of a given sample will increase with the length, but decrease with greater cross-sectional area. Resistance is measured in ohms. Length over area has units of \(1/\)distance. To end up with ohms, resistivity must be in the units of "ohms \(\times\) distance" (SI ohm-metre, US ohm-inch).

In a hydraulic analogy, increasing the cross-sectional area of a pipe reduces its resistance to flow, and increasing the length increases resistance to flow (and pressure drop for a given flow).

- **Metals** have high electrical conductivity and consist of positive ions in a crystal lattice surrounded by delocalized electrons
- **Insulators** (also called dielectrics) resist the flow of electric current
• **Semiconductors** have electrical resistivity between metals and insulators, which is temperature dependent.

Resistivity Range of Materials

1 Ω-m = 100 Ω-cm

Resistivity of various materials

- A **conductor** such as a **metal** has high conductivity and a low resistivity.
- An **insulator** like **glass** has low conductivity and a high resistivity.
The conductivity of a semiconductor is generally intermediate, but varies widely under different conditions, such as exposure of the material to electric fields or specific frequencies of light, and, most important, with temperature and composition of the semiconductor material.

The degree of doping in semiconductors makes a large difference in conductivity. To a point, more doping leads to higher conductivity. The conductivity of a solution of water is highly dependent on its concentration of dissolved salts, and other chemical species that ionize in the solution. Electrical conductivity of water samples is used as an indicator of how salt-free, ion-free, or impurity-free the sample is; the purer the water, the lower the conductivity (the higher the resistivity). Conductivity measurements in water are often reported as specific conductance, relative to the conductivity of pure water at 25 °C. An EC meter is normally used to measure conductivity in a solution.

This table shows the resistivity, conductivity and temperature coefficient of various materials at 20 °C (68 °F).

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ [Ω·m] at 20 °C</th>
<th>σ [S/m] at 20 °C</th>
<th>Temperature coefficient [K⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1.59×10⁻⁸</td>
<td>6.30×10⁷</td>
<td>0.0038</td>
</tr>
<tr>
<td>Copper</td>
<td>1.68×10⁻⁸</td>
<td>5.96×10⁷</td>
<td>0.0039</td>
</tr>
<tr>
<td>Annealed Copper</td>
<td>1.72×10⁻⁸</td>
<td>5.80×10⁷</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>2.44×10⁻⁸</td>
<td>4.32×10⁷</td>
<td>0.0034</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.82×10⁻⁸</td>
<td>3.5×10⁷</td>
<td>0.0039</td>
</tr>
<tr>
<td>Calcium</td>
<td>3.36×10⁻⁸</td>
<td>2.98×10⁷</td>
<td>0.0041</td>
</tr>
<tr>
<td>Tungsten</td>
<td>5.60×10⁻⁸</td>
<td>1.79×10⁷</td>
<td>0.0045</td>
</tr>
<tr>
<td>Zinc</td>
<td>5.90×10⁻⁸</td>
<td>1.69×10⁷</td>
<td>0.0037</td>
</tr>
<tr>
<td>Nickel</td>
<td>6.99×10⁻⁸</td>
<td>1.43×10⁷</td>
<td>0.006</td>
</tr>
<tr>
<td>Lithium</td>
<td>9.28×10⁻⁸</td>
<td>1.08×10⁷</td>
<td>0.006</td>
</tr>
<tr>
<td>Iron</td>
<td>1.0×10⁻⁷</td>
<td>1.00×10⁷</td>
<td>0.005</td>
</tr>
<tr>
<td>Platinum</td>
<td>1.06×10⁻⁷</td>
<td>9.43×10⁶</td>
<td>0.00392</td>
</tr>
<tr>
<td>Tin</td>
<td>1.09×10⁻⁷</td>
<td>9.17×10⁶</td>
<td>0.0045</td>
</tr>
<tr>
<td>Lead</td>
<td>2.2×10⁻⁶</td>
<td>4.55×10⁶</td>
<td>0.0039</td>
</tr>
<tr>
<td>Titanium</td>
<td>4.20×10⁻⁷</td>
<td>2.38×10⁶</td>
<td>X</td>
</tr>
<tr>
<td>Manganese</td>
<td>4.82×10⁻⁷</td>
<td>2.07×10⁶</td>
<td>0.000002</td>
</tr>
<tr>
<td>Constantan</td>
<td>4.9×10⁻⁷</td>
<td>2.04×10⁶</td>
<td>0.000008</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>6.897×10⁻⁷</td>
<td>1.450×10⁶</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>9.8×10⁻⁷</td>
<td>1.02×10⁶</td>
<td>0.0009</td>
</tr>
<tr>
<td>Nichrome</td>
<td>1.10×10⁻⁶</td>
<td>9.09×10⁵</td>
<td>0.0004</td>
</tr>
<tr>
<td>Carbon (amorphous)</td>
<td>5×10⁻¹⁴ to 8×10⁻⁴</td>
<td>1.25 to 2×10⁻¹</td>
<td>−0.0005</td>
</tr>
<tr>
<td>Carbon (graphite)</td>
<td>2.5×10⁻⁶ to 5.0×10⁻⁶ /basal plane</td>
<td>2 to 3×10⁻⁵ /basal plane</td>
<td>3.3×10⁻⁵ /basal plane</td>
</tr>
<tr>
<td>Carbon (diamond)</td>
<td>1×10⁻¹²</td>
<td>~10⁻¹³</td>
<td></td>
</tr>
<tr>
<td>Germanium</td>
<td>4.6×10⁻¹</td>
<td>2.17</td>
<td>−0.048</td>
</tr>
<tr>
<td>Sea water</td>
<td>2×10⁻¹</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>Drinking water</td>
<td>2×10⁻¹⁴ to 2×10⁻¹⁷</td>
<td>5×10⁻⁴ to 5×10⁻²</td>
<td></td>
</tr>
</tbody>
</table>
The effective temperature coefficient varies with temperature and purity level of the material. The 20 °C value is only an approximation when used at other temperatures. For example, the coefficient becomes lower at higher temperatures for copper, and the value 0.00427 is commonly specified at 0 °C. For further reading.

The extremely low resistivity (high conductivity) of silver is characteristic of metals. "The metallic substances differ from all other materials by the fact that the outer shells of their atoms are bound rather loosely, and often let one of their electrons go free. Thus the interior of a metal is filled up with a large number of unattached electrons that travel aimlessly around like a crowd of displaced persons. When a metal wire is subjected to electric force applied on its opposite ends, these free electrons rush in the direction of the force, thus forming what we call an electric current." More technically, the free electron model gives a basic description of electron flow in metals.

**Temperature dependence**

In general, electrical resistivity of metals increases with temperature, while the resistivity of intrinsic semiconductors decreases with increasing temperature. In both cases, electron–phonon interactions can play a key role. At high temperatures, the resistance of a metal increases linearly with temperature. As the temperature of a metal is reduced, the temperature dependence of resistivity follows a power law function of temperature. Mathematically the temperature dependence of the resistivity $\rho$ of a metal is given by the Bloch–Grüneisen formula:

$$\rho(T) = \rho(0) + A \left( \frac{T}{\Theta_R} \right)^n \int_0^{\Theta_R/T} \frac{x^n}{(e^x - 1)(1 - e^{-x})} \, dx$$

where $\rho(0)$ is the residual resistivity due to defect scattering, $A$ is a constant that depends on the velocity of electrons at the Fermi surface, the Debye radius and the number density of electrons in the metal. $\Theta_R$ is
the Debye temperature as obtained from resistivity measurements and matches very closely with the values of Debye temperature obtained from specific heat measurements. \( n \) is an integer that depends upon the nature of interaction:

1. \( n=5 \) implies that the resistance is due to scattering of electrons by phonons (as it is for simple metals)
2. \( n=3 \) implies that the resistance is due to s-d electron scattering (as is the case for transition metals)
3. \( n=2 \) implies that the resistance is due to electron–electron interaction.

As the temperature of the metal is sufficiently reduced (so as to 'freeze' all the phonons), the resistivity usually reaches a constant value, known as the residual resistivity. This value depends not only on the type of metal, but on its purity and thermal history. The value of the residual resistivity of a metal is decided by its impurity concentration. Some materials lose all electrical resistivity at sufficiently low temperatures, due to an effect known as superconductivity.

An even better approximation of the temperature dependence of the resistivity of a semiconductor is given by the Steinhart–Hart equation:

\[
1/T = A + B \ln(\rho) + C(\ln(\rho))^3
\]

where \( A, B \) and \( C \) are the so-called Steinhart–Hart coefficients.

This equation is used to calibrate thermistors.

In non-crystalline semi-conductors, conduction can occur by charges quantum tunnelling from one localised site to another. This is known as variable range hopping and has the characteristic form of \( \rho = Ae^{T−1/n} \), where \( n=2,3,4 \) depending on the dimensionality of the system.

### Resistivity density products

In some applications where the weight of an item is very important resistivity density products are more important than absolute low resistivity- it is often possible to make the conductor thicker to make up for a higher resistivity, and then a low resistivity density product material (or equivalently a high conductance to density ratio) is desirable. For example, for long distance overhead power lines— aluminum is frequently used rather than copper because it is lighter for the same conductance.

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity [( n\Omega \cdot m )]</th>
<th>Density [g/cm³]</th>
<th>Resistivity-density product [( n\Omega \cdot m \cdot g/cm³ )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium</td>
<td>47.7</td>
<td>0.97</td>
<td>46</td>
</tr>
</tbody>
</table>
Lithium 92.8  0.53  49
Calcium 33.6  1.55  52
Potassium 72.0  0.89  64
Beryllium 35.6  1.85  66
Aluminum 26.50  2.70  72
Magnesium 43.90  1.74  76.3
Copper 16.78  8.96  150
Silver 15.87  10.49  166
Gold 22.14  19.30  427
Iron 96.1  7.874  757

Silver, although it is the least resistive metal known, has a high density and does poorly by this measure. Calcium and the alkali metals make for the best products, but are rarely used for conductors due to their high reactivity with water and oxygen. Aluminium is far more stable. And the most important attribute, the current price, excludes the best choice: Beryllium.

**Electrical mobility**

- **Electrical mobility** is the ability of charged particles (such as electrons or protons) to move through a medium in response to an electric field that is pulling them. The separation of ions according to their mobility in gas phase is called Ion mobility spectrometry, in liquid phase it is called electrophoresis.

The **drift velocity** is the average velocity that a particle, such as an electron, attains due to an electric field. It can also be referred to as Axial Drift Velocity since particles defined are assumed to be moving along a plane. In general, an electron will 'rattle around' in a conductor at the Fermi velocity randomly. An applied electric field will give this random motion a small net velocity in one direction.

In a semiconductor, the two main carrier scattering mechanisms are ionized impurity scattering and lattice scattering.

Because current is proportional to drift velocity, which is, in turn, proportional to the magnitude of an external electric field, Ohm's law can be explained in terms of drift velocity.

Drift velocity is expressed in the following equations: 

\[ J_{\text{drift}} = \sigma \cdot v_{\text{avg}} \]

where \( J_{\text{drift}} \) is the current density, \( \sigma \) is charge density in units \( \text{C/m}^2 \), and \( v_{\text{avg}} \) is the average velocity of the carriers (drift velocity);

\[ v_{\text{avg}} = \mu \cdot E \]

where \( \mu \) is the electron mobility in \( \text{m}^2/\text{[V.s]} \) and \( E \) is the electric field in \( \text{V/m} \).
Theory

When a charged particle in a gas or liquid is acted upon by a uniform electric field, it will be accelerated until it reaches a constant drift velocity according to the formula:

\[ v_d = \mu E \]

where

- \( v_d \) is the drift velocity (m/s)
- \( E \) is the magnitude of the applied electric field (V/m)
- \( \mu \) is the mobility (m^2/(V.s))

In other words, the electrical mobility of the particle is defined as the ratio of the drift velocity to the magnitude of the electric field:

\[ \mu = \frac{v_d}{E} \]

Electrical mobility is proportional to the net charge of the particle. This was the basis for Robert Millikan's demonstration that electrical charges occur in discrete units, whose magnitude is the charge of the electron.

Electrical mobility of spherical particles much larger than the mean free path of the molecules of the medium is inversely proportional to the diameter of the particles; for spherical particles much smaller than the mean free path, the electrical mobility is inversely proportional to the square of the particle diameter.

**Mobility in gas phase**

Mobility is defined for any species in the gas phase, encountered mostly in plasma physics and is defined as:

\[ \mu = \frac{q}{mv_m} \]

where

- \( q \) is the charge of the species,
- \( v_m \) is the momentum transfer collision frequency, and
- \( m \) is the mass.

Mobility is related to the species' diffusion coefficient \( D \) through an exact (thermodynamically required) equation known as the Einstein relation:
\[ \mu = \frac{q}{k T} D, \]

where

- \( k \) is the Boltzmann constant,
- \( T \) is the gas temperature, and
- \( D \) is a measured quantity that can be estimated. If one defines the mean free path in terms of momentum transfer, then one gets:

\[ D = \frac{\pi}{8} \lambda^2 n. \]

But both the momentum transfer mean free path and the momentum transfer collision frequency are difficult to calculate. Many other mean free paths can be defined. In the gas phase, \( \lambda \) is often defined as the diffusional mean free path, by assuming a simple approximate relation is exact:

\[ D = \frac{1}{2} \lambda v, \]

when \( v \) is the root mean square speed of the gas molecules:

\[ v = \sqrt{\frac{3 k T}{m}} \]

where \( m \) is the mass of the diffusing species. This approximate equation becomes exact when used to define the diffusional mean free path.

**Applications**

Electrical mobility is the basis for electrostatic precipitation, used to remove particles from exhaust gases on an industrial scale. The particles are given a charge by exposing them to ions from an electrical discharge in the presence of a strong field. The particles acquire an electrical mobility and are driven by the field to a collecting electrode.

Instruments exist which select particles with a narrow range of electrical mobility, or particles with electrical mobility larger than a predefined value. The former are generally referred to as "differential mobility analyzers". The selected regardless of whether it is actually spherical. Mobility is often identified with the diameter of a singly charged spherical particle, thus the "electrical-mobility diameter" becomes a characteristic of the particle,