Comparative Study between Joint Space and Cartesian Space Path Planning for Two-Link Robot Manipulator using Fuzzy Logic

Dr. Firas A. Raheem, Ivan I. Gorial

1Control and Systems Engineering department, University of Technology, Baghdad, Iraq

email: E-mail:dr.firas7010@yahoo.com

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Abstract – This paper focuses on the comparison of two proposed fuzzy logic-based path planning systems for a 2-DOF robot manipulator. The first system is joint space path planning and the second system is Cartesian space path planning. The proposed planning systems were composed of several separate fuzzy units which individually control each manipulator joint. For the 1st system, the main inputs of the two fuzzy blocks were the current joint position and the difference in joint angle between the goal and the current positions. For the 2nd system the main inputs were the new x-axis error and the current x-axis value of the robot end-effector for the first fuzzy block, and the new y-axis error and the current y-axis value of the robot end-effector for the second fuzzy block. The objectives were to move the arm from the start configuration to the goal configuration. The comparison of the simulation results shows clearly that the results of the second system is better and the robot reached the goal configuration in the two cases successfully with relatively small error in the order of (0.00041775 m in x-axis; and $-2.8386 \times 10^{-005}$ m in y-axis) for the second system and in the order of 0.014087 and $-0.040168$ degree in terms of joint angular positions $\theta_1$ and $\theta_2$ respectively for the first system.

Keywords – Robot Manipulator, Joint Space, Cartesian Space, Path Planning, Fuzzy Logic.
1. Introduction

A robot manipulator is a movable chain of links interconnected by joints. One end is fixed to the ground and a hand or end effector that can move freely in space is attached at the other end. Most robotic manipulators are strong, rigid with powerful motors, strong gearing systems, and very accurate models of the dynamic response. A robot manipulator is a device designed to perform efficiently very complex tasks in a cluttered environment [1-3].

Fuzzy set theory is “a body of concepts and techniques that give a form of mathematical precision to human cognitive processes” [4]. The advantages of using fuzzy logic in robotics are not only fast response, low cost and good real-time ability, but also it is not necessary to know the exact model of the object or process to be controlled [5]. There are several robot motion planning methods such as the methods that use artificial potential field, configuration space and fuzzy logic. The artificial potential field method needs the accurate information of the obstacles so it cannot be applied for moving objects or inaccurate information. The configuration space method maps the obstacles into the C-space. The computation burden is huge. It would fail to meet the real-time requirements for robot motion planning [6-9].

Many researchers have utilized the fuzzy capabilities in control and path planning of robot motion. Fu Yi-li et al showed a robot fuzzy motion planning approach in unknown environments for three-degree industrial robots through simulation [4].


In this paper a comparative analysis between joint space and Cartesian space path planning systems using fuzzy logic approach is proposed for 2-DOF industrial robots operating. The effectiveness of the proposed approach is verified through simulations.

2. Proposed Methods

Two fuzzy planning systems are suggested in this paper. The first system is a joint space path planning using fuzzy logic. This planning system consists of two blocks for two-link robot planar as example. Each fuzzy block (FB) plans the motion of each robot link separately. The first block for the first link produces $\Delta \theta_1(i+1)$ depending on the first input $\Delta \theta_{1g}(i+1)$ the error between the goal value and the current value of $\theta_1$ and on the current value $\theta_1(i)$ which represents the second input to the FB1. Similarly the output of the second fuzzy block is $\Delta \theta_2(i+1)$ depending on the first input $\Delta \theta_{2g}(i+1)$, the error between the goal value and the current value of $\theta_2$ and on the current value $\theta_2(i)$ which represents the second input to the FB2. Figure 1
shows system of the proposed motion planning for a two-link robot.

![Diagram](image)

Figure 1. Fuzzy system for on-line planning of a two-link robot manipulator.

The fuzzy rules are used for every fuzzy block to produce the proper output using the Mamdani method. For example, the rules of fuzzy block 1 for first link will be as:

If (Δθ₁₈ is NS) and (θ₁ is HR₁) then (Δθ₁ is NB);
If (Δθ₁₈ is PS) and (θ₁ is HR₁) then (Δθ₁ is PS).

Using a case study called "Rotradi Robot", robot with a 2.5 ms sample period, the length of 1st link (L₁) = the length of 2nd link (L₂) = 0.45 m, for two-link servo maximum velocity rev/s equals 1 and 2 respectively [13-18]. When applying fuzzy algorithm for the FB1 and FB2 respectively we found that the fuzzy membership function design for every fuzzy block is shown in the Figures (2-3). In these figures the design was made for the case study in the computer modeling.

![Diagram](image)

Figure 2. Fuzzy membership functions FB1 for the first robot link.

![Diagram](image)

Figure 3. Fuzzy membership functions FB2 for the second robot link.

Where:

<table>
<thead>
<tr>
<th>NB</th>
<th>PM</th>
<th>VA</th>
</tr>
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<tbody>
<tr>
<td>Neg. Big</td>
<td>Positive Medium</td>
<td>Vertical Above</td>
</tr>
<tr>
<td>NM</td>
<td>HR₂</td>
<td>HL</td>
</tr>
<tr>
<td>Neg. Medium</td>
<td>Horizontal Right2</td>
<td>Horizontal Left</td>
</tr>
<tr>
<td>NS</td>
<td>PB</td>
<td>VB</td>
</tr>
<tr>
<td>Neg. Small</td>
<td>Positive Big</td>
<td>Vertical Below</td>
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<tr>
<td>PS</td>
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<tr>
<td>Posit. Small</td>
<td>Horizontal Right1</td>
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Figure 4. shows Surface View of FB1 and FB2 respectively.

![Surface View: (a) FB1 and (b) FB2](image)

The second system is a Cartesian space path planning. This system suggests the use of fuzzy logic for solving the problem of Cartesian space path planning. The suggested fuzzy system consists of two blocks for two-link robot planar as example. Each fuzzy block plans the motion of each robot link separately. The FB1 for the 1st link produces: \( \Delta x(i+1) \) which represents the required change in x-axis for planning the robot motion to the next point at \((i+1)\) depending on the new y-axis error \(e_y(i + 1)\) and the current value \(y(i)\) of the robot end-effector position.

A very basic problem in the study of mechanical manipulation is called forward kinematics. This is the static geometrical problem of computing the position and orientation of the end-effector of the manipulator. Specifically, given a set of joint angles the forward kinematic problem is to compute the position and orientation of the tool frame relative to the base frame. Also the inverse kinematics is the determination of all possible and feasible sets of joint variables which would achieve the specified positions and orientations of the manipulator’s end-effector with respect to the base frame [2, 19].

In our suggested planning system, a block of the inverse kinematics (IK) is needed to generate the new robot joint variable \((\theta_1(i+1), \theta_2(i+1))\) generated from the new Cartesian point \((x(i + 1), y(i + 1))\).

\[
D = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}, \quad (1)
\]

\[
\theta_2 = \text{atan2}(D, \pm \sqrt{1 - D^2}), \quad (2)
\]

\[
\theta_1 = \text{atan2}(x, y) - \text{atan2}(L_2 \cos(\theta_2), L_2 \sin(\theta_2)). \quad (3)
\]

Also the block of the forward kinematics (FK) is needed to convert the current joint variables \((\theta_1(i), \theta_2(i))\) to the desired current position of the robot end-effector which represents the main feedback in our suggested planning system.
x = L₁ cos(θ₁) + L₂ cos(θ₁ + θ₂),
y = L₁ sin(θ₁) + L₂ sin(θ₁ + θ₂).  \hspace{1cm} (4)

Where:

θ₁: Joint angles.
θ₂: Link length.
L: Link length.

Figure 5 shows the system of the proposed motion planning for a two-link robot. Fuzzy membership function design for every FB is shown in the Figures (6-7). In these figures the design was made for the case study in the computer modeling.

P: Positive,  Z: Zero,  N: Negative

The rules are used for every fuzzy block to produce the proper output using the k Mamdani method. For example, the rules of fuzzy block 1 for the first link will be as:

If (ex(i + 1)) is N) and (x(i) is NB) then (Δx(i + 1)) is NM);
If (ex(i + 1) is Z) and (x(i) is NM) then (Δx(i + 1)) is Z).

Where:

Figure 8 shows Surface View of FB1 and FB2 respectively.
The most popular methods to calculate the fuzzy intersection (fuzzy-AND) operation according the fuzzy rules are the minimum and product operators. The final output of every fuzzy block can be computed using the center of gravity (COG) defuzzification method over all rules [20].

\[ \tau_j = \frac{\sum_{i=1}^{n} \sigma_{ij} \cdot \tau_{ij}}{\sum_{i=1}^{n} \sigma_{ij}}. \]  

(5)

3. Computer Modeling and Results

Computer modeling and simulation have been done to test the overall system of Figure 1. The length of the two-link robot arm \( L_1 = L_2 = 0.45m \) is used for this model. The robot has to move from the start configuration \( (\theta_1 = 170^0, \theta_2 = 30^0) \) to the goal configuration \( (\theta_1 = 10^0, \theta_2 = 35^0) \). Figure 9 shows the results \( (\Delta \theta_{1g}, \Delta \theta_{2g}, \theta_1, \theta_2, \Delta \theta_1, \Delta \theta_2) \) of the motion planning. The error in reaching the goal after (363) program iterations was:

\[ \Delta \theta_{1g}(i + 1) = 0.014087 \text{ degree} \]
\[ = 2.458645317 \times 10^{-4} \text{ rad}; \]
\[ \Delta \theta_{2g}(i + 1) = -0.040168 \text{ degree} \]
The result of Joint space path planning using fuzzy logic of the two-link robot manipulator is shown in Figure 10.

Computer modeling and simulation has been done to test the overall system of Figure (5). The robot has to move from the start point \( (x = -0.87 \text{ m}, y = -0.07 \text{ m}) \) to the goal point \( (x = 0.76 \text{ m}, y = 0.39 \text{ m}) \). Figure (11) shows the results \((\Delta x, \Delta y, e_x, e_y, \theta_1, \theta_2, x \text{ and } y)\) of the motion planning.

The error in reaching the goal after \((280)\) program iterations was:

\[ e_x(i + 1) = 0.00041775 \text{ m} \quad \text{and} \quad e_y(i + 1) = -2.8386 \times 10^{-005} \text{ m}. \]

Moreover, no oscillatory motion of robot end-effector has been observed or recorded near the goal point.
The result of Cartesian space path planning using fuzzy logic of the two-link robot manipulator is shown in Figure 12.
4. Conclusions

Robot manipulator path planning has been studied using a fuzzy logic technique from the following sides: Joint space path planning and Cartesian space path planning, because of the difference in the planning, according to planning theory, due to presence of difference in sense, logic, and solution.

The suggested joint space planning system using fuzzy logic gives good results since the robot has reached the goal configuration with relatively small error:

\[(0.014087 \text{ degree for } \theta_1 \text{ and } -0.04168 \text{ degree for } \theta_2)\].

One may suggest neural network system to minimize this planning error in a future work study.

The suggested Cartesian space path planning system gave good results in terms of planning error. At the goal point and after (280) program iterations, the Cartesian error reported was:

\[e_x(i + 1) = 0.00041775 \text{ m}\]
\[e_y(i + 1) = -2.8386 \times 10^{-005} \text{ m}\]

Moreover, no oscillatory motion of robot end-effector has been observed or recorded near the goal point.

It has been noticed that the Cartesian space path planning fuzzy system gave better results due to less error of reaching of the goal point, less number of program iterations and no oscillatory motion of robot end-effector observed or recorded near the goal point.

Joint space system was distinguished from Cartesian space system because it is easier and does not need inverse kinematics, while Cartesian space path planning needs inverse kinematics. Furthermore, the difficulty in choosing one of the solutions sometimes results in a problem.

References


