Design of PSO Based Robust Blood Glucose Control in Diabetic Patients

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Abstract – In this paper, the design of robust blood glucose controller in diabetes using H-infinity technique is presented. The Particle Swarm Optimization (PSO) method is used to tune the specific structure controller parameters subject to H-infinity constraints. The Bergman model is used to represent the artificial pancreas. This model is one of the more widely used models of the effect of insulin infusion and glucose inputs on the blood glucose concentration. The results show the effectiveness of the designed controller in controlling the behavior of glucose deviation to a sudden rise in the blood glucose. The proposed controller can effectively attenuate the blood glucose deviation to 0.15. This value of attenuation makes the proposed controller superior to the other controllers in previous works. Matlab 7.11 is used to demonstrate the simulation results.

Keywords – Type I diabetes, PSO, Robust control, H-infinity control.
1. Introduction

Many innate feedback control loops are found in the human body. One of these loops is the loop that regulates the blood glucose by the production of insulin by the pancreas. People with diabetes have a reduced capability of producing insulin. Type I diabetes mellitus patients cannot produce any insulin and must administer insulin shots several times a day to help regulate their blood glucose level. A typical patient is then serving as a control system [1]. On the other hand, any patient that suffers from diabetes and not properly receives the insulin cure can lead to complications such as nerve damage, brain damage, amputation and eventually death. In the human body, the normal blood glucose concentration level varies in a narrow range (70-110) mg/dL. The diabetes is diagnosed if for some reason the human body is unable to control the normal glucose-insulin interaction [2]. The diabetes can be classified into four types: Type I which is known as insulin dependent diabetes mellitus, Type II which is known as insulin independent diabetes mellitus, gestational diabetes and the genetic deflections [3].

The beta cells are responsible for producing the insulin, which is essential for uptake of glucose in muscles and storage in the liver. In diabetes, beta cells are destroyed and the human body will be unable to control the blood glucose level. For this reason, the blood glucose must be regulated by injecting the insulin [4]. Therefore, several approaches have been proposed for the design of closed loop glucose regulation system, which requires three components, which are: glucose sensor, insulin pump and a control algorithm for determining the necessary insulin dosage based on the glucose measurements [2]. Fig. 1 shows the block diagram of closed loop glucose regulation system.

![Figure 1: Closed loop insulin regulation system block diagram](image)

In this paper, a Particle Swarm Optimization (PSO) method is used to tune a specific structure controller subject to robust H-infinity constraints. The Bergman model is used in this work to represent the pancreas biological system. This model is the widely used mathematical model due to its simplicity.

2. Particle Swarm Optimization (PSO) method

In this section an overview of the Particle swarm optimization (PSO) method is presented. It is one of the powerful optimization methods with high efficiency in comparison to other methods. It is inspired from studies of social behaviour among ants and birds. The PSO mechanism is initialised with a population of random solutions and searches for optima by updating generations. The potential solutions of PSO are called “particles”, which fly through the problem space by following the current optimum particles. Each particle represents a candidate solution to the problem at hand. The particles always change their positions by flying around in a multidimensional search space until computational limitations are exceeded. The particles keep track of their coordinates in the problem space, which are associated with the best solution (best fitness) they have achieved so far. The best particle in the population is typically
denoted by the global best, while the local best denotes the best position that has been visited by the current particle. The global best individual connects all members of the population to one another. That is, each particle is influenced by every best performance of any member in the entire population. The (local best) individual is conceptually seen as the ability for particles to remember past personal success. The particle swarm optimization concept involves, at each time step, changing the velocity of each particle towards its global best and local best locations [5], [6], [7].

The velocity of each particle is adjusted according to its own flying experience and the flying experience of other particles. The \( i \)-th particle is represented as \( x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d}) \) in the \( d \)-dimensional space. The best previous position of the particle, \( i \) is recorded and represented as: \( x_i^b = (x_{i,1}^b, x_{i,2}^b, \ldots, x_{i,d}^b) \).

The index of best particle among all of the particles in the group is gbest. The velocity for particle \( i \) is represented as: \( v_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d}) \).

The modified velocity and position of each particle can be calculated using the current velocity and the distance from \( x_{i,d}^b \) to gbest as shown in the following equations [8]-[10]:

\[
\begin{align*}
    v_i^{k+1} &= h \times v_i^k + c_1 \times \text{rand} \times (x_i^b - x_i^k) + c_2 \times \text{rand} \times (x_{gbest}^b - x_i^k) \\
    x_i^{k+1} &= x_i^k + v_i^{k+1}
\end{align*}
\]

(1)

(2)

where \( v_i^k \) is the particle velocity, \( x_i^k \) is the current particle position, \( h \) is the inertia weight factor, \( x_i^b \) is the best previous position of the \( i \)-th particle, \( x_{gbest}^b \) is the best particle among all the particles in the population, \( \text{rand} \) is a random function between 0 and 1, \( c_1 \) and \( c_2 \) are acceleration constants, \( k \) is the pointer of iterations, \( i=1,2,3,\ldots,N \), where \( N \) is the number of particles.

Nowadays, the PSO algorithm has been widely used to solve non-linear and multi-objective problems such as optimisation of weights of neural network, electrical utility, computer games and mobile robots. It is required only a few lines of computer code to realize PSO algorithm. Also, it is characterized as a simple, easy to implement, and computationally efficient algorithm [11].

3. Diabetic Mathematical Model

An adequate model is necessary to design an appropriate control. Several models that describe the insulin-glucose regulatory system and close to physiological process have been appeared for Type I diabetes patients. For example, the most complex diabetic model proved to be the 19-th order Sorensen model [2]. On the other hand, one of the more widely used models of the effect of insulin infusion and glucose inputs on the blood glucose concentration is known as the Bergman’s three state minimal patient model. This model is described by the following differential equations [1], [3]:

\[
\begin{align*}
    \frac{dG}{dt} &= -p_1G - X(G + G_b) + \frac{G_{meal}}{V_i} \\
    \frac{dX}{dt} &= -p_2X + p_3I \\
    \frac{dI}{dt} &= -n(I + I_b) + \frac{U}{V_i}
\end{align*}
\]

(3)

Where \( G \) and \( I \) represent the deviation in blood glucose and insulin concentrations respectively. \( X \) is proportional to the insulin concentration in a remote compartment. \( G_{meal} \) and \( U \) are the meal disturbance input of glucose and the manipulated insulin infusion rate respectively. The parameters \( p_1, p_2, p_3, n \) and \( V_i \) are the blood volume. \( G_b \) is the “basal” base line or steady state.
value of blood glucose and $I_b$ is the insulin concentration.

A linear state space model can be developed for use in control system design as [1]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -p_1 & G_b & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} V_1 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

(4)

where

$x_1 = G_y, x_2 = X, x_3 = I, u = U - U_b, d = G_{meal}$ and

Combining the Laplace transformations of (4) allows the operating point dependant transfer function model of the open loop system to be written as:

$$y(s) = G_p(s)u(s) + G_d(s)d(s)$$

(5)

Where $G_p(s)$ is the transfer function of the system from the deviation in blood glucose to the manipulated insulin infusion $G_d(s)$ is the transfer function from the deviation in blood glucose to the meal disturbance input of glucose.

It is important to refer that the states, input and output variables are defined in deviation form. The set of parameters that are used for the modeled diabetic in (4) are given in Table 1. Since the concentrations are in mmol/liter, and the glucose disturbance has units of grams, the conversion factor of 5.5556 mmol/g must be applied to the $G_{meal}$. Also, it is more common to work with glucose concentration units of mg/diciliter rather than mmol/liter. Since the molecular weight of glucose is 180 g/mol, we must multiply the glucose state (mmol/liter) by 18 to obtain the measured glucose output in (mg/diciliter).

**Table 1: Diabetic model parameters [1].**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_y$(mmol/liter)</td>
<td>4.5</td>
</tr>
<tr>
<td>$I_b$(mU/liter)</td>
<td>4.5</td>
</tr>
<tr>
<td>$V_1$(liter)</td>
<td>12</td>
</tr>
<tr>
<td>$p_1$(min$^{-1}$)</td>
<td>0.000001</td>
</tr>
<tr>
<td>$p_2$(min$^{-1}$)</td>
<td>0.02</td>
</tr>
<tr>
<td>$p_3$(mU/liter)</td>
<td>0.000013</td>
</tr>
<tr>
<td>$n$(min$^{-1}$)</td>
<td>5/54</td>
</tr>
</tbody>
</table>

### 4. Controller Design

Several types of control algorithms for blood glucose regulation have been developed in literature. Some of these algorithms are PID and Fuzzy logic controllers [12], Back stepping based PID controller [3], Signal correction technique [13] and model predictive control [4]. However, in this work the design of specific structure controller is proposed. The parameters of the proposed controller are tuned using PSO method by minimizing the cost function subject to $H_\infty$ constraints to design a robust controller. $H_\infty$ is one of the best known techniques for robust control and it is an effective method for attenuating disturbances and noise that appear in the system [14].

In this work, the robust controller is designed so that $H_\infty$-norm from the input disturbance $d$ ($G_{meal}$) to the outputs ($e_1$ and $e_2$) is minimized. $e_1$ and $e_2$ are the weighted error signal and weighted control signal respectively. Fig. 2 shows the block diagram of the controlled system with weights and the proposed controller $G_c(s)$.

The augmented plant $P$ is derived to be:

$$\begin{bmatrix} e_1 \\ e_2 \\ e \end{bmatrix} = \begin{bmatrix} W_1W_2G_d & -W_2G_p \\ 0 & W_2 \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix} \begin{bmatrix} 0 \\ W_1G_d \end{bmatrix}$$

(6)
where \( u \) is the control signal, \( W_1, W_2 \) and \( W_3 \) are performance weighting function, control weighting function and disturbance filter respectively.

The lower linear fractional transformation of the augmented plant and controller can be described by:

\[
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2
\end{bmatrix} = F(P, G_c) = \begin{bmatrix}
W_1W_3G_pS \\
W_2W_3G_pG_cS
\end{bmatrix} [d]
\]  

(7)

where \( S = (1 + G_p(s)G_c(s))^{-1} \) is the sensitivity function.

The PSO method is used to tune the parameters of the controller \( G_c(s) \) and the parameters of the selected performance weighting function \( W_1 \) by minimizing the cost function in (8).

On the other hand, the general formulation for the controller structure is described by:

\[
G_c(s) = \frac{a_0s^m + a_{m-1}s^{m-1} + \ldots + a_0}{b_0s^m + b_{m-1}s^{m-1} + \ldots + b_0}
\]  

(9)

where \( m \) represents the selected controller order. In this work, the selected order is six order which is a very large search domain for the required robust controller. The obtained optimal controller using PSO from the search space by minimizing the cost function in (8) is:

\[
G_c(s) = \frac{-0.0008267s^3 - 0.1673s^2 - 84.39s^2 - 12.66s - 0.4537}{22.18s^3 + 141.6s^2 + 210.1s^2 + 123.6s + 28.99}
\]  

(10)

The selected structure for the performance weighting function is a 2\textsuperscript{nd} order structure and the obtained performance weighting function using PSO algorithm is:

\[
W_1(s) = \frac{0.000385s^2 + 0.3267s + 770}{0.077s^3 + 5.55s + 10000}
\]  

(11)

The control weighting function and disturbance filter are selected by trial and error to be:

\[
W_2(s) = 0.01 \quad \text{and} \quad W_3(s) = \frac{0.001}{0.4s + 1}
\]  

(12)

The parameters that have been used for carrying out the design of robust controller using PSO are: population size=10, inertia weight factor \( h=2 \), \( c_1 = c_2 = 2 \), the maximum iteration=100 and the number of function evaluations is 1000. The flowchart of PSO based robust controller design is shown in Fig. 3.

5. Simulation Results and Discussion

The PSO based robust controller was designed to regulate the blood glucose level in Type I diabetes. Fig. 4 shows the
meal disturbance function, which represents the exogenous glucose infusion. This function has zero initial conditions, easily implemented and physiologically accurate function [15]. Fig. 5 shows the resulting frequency characteristics of the sensitivity function $S$ compared with the inverse of the performance weighting function. It is shown that the sensitivity function has magnitude less than the magnitude of the inverse of weighting function for all frequencies. This means that the robust performance has been achieved using the designed controller. The frequency characteristics of the resulting controller can be shown in Fig. 6. It is clear that the resulting controller has low magnitude, which satisfies the requirements of low control effort. Fig. 7 illustrates the insulin infusion rate. The blood glucose deviation response is shown in Fig. 8. It is shown that the proposed controller effectively improved the blood glucose response where a maximum peak of 0.15 has been obtained. Finally, to show the effectiveness of the proposed controller, the obtained results have been compared to those achieved by previous works that used different control approaches as shown in Table 2.

6. Conclusions

In this paper, the design of robust controller for regulating the blood glucose deviation for Type I diabetes patients. The Bergman’s model of the effect of insulin infusion and glucose inputs on the blood glucose concentration was used as an artificial pancreas. The Particle Swarm Optimization (PSO) method was used for obtaining the optimal parameters of the proposed controller by minimizing a cost function subject to $H_{in}$ constraints. It was shown that the PSO method is a powerful optimization method for solving multi-objective problems with only a few lines of computer code, easy to implement and computationally efficient method. On the other hand, it can be concluded that the proposed controller effectively regulate the blood glucose with a maximum peak of 0.15 which is a more desirable attenuation for glucose deviation. Finally, the superiority of the proposed controller to the controllers that have been proposed in the previous works was made very clear in this work.

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**Figure 3: Flowchart of PSO based robust controller design.**
Figure 4: Disturbance meal (exogenous glucose infusion) function.

Figure 5: The resulting sensitivity function (solid) compared to the inverse performance weighting function (dotted).
Figure 6: Frequency characteristics of the resulting robust controller.

Figure 7: The insulin infusion rate.

Figure 8: Controlled blood glucose deviation.
References


