Nonlinear Dynamical Fuzzy Control Systems Design with Matching Conditions

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Abstract.
In this paper, a new approach of fuzzy control for continuous nonlinear dynamical systems is developed based on the framework of Takagi-Sugeno fuzzy model and a common controller for all the local models generated by fuzzifying the nonlinear dynamical system.

Some theorems that give sufficient conditions for checking the simultaneous stabilization of fuzzy control systems via a common quadratic Lyapunov function have been presented and developed; their theoretical aspects have also been proved and discussed.

Design algorithms, illustrative examples and graphs have been presented to show effectiveness of the approach.

Introduction.
In many physical and engineering systems, engineers are hindered by strong nonlinearity from successful application of linear control theory. In the past few decades, as the interest in fuzzy systems has increased, researchers have considered the stability analysis of these fuzzy using a variety of modeling and control frameworks. One of the frameworks that have attracted a great deal of attention is the so called fuzzy Takagi-Sugeno (T-S) model originating from fuzzy identification (T. Takagi and M. Sugeno 1985).

The popularity of the T-S model arises not only from its simplicity, but also from the idea that local dynamics of a nonlinear plant can be represented by different fuzzy rules (linear Models) in T-S model. Recent results have also shown that T-S model can be a universal approximator of any smooth nonlinear dynamic systems (C. Fantuzzi and R. Ruvatti 1996, H. Ying 1998, J. Li, H. O. Wang, D. Niemann and K. Tanaka 2000). These results differ from the commonly held view that a T-S model has only limited capability in representing nonlinear system. Moreover, it has been found out that many nonlinear systems can be represented exactly through sectorization by T-S models with only a few rules. From the point of view of system analysis, T-S model are appealing as well since the stability and performance characteristics of the system can be analyzed using a Lyapunov approach (K. Tanaka and M Sugeno 1992, K. Tanaka and M. Sano 1993, S. G. Cao and N. W. Rees 1995, S. G. Cao, N. W. Rees and G. Feng 1996, G. Kang, J. Zhao, V. Wertz and R. Gorez 1996, W. Lee and M. Sugeno 1998).

In this paper a theorem labeled theorem (1) has been developed, which describes a fuzzy controller that simultaneously quadratically stabilize the multi-input systems whose $i^{th}$ local model is $(A_i, B)$ via a common Lyapunov function $V(x) = x^T W^{-1} x$.

Also we discussed the simultaneous quadratic stabilizability of the uncertain multi-input fuzzy dynamical systems whose $i^{th}$ local model $(A + A_i, B + B_i)$, for that a theorem labeled theorem (2) has been developed as an extension to theorem (1), which describes a fuzzy controller that simultaneously quadratically stabilize the above uncertain multi-input fuzzy dynamical systems via a common Lyapunov function $V(x) = x^T T^T W^{-1} T x$, using T-S fuzzy model and suitable coordinates transformation.

Afterwards a step by step design algorithm has been suggested. A practical example and illustration graphs for the local systems are presented.
**Stability Analysis of Takagi-Sugeno Fuzzy Systems:**

In this section we consider the nonlinear analysis of fuzzy control systems where the plant and controller are Takagi-Sugeno fuzzy systems. The main feature of the Takagi-Sugeno fuzzy model (T-S) is the expression of each dynamic by a fuzzy implication (rule). The overall fuzzy model of the system is achieved by fuzzy interpolation of these linear systems models. We shall be concerned in the study of the continuous case of The (T-S) fuzzy model which has following form, [Jing, 2000]:

\begin{align}
\text{Rule } i: & \text{ If } x_1(t) \text{ is } F_j^i \text{ and } x_2(t) \text{ is } F_2^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } F_p^i \\
\text{Then } & \dot{x}(t) = A_i x(t) + B_i u(t) \tag{1.1}
\end{align}

where \( x^T(t) = [x_1(t), x_2(t), \ldots, x_n(t)] \), \( u^T(t) = [u_1(t), u_2(t), \ldots, u_m(t)] \), \( i = 1, 2, \ldots, r \) and \( r \) is the number of the If-Then rules, \( x_1(t) \) are some fuzzy variables, \( F_j^i \) are fuzzy sets, \( \dot{x}(t) = A_i x(t) + B_i u(t) \) from the \( i^{th} \) If-Then rule "(\( A_i, B_i \)) is called the \( i^{th} \) local model".

Using Singleton fuzzification, product inference and Center-of-Gravity Defuzzification method, the expression of the (T-S) fuzzy model (1.1) takes the final state (1.2) as follows:

\begin{align}
\dot{x}(t) &= \sum_{i=1}^{r} h_i(x) (A_i x(t) + B_i u(t)) \\
&= \sum_{i=1}^{r} h_i(x) \
\end{align}

where \( h_i(x(t)) = \Phi_i(t) / \sum_{j=1}^{r} \Phi_j(t) \), \( \Phi_i(t) = \prod_{j=1}^{p_i} F_j^i(x_j(t)) \), and \( \Pi \) stands for the product operation.

**Lemma (1) "Finsler's Lemma":**

If \( x^T A x > 0 \), whenever \( x^T B x = 0 \) where \( A \) and \( B \) are symmetric constant matrices, then there exists a real scalar \( \delta > 0 \) such that \( A + \delta BB^T > 0 \), [Jacobson, 1977].

**Definition (1)**

Suppose that there exists a matrix \( P > 0 \) which satisfies \( PA_i + A_i^T P < 0 \); \( i = 1, 2, \ldots, r \). Then the quadratic function \( V(x) = x^T P x \) is called a common Lyapunov function for the family of asymptotically stable linear systems \( \dot{x} = A_i x \), \( i = 1, 2, \ldots, r \), [Daniel, 2003].

**Definition (2)**

The linear system \( \dot{x}(t) = A x(t) + B u(t) \) where \( A \in \mathbb{R}^{n \times n}, \ B \in \mathbb{R}^{n \times m}, \ x \in \mathbb{R}^n \) is the state, and \( u(t) \in \mathbb{R}^m \) is the control input, is said to be quadratically stabilizable if there exists a continuous feedback control \( u(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^m \) with \( u(0) = 0 \), \( n \times n \) positive definite symmetric matrix \( P \), and a constant \( \alpha > 0 \) such that the following condition holds: the Lyapunov derivative corresponding to the closed-loop system \( \dot{x} = A x + B u(t) \) satisfies the inequality:

\begin{align}
\dot{V}(x) &= x^T [A^T P + P A] x + 2 x^T P B u \leq -\alpha \| x \|^2 \
&= \| x \|^2 \tag{1.3}
\end{align}

for all nonzero state \( x \in \mathbb{R}^n \) and \( t \geq 0 \). In this inequality, \( \| \cdot \| \) standard Euclidian norm. Then it admits a Lyapunov function \( V(x) = x^T P x \), [Jan, 1988].
Definition (3)
Consider the family of multi-input systems \((A_i, B), i=1,2,\ldots, r\), described by the state equation \(\dot{x} = A_i x + B u\) where \(A_i \in \mathbb{R}^{n \times n}\) and \(B \in \mathbb{R}^{n \times m}\), \(x \in \mathbb{R}^n\) is the state variables, \(u \in \mathbb{R}^m\) is the control. The set of systems is simultaneously quadratically stabilizable if there exist a matrices \(W = W^T > 0\) and \(K\) such that:
\[
x^T [(A_i - BK)^T W^{-1} + W^{-1} (A_i - BK)] x < 0
\]
for every \(x \in \mathbb{R}^n, i=1,2,\ldots, r\) holds [Dasgupta, 1998].

Remark (1):
If the condition of definition (3) is satisfied, then there exists a common quadratic Lyapunov function \(V(x) = x^T W^{-1} x\), and a single state feedback \(u = -K x\) such that the derivative of \(V(x)\) along the trajectory of each system is negative definite [Peres, 1989].

Lemma (2):
A family of plants \(\dot{x} = A_i x + B u\) is simultaneously quadratically stabilizable iff \(\exists W = W^T > 0\) such that for each \(i\), \(x^T (A_i W + WA_i^T) x < 0, i=1,2,\ldots, r\) holds [Peres, 1989].

Remark (2):
If lemma (2) holds, then \(V(x) = x^T W^{-1} x\) is a quadratic Lyapunov function for the closed-loop systems with control law \(u = -k x = -\frac{\gamma}{2} B^T W^{-1} x\) and there exist a set of scalars \(\{\gamma_i\}\) for a given matrix \(W\) such that \(A_i W + W A_i^T - \gamma_i B B^T < 0, \forall i, \gamma \geq \max \{\gamma_i\}\) [El-Tahir, 2001].

(2) Block controllable companion form:
A linear time-invariant system can be described by state equations in general coordinates as follows:
\[
\dot{x} = A x + B u
\]
where \(x \in \mathbb{R}^n, u \in \mathbb{R}^m, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}\). If the rank of the block controllability test matrix:
\[
\rho(A, B) = [B; AB; A^2 B; \cdots; A^{n-1} B]
\]
is \(n\), then the system in (1.5) is completely block controllable and can be transformed into block controllable companion form
\[
\dot{z} = A_c z + B_c u
\]
where
\[
A_c = T A T^{-1} = \begin{bmatrix}
0_m & I_m & 0_m & \cdots & 0_m & 0_m \\
0_m & 0_m & I_m & \cdots & 0_m & 0_m \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_m & 0_m & 0_m & \cdots & 0_m & I_m \\
-A_n & -A_{n-1} & -A_{n-2} & \cdots & -A_2 & -A_1
\end{bmatrix}
\]
\[
B_c = T B = \begin{bmatrix}
0_m & 0_m & \cdots & 0_m & I_m
\end{bmatrix}^T
\]
where $I_m$ is the identity matrix of order $m$ and $z = Tx$ provided that the matrix $T$ is nonsingular matrix. The similarity transformation matrix $T$ is given by:

$$T = [\sigma : \sigma A : \sigma A^2 : \cdots : \sigma A^{n-1}]^T$$

(1.10)

where $\sigma = B_c^T \rho^{-1}(A, B)$ [Leang, 1983].

**Lemma (3):**

Consider the nominal systems $\dot{x} = A_i x + Bu$ with the pair $(A_i, B)$ are in the block companion form. Where $A_i$ and $B$ have the structure:

$$A_i = 
\begin{bmatrix}
0_m & I_m & 0_m & \cdots & 0_m & 0_m \\
0_m & 0_m & I_m & \cdots & 0_m & 0_m \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_m & 0_m & 0_m & \cdots & 0_m & I_m \\
-\alpha_i & -\alpha_i & 0_m & \cdots & -\alpha_i & -\alpha_i \\
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22} \\
\end{bmatrix},
B = 
\begin{bmatrix}
0_m \\
I_m \\
\end{bmatrix}
$$

where $A_{11} \in \mathbb{R}^{n-m \times m \times n-m}$, $A_{12} \in \mathbb{R}^{n-m \times m}$, $A_{21} \in \mathbb{R}^{m \times n-m}$, $A_{22} \in \mathbb{R}^{m \times m}$ and $B \in \mathbb{R}^{n \times m}$. With control law $u = -\frac{\gamma}{2} B^T W^{-1} x$, then $\exists W = W^T > 0$ which guarantee the simultaneous quadratic stabilizability of the nominal systems $\dot{x} = A_i x + Bu$ for each $i$. (i.e. $V^T (A_i W + W A_i^T) V < 0$, $\forall i \in [1, 2, \ldots, r]$ is presented) [Peres, 1989], [El-Tahir, 2001].

**Theorem (1):**

Consider the nonlinear dynamical model $\dot{x}(t) = f(x(t)) + g(x(t))u(t)$ where $x(t)$ is the state vector, $u(t)$ is the control input vector, $i = 1, 2, \ldots, r$.

Using singleton fuzzification to have the (T-S) fuzzy model:

Rule $i$: If $x_1(t)$ is $F_1^i$ and $x_2(t)$ is $F_2^i$ and $\cdots$ $x_n(t)$ is $F_p^i$

Then $\dot{x}(t) = A_i x(t) + Bu(t)$

Using product inference and Center-of-Gravity defuzzification method we have the following model:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x)(A_i x(t) + Bu(t))$$

with the controllable pair $(A_i, B)$ as the $i^{th}$ local model, $A_i$ is in block controllable companion form for $i = 1, 2, \ldots, r$. And linear state feedback $u = -\frac{\gamma}{2} B^T W^{-1} x$ where $\gamma > 0$ is sufficiently large positive scalar and the matrix $W = W^T > 0$.

Assume that $\sum_{i=1}^{r} h_i(x) = 1$, $h_i(x) \geq 0$, $i = 1, 2, \ldots, r$ then the model (1.11) is simultaneously quadratically stabilizable via a common Lyapunov function $V(x) = x^T W^{-1} x$ such that $A_i W + W A_i^T < 0$, $\forall i$. 

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Proof:
Consider the common quadratic Lyapunov function \( V(x) = x^T W^{-1} x \), the Lyapunov function derivative along the trajectories of (1.11) yields:
\[
\dot{V}(x) = x^T W^{-1} \left[ \sum_{i=1}^{r} h_i(x) \left( A_i x + B u \right) \right] + \left[ \sum_{i=1}^{r} h_i(x) \left( A_i x + B u \right) \right]^T W^{-1} x
\]
(1.12)
and since linear state feedback control is suggested to have the form \( u = -\frac{\gamma}{2} B^T W^{-1} x \) for arbitrary positive constant \( \gamma \), then equation (1.12) becomes:
\[
\dot{V}(x) = x^T \left[ \sum_{i=1}^{r} h_i(x) \left[ W^{-1} \left( A_i - \frac{\gamma}{2} BB^T W^{-1} \right) + \left( A_i - \frac{\gamma}{2} BB^T W^{-1} \right)^T W^{-1} \right] \right] x
\]
(1.13)
Since \( W \) is Symmetric matrix then \( (W^{-1})^T = (W^T)^{-1} = W^{-1} \) and (1.13) becomes:
\[
\dot{V}(x) = x^T \left[ \sum_{i=1}^{r} h_i(x) \left[ W^{-1} \left( A_i W + W A_i^T - \gamma BB^T \right) W^{-1} \right] \right] x
\]
Since by assumption \( \sum_{i=1}^{r} h_i(x) = 1 \), then \( \dot{V}(x) = x^T \left[ \sum_{i=1}^{r} h_i(x) \left( W^{-1} \left( A_i W + W A_i^T - \gamma BB^T \right) W^{-1} \right) \right] x \), and we have \( A_i W + W A_i^T - \gamma BB^T < 0 \), \( \forall i \), then by remark (2), one can always find \( \gamma \geq 0 \) such that \( A_i W + W A_i^T - \gamma BB^T < 0 \). Let \( A_i W + W A_i^T - \gamma BB^T = -Q \), then:
\[
\dot{V}(x) = x^T \left( \sum_{i=1}^{r} h_i(x) W^{-1} (-Q) W^{-1} \right) x \leq -x^T M x \leq 0
\]
where \( M = W^{-1} (-Q) W^{-1} \).
Which completes the proof of the theorem.

(3) Design Algorithm:
The following is a suggested algorithm for the fuzzy system (1.11) using theorem (1) has been discussed in order to find such a stabilizing controller. This algorithm is divided into two categories; the first is to compute the linear state feedback stabilizing controller. While the second is to compute the common Lyapunov function that will stabilize the fuzzy model:

1. Approximating the nonlinear model by linear local fuzzy models (In our work we shall use singleton fuzzification):
   - Rule i: If \( x_1(t) \) is \( F^i_1 \) and \( x_2(t) \) is \( F^i_2 \) and \( \cdots \) \( x_n(t) \) is \( F^i_p \)
   - Then \( \dot{x}(t) = A_i x(t) + B u(t) \)
   \[
   \text{Rule i: If } x_1(t) \text{ is } F^i_1 \text{ and } x_2(t) \text{ is } F^i_2 \text{ and } \cdots x_n(t) \text{ is } F^i_p
   \]
   (1.14)
   where \( i = 1, 2, \ldots, r \), \( (A_i, B) \) is the \( i \)th local model, \( A_i \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), and the membership functions of the fuzzy sets \( F^i_j, i = 1, 2, \ldots, r \), are defined suitably according to the decision maker depending on the system.
2. Check the controllability of the pairs \((A_i, B)\) and whether \(A_i\) is in the companion form \(\forall i\), if not one can always transform it (see section 2). And let it be partitioned according to lemma (3).

3. Using Center-Of-Gravity Defuzzification method, we connect all local models produced by fuzzification; into one global model (see section 1).

4. Calculate the matrix \(M\) such that the Matrix \(A_{11} + A_{12}M\) is stable. We shall suggest using the Pole-Placement method to calculate \(M\).

5. Calculate the positive definite symmetric matrix \(W\) defined by:

\[
W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix}
\]

where \(W_1 \in \mathbb{R}^{n-m \times n-m}\) is \(W_1 = W_1^\top > 0\) computed by solving the Lyapunov equation:

\[
(A_{11} + A_{12}M)W_1 + W_1(A_{11} + A_{12}M)^\top = Q_o
\]

and \(Q_o \in \mathbb{R}^{n-m \times n-m}\) is any negative definite symmetric matrix, \(W_2 \in \mathbb{R}^{n-m \times m}\) is computed from \(W_2 = (MW_1)^\top\), and \(W_3 \in \mathbb{R}^{m \times m}\) is computed from \(W_3 = N + W_2^\top W_1^{-1} W_2\) where \(N\) is any positive definite symmetric matrix.

6. Calculate the scalar \(\gamma\) which satisfies \(\gamma \geq \max(\gamma_i)\). Define

\[
Q^i = A_i W + W A_i^\top - \gamma_i B B^\top = \begin{bmatrix} Q_o & q_i \\ q_i^\top & q_{2i} - \gamma_i I_m \end{bmatrix}
\]

Where \(Q_o\) is any negative definite matrix (defined in (1.16)), \(\gamma_i\) are calculated so as to satisfy \(q_{2i} - \gamma_i I_m - q_i^\top Q_o^{-1} q_i < 0\), and \(q_i, q_{2i}\) are calculated using (1.18), (1.19):

\[
q_i = A_{11} W_2 + A_{12} W_3 + W_1(A_{21}^\top)^\top + W_2 A_{22}^i
\]

\[
q_{2i} = A_{21}^i W_2 + A_{22}^i W_3 + W_2^\top (A_{21}^i)^\top + W_3 (A_{22}^i)^\top
\]

7. Calculate the linear state feedback \(u = -\frac{\gamma}{2} B^\top W^{-1} x\) and substitute it in original model and in the global model in (equation 1.11).

8. Calculate the common Lyapunov function \(V(x) = x^\top W^{-1} x\).

**Example (2.1):** Consider the following nonlinear dynamical model,

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= 17.294117 \sin(x_3) - 0.0884 x_2^2 \sin(2x_3) - 0.17647 u
\end{align*}
\]

1. Notice that the sine function on the two operating points of the systems has the property that

\[
\sin(x) \equiv x \quad \text{when} \quad x \equiv 0
\]

\[
\sin(x) = \frac{2}{\pi} x \equiv 1 \quad \text{when} \quad x = \frac{\pi}{2}
\]
Then the state equation of the nonlinear model is simplified by the following linear systems. From (1) we have:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{pmatrix}
= \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 17.294117 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
-0.17647
\end{pmatrix}u
\]

(3)

From (2) we have:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{pmatrix}
= \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 17.294117 \frac{2}{\pi} & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
-0.17647
\end{pmatrix}u
\]

(4)

Now, from (3) and (4), we have the following linear local fuzzy models:

Rule 1: if \((x_3)\) is about \(0\) then \(\dot{x} = A_1 x + Bu\)

Rule 2: if \((x_3)\) is about \(\pi/2\) then \(\dot{x} = A_2 x + Bu\)

where the membership functions for the fuzzy sets \(F_1^1 = (x_3)\) is about \(0\), \(F_1^2 = (x_3)\) is about \(\pi/2\), are chosen respectively, \(F_1^1(x_3(t)) = \exp\left(-\frac{\pi}{2} x_3^2(t)\right)\), \(F_1^2(x_3(t)) = 1 - \exp\left(-\frac{\pi}{2} x_3^2(t)\right)\).

And:

\[
A_1 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 17.294117 & 0
\end{pmatrix}, A_2 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 11.005347 & 0
\end{pmatrix}, B = \begin{pmatrix}
0 \\
0 \\
0 \\
-0.17647
\end{pmatrix}.
\]

2. Using the condition in (1.6), then the pairs \((A_1, B)\) and \((A_2, B)\) are controllable. Comparing with equation (1.8), one can see that \(A_1\) and \(A_2\) are in the block companion form. So the Partitioning will be:

\[
A_{11} = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}, A_{12} = \begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}, A_{21} = \begin{pmatrix}
0 & 0 & 17.294117 \\
0 & 0 & 11.005347 \\
0 & 0 & 0
\end{pmatrix}, A_{22} = (0).
\]

3. Defuzzification to global model:

\[
\dot{x} = \frac{\sum h_1(t)(A_1 x + Bu)}{\sum h_1(t)}, M_1(t) = F_1^1(x_3(t)) = \exp\left(-\frac{\pi}{2} x_3^2(t)\right), h_1(x_3(t)) = \exp\left(-\frac{\pi}{2} x_3^2(t)\right),
\]

\[
M_2(t) = F_1^2(x_3(t)) = 1 - \exp\left(-\frac{\pi}{2} x_3^2(t)\right), h_2(x_3(t)) = 1 - \exp\left(-\frac{\pi}{2} x_3^2(t)\right),
\]

then we have the following dynamical system:

\[
\dot{x} = \exp\left(-\frac{\pi}{2} x_3^2(t)\right)(A_1 x + Bu) + \left[1 - \exp\left(-\frac{\pi}{2} x_3^2(t)\right)\right](A_2 x + Bu).
\]

4. Calculating the matrix \(M\) (Poles-Placement): \(M = (-24 \ -26 \ -9)\).
5. First we shall proceed in the procedure of finding the matrix $W$ where $W_1 \in \mathbb{R}^{3 \times 3}$, $W_2 \in \mathbb{R}^{3 \times 1}$, $W_3 \in \mathbb{R}^{1 \times 1}$. $Q_o$ is chosen to be:

$$Q_o = \begin{pmatrix}
-0.001 & 0.003 & 0 \\
0.003 & -0.01 & 0.001 \\
0 & 0.001 & -0.004
\end{pmatrix},$$

hence $W_1 = \begin{pmatrix}
0.0002155 & -0.0005 & 0.0003143 \\
-0.0005 & 0.0026857 & -0.005 \\
0.0003143 & -0.005 & 0.0138286
\end{pmatrix}$, and $W_2 = \begin{pmatrix}
0.0002155 & -0.0005 & 0.0003143 & 0.0049993 \\
-0.0005 & 0.0026857 & -0.005 & -0.0128282 \\
0.0003143 & -0.005 & 0.0138286 & -0.0020006 \\
0.0049993 & -0.0128282 & -0.0020006 & 1.2315554
\end{pmatrix}$, chose $N = 1$ then $W_3 = (1.2315554)$, Finally,

$$W = \begin{pmatrix}
0.0002155 & -0.0005 & 0.0003143 & 0.0049993 \\
-0.0005 & 0.0026857 & -0.005 & -0.0128282 \\
0.0003143 & -0.005 & 0.0138286 & -0.0020006 \\
0.0049993 & -0.0128282 & -0.0020006 & 1.2315554
\end{pmatrix}$$

6. The scalar $\gamma$, for $A_1$:

$$q_1 = \begin{pmatrix}
-0.00739 \\
-0.08847 \\
1.47071
\end{pmatrix}, \quad q_{21} = (-0.0692), \quad \gamma_1 = 628.81612$$

For $A_2$:

$$q_2 = \begin{pmatrix}
-0.00937 \\
-0.05703 \\
1.38374
\end{pmatrix}, \quad q_{22} = (-0.04403), \quad \gamma_2 = 569.42011$$

hence $\gamma = 628.81612$.

7. The calculation of the linear state feedback $u = -\frac{\gamma}{2} B^T W^{-1} x$:

$$u = (1331.6061684 \quad 1442.5733491 \quad 499.3523131 \quad 55.4835903)x.$$
Graph (1)
The state-space components versus time of the first local system with initial condition $(0, 0, \frac{7\pi}{36}, 0)$.

Graph (2)
The state-space components versus time of the second local system with initial condition $(0, 0, \frac{16\pi}{36}, 0)$. 
Graph (3)
(a) The controller at the 1st local system. (b) The controller at the 2nd local system. (c) The common Lyapunov function at the 1st local system. (d) The common Lyapunov function at the 2nd local system.

Graph (4)
The membership function of the fuzzy inferred set ($x_3$ is about 0) versus $x_3$ and the time.

Graph (5)
The membership function of the fuzzy inferred set ($x_3$ is about $\pi$/2) versus $x_3$ and the time.
(4) Matching Conditions:

The matching conditions are preconditions, which constrain the manner in which the uncertainty is permitted to enter into the dynamics. The satisfaction of matching conditions is sufficient for stabilizability. Consider the systems of the following form \( \dot{x} = (A + A_i)x(t) + (B + B_i)u(t) \), now \( A_i, B_i \) are said to satisfy matching conditions if there exists matrices \( E_i, D_i \) such that \( A_i = BE_i, B_i = BD_i \) where \( i \in I = \{1, 2, \ldots, r\} \) [Petersen, 1987].

Lemma (4):

If \( A \) is an \( n \times n \) positive definite nonsingular real symmetric matrix, then there exist a nonsingular matrix \( S \) such that \( A = SS^T \) [Ogata, 1967].

The following theorem has been developed to design a suitable controller for some nonlinear fuzzy dynamical systems.

Theorem (2):

Consider the nonlinear dynamical model:
\[
\dot{x}(t) = f(x(t)) + g(x(t))u(t)
\]
where \( x(t) \) is the state vector, \( u(t) \) is the control input vector. Using singleton fuzzification to have the (T-S) fuzzy model:

Rule \( i \): If \( x_1(t) \) is \( F_1^i \) and \( x_2(t) \) is \( F_2^i \) and \( \cdots \) \( x_n(t) \) is \( F_p^i \)

Then \( \dot{x}(t) = (A + A_i)x(t) + (B + B_i)u(t) \)

(1.21)

where \( i = 1, 2, \ldots, r \) is the number of the rules, \( j = 1, 2, \ldots, p \) is the number of the inferred fuzzy sets. Let \( F_j^i(x_j(t)) \) be the membership function of the inferred fuzzy sets \( F_j^i \) and \( h_i(x) \) to be find as shown in section (1). Using product inference and Center-of-Gravity defuzzification method on (1.21), we have the following model:

\[
\dot{x}(t) = \frac{\sum_{i=1}^{r} h_i(x)((A + A_i)x(t) + (B + B_i)u(t))}{\sum_{i=1}^{r} h_i(x)}
\]

(1.22)

with controllable nominal system \((A, B)\) and \((A + A_i, B + B_i)\) as the controllable \( i \)th local model, \( i = 1, 2, \ldots, r \). Let \( A_i = BE_i, B_i = BD_i \) \( \forall i \in I \) and \( D_i > 0 \) be the matching conditions. Define linear state feedback \( u(t) = u_s(t) + u_1(t) = (K_o + K_1)x(t) \) where \( u_s(t) \) is selected such that the nominal model \( \dot{x}(t) = Ax(t) + Bu(t) \) is asymptotically stable and \( u_1(t) = K_1x(t) = -\frac{\gamma}{2}(TB)^T W^{-1}Tx(t) \) where \( \gamma > 0 \) is sufficiently large positive scalar, \( T \) is any transformation matrix selected as described in section (2), and \( W = W^T > 0 \) such that \( \bar{M}_i W + WM_i^T < 0 \) \( \forall i \in I \) where \( \bar{A} = A + BK_o \) and \( \bar{M}_i = T\bar{A}\bar{T}^{-1} + T[A_i + B_iK_o]T^{-1} \). Assume that \( \sum_{i=1}^{r} h_i(x) = 1, h_i(x) \geq 0 \) then the model (1.22) is simultaneously quadratically stabilizable via a common Lyapunov function \( V(x) = z^TW^{-1}z \).
Proof:
Define the coordinates transformation for the model (1.22) to be \( z(t) = T x(t) \) and substitute it in model (1.22) yields:
\[
\dot{z}(t) = (T x(t)) = \dot{T} x(t) = T \sum_{i=1}^{r} h_i(x(t)) \left[ (A + A_i) x(t) + (B + B_i) u(t) \right] / \sum_{i=1}^{r} h_i(x(t)) \tag{1.23}
\]
Now, consider the linear state feedback \( u(t) = (K_o + K_1) x(t) \) then (1.23) becomes:
\[
\dot{z}(t) = T \sum_{i=1}^{r} h_i(x(t)) \left[ (A + A_i) x(t) + (B + B_i)(K_o + K_1) x(t) \right] / \sum_{i=1}^{r} h_i(x(t)) \tag{1.24}
\]
Since \( x(t) = T^{-1} z(t) \) and the matching conditions \( A_i = B E_i, \ B_i = B D_i, \ \forall i \in I \), then:
\[
\dot{z}(t) = \sum_{i=1}^{r} h_i(z(t)) \left[ T A \dot{A} T^{-1} z + T(A_i + B_i K_o) T^{-1} z + T B K_i T^{-1} z + T B D_i K_i T^{-1} z \right] / \sum_{i=1}^{r} h_i(z(t)) \tag{1.25}
\]
Let \( \bar{F} = T \bar{A} T^{-1} \) and let it be partitioned as follows:
\[
\bar{F} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}
\]
where \( F_{11} \in \mathbb{R}^{n \times n}, \ F_{12} \in \mathbb{R}^{n \times m}, \ F_{21} \in \mathbb{R}^{m \times n}, \ F_{22} \in \mathbb{R}^{m \times m} \), Let \( F^i = T (A_i + B_i K_o) T^{-1} \), and define \( \bar{M}_i = \left( \bar{F} + F^i \right) \) and let it be partitioned as follows:
\[
\bar{M}_i = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} + F_{21}^i & F_{22} + F_{22}^i \end{bmatrix},
\]
Let \( G = T B, \ G_i = T B D_i \) and \( u^*(t) = K_i T^{-1} z \) then:
\[
(G + G_i) u^*(t) = (T B + T B D_i) K_i T^{-1} z = T B (I_m + D_i) K_i T^{-1} z = T B C_i K_i T^{-1} z,
\]
where \( C_i = I_m + D_i \), since \( D_i > 0, \ I_m > 0 \) implies \( I_m + D_i > 0 \equiv C_i > 0 \), then (1.24) will become:
\[
\dot{z}(t) = \sum_{i=1}^{r} h_i(z(t)) \left[ \bar{M}_i z(t) + G C_i u^*(t) \right] / \sum_{i=1}^{r} h_i(z(t)) \tag{1.25}
\]
Now, consider the Lyapunov function \( v(z(t)) = z^T W^{-1} z \) its differentiation yields:
\[
\dot{v}(z(t)) = z^T W^{-1} \dot{z} + \dot{z}^T W^{-1} z
\]
\[
= z^T W^{-1} \left[ \sum_{i=1}^{r} h_i(z) \left( \bar{M}_i z + G C_i U^* \right) \right] + \left[ \sum_{i=1}^{r} h_i(z) \left( \bar{M}_i z + G C_i U^* \right) \right]^T \times \frac{\sum_{i=1}^{r} h_i(z)}{\sum_{i=1}^{r} h_i(z)} \times W^{-1} z \tag{1.26}
\]
Since the linear state feedback is \( u^* = -\frac{\gamma}{2} G^T W^{-1} z \) where \( \gamma > 0 \) is sufficiently large positive scalar, then (1.26) becomes:
\[
\dot{v} = z^T \left[ \sum_{i=1}^{r} h_i(z) \left( W^{-1} \left( \bar{M}_i - \gamma G_i G^T W^{-1} \right) + \left( \bar{M}_i - \gamma G_i G^T W^{-1} \right)^T W^{-1} \right) \right] z.
\]

Since \( W^{-1} = W^{-1} \) “Symmetric”, then:

\[
\dot{v} = z^T \left[ \sum_{i=1}^{r} h_i(z) \left( W^{-1} \left( \bar{M}_i W + W \bar{M}_i^T - \gamma GL_i L_i^T G^T \right) W^{-1} \right) \right] z.
\]

C_i > 0 then by lemma(4) \( C_i = L_i L_i^T > 0 \ \forall i \), and the Lyapunov function derivative becomes:

\[
\dot{v} = z^T \left[ \sum_{i=1}^{r} h_i(z) \left( W^{-1} (-Q) W^{-1} \right) \right] z,
\]

where \( \bar{M}_i W + W \bar{M}_i^T - \gamma GL_i L_i^T G^T < 0 \ \forall i \in I \).

Now, let \( \bar{M}_i W + W \bar{M}_i^T - \gamma GL_i L_i^T G^T = -Q \), then:

\[
\dot{v} = z^T \left[ \sum_{i=1}^{r} h_i(z) \left( W^{-1} (-Q) W^{-1} \right) \right] z,
\]

and \( M = W^{-1} (-Q) W^{-1} \), then \( \dot{v} = z^T \left[ \sum_{i=1}^{r} h_i(z) \left( W^{-1} (-Q) W^{-1} \right) \right] z < -z^T M z < 0 \).

And this completes the proof of the theorem.

**Design Algorithm:**

Using what have been presented in the first and second chapter, above, and more from the literature, we shall suggest a step by step design algorithm for the system under discussion to achieve the stabilizability of model (1.22):

1. **Approximating the nonlinear model by linear local fuzzy models** (In our example we shall use singleton fuzzification):

   Rule \( i \): If \( x_1(t) \) is \( F_{1i} \) and \( x_2(t) \) is \( F_{2i} \) and \( \cdots x_n(t) \) is \( F_{ni} \)

   Then \( \dot{x} = (A + A_i)x(t) + (B + B_i)u(t) \) \hspace{1cm} (1.27)

   where \( (A + A_i, B + B_i) \) is the \( i \)-th local model where \( A, A_i \in \mathcal{R}^{n \times n}, \ B, B_i \in \mathcal{R}^{n \times m} \).

2. Using Center-Of-Gravity Defuzzification method we connect all local models produced by fuzzification into one global model:

   \[
   \dot{x}(t) = \frac{\sum_{i=1}^{r} h_i(x)((A + A_i)x(t) + (B + B_i)u(t))}{\sum_{i=1}^{r} h_i(x)} \hspace{1cm} (1.28)
   \]
where \( h_i(x(t)) = \frac{\Phi_i(t)}{\sum_{j=1}^{r} \Phi_j(t)} \), \( \Phi_i(t) = \prod_{j=1}^{p} F_j^i(x_j(t)) \), and \( F_j^i(x_j(t)) \) is the membership function of the fuzzy set \( F_j^i \).

3. Check for matching conditions \( A_1 = BE_i \) and \( B_1 = BD_i \), where \( E_i \) are \((m_i \times n_i)\) constant matrices and \( D_i \) are \((m_i \times m_i)\) constant matrices.

4. Construct a matrix \( K_o \) such that the nominal system \((A + BK_o)x(t) = \bar{A}x(t)\) is asymptotically stable "Use pole-placement".

5. Choose an \( n \times n \) invertible transformation matrix \( T \) selected as described in section (2).

6. Form the matrices \( \bar{F} = T\bar{A}T^{-1}, \bar{G} = TB \), and \( \bar{F}^i = TA_iT^{-1}, i \in I \).

7. Construct the matrices \( M_i = \bar{F} + \bar{F}^i \) and partition them as follows:

\[
M_i = \begin{bmatrix}
M_{i1} & M_{i2} \\
M_{i1}^T & M_{i2}^T
\end{bmatrix}
\]

where \( M_{i1} \in \mathbb{R}^{n \times m \times n \times m}, M_{i2} \in \mathbb{R}^{n \times m \times n \times m}, M_{i1}^i \in \mathbb{R}^{n \times m \times n \times m}, \) and \( M_{i2}^i \in \mathbb{R}^{n \times m \times n \times m}, i \in I \).

8. Calculate matrix \( N \) such that the Matrix \( M_{i1} + M_{i2} \) is stable. "Use pole-placement ".

9. Calculate the positive definite symmetric matrix \( W \) defined by:

\[
W = \begin{bmatrix}
W_1 & W_2 \\
W_2^T & W_3
\end{bmatrix}
\]

where \( W_1 \in \mathbb{R}^{n \times n \times n \times n} \) is \( W_1 = W_1^T > 0 \) computed by solving the Lyapunov equation that has the form \((M_{i1} + M_{i2})W_1 + W_1(M_{i1} + M_{i2})^T = Q_o \), where \( Q_o \in \mathbb{R}^{n \times n \times n \times n} \) is any negative definite symmetric matrix, \( W_2 \in \mathbb{R}^{n \times m \times n \times m} \) is computed from \( W_2 = (NW_1)^T \) and \( W_3 \in \mathbb{R}^{n \times m \times n \times m} \) is computed from \( W_3 = P + W_2^T W_1^{-1} W_2 \) where \( P = P^T > 0 \).

10. Calculate the scalar \( \gamma \) which satisfies \( \gamma \geq \max(\gamma_i) \). Define:

\[
Q^i = M_iW + WM_i^T - \gamma_iGG^T = \begin{bmatrix} Q_o & q_i \\ q_i^T & q_{2i} - \gamma_iI_{m} \end{bmatrix}
\]

Where \( Q_o \) is any negative definite matrix (defined in step(9)), \( \gamma_i \) are calculated so as to satisfy \( q_{2i} - \gamma_iI_{m} - q_i^T Q_o^{-1} q_i < 0 \), where \( q_i = M_{i1}W_2 + M_{i2}W_3 + W_1(M_{i2}^i)^T + W_2M_{i2}^i \) and \( q_{2i} = M_{i2}^iW_2 + M_{i2}^iW_3 + W_2^T (M_{i2}^i)^T + W_3(M_{i2}^i)^T \).

11. Calculate the linear state feedback \( u(t) = u_o(t) + u_1(t) = (K_o + K_1)x(t) \) where \( K_o \) as in step (3) and \( K_1 = -\frac{\gamma}{2}(TB)^T W^{-1} T \).

12. Calculate the common Lyapunov function \( v(x) = (Tx)^T W^{-1} (Tx) \).
Example (2): Consider the dynamical system described by the following equation:
\[ \dot{x} = f(x) + g(x)u \] (1.32)
where
\[
f(x) = \begin{bmatrix}
    x_2 \\
    x_1 + 0.31831x_3 + \pi x_1 \frac{\sin(x_3)}{x_3} + \frac{1}{5} \pi \sin(x_3) \\
    x_4 \\
    0.63662 x_1 + 0.95493 x_3 + \frac{3}{2} \pi x_1 \frac{\sin(x_3)}{x_3} + \frac{1}{2} \pi \sin(x_3)
\end{bmatrix}, \quad g(x) = \begin{bmatrix}
    0 \\
    0.30329 \pi e^{x_3} \\
    0 \\
    0.45493 \pi e^{x_3}
\end{bmatrix}.
\]

1. Fuzzifying the system along the desired operating points yields (recall the properties of the sine function in example (1)):
   Rule 1: if \( x_3 \) is near 0 then \( \dot{x} = (A + A_1)x + (B + B_1)u \)
   Rule 2: if \( x_3 \) is near \( \frac{\pi}{2} \) then \( \dot{x} = (A + A_2)x + (B + B_2)u \)

   where
   \[
   A = \begin{bmatrix}
       0 & 1 & 0 & 0 \\
       0 & 0.31831 & 0 & 0 \\
       0.63662 & 0 & 0.95493 & 0 \\
   \end{bmatrix}, \quad A_1 = \begin{bmatrix}
       0 & 0 & 0 & 0 \\
       3.14159 & 0 & 1.0472 & 0 \\
       4.71239 & 0 & 1.5708 & 0 \\
   \end{bmatrix}, \quad A_2 = \begin{bmatrix}
       2 & 0 & 0 & 0 \\
       0 & 0 & 0 & 0 \\
       3 & 0 & 1 & 0 \\
   \end{bmatrix},
   \]
   \[
   B = \begin{bmatrix}
       0 \\
       -1.0472 \\
       -1.5708
   \end{bmatrix}, \quad B_1 = \begin{bmatrix}
       0 \\
       2.58342 \\
   \end{bmatrix}, \quad B_2 = \begin{bmatrix}
       0 \\
       3.87513
   \end{bmatrix}.
   \]

2. Defuzzificating to global model; The Fuzzy sets are \( F_1^1 = (x_3 \) is near 0), \( F_1^2 = (x_3 \) is near \( \frac{\pi}{2} \)), the membership functions respectively are:
   \[ F_1^1(x_3(t)) = \frac{0.1}{0.3x^4 + 0.1}, \quad F_1^2(x_3(t)) = 1 - \frac{0.1}{0.3x^4 + 0.1}. \]
   The functions \( \Phi_1, \ h_1 \) and the global model are:
   \[ \Phi_1(t) = \frac{0.1}{0.3x^4 + 0.1}, \quad \Phi_2(t) = 1 - \frac{0.1}{0.3x^4 + 0.1}, \quad h_1 = \Phi_1, \ h_2 = \Phi_2, \]
   \[ \dot{x}(t) = \sum_{i=1}^{2} h_i(x)(A + A_i)x(t) + (B + B_i)u(t) \] (1.33)

3. The constant matrices \( E_i \) are \( E_1 = (1.5708 \ 0 \ 0.5236 \ 0), \ E_2 = (1 \ 0 \ 0.33333 \ 0) \)
   and \( D_i \) are \( D_1 = -\frac{\pi}{6}, \ D_2 = 1.29171 \).

4. Construct the matrix \( K_o \) (Poles-Placement):
   \[ K_o = (-4807.31213 \ -3315.59241 \ 3144.55645 \ 2203.06161). \]
5. The transformation matrix $T$ is chosen to be:

$$
T = \begin{bmatrix}
1.27324 & 0 & 0 & 0 \\
0 & 0 & 2.22816 & 0 \\
3.05577 & -3.81971 & 4.45633 & 2.54647 \\
-0.4 & 0.5 & -0.58333 & 0
\end{bmatrix}, \quad T^{-1} = \begin{bmatrix}
0.7854 & 0 & 0 & 0 \\
0.62832 & 0.5236 & 0 & 2 \\
0 & 0.4488 & 0 & 0 \\
0 & 0 & 0.3927 & 3
\end{bmatrix}.
$$

6. The matrix $G$ is $G = (0 \ 0 \ 0 \ 1)^T$, the matrix $F$ is:

$$
F = \begin{bmatrix}
0.8 & 0.666666 & 0 & 2.54648 \\
-0.000001 & -0.000011 & 0.8749995 & 6.684489 \\
0.251825 & 2.148895 & 1.741352 & 19.480767 \\
-5858.760165 & -324.906457 & 864.914284 & -24.541341
\end{bmatrix},
$$

The matrices $F^i$ are:

$$
F^1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-0.000002337 & -0.0000235 & 0 & 0 \\
1.233699067 & 0.234992101 & 0 & 0
\end{bmatrix},
$$

$$
F^2 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-0.000007854 & -0.00000721 & 0 & 0 \\
0.785397883 & 0.149601016 & 0 & 0
\end{bmatrix}.
$$

7. The matrices $M_i$ are:

$$
M_1 = \begin{bmatrix}
0.8 & 0.66667 & 0 & 2.54648 \\
0 & -0.00001 & 0.875 & 6.68449 \\
0.25182 & 2.14889 & 1.74135 & 19.48077 \\
-5857.52647 & -324.67147 & 864.91428 & -24.54134
\end{bmatrix},
$$

$$
M_2 = \begin{bmatrix}
0.8 & 0.66667 & 0 & 2.54648 \\
0 & -0.00001 & 0.875 & 6.68449 \\
0.25182 & 2.14889 & 1.74135 & 19.48077 \\
-5857.97477 & -324.75686 & 864.91428 & -24.54134
\end{bmatrix},
$$

And the partitions are as follows:

$$
M_{11} = \begin{bmatrix}
0.8 & 0.66667 & 0 \\
0 & -0.00001 & 0.875 \\
0.25182 & 2.14889 & 1.74135
\end{bmatrix}, \quad M_{12} = \begin{bmatrix}
2.54648 \\
6.68449 \\
19.48077
\end{bmatrix}, \quad M_{22} = -24.54134,
$$

$$
M_{21} = (-5857.52647 \ -324.67147 \ 864.91428),
$$

$$
M_{21} = (-5857.97477 \ -324.75686 \ 864.91428).
$$

8. Calculating the matrix $N$ (Poles-Placement):

$$
N = (-186499171 \ 13668943 \ 19567623).
$$
9. Now we shall proceed in the procedure of finding the matrix $W$:

$$W = \begin{pmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{pmatrix}$$

where $W_1 \in \mathbb{R}^{3 \times 3}$, $W_2 \in \mathbb{R}^{3 \times 1}$, $W_3 \in \mathbb{R}^{1 \times 1}$. The Lyapunov equation

$$(M_{11} + M_{12} N)W_1 + W_1 (M_{11} + M_{12} N)^T = Q_o,$$

where $Q_o$ is chosen to be:

$$Q_o = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -10 & 3 \\ 0 & 3 & -4 \end{pmatrix},$$

Hence $W_1 = \begin{pmatrix} 6046.1477426 & 15949.608694 & 46453.252038 \\ 15949.608694 & 42080.707078 & 122538.43585 \\ 46453.252038 & 122538.43585 & 356912.026115 \end{pmatrix}$,

$$W_2 = (N W_1)^T = \begin{pmatrix} -6075.2885118 \\ -16041.029944 \\ -46021.464984 \end{pmatrix},$$

$W_3 = P + W_2^T W_1^{-1} W_2$ where $P = (583.2738711)^2$.

Finally,

$$W = \begin{pmatrix} 6046.1477426 & 15949.608694 & 46453.252038 & -6075.2885118 \\ 15949.608694 & 42080.707078 & 122538.43585 & -16041.029944 \\ 46453.252038 & 122538.43585 & 356912.026115 & -46021.464984 \\ -6075.2885118 & -16041.029944 & -46021.464984 & 133000 \end{pmatrix},$$

10. The calculations of the scalar $\gamma$:

$$q_1 = \begin{pmatrix} 56450.9529435 \\ 140019.793719 \\ 416651.625895 \end{pmatrix},$$

$$q_2 = \begin{pmatrix} 52378.5278241 \\ 129276.312564 \\ 385363.0759697 \end{pmatrix},$$

hence $\gamma = 131832190162.193$.

11. The feedback gain matrix:

$$K_o = (-4807.31213 -3315.59241 3144.55645 2203.06161),$$

$$K_1 = (-201035340129.999 -84650518141.0083 133229801355.076 56395878441.0714),$$

$$K = (-201035344937.331 -84650521456.6007 133229804499.633 56395880644.1333),$$

$$u = -201035344937.331 x_1 -84650521456.6007 x_2 +133229804499.633 x_3 +56395880644.1333 x_4,$$

Substitute $u$ in (1.32) yields:

$$\dot{x} = Ax + f(x)$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0.31831 & 0 \\ 0 & 0 & 0 & 1 \\ 0.63662 & 0 & 0.95493 & 0 \end{pmatrix},$$

$$f(x) = \begin{pmatrix} 0 \\ g_1(x) + g_2(x) \\ 0 \\ g_3(x) + g_4(x) \end{pmatrix},$$

$$\gamma = 131832190162.193.$$
\[ g_1(x) = \frac{\pi x_1 \sin(x_3)}{x_3} + \frac{\pi \sin(x_3)}{3}, \quad g_2(x) = 0.30329\pi e^{x_3}(-20103534497.331x_1 - 8465052146.6007x_2 + 13322980499.633x_3 + 5639588064.133x_4) \]
\[ g_3(x) = \frac{3\pi x_1 \sin(x_3)}{2x_3} + \frac{\pi \sin(x_3)}{2}, \quad g_4(x) = 0.45493\pi e^{x_3}(-20103534497.331x_1 - 8465052146.6007x_2 + 13322980499.633x_3 + 5639588064.133x_4) \]

Substitute \( u \) in (1.33) yields:
\[
\dot{x} = \begin{bmatrix} x_2 \\ f_1(x) + f_2(x) \\ x_4 \\ f_3(x) + f_4(x) \end{bmatrix}
\]

where
\[
f_1(x) = (-191546476652.12833x_1 - 80655016843.84915x_2 + 126941357728.61583x_3 + 5373995077.72992x_4) \left( \frac{1}{3x_3^4 + 1} \right),
\]
\[
f_2(x) = (-2764288262068.70995x_1 - 11639666679163.83834x_2 + 1831944451622.07859x_3 + 775458021785.79622x_4) \left( \frac{x_3^4}{3x_3^4 + 1} \right),
\]
\[
f_3(x) = (-287319714979.05587x_1 - 120982525265.77372x_2 + 190412036593.40121x_3 + 80600992616.59488x_4) \left( \frac{1}{3x_3^4 + 1} \right),
\]
\[
f_4(x) = (-4146432393105.65506x_1 - 1745950018745.75751x_2 + 2747916677434.55027x_3 + 1163187032678.69433x_4) \left( \frac{x_3^4}{3x_3^4 + 1} \right).
\]

12. The common Lyapunov function \( V(x) = (Tx)^TW^{-1}(Tx) \), where:
\[
V(x) = 5433.733593x_1^2 + 4574.305407x_1x_2 - 702.071807x_1x_3 - 3047.503693x_1x_4 + 963.036752x_2^2 - 3031.476018x_2x_3 - 1283.192859x_2x_4 + 2386.473316x_3^2 + 2019.636543x_3x_4 + 427.445763x_4^2
\]
Graph (9)
The state-space components versus time of the first local system with initial condition \((0, 0, \pi/12, 0)\).

Graph (10)
The state-space components versus time of the second local system with initial condition \((0, 0, 5\pi/12, 0)\).
Graph (11)
(a) The controller at the 1st local system. (b) The controller at the 2nd local system. (c) The common Lyapunov function at the 1st local system. (d) The common Lyapunov function at the 2nd local system.

Graph (12)
The membership function of the fuzzy inferred set ($x_3$ is near 0) versus $x_3$ and the time.

Graph (13)
The membership function of the fuzzy inferred set ($x_3$ is near $\pi/2$) versus $x_3$ and the time.
Conclusions and Future Studies.

From the calculations we can see the flexibility in the choice of the operating points, the poles in the pole-placement method, and the matrix $Q$, that enables us to manipulate the plant to give results with desired characteristics, and that this approach is effective.

A future studies can be studying the effect of the nonsatisfaction of the matching condition and how to deal with the system in that case, applying the ideas and theorems to discrete time nonlinear dynamical systems or to chaotic dynamical systems.

References.


تصميم أنضمة السيطرة الضبابية الدينامية غير الخطية

الملخص

قد تم بناء أنضمة السيطرة الضبابية الدينامية غير الخطية في جامعتي بغداد، النهر، كلية الحاسوب، طبائع الرياضيات، قسم

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