Study of Natural Convection in a Porous Trapezoidal Cavity with a Square Body at the Center of the Enclosure

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ABSTRACT
Steady state free convection heat transfer in a two dimensional trapezoidal enclosure filled with a fluid-saturated porous medium with a square solid body located at the center of cavity, is performed in this study. The bottom wall of the cavity was heated with a sinusoidal temperature distribution $\theta = \lambda (1 - \cos(2\pi X))$, the inclined walls are insulated (adiabatic) and the top wall is maintained at $\theta = 0$. To obtain the effects of the presence of a square body on heat transfer and fluid flow inside the enclosure, three different temperature boundary conditions were applied for the body as cold ($\theta_{body} = 0$), heated ($\theta_{body} = 1$) and adiabatic ($\frac{\partial \theta}{\partial n} = 0$) at different $Ra$ numbers. In this study, the governing equations were solved numerically using finite element software package (FLEXPDE). Results for the mean Nusselt number, $Nu_m$, the contour maps of the streamlines and isotherms are presented. It is observed that fluid flow and temperature fields strongly depend on thermal boundary conditions of the body. A comparison of the flow field, isotherm field and averaged Nusselt number, $Nu_m$ with previous work, which revealed a good agreement.

Keywords: Steady State, Free Convection, Porous Medium, Trapezoidal Enclosure.

دراسة الحلم الحر داخل فجوة على شكل شبه محرف مملوء بالمادة المتسلمة مع جسم مربع في مركز الغلاف

الخلاصة
لقد تم في هذه البحث دراسة عملية انتقال الحرارة بالحلم الحر لحالة الاستقرار داخل غلاف معين ذو نصف الأبعاد ملعو بأشكال متسامى مع جسم صلب مربع الشكل موضوع في مركز الغلاف. الجدار السفلي للغلاف مسخ بدرجة حرارة غير متزنة منتظمة ممثلة بالمعادلة $\theta = 0$, الجدرين المائلان معلزوة أما الجدار العلوي فقد ثبت عند درجة حرارة $\lambda (1 - \cos(2\pi X))$. للحصول على تأثير وجود الجسم المربع على انتقال الحرارة وجريان المائع داخل الغلاف، ثلاث شروط حدية مختلفة طبقت على الجسم، واردة، ساخنة ومؤلمة عند قيم مختلفة من $Ra$ رايلي في هذه الدراسة. استخدمت المعادلات الفسيولوجية (FLEXPDE) لحل المعادلات المطورة الحاكمة بعملية انتقال الحرارة عددياً بطريقة العناصر المحددة. النتائج تمثل معدات رقم سخت خيوط الجريان وخطوط التدحرج. لوحظ بأن مجالات درجات الحرارة والجريان تعتمد $Nu_m$
INTRODUCTION

The heat and fluid flow in fluid-saturated porous media is an important problem in engineering and it has gained significant attention over the last three decades. Applications of porous media can be found in grain storage, chemical catalytic reactors, geophysical problems, solar collectors, heat exchangers, etc. Natural convection under different thermal boundary conditions in a porous media has been extensively studied in the past years. Basak et al.

Sathiymoorthy et al.[2], studied numerically the natural convection flows in a square cavity filled with a porous matrix using penalty finite element method for uniformly and non-uniformly heated bottom wall, and adiabatic top wall maintaining constant temperature of cold vertical walls. They used Darcy–Forchheimer model to simulate the momentum transfer in the porous medium. The numerical procedure is adopted in their study over a wide range of parameters (Rayleigh number $Ra$, $10^3 \leq Ra \leq 10^6$, Darcy number $Da$, $10^{-3} \leq Da \leq 10^{-1}$ and Prandtl number $Pr$, $0.2 \leq Pr \leq 100$). Numerical results are presented in terms of stream functions, temperature profiles and Nusselt numbers. They found that the heat transfer is primarily due to conduction for $Da \leq 10^{-3}$ irrespective of $Ra$ and $Pr$.

Natural convection in a rectangular/square enclosure (horizontal or inclined) filled with a fluid-saturated porous medium under different temperature or heat flux boundary conditions has been extensively analyzed in earlier studies by Oztop.
[3], Varol et al. [4], Zhao et al. [5] and Mansour et al. [6] etc. In most of these studies, isothermal boundary conditions were applied to the side walls of the rectangular/square enclosures. Varol et al. [7,8] analyzed steady-state free convection heat transfer in a right-angle triangular enclosure with different thermal boundary conditions in a porous media using finite difference technique and Darcy law to write the equations of porous media. Their study was performed for different aspect ratios ($0.25 \leq AR \leq 1.0$) and Rayleigh numbers ($100 \leq Ra \leq 1000$). They observed that heat transfer is increased with the decreasing of aspect ratio and multiple cells are formed at high Rayleigh numbers. Varol et al. [9] used a finite difference technique to study the natural convection in porous media for a right-triangular enclosure with a square body located far from the origin with the distance of 0.3 in both directions. Four different temperature boundary conditions were applied for the body as heated, cooled, neutral and adiabatic at different $Ra$ numbers and they observed that fluid flow and temperature fields strongly depend on thermal boundary conditions of the body. Basak et al. [10] studied numerically the phenomena of natural convection in an isosceles triangular enclosure filled with a porous matrix by using a penalty finite element analysis with bi-quadratic elements for solving the Navier-Stokes and energy balance equations.

The phenomena of natural convection in a trapezoidal enclosure of uniform and non-uniform heating of bottom wall filled with porous matrix has been studied numerically by using a penalty finite element analysis with bi-quadratic elements studied by Basak et al. [11,12]. They found that non-uniform heating of the bottom wall produces greater heat transfer rate at the center of the bottom wall than uniform heating case for all Rayleigh and Darcy numbers. Varol et al. [13] investigated the steady free convection flow in a two-dimensional right angle trapezoidal enclosure filled with a fluid-saturated porous medium. Flow and heat transfer characteristics are studied for a range of parameters: the Rayleigh number, $Ra$, 100$\leq Ra \leq 1000$; and the aspect ration, $AR=0.25$, 0.50 and 0.75. Their results indicate that there exist significant changes in the flow and temperature fields as compared with those of a differentially heated square porous cavity and These results lead, in particular, to the prediction of a position of minimum heat transfer across the cavity, which is of interest in the thermal insulation of buildings and other areas of technology.

The present study deals with a steady natural convection flow in a porous trapezoidal with a square body located at the center of enclosure based on Darcy equation model as shown in Figure(1). Where the bottom wall is heated non-uniformly and vertical walls are adiabatic whereas the top wall is cold. To obtain the effects of the presence of a square body on heat transfer and fluid flow inside the enclosure, three different thermal boundary conditions were applied, the body is cold, heated and adiabatic at different $Ra$ numbers. The finite element method with software package (FlexPDE) [14] has been used to solve the governing equations for flow and temperature fields.
GOVERNING EQUATIONS

In order to obtain the governing equations of the two-dimensional laminar flow of an incompressible fluid, some assumptions were made as following [7-9]:

- The properties of the fluid and the porous media are constant.
- The boundaries of cavity are impermeable.
- The flow is described by Darcy equation model and that the Boussinesq approximation is valid.
- The viscous drag and inertia terms of the momentum equations are negligible.

With using the above assumptions, the dimensional form of the continuity, momentum and energy equations can be written as follows, [9,15]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots \ (1)
\]

\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g \beta K}{\nu} \frac{\partial T}{\partial x} \quad \ldots \ (2)
\]
The following stream functions are defined as:

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]  \hspace{1cm} \text{(4)}

Using the stream functions (4) eqs. (1)–(3) can be written in non-dimensional forms as follows:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  \hspace{1cm} \text{(5)}

\[ \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \]  \hspace{1cm} \text{(6)}

\[ \frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \]  \hspace{1cm} \text{(7)}

The non-dimensional parameters are given below as:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \Psi = \frac{\psi}{\alpha}, \quad Ra = \frac{g \beta K (T_h - T_c) L}{\nu \alpha} \]  \hspace{1cm} \text{(8)}

**Boundary Conditions**

The boundary conditions for the considered model are depicted on the physical model as shown in Figure (1). In this model, \( u=v=0 \) for all solid boundaries of trapezoidal enclosure and the body.

For all solid walls, boundary conditions are given as \( \Psi=0 \):

- On the bottom wall (hot wall); \( \theta = \lambda (1 - \cos(2\pi X)) \)
- On the side walls (adiabatic); \( \partial \theta / \partial n = 0 \)
- On the top wall (cold); \( \theta = 0 \)

On the body,

- Cold body (case 1); \( \theta_{\text{body}} = 0 \)
- Heated body (case 2); \( \theta_{\text{body}} = 1 \)
- Adiabatic body (case 3); \( \frac{\partial \theta}{\partial n} = 0 \)

Where \( \lambda \) the non-dimensional sinusoidal temperature distribution on the bottom wall and we take \( \lambda = 0.5 \).
Heat Transfer Calculations

Heat transfer process can be represented by calculating the local Nusselt number $Nu_L$ and the mean Nusselt number $Nu_m$ which are given by the following equations:

$$Nu_L = \frac{\partial \theta}{\partial n} \quad \ldots \ (9)$$

$$Nu_m = \frac{1}{S} \int_0^S Nu_L \, dS \quad \ldots \ (10)$$

Where $n$ denotes the normal direction on a plane and $S$ is the length of the heated wall.

NUMERICAL SOLUTION

The code FlexPDE [14] was used to perform finite element method to analyze the steady natural convection in a porous trapezoidal cavity with a square body located at the center of enclosure such that this code is relied in the solution of the nonlinear system of equations (6) and (7). Hence, the continuity equation (5) is used to check the error of the solution throughout the grids of domain.

Software Validation

To check the validation of software; three grid densities were examined $10^{-3}$, $10^{-4}$ and $10^{-5}$. The grid dependency were checked with the continuity equation and the obtained results show a reasonable validation of the velocity distribution for each grid density. i.e example, the gridded domain for the trapezoidal enclosure with square body located at the center of enclosure at $Ra=500$ is shown in Figure (2a) and the distribution of the values of $(\partial U/\partial X + \partial V/\partial Y)$ over the domain, is presented in Figure (2b).

![Figure( 2) For 10^{-5} accuracy (a) grid distribution over the domain( b) validation of continuity](image)
Code Validation

In order to assess the accuracy of the numerical procedure, the calculation for a triangular enclosure filled with porous medium at $Ra=500$ were performed. The thermal boundary condition of the bottom wall is the non-uniform temperature $	heta = \lambda(1 - \cos(2\pi X))$ while the vertical and the inclined walls are isotherm at $\theta = 0$. The program was tested by comparing the average Nusselt number (at the hot wall), the maximum and minimum values of stream functions with a theoretical work performed by [8] as shown in Table-1. The streamlines and isotherms plots at $Ra=500$ are presented in Figure (3) as another comparison. The results show a very good agreement and from these comparisons it can be decided that the current code can be used to predict the flow field of the present problem.

Table (1) Comparison of mean Nusselt number, the maximum and minimum values of stream functions with Ref. [8].

<table>
<thead>
<tr>
<th></th>
<th>At $Ra=100$</th>
<th></th>
<th>$\Psi_{\text{max.}}$</th>
<th>$\Psi_{\text{min.}}$</th>
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<tbody>
<tr>
<td>Ref. [8]</td>
<td>Nu</td>
<td>$\Psi_{\text{max.}}$</td>
<td>$\Psi_{\text{min.}}$</td>
<td></td>
</tr>
<tr>
<td>Present study</td>
<td>2.83951</td>
<td>2.56</td>
<td>-1.89</td>
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<tr>
<td></td>
<td>3.14928</td>
<td>2.56</td>
<td>-1.90</td>
<td></td>
</tr>
<tr>
<td>At $Ra=500$</td>
<td></td>
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</tr>
<tr>
<td>Ref. [8]</td>
<td>Nu</td>
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<td></td>
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<td></td>
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</table>
RESULTS AND DISCUSSION

The results for natural convection has been performed in a trapezoidal porous enclosure with inserted solid square body located at the center of cavity at three different thermal boundary conditions, cold, heated and adiabatic were considered. The governing parameters which effectively affect the flow and heat transfer are Rayleigh number $Ra$ and the thermal boundary conditions for the body.

Figures from (4) to (6) display the stream function and isotherm contours for different values of $Ra$ with different thermal boundary conditions of the square body. Results of flow fields are represented by streamlines with equal increment $\Delta \psi$ and temperature distributions are represented by isotherms with equal increment $\Delta T$. Figure (4) shows the streamlines (on the left), isotherms (on the right) in a porous trapezoidal enclosure with cold square body (case 1). It is observed that the cavity is heated from the bottom wall and cooled from the top wall as well as the cold square solid body located at the center of cavity as
indicated in Figure (1). Thus, the fluid near the bottom wall is hotter than the fluid near the bottom side of the cold body and the fluid near the top side of the cold body is hotter than the fluid near the cold wall of cavity due to the conduction through the solid body. A density difference is formed due to this temperature difference. As a result, fluid rises from the middle portion of the bottom wall of cavity and the fluid flows downwards from the top wall. The strength of moving flow from the bottom wall is higher than that from the top wall. Thus four cells are formed which are symmetrical about the middle vertical axis of the bottom wall. In the case of a relatively low Ra number, Ra =100, the flow strength is very weak and the maximum value of the stream function is found to be 0.97. In this case, the conduction mode of heat transfer become stronger and isotherms are almost parallel to each other. Also, this distribution indicates the conduction mode of heat transfer. When increasing Ra number, the value of stream function increases and flow field becomes very complex due to domination of free convection. The extreme value of stream function is obtained as 8.81 at Ra =900.

Isotherms are moved to almost parallel to each other in low Ra number due to domination of conduction regime. These isotherms are presented in Figure (4a). However, as convection heat transfer becomes stronger, the plumelike distribution in the middle part of the heated bottom wall of the enclosure is formed and it can be shown in Figures (3b) and (3c).

Figure (5) shows the results of the flow and temperature fields for the case of heated square body (case 2), for Ra =100, 500 and 900 respectively. In this case double symmetric circulation cells were formed in different rotating directions, starting from the middle of the bottom wall of the cavity for all Ra numbers considered. It can be seen that for low value of Ra number Ra=100, the location of each cell center at the mid of cavity, but when increasing Ra the cell center is moved towards the upper part of the cavity as shown in Figure (5b) and (5c). The strength of the fluid is increased with the increasing of Ra number because of the domination of convection mode of heat transfer. When the results of this case are compared with that of cold body (case 1) a higher flow strength is obtained for heated body (case 2) due to increasing of heat transfer to the fluid as a result of inserting heated body. We can also see from the isotherm lines that they are distributed almost parallel to each other near the cold wall of cavity and a plumelike distribution is formed from the middle part also it can be observed from this figure that the isotherm lines are skewness toward the inclined walls near the bottom corner of cavity this is because of the wall inclination. With increasing Ra number the plumelike distribution which generated from the bottom wall of cavity and from the heated square body will occupy the whole space due to the increasing of the convection heat transfer.

Figure (6) illustrates the streamlines (on the left) and the isotherms (on the right) in the porous trapezoidal enclosure with adiabatic square solid body located at the center of cavity (case 3) for different values of Ra numbers, Ra=100, 500 and 900. In this case four cells are formed which are symmetrical about the middle vertical axis of the bottom wall of cavity for all Ra numbers considered. The cells are obtained near the top wall of cavity then grow and strengthen the other cells. It is clearly evident from the isotherms that at low value of Ra number the conduction mode of heat transfer dominant to convection but with increasing Ra number the convection mode heat transfer becomes dominates.
Effects of $Ra$ number on local Nusselt number ($Nu_L$) along the bottom wall of the trapezoidal cavity at different boundary conditions of the solid square body are seen in Figure (7). This figure shows that the values of ($Nu_L$) increase with increasing $Ra$ number and the variation of ($Nu_L$) is symmetrical due to the symmetry in the temperature field for all values of $Ra$ number. Negative values of ($Nu_L$) are appears near the corners of the bottom wall since the direction of the heat transfer is reversed in this region while positive values of ($Nu_L$) is observed in the main part of the bottom wall depends on the recirculation intensity. It is clear from this figure that the value of ($Nu_L$) is small at the middle of the bottom wall due to the reduction in the recirculation intensity. Also it can be seen that the heat transfer process between the heat surface of the bottom wall of cavity and the fluid for adiabatic body case is smaller than the other cases (cold body and heated body) this is because the thermal resistance occurs in the system. It means that the boundary conditions for the concentric solid square body plays an extremely important role on heat transfer.

Figure (8) shows the variation of ($Nu_L$) for four sides of the square body at cold and hot boundary conditions at $Ra=500$. As can be seen from the figures, when cold and hot boundary conditions are applied to the body, U-shaped variations are obtained for all sides. It means that at the middle of each side of the body, a minimum value is formed. The same values are obtained for the right and left walls of the body due to the symmetry of the isotherm lines along vertical axis i.e.(the same distance between the body and inclined walls of cavity). Also it can be seen that for the two cases, cold and hot boundary conditions of the body the heat transfer process is enhanced near the bottom side of the solid body due to the growth of the thermal boundary layer in this region.

Figure (9) shows the variation of surface-averaged Nusselt numbers for each side of the body at different boundary conditions. It can be seen that also, for the two cases (cold body and hot body) the values of mean Nusselt number for the bottom side of the body are greater than that of the other sides. This is because of the reduction of the heat losses from the bottom side of square body as compared with the other sides, while the average Nusselt number has the same value for the right and left sides of the body, due to the equal distances between the body and the inclined walls of the cavity.

Finally, surface-averaged Nusselt numbers at the hot bottom wall for different $Ra$ number are presented in Figure (10) for different boundary conditions. As seen from this figure that the mean Nusselt number for cold body (case 1) is higher than that of heated body (case 2) and for the adiabatic body (case 3). The heat transfer process increases when increasing the $Ra$ number for all cases due to the dominant of convection mode of heat transfer.
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Figure (4) Streamlines (on the left) and isotherms (on the right) for cooled body case \( \theta_{body} = 0 \) at different values of Rayleigh number.

(a) \( Ra = 100 \)

(b) \( Ra = 500 \)

(c) \( Ra = 900 \)
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Figure (5) Streamlines (on the left) and isotherms (on the right) for heated body case $\theta_{body}=1$ at different values of Rayleigh number.

(a) $Ra=100$

(b) $Ra=500$

(c) $Ra=900$
Figure (6) Streamlines (on the left) and isotherms (on the right) for adiabatic body case $\left(\frac{\partial \theta}{\partial n}\right)_{body} = 0$ at different values of Rayleigh number.
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Figure (7) Variation of local Nusselt number along the hot bottom wall, (a) $Ra=100$, (b) $Ra=500$ and (c) $Ra=900$. 

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Figure (8) Variation of surface local Nusselt number along the body wall at different surfaces of the body at Ra=500 , (a) $\theta_{\text{body}}=0$ and (b) $\theta_{\text{body}}=1$. 
Figure (9) Surface averaged Nusselt number as a function of Rayleigh numbers at different surfaces of the body, (a) $\theta_{body}=0$ and (b) $\theta_{body}=1$. 
CONCLUSIONS

The present study was performed to investigate the flow and temperature field when a body with different boundary conditions is inserted in a trapezoidal porous enclosure. The governing parameters are the Rayleigh number, $Ra$ and the BCs of the square body. Based on the obtained results, it can be concluded:

- Heat transfer is increased with the increasing of $Ra$ number. It is observed that the strength of the fluid is also increased with increasing of $Ra$ number due to the domination of convection mode of heat transfer.
- The fluid flow and temperature fields strongly depend on the thermal boundary conditions of the body, i.e., the presence of the body obstructs the flow and temperature fields.
- The highest heat transfer was obtained at the bottom of the cold and heated side of the body but the lowest heat transfer at top side while the same heat transfer was obtained at the right and left sides of the cooled and heated body.
- The flow strength increased with the increasing of $Ra$ number for all cases and the highest flow strength was formed for the case of heated body.
- The highest surface-averaged Nusselt number was obtained in the case of cold body insertion.

REFERENCES

[2]. Sathiyamoorthy, M., Basak, T., Roy, S. and Pop, I., " Steady natural convection flow in a square cavity filled with a porous medium for linearly heated