Simulation of Behavior of Plate on Elastic Foundation under Impact Load by the Finite Element Method

ABSTRACT

Transient response and vibrations of an elastic plate resting on sandy soil are presented. Plates are commonly used structural elements and are subjected to a wide variety of static and dynamic loads. Such studies are of particular interest in analytical investigations related to structural foundation on soil media. The influence of impact induced high strain-rates within the structure, which causes property changes in all used materials, has to be regarded according to experimental results.

The main objective of the present paper, using the finite element approach through ANSYS program is the simulation of the dynamic response of the foundation under impact load. As a case study, previous experimental work included application of a dynamic load generated by dropping a steel ball (38.1 mm in diameter, 2.22 N in weight) from a height of 609.6 mm onto an aluminum target plate (203.2 mm in diameter, 12.7 mm thick) placed on top of a sand medium. The impact load is defined as a product of the loading magnitude and a time varying function which is assumed to be a Hanning's function for a monopeak, smooth-shaped curve. The problem is discretised by using four types of elements; Solid 45 to model the soil, Shell 63 to model the aluminum plate and Target 170 and Contact 174 are used to model the contact between the plate and soil. Shell 63 (elastic shell) has both bending and membrane capabilities.

It is noticed that the finite element analysis agrees well with the experimental results throughout the entire range of behavior, and the difference in the ultimate displacement is about 6.2%. It can be concluded that ANSYS program is well suited for impact analyses of soil and structural dynamics problems in the non-linear range.

Keywords: Plate, Elastic Foundation, Impact, Finite Elements.
INTRODUCTION

The dynamic response of an elastic plate resting on sand is a practical topic for investigation that covers a wide range of applications. First, the plate can be viewed as a footing of a structure, which has significance for foundation-vibration studies. Secondly, the finite plate response due to an impact force is a fundamental problem in structural design. Moreover, investigating the impact force itself is also important and interesting since an accurate knowledge of the live load on the impacted structure enables a better design and prediction of the damage to the structure. Finally, it is important to understand fully the impact loading transmission through the soil.

Impact may be defined as a collision between two bodies which occurs in a very short interval of time during which the two bodies exert on each other relatively large forces, called impact loads, which depend on velocity, mass, shape, elastic and plastic properties of the collided bodies. Impact load may be applied to many structures which
Have been designed only to resist their own dead loads in addition to the conventional static live loads. It is useful to check the impact resistance of structures which has been designed to resist static loads. Some structures such as shelters and buildings of nuclear plant must be designed to resist impact loads.

**PREVIOUS STUDIES**

In close analytical, experimental, or numerical study for concrete targets under impact load, many researchers presented the impact force time-history which showed the duration, the value, and the shape of the load:

**Experimental methods**

Hussain (1987) presented experimental tests on simply supported reinforced concrete slabs of dimensions (500 x 500 x 20 mm) subjected to both impact loading from a falling mass and static loading. The independent test variables were the falling mass, the head shape of the falling mass and the height of drop. A theoretical analysis based on the numerical solution of the impact integral equation was carried out to calculate the impact force and deflection time histories. The main results indicated that the theoretical maximum transient central deflection of the slabs was found in a good agreement with the experimental deflection, the calculated impact duration was longer for heavier falling masses, and the applied kinetic energy required to cause slab failure was about (2-3) times the static strain energy absorption capacity of the slab.

Sayhood (1988) tested experimentally forty-nine simply supported reinforced concrete beams subjected to both impact loading from a falling mass and static loading at midspan. The dimensions of the beams were (60 x 120 x 1000 mm) and (30 x 60 x 500 mm). A theoretical analysis based on the numerical solution of the beam impact integral equation was carried out to determine the impact force and deflection-time histories. A good agreement was found between the theoretical and experimental results. Moreover, a reasonable agreement was found between the experimental results of the two beam sizes, giving evidence about the possibility of using small models to represent full prototype larger structures.

Baidya (2004) presented an experimental study on natural frequency of a foundation on a layered soil system subjected to dynamic loading. It was observed that the natural frequency of the system decreases due to presence of the soft layer at the top whereas it increases due to presence of stiff layer at the top. It was also observed that presence of a soft layer at top is more dangerous than at depth.

Zhang et al. (2008) developed a new drop weight impact machine to investigate the impact behavior of concrete and concrete structures. The time delay during the test between the impact force and the reaction force was due to it takes some time for the relatively great changes in contact resistance caused by the specimen deformation. Most of the impact force was used to maintain the balance with the inertia force, and only a small portion of the impact force is actually used to deform and fracture the specimen. The high strength concrete was a loading rate sensitivity material, it resisted higher load and absorbs more energy under impact loading than under static loading.
Numerical methods

Hinton (1988) analyzed reinforced concrete plates and shells under dynamic loading. Three dimensional isoparametric elements with 20 nodes were used to simulate the concrete. A modified elastic-plastic constitutive model was adopted to represent concrete behavior. Newmark implicit time scheme was used to solve nonlinear dynamic equations. Results were compared with other models and were found in good agreement.

Miyamoto et al. (1991) used Drucker-Prager to model concrete when they researched the analytical and numerical failure modes of reinforced concrete slabs subjected to impulse loads. They examined the rate of dynamic load, the maximum deflection in the centre, and the impulse load-midspan deflection curves from zero to ultimate load, the propagation of cracks through the cross section, and the pattern at failure. The numerical failure mode was determined as well as the load rating in failure mode and the distribution of cracking while they carried out a comprehensive study in this field they could not simulate steel reinforcement under impact loading, the density of cracking in the critical region, and complete failure process from crushing in the compression zone to cracking in the tensile region.

Saha (1997) studied the dynamic stability of a rectangular plate on elastic foundation subjected to uniform dynamic loads and supported on completely elastically restrained boundaries. In that study, non-homogeneous foundation consisted of two regions having different stiffnesses but symmetric about the centre lines of the plate. The equation governing the small amplitude motion of the system was derived by a variation method. The effects of stiffness and geometry of the foundation were also studied, addition to boundary conditions, static load factor, in-plane load ratio and aspect ratio on the stability boundaries of the plate for first- and second-order simple and combination resonance.

Tee (2005) studied the dynamic response of a finite circular plate resting on sand by using ABAQUS program. In the analysis, a free-drop impact system was considered to generate the dynamic loading on the plate free surface. Two finite element models were built, one with a slide line underneath the target plate and another one without the slide line. The numerical results of the finite element method for the radial strain at the bottom of the target plate were compared with the experimental measurement. The numerical results showed good agreement with the experimental results.

Muslih (2007) studied the behavior of rectangular slabs with different boundary conditions and subjected to impact loading caused by falling mass. The model slabs were of dimensions (500 x 500 x 20) mm and the independent variables were the falling mass, the height of drop and the deformation constant. Also, the effects of moment of inertia were discussed. Theoretical analysis based on the numerical solution of the slab impact integral equation was carried out to determine the impact force and deflection time histories, the strain energy absorbed by the slabs and the maximum bending moment. Effect of slab boundary conditions on impact response of slab was also discussed. The theoretical results obtained from the analysis were
compared with experimental and theoretical works previously done, and showed good agreement with experimental results.

Civalek et al. (2007) carried out an investigation to introduce the numerical solution of geometrically nonlinear dynamic problem of rectangular plates resting on elastic foundation. Winkler-Pasternak two-parameter foundation model was considered. The effects of Winkler and Pasternak foundation parameters on the dynamic response of plates had been investigated. Four types of loadings, namely, a uniform step load of infinite duration, sinusoidal loading of finite duration \( t = 0.16 \) sec., N-shaped pulse load of finite \( (t = 0.2 \) sec.), and triangular load of finite \( (t = 0.16 \) sec.) duration, have been considered. It appeared that the shear parameter \( G \) of the Pasternak foundation and stiffness parameter \( K \) of the Winkler foundation have a significant influence on the dynamic response of the plates. The effect of Winkler parameter \( K \), on the displacements was greater than the Pasternak parameter, \( G \). However, the step load of infinite duration had bigger effect on the dynamic response of the rectangular plates on elastic foundation compared with the other dynamic loads which were considered in their study.

Chen and Gurdal (2011) tried to determine three dimensional stresses in an infinite orthotropic plate on an elastic foundation subjected to a transverse point load as shown in Figure (1). A three-dimensional stress distribution in the vicinity of the applied load was sought without considering the friction between the plate and the foundation. Based on the assumption of a uniform stress distribution for the applied load, a double Fourier transform technique was employed to solve the problem in the transform domain. The Gaussian integration scheme was used to carry out the inverse transformation to obtain the real stress components. It was found that the in-plane normal stresses under the loading area at the top surface can be several times larger in compression compared to the average applied stress. For the material properties considered, the normal stress in the minor principal material direction is slightly larger than the applied stress, whereas it was more than four times larger in the major principal material direction. The maximum magnitude of a through-the-thickness shear stress component is achieved just outside the loading area, and has a larger magnitude at points closer to the top surface than the middle surface.

Based on the literature reviewed, it is found that many literatures are reported on nonlinear behaviour of soil and concrete, and considerable amount of literatures are reported on the three-dimensional nonlinear analysis of plate on elastic foundation, but very few studies adopted the dynamic response of soil-structure interaction problems. The present research aims to analyze the foundation by three-dimensional nonlinear finite element method using ANAYS 11 finite element software, in which both the structure and its supporting soil are modeled as continua. The foundation and soil are discretised by eight noded brick elements. The soil is modeled as elasto-plastic material. The finite element procedure is used to analyze the foundation subjected to concentrated impact load.
FINITE ELEMENT FORMULATION FOR DYNAMIC ANALYSIS

The finite element method is one of the techniques used for numerical solutions in the field of ordinary differential equations. The general equation of motion for a structural system is given by Zienkiewicz and Taylor (2005):

\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F_a\}
\]  ... (1)

where:

- \([M]\) = structural mass matrix,
- \([C]\) = structural damping matrix,
- \([K]\) = structural stiffness matrix,
- \([\ddot{u}]\) = nodal acceleration vector,
- \([\dot{u}]\) = nodal velocity vector,
- \([u]\) = nodal displacement vector, and
- \([F_a]\) = applied load vector.

All points in the structure are moving at the same known frequency, however, not necessarily in phase. Also, it is known that the presence of damping causes phase shifts. Therefore, the displacements may be defined as:

\[
[u] = \{u_{\text{max}} e^{i\phi}\} e^{i\Omega t}
\]  ... (2)

\[u_{\text{max}}\] = maximum displacement ,
\[i = \sqrt{-1}\ , \]
\[\Omega = \text{imposed circular frequency (radians/time)} = 2\pi f\ , \]
\[f = \text{imposed frequency (cycles/time)} , \]
\[t = \text{time} , \]
\[\Phi = \text{displacement phase shift (radians)} .\]

Note that \(u_{\text{max}}\) and \(\Phi\) may be different at each degree of freedom. The use of complex notation allows a compact and efficient description and solution of the problem. Equation (2) can be rewritten as:

\[
[u] = \{u_{\text{max}} (\cos\Phi + i \sin\Phi)\} e^{i\Omega t}
\]  ... (3)

Or as:

\[
[u] = ([u_1] + i[u_2]) e^{i\Omega t}
\]  ... (4)

Where:
\[ \{ u_1 \} = \{ \text{umax \ cos \ } \Phi \} = \text{real displacement vector} , \]
\[ \{ u_2 \} = \{ \text{umax \ sin \ } \Phi \} = \text{imaginary displacement vector}. \]

The force vector can be specified analogously to the displacement:

\[ \{ F \} = \{ F_{\text{max}} e^{i\psi} \} e^{i\Omega t} \]  \( \ldots (5) \)

\[ \{ F \} = \{ F_{\text{max}} (\cos \psi + i \sin \psi) \} e^{i\Omega t} \]  \( \ldots (6) \)

\[ \{ F \} = \{ (F_1) + i(F_2) \} e^{i\Omega t} \]  \( \ldots (7) \)

Where:

- \( F_{\text{max}} \) = force amplitude,
- \( \psi \) = force phase shift (radians),
- \( \{ F_1 \} = \{ F_{\text{max}} \cos \psi \} = \text{real force vector} \), and
- \( \{ F_2 \} = \{ F_{\text{max}} \sin \psi \} = \text{imaginary force vector} \).

Substituting Equation (4) and Equation (7) into Equation (1) gives:

\[ (-\Omega^2[M] + i\Omega[C] + [K])(\{ u_1 \} + i\{ u_2 \}) e^{i\Omega t} = ((F_1) + i(F_2)) e^{i\Omega t} \]  \( \ldots (8) \)

The dependence on time (ei\Omega t) is the same on both sides of the equation and may therefore be removed:

\[ \begin{bmatrix} [K] - \Omega^2 [M] + i\Omega [C] \end{bmatrix} (\{ u_1 \} + i\{ u_2 \}) = \{ F_1 \} + i\{ F_2 \} \]  \( \ldots (9) \)

**COMPUTER PROGRAM**

ANSYS (V 11) is used for analysis throughout this work. ANSYS (V 11) program can be applied to a wide variety of engineering applications. It provides the possibility to simulate the super-structure together with its foundation and foundation soil along all its profile required to be considered. The program contains many routines, all the main purposes of achieving a solution to an engineering problem by the finite element method.

One of the main advantages of ANSYS is the integration of the three phases of finite element analysis: pre-processing phase, solution phase and post-processing phase (Bachachi, 2007).
ANSYS STRUCTURAL TRANSIENT ANALYSIS

Practical engineering problems having a non-cyclic transient loading can be solved using the transient analysis in ANSYS.

Transient dynamic analysis (sometimes called time-history analysis) is a technique used to determine the dynamic response of a structure under the action of any general time-dependent loads. This type of analysis to determine the time-varying displacements, strains, stresses, and forces in a structure as it responds to any combination of static, transient, and harmonic loads. The time scale of the loading is such that the inertia or damping effects are considered to be important (Madenci and Guven, 2006). Transient dynamic analysis is used in the design of (Moaveni, 1999):

- Structures subjected to shock loads, such as automobile doors and bumpers, building frames, and suspension systems.
- Structures subjected to time-varying loads, such as bridges, earth moving equipment, and other machine components.
- Household and office equipment subjected to “bumps and bruises,” such as cellular phones, laptop computers, and vacuum cleaners.

DEFINITION OF ELEMENT TYPES

The ANSYS element library contains (165) different element types. Each element type has a unique number and prefix that identifies the element category. BEAM4, PLANE42, SOLID96…etc.

In the context of finite element method, an interface element is used in order to account for the relative motions and associated deformation modes at the interface between two materials or structural components. Therefore, a contact element can effectively be used to describe the discontinuity in structural members (cracks, joints, composite beams, soil-structure interaction) (Hussien, 2007).

In this study, (CONTACT 174) and (TARGET 170) elements are used to model the interface between the soil and the foundation.

DEFINITION OF REAL CONSTANTS

Element real constants are properties, which depend on the element type, such as cross-section, area, moment of inertia, initial strain and thickness …etc. In the analysis, real constant set 1 is used for solid 65 element, which requires real constants for smeared reinforcement in the three directions x, y and z (ANSYS Manual V11, 2007).

DRUKER-PRAGER MODEL

Druker-Prager (DP) theory is used for soil as yield criterion which is applicable to granular (frictional) material such as soil, Drucker – Prager yield criterion can be used with either an associated or nonassociated flow rule. The yield surface does not change with progressive yielding, hence there is no hardening rule and the material is elastic – plastic (Al-Kinani, 2004).

The input data consists of only two constants:
- The cohesion value (c).
- The angle of internal friction (ϕ).

In this study, the Drucker Prager (DP) theory is dependent to define the yield criterion of the soil.

**DESCRIPTION OF THE PROBLEM**

Chen and Chen (2011) studied the dynamic load generated by dropping a steel ball (38.1 mm in diameter, 2.22 N in weight) from a height of 609.6 mm onto an aluminum target plate (203.2 mm in diameter, 12.7 mm thick) placed on top of a sand medium. The impact test setup is shown in Figure (2). The impact duration is measured using a voltage jump and a typical impact duration is about 0.204 ms.

The loading mechanism used in the described experiment to generate a dynamic loading into the soil medium is a low impact system. The low-velocity impact can be modeled as a point load applied at the center of the target plate. The impact load \( P(t) \) can be defined as a product of the loading magnitude and a time varying function:

\[
P(t) = P_0 f(t)
\]

where the time function is assumed as a Hanning's function for a monopeak, smooth-shaped curve (for a duration of \( T_0 \)):

\[
f(t) = 0.5 - 0.5 \cos \left( \frac{2\pi t}{T_0} \right)
\]

Where \( T_0 = \) impact duration.

The peak amplitude \( P_0 \) of the loading can be calculated as:

\[
P_0 = \frac{M_b (2gh_c)^{0.5}}{\int_0^{T_0} f(t) dt}
\]

Where:
- \( M_b \) = mass of the ball; \( h_c \) = dropping height; and \( g \) = gravitational acceleration.

For a 609.6 mm dropping height, the peak amplitude \( P_0 \) is calculated as 7682 N with a measured duration of 0.204 ms. The loading function is shown in Figure (3).

The problem is discretised by using three types of elements; solid 45 to model the soil, SHELL 63 to model the aluminum plate and target170 and contact174 are used to model the contact between the plate and soil.

SHELL 63 (Elastic Shell) has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes (Manual of ANSYS V11, 2007).

The steps of creating model are:
- Specifying volumes of aluminum and soil by (shell 63) and (solid 45).
Meshing the model, Figure (4).

Specifying interface model which uses pair contact elements (target 170 and contact 174) used to represent the contact between aluminum and soil, Figure (5).

Applying loads and boundary conditions.

The parameters of this model are as follows:

Aluminum property parameters are given in Table (1), while the soil parameters are given in Table (2). The interface property parameters are shown in Table (3).

After creating the model and entering all associated model parameters, the analysis is performed. ANSYS divides the load into a number of sub-steps and perform the iteration for each sub-step until reaching the convergence. The deformed shape is shown in Figure (6).

From Figure (7), it can be noted that the finite element analysis agrees well with the experimental results throughout the entire range of behavior. The figure shows that the difference in the ultimate displacement is about 6.2%, and the peak displacements from the experiments are longer than those of the numerical results. It is noticed that the displacement decays rapidly due to material damping.

CONCLUSIONS

The finite element method can be used to model wave propagation through soil, and the current assumption for material properties of sand and concrete is a good approximation. In addition, selection of elements and interface revealed good idealization of the problem.

Displacements and loadings obtained at the center of the plate from finite element analysis are found comparable in both magnitudes and durations to those obtained from experiment, i.e. the accuracy which can be expected from finite element analysis when simulating this kind of problems is accepted.

REFERENCES


Simulation of Behavior of Plate on Elastic Foundation under Impact Load by the Finite Element Method

Table (1) Aluminum property parameters (Tee, 2005).

<table>
<thead>
<tr>
<th>Aluminum parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Es (MPa)</td>
<td>Young’s modulus of elasticity</td>
<td>68928</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson’s ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>density</td>
<td>2821</td>
</tr>
</tbody>
</table>

Table (2) Soil parameters (Chen and Chen, 2011).

<table>
<thead>
<tr>
<th>soil parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Es (MPa)</td>
<td>1100</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>1760</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>( \phi )</td>
<td>35°</td>
</tr>
</tbody>
</table>

Table (3) Interface element parameters (Manual of ANSYS V11, 2007).

<table>
<thead>
<tr>
<th>Interface parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>Coefficient of friction</td>
<td>0.6*</td>
</tr>
</tbody>
</table>

*assumed

Figure (1) Plate configuration and loading in an infinite orthotropic plate on elastic foundation (Chen and Gurdal, 2011).
Figure (2) Experimental setup used by Chen and Chen (2011).

Figure (3) Loading function used by Chen and Chen (2011).
Figure (4) Mesh formation for model of the verification problem.

Figure (5) Interface elements in ANSYS for contact area between the Aluminum and soil.
Figure (6) Deformed shape of the plate as predicted by ANSYS (V11), at time=1 msec., max. displ. = 0.01 mm.

Figure (7) Comparison between displacement time relationship obtained experimentally and by finite element analysis.