Numerical Investigation of Energy Storage in Packed Bed of Cylindrical Capsules of PCM

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ABSTRACT
A theoretical study of storage thermal energy using capsulated cylinders filled with phase change material PCM is performed. These cylinders are arranged in-line in the direction of heat transfer fluid. The energy equations of fluid (water) and PCM capsules are solved numerically using finite volume method with heat capacity method for phase change of PCM. The effect of Reynolds number and the ratio of pitch to diameter of the cylinders on the temperature distribution and melt fraction are presented. The results show that the increment of both Reynolds number and the ratio of the pitch to diameter gives decrement in the final time of melting of PCM in the cylinders.

Keywords: Energy Storage, PCM, Finite Volume Method, Cylindrical Capsule

NOMENCLATURE
A area $m^2$  
c specific heat $J/kg K$
D  diameter of cylinder  m
Fo  Fourier Number (dimensionless time)
h  heat transfer coefficient  W / m² K
H  heat diffusion of PCM  J/kg
k  thermal conductivity  W / m K
l  the pitches between cylinders  m
L  length of domain  m
N  number cylinders in each column
Nu  Nusselt number
Pr  Prandtl number
r  radial coordinate of cylinder  m
Re  Reynolds number
Ste  Stephan number
t  time  sec
T  temperature  °C
u  velocity  m/s
V  volume  m³
x  axial coordinate in the direction of flow  m

GREEK SYMBOLS
α  thermal diffusivity  m² / s
θ  dimensionless temperature
δT  temperature difference
μ  viscosity  kg / m s
ρ  density  kg / m³

SUBSCRIPTS
f  fluid
i  initial, local in the x-direction
in  inlet
l  liquid state of PCM, longitudinal
m  melting
N  north neighboring node
n  north control volume (cell face)
p  phase change material
P  central node
ps  solid state of PCM
S  south neighboring node
s  south control volume (cell face), solid
t  transverse
x  x-direction
INTRODUCTION

T
hermal storage with PCM have a wide space in the domestic and practical applications. Phase change materials have a large melting heat, which can be released or stored at solidification or melting. The latent heat thermal energy storage systems using PCM capsules are used for applications such as domestic hot water heating system using water, storage system for air conditioning, solar thermal energy storage and waste heat recovery systems. The PCM melting temperature should be within the operating temperature of the thermal system, and the latent heat should be high enough to store a large quantity of energy in relatively small volume. Low melting temperature PCMs (Tm<35°C) are typically used in refrigeration systems, whereas high melting temperature PCMs (Tm>50°C) are often used in solar systems. A review for large number of PCMs that melt and solidify at a wide range of temperatures, making them attractive in a number of application is introduced by Sharma et al. [1] 2009. Also, they summarized the investigation and analysis of the available thermal energy storage systems incorporating PCMs for use in different applications. There are several studies that investigate the spherical packed bed latent heat thermal energy storage system or granules containing PCM [2,3,4,5 &6]. Also, phase change around a finned represents another type of storages energy using PCM [7&8]. Alawadhi [9], 2009, simulated transient laminar flow past an in-line cylinders array containing phase change material (PCM) using finite element method. The results indicated that the Reynolds number has a significant effect on the PCM melting time, whereas the pitch to the diameter ratio has an insignificant effect. The fixed moving front in the a horizontal circular cylinder of constant wall temperature and in the presence of the natural convection is presented by Ismail and Silva [10] 2003. They used implicit scheme for the momentum and energy equations and an explicit scheme for the energy balance at the interface. Rathod and Banerjee [11] 2011, investigated numerically the freezing time of PCM packed in three different shaped containers viz. rectangular, cylindrical and cylindrical shell using enthalpy method. Results showed that for the same mass of PCM and surface area of heat transfer, cylindrical shell container takes the least time for freezing the PCM. Also, Ismail and Moraes [12], 2009, studied experimentally and numerically the solidification of different phase change materials (PCM) encapsulated in spherical and cylindrical shell of different materials and diameters subjected to constant surface temperature.

The aims of the present model are: to investigate the energy storage in the cylindrical capsules of PCM using heat capacity method and solve the energy balance equation of heat transfer fluid and energy equation of cylindrical capsules PCM, and to find the parameters that affect on this process.

MATHEMATICAL MODELING

The present model represents a rectangular container filled with capsulated cylindrical rods of PCM. The rods has diameter D and length L and the number of cylinders in each row and column equal to N=50 as shown in Fig.1. Also, the longitudinal pitch is equal to transverse pitch \( S_l = S_t = l \). The heat transfer fluid HTF is the water and flow with uniform inlet velocity \( u_{in} \) and the inlet temperature \( T_{in} \), and to simplify the analysis one column is taken in this study in the direction of flow because the symmetry of columns.
The natural convection is negligible because the PCM cylinders are relatively small, also the conductive resistance of solid surface (wall) of cylinders is negligible. The energy balance equation of heat transfer fluid and energy equation of PCM cylinders are solved numerically using finite volume method. The latent heat of phase change of PCM is accounted for by making the capacity of PCM a function of temperature which called heat capacity method. Also, the melting of PCM is occurred in the range of temperature $T_m - \delta T_m$ to $T_m + \delta T_m$.

Assumptions
1-Constant properties of the heat transfer fluid HTF.
2-The insulated tank contains cylindrical rods of phase change materials.
3-Axial variation of HTF temperature (1D); that means the temperature of HTF is independent of radial position. Also, The row of cylinder has been divided into N elements (small elements, $N = L/l; l = \Delta x$), these elements can be assumed to has a single temperature for HTF.
4-The radial variation of PCM temperature (1D) only (the length of cylinder is long, and symmetry with angular direction).
5-No heat generation and dissipation

According to the above assumptions the governing equations are:
1-The energy balance of heat transfer fluid,

$$-u_m A_x \rho_f c_f \frac{\partial T_f}{\partial x} \Delta x = h \pi D.1 (T_f - T_p) + \rho_f V_f c_f \frac{\partial T_f}{\partial t}$$

... \(1\)

2-The energy equation of PCM,

$$\rho_p c_p \frac{\partial T_p}{\partial t} = k_p \left[ \frac{\partial^2 T_p}{\partial r^2} + \frac{1}{r} \frac{\partial T_p}{\partial r} \right]$$

... \(2\)

Where:
- the density of PCM $\rho_p$ is constant.

and \([13]\)

$$c_p = \begin{cases} c_s & T_p < T_m - \delta T_m \\ \frac{c_s + c_l}{2} + \frac{H}{2\delta T_m} (T_m - \delta T_m \leq T_p \leq T_m + \delta T_m) & T_m - \delta T_m \leq T_p \leq T_m + \delta T_m \\ c_l & T_p > T_m + \delta T_m \end{cases}$$

... \(3a\)
Numerical Investigation of Energy Storage in Packed Bed of Cylindrical Capsules of PCM

\[
\begin{align*}
   k_p = \begin{cases} 
   k_s & T_p < T_m - \delta T_m \\
   k_s + \frac{k_i - k_s}{2\delta T_m} [T_p - T_m + \delta T_m] & T_m - \delta T_m \leq T_p \leq T_m + \delta T_m \\
   k_i & T_p > T_m + \delta T_m
   \end{cases}
\end{align*}
\]

\[T_f = T_p = T_i \text{ at } t = 0 \text{ and } 0 < x < L \text{ and } 0 < r < D / 2\]

and the initial conditions:

and the boundary conditions can be written as,

for heat transfer fluid

\[T_f = T_{in} \text{ at } x = 0 \text{ (inlet)}\]

for PCM,

\[T_p = T_p \bigg|_{r=D/2}\]

\[\frac{\partial T_p}{\partial r} = 0 \text{ at } r = 0\]

using the following dimensionless form

\[R = \frac{r}{D}, \quad X = \frac{x}{D}, \quad \Theta = \frac{T - T_m}{T_m - T_m}, \quad F_0 = \frac{\alpha_{ps} t}{D^2},\]

\[\text{Re} = \frac{u_in \rho_f D}{\mu_f}, \quad \text{Pr} = \frac{\mu_f c_f}{k_f}, \quad \alpha_f = \frac{k_f}{\rho_f c_f},\]

\[C = \frac{\rho_p c_p}{\rho_{ps} c_{ps}}, \quad K = \frac{k_p}{k_{ps}}, \quad \epsilon = \frac{\pi D^2 / 4}{l^2},\]

\[K_f = \frac{k_f}{k_{ps}}, \quad \alpha_{ps} = \frac{k_{ps}}{\rho_{ps} c_{ps}}, \quad \alpha_r = \frac{\alpha_f}{\alpha_{ps}}\]

\[C_i = \frac{\rho_i c_i}{\rho_{ps} c_{ps}}, K_i = \frac{k_i}{k_{ps}}, \quad \text{Ste} = \frac{c_{ps} (T_{in} - T_m)}{H},\]

\[\text{Nu} = \frac{hD}{k_f} \quad \text{and} \quad l = \frac{L}{N}.\]

The governing equations in the dimensionless form are:

- Heat transfer fluid

\[\frac{\partial \Theta_f}{\partial F_0} = -\text{Re} \text{ Pr} \alpha_f \frac{\partial \Theta_f}{\partial X} - \frac{4 \text{ Nu} \alpha_r \epsilon}{1 - \epsilon} (\Theta_f - \Theta_p)\]

\[\ldots (4)\]
Where $Nu$: Nusselt number for flow over cylinder [14]

$$Nu = C_1 C_2 \frac{Re_{D,\text{max}}^n}{Pr} (Pr/Pr_w)^{0.25}$$  \hspace{1cm} \text{(5)}$$

where

- $Pr_w$: Prandtl number at the wall temperature of cylinder capsule of PCM.
- and [14]

- $C_1 = 1.0$ for $N > 16$, $Pr/Pr_w = 1,$
- $m = 0.36$, $n = 0.5$ and $C_2 = 0.52$

for $10^2 < Re < 10^3$

$$Re_{D,\text{max}} = \frac{u_{\text{max}} \, \rho_f \, D}{\mu_f}$$

$$u_{\text{max}} = \frac{l}{l-D} u_{\text{in}}$$  \hspace{1cm} \text{(6)}$$

Where:

- $u_{\text{max}}$: maximum velocity over the surface of cylinder.
- Phase change material PCM

$$\frac{\partial \theta_p}{\partial \Omega} = \frac{K}{C} \left( \frac{\partial^2 \theta_p}{\partial R^2} + \frac{1}{R} \frac{\partial \theta_p}{\partial R} \right)$$

Where:

$$C = \begin{bmatrix} 1 & \theta_p < \theta_m - \theta_m \\ \frac{1 + C_1}{2} + \frac{1}{2 Ste \theta_m} & \theta_m - \theta_m \leq \theta_p \leq \theta_m + \theta_m \\ C_1 & \theta_p > \theta_m + \theta_m \end{bmatrix}$$

\hspace{1cm} \text{\textbf{...(7a)\textbf{)}}}

and

$$K = \begin{bmatrix} 1 & \theta_p < \theta_m - \theta_m \\ 1 + \frac{K_1 - 1}{2 \delta T_m} [\theta_p + \delta \theta_m] & \theta_m - \theta_m \leq \theta_p \leq \theta_m + \delta \theta_m \\ K_1 & \theta_p > \theta_m + \delta \theta_m \end{bmatrix}$$

\hspace{1cm} \text{\textbf{...(7b)\textbf{)}}}
where:
\[
\delta \theta_m = \frac{\delta T_m}{T_{in} - T_m}
\]

The initial and boundary conditions in the nondimensional form are:
Initial conditions
\[
\theta_f = \theta_p = \theta_i \text{ at } Fo = 0 , \ 0 < X < L/D
\]
and \(0 < R < 1/2\)

the boundary condition for heat transfer fluid;
\[
\theta_f = 1 \text{ at } X = 0 \ (inlet)
\]
for PCM,
\[
\theta_p = \theta_p \bigg|_{R=1/2}
\]
\[
\frac{\partial \theta_p}{\partial R} = 0 \text{ at } R = 0
\]

**NUMERICAL SOLUTION**

The unsteady equation of the cylindrical capsulated of PCM is solved simultaneously with the energy balance equation of heat transfer fluid using explicit method for time with finite volume method [15] applying backward difference for first order spatial differential and central difference for second order spatial differential. The discretization equations of PCM is,
\[
\theta_{p,n+1} = A_p \theta_{p,n} + A_N \theta_{p,n} + A_S \theta_{p,n}
\]
\[
\ldots(8)
\]

Where:
\[
A_p = 1 + \frac{K \Delta Fo}{C \Delta V} \left[ -\frac{A_n}{\delta R} - \frac{A_i}{2R_p} + \frac{A_s}{2R_s} \right]
\]
\[
A_N = \frac{K \Delta Fo}{C \Delta V} \left[ \frac{A_n}{2R_n} + \frac{A_n}{2R_n} \right]
\]
\[
A_S = \frac{K \Delta Fo}{C \Delta V} \left[ \frac{A_s}{\delta R} - \frac{A_s}{2R_s} \right]
\]

Where: (as shown in the Figure.1)
\[
A_n = 2\pi R_n \cdot 1
\]
\[
A_i = 2\pi R_s \cdot 1
\]
\[
\Delta V = 2\pi R_p \cdot 1 \cdot \delta R
\]

from the heat balance at the surface of cylinder between the HTF and cylindrical container of PCM obtain,
\[ \theta_{p,NJ} = \frac{(Nu K_f \delta R/2K)\theta_{f,i} + \theta_{p,NJ-1}}{1 + (Nu K_f \delta R/2K)} \]  

...(9)

Where NJ: number of nodes in radial direction, (at the wall of cylindrical capsule).

At the center, applying L' Hospital’s rule [16], the Eq.6 becomes,

\[ \frac{\partial \theta}{\partial Fo} = \frac{2K}{C} \left( \frac{\partial \theta}{\partial R} \right) \]  

...(10)

and applying finite volume method becomes,

\[ \theta_{p,2}^{n+1} = A_P \theta_{p,2}^n + A_N \theta_{p,3}^n \]  

...(11)

Where:

\[ A_P = 1 + \frac{K \Delta Fo}{C \Delta V} \left[ -A_n \right] \]

\[ A_N = 2K \Delta Fo \left[ \frac{A_n}{\delta R} \right] \]

The rate of storage energy of heat transfer fluid is small in the control volume, therefore the energy balance of heat transfer fluid at an instant of time becomes [5],

\[ \frac{Re Pr \alpha_f}{1-\varepsilon} \frac{\partial \theta}{\partial X} + \frac{4 Nu \alpha_e}{1-\varepsilon} (\theta_f - \theta_p) = 0 \]  

...(12)

Simplifying and integration equation (12) gives,

\[ \theta_{f,i+1} = \theta_{p,i} + (\theta_{f,i} - \theta_{p,i}) e^{-\frac{4NuX}{Re Pr}} \]  

...(13)

where :

\[ \theta_{p,i} : \text{the surface temperature of PCM, at } i \text{ axial local.} \]

The resulting algebraic equations (8 & 9) are linear and solved using explicit method.

**STABILITY OF NUMERICAL SOLUTION**

For stability the solutions of governing equations, all coefficients must be positive in the discretized energy equation 8. The size of the network chosen affects the accuracy of the estimated derivatives and, therefore, the accuracy of the result. In general, the smaller the intervals \( \delta R \) and \( \Delta Fo \), the more accurate is the result. The coefficients of the temperatures in descritization equations indicate the magnitude of the effect of the various temperatures on \( \theta_{p,p}^{n+1} \), it must be concluded that none of these coefficients may be negative. A negative coefficient would mean that a high temperature at any point, or in its vicinity, would tend to produce a lower subsequent temperature at that point. This often leads to instability in the numerical procedure. It may cause temperatures to oscillate, giving an unreasonable physical behavior, therefore, a coefficient of zero is the
minimum acceptable value. The coefficient of $\theta_{p,P}$ must be equal to or greater than zero, therefore, $\Delta F_0$ can be determined as follow:

To solve equation (8) must be:

$$A_p = 1 + \frac{K \Delta F_0}{C \Delta V} \left[ -\frac{A_n}{\delta R} - \frac{A_s}{\delta R} + \frac{A_n}{2R_n} - \frac{A_s}{2R_s} \right] \geq 0$$

and

$$\Delta F_0 \leq \frac{C \Delta V}{K} \left[ -\frac{A_n}{\delta R} - \frac{A_s}{\delta R} + \frac{A_n}{2R_n} - \frac{A_s}{2R_s} \right]$$

RESULTS AND DISCUSSION

To check the validity of present model, the results of the present model are compared with the results of Alawadhi [9]. The comparison based on the total dimensionless time $F_0$ of row of six cylinders exposed to stream of air $(Pr=0.7)$ and the cylinders are capsulated with n-octadecane as PCM Table (1).

Table (2) shows this comparison which gives the acceptable agreement.

The results are presented as dimensionless temperature distribution of PCM $\theta_p$ and melt fraction MF with dimensionless time (Fourier number). The water is used as heat transfer fluid $(Pr=5.83)$ and n-octadecane [9] as PCM. Table 1 shows the properties of PCM. The number of cylinders of PCM at each column are 50 and the dimensionless temperatures for inlet heat transfer fluid, melting of PCM and initial of both fluid and PCM are: $\theta_{in}=1$, $\theta_m = -\delta \theta_m$ to $\delta \theta_m$ $(\delta \theta_m = 0.01)$ and $\theta_i = -1$ respectively.

Also, Stephan number for all results is equal to Ste=0.3179.

Figure (2a&b) shows the variation of dimensionless temperature of PCM $\theta_p$ with dimensionless time $F_0$ for cylinders 5, 25 and 50 and final $F_0=0.1662$, 0.5194 at the radial local $R=0.0921$ near the center of cylinders and $Re=200$, $I/D=2.0$. This figure shows that the temperature of PCM is increased gradually with time (sensible heat range for solid PCM) and then approximately remains constant in the latent heat range and then increased rapidly to reach to the inlet temperature (sensible heat range of liquid PCM). Also it appears that the cylinder5 is converted to liquid faster than cylinder25 and cylinder50 because the first cylinders are exposed to hotter heat transfer fluid than the last cylinders. Increment the $F_0$ from 0.1662 to 0.5194 gives more time to change the PCM from solid to liquid. The same behaviors of Fig.2 are happened with Figure (3a&b) which illustrates the variation of the melt fraction MF of the PCM with dimensionless time $F_0$ for cylinders 5, 25 and 50 and final $F_0=0.1662$, 0.5194 at the radial local $R=0.0921$ near the center of cylinders and $Re=200$, $I/D=2.0$. It can be seen that the MF of cylinder50 is reached to 1 at the last.
Figure (4a&b) presents the effect of Reynolds number (Re= 200, 500 and 1000) on the variation of dimensionless temperature of the PCM $\theta_p$ and the variation of the melt fraction MF with dimensionless time Fo for cylinders 25 at R=0.0921 and $l/D=2.0$. It is clear that the increment of the Reynolds number gives decrement in the final time to melt the PCM because of increasing the velocity and then the mass flow rate of heat transfer fluid with increasing Re.

Figure (5a&b) presents the effect of the ratio $5.22, 5.1$ and $D_l$ on the variation of dimensionless temperature $\theta_p$ and the variation of the melt fraction MF with dimensionless time Fo for cylinders 25 at R=0.0921 and Re=200. The figure shows that the increment of $l/D$ leads to decrease in the final time to melt the PCM which attributed to increase the total mass flow with increasing the pitch.

Finally, the distribution of dimensionless temperature of PCM with R is presented in the Figure (6). For cylinders 5, 25 and 50 and Fo=0.1662, Re=200 and $l/D=2$. It is clear from the Figure that the minimum temperature at the center of cylinders R=0 and the temperature increased toward the outer surface of cylinders.

CONCLUSIONS

The energy storage in cylindrical capsules is investigated numerically and the governing equations of heat transfer fluid and PCM are solved applying finite volume method and heat capacity method for PCM. It can concluded from this study:

1- The first cylinders in the column is melt faster than the last cylinders in the column, and the final time is computed depending on the complete melting of last cylinder (50).

2- The temperature of PCM is hotter in the center of PCM cylinder and colder in the outer surface.

3- Increment of Reynolds number gives the decrement in the total time of melting PCM.

4- The ratio $l/D$ gives behavior similar to Reynolds number.

REFERENCES


Table (1) Properties of PCM.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting temperature $T_m$ °C</td>
<td>47</td>
</tr>
<tr>
<td>Heat of fusion $H$ kJ/kg</td>
<td>243.5</td>
</tr>
<tr>
<td>Specific heat capacity $c$ kJ/kg.K (solid)</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>4.3 (liquid)</td>
</tr>
<tr>
<td>Thermal conductivity $k$ W/m.K (solid)</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.24 (liquid)</td>
</tr>
<tr>
<td>Density $\rho$ kg/m$^3$</td>
<td>800 (solid and liquid)</td>
</tr>
</tbody>
</table>

Table (2) the comparison of the present results with results of Alawadhi [9].

<table>
<thead>
<tr>
<th>No. of cylinder</th>
<th>Cylinder1</th>
<th>Cylinder2</th>
<th>Cylinder3</th>
<th>Cylinder4</th>
<th>Cylinder5</th>
<th>Cylinder6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re=250</td>
<td>present</td>
<td>0.376051</td>
<td>0.38933</td>
<td>0.418022</td>
<td>0.454696</td>
<td>0.509239</td>
</tr>
<tr>
<td>$l/D = 2$</td>
<td>Alwadhi</td>
<td>0.311442</td>
<td>0.410931</td>
<td>0.470408</td>
<td>0.519071</td>
<td>0.551513</td>
</tr>
<tr>
<td>Re=1000</td>
<td>present</td>
<td>0.205987</td>
<td>0.211077</td>
<td>0.224166</td>
<td>0.242344</td>
<td>0.267069</td>
</tr>
<tr>
<td>$l/D = 1.5$</td>
<td>Alwadhi</td>
<td>0.202221</td>
<td>0.270349</td>
<td>0.302791</td>
<td>0.331989</td>
<td>0.351454</td>
</tr>
</tbody>
</table>

Figure (1) Sketch of the physical model (side view).
Figure (2) The variation of temperature of PCM $\theta_p$ with dimensionless time (Fo), at $Re=200$ and $l/D=2$. For the total dimensionless time, a: $Fo=0.1662$, b: $Fo=0.5194$. 
Figure (3) The variation of melt fraction $MF$ of PCM with dimensionless time ($Fo$), at $Re=200$ and $l/D = 2$. For the total dimensionless time, a: $Fo=0.1662$, b: $Fo=0.5194$. 
Figure (4 a) The variation of temperature $\theta_p$ of PCM with dimensionless time ($Fo$), (b) The variation of melt fraction MF of PCM with dimensionless time ($Fo$), for different values of Reynolds numbers and $l/D = 2$. 
Figure (5a) The variation of temperature $\theta_p$ of PCM with dimensionless time (Fo), (b) The variation of melt MF fraction of PCM with dimensionless time (Fo), for different values of $l/D$ and Re=200.
Figure (5) The variation of temperature $\theta_p$ of PCM with radial locations $R$, at $Fo=0.1662$, $l/D = 2$ and $Re=200$. 