The Dynamic Coefficients and Elastic Deformation with Thermal Effect For Cylindrical Pivot Tilting 5-Pad Bearing

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Abstract

The paper describes the results of theoretical study of the tilting pad bearing consist of five pads each pad can be tilting 0.2° about a cylindrical pivot at high speed of rotating journal. The total oil film thickness includes elastic deformation with thermal effect had been estimated. By using finite element method to solve the Reynolds equation for dynamic load, from this solution generated pressure, components of oil film forces and moments are expressed as functions of journal co-ordinates, dynamic coefficients (stiffness and damping coefficients) these coefficients of oil film can be determined by differentiation of the oil film force finally elastic deformation of pad surface was obtained. This paper also includes the study of many parameters such as pivot offset, preload factor, direction of load (load on pad and load between load) and dynamic coefficients were presented as functions of Sommerfeld number.

Results show that good bearing performance when the pad preload factor is increase and pivot offset decreases.

Keywords: Tilting pads bearing, Cylindrical pivot, Dynamic coefficients, Elastic deformation, Pivot offset

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Notation

- $C_p$ Radial pad clearance $C_p = (R_p - r)$ m
- $C_r$ Radial bearing clearance $C_r = (R - r)$ m
- $D$ Diameter of bearing m
- $d_{L-P}$ Distance between the leading edge of the pad and the pivot m
- $e$ Journal eccentricity m
- $f_p$ Fractional angular position of the pad pivot degree
- $H$ Heat transfer coefficient Wm$^{-2}$C$^{-1}$
- $h$ Oil film thickness is measured at any angle ($\theta$) m
- $h_{min}$ Minimum inlet oil film thickness m
- $h_{in}$ Inlet oil film thickness for current pad m
- $h_{rad}$ Oil film thickness due to radial displacement m
- $O_j$ Center of journal
- $O_b$ Center of bearing
- $O_p$ Point pre-tilt position of the pad center of curvature
- $O_p^*$ Point post-tilt position of the pad center of curvature
- $PF$ Preload factor
- $L$ Axial pad length m
- $NPE$ Number of nodes per element
- $R$ Bearing radius m
- $r\Delta_p$ Pad arc length m
- $r$ Journal radius m
- $R_p$ Pad radius of curvature m
- $S$ Sommerfeld number
- $u_j$ Journal surface speed m/s
- $t_p$ The distance between the pivot point and the surface of the pad m
- $W$ Load kN
- $\Delta \theta$ Increment in pad angle degree
- $\Delta$ Total angular length of the pad degree
- $\mu_0$ Lubricant supply viscosity N/m$^2$ sec.
- $\theta_c$ The angle to the line-of-centers which connects $O_b$ to $O_j$ degree
- $\theta$ Circumferential angle degree
\( \theta_s \) \hspace{1cm} \text{Start angle of the pad}\degree

\( \theta_e \) \hspace{1cm} \text{End angle of the pad and}\degree

\( \delta \) \hspace{1cm} \text{Tilting angle}\degree

\( \omega \) \hspace{1cm} \text{Angular speed}\ rad/sec

**Introduction**

Dynamics of high speed rotating machinery depend strongly on journal bearing types. Tilting pad journal bearings (TPJB) are dominant as shaft support in such machinery, tilting pad bearings can be classified into the following categories based on the type of pivots they employ [1]:

(i) Point contact (Sphere in cylinder).

(ii) Line contact (Rocker back or axial pin type of pivot) (iii) Surface contact (Ball in Sphere). It may be noted that they are arranged in the increasing order of load carrying capacity based on pivot strength; two distinct factors limit the load carrying capacity of a tilting pad bearing, viz the maximum pad metal/oil film temperature and the pivot stresses. Tilting pad journal bearings can either be of a load on pad (LOP) or of a load between pads (LBP) configuration [2]. Waldemar (2005) [3], analyzes the variations of the stiffness and damping characteristics for the tilting pad journal bearings with the frequency of excitation and describes the analytical and experimental techniques used to evaluate these properties. The dynamic characteristics of tilting 12-pads journal bearing were also studied by Olszewsski and etal (2003) [4], this study led to the following conclusions: the number of tilting-pads can be basically 3 to 5 depending on the required operating parameters of rotating machine. Large number of pads allowed good cooling of bearing and the load capacity increases with the increase of relative clearance of pad. Stanislaw and Zygmunt (2003) [5], the paper introduces the results of calculation of dynamic coefficients of the tilting pads journal bearing characterized by asymmetric support of pads at different temperature of supplied oil as well as at different thermal conditions between the pads bearing.

This paper analyzes the variations of the load carrying capacity, dynamic coefficients (stiffness and damping coefficients) and elastic deformation with thermal effect for the tilting 5-pad journal bearings having cylindrical pivot, also study the effects of the preload factor, pivot offset and the load directions on the pads.

**Tilting Pad Bearing Geometry**

Figure (1) show the configurations for tilting pad bearing geometry with of five shoes, configuration (a) show load between pad (LBP) and (b) show load on
pad (LOP). Each pad is able to rotate about a pivot thus attaining its own equilibrium position, with a strongly converging film region for each loaded pad. Tilting pad bearing have no cross-coupled stiffness for certain load conditions and offer an operation free of sub synchronous whirl [2]. Table (1) shows the tilting pads bearing details using in this paper.

**Total Oil Film Thickness with Cylindrical Pivot**

A single pad of a TPJB with a cylindrical pivot can be illustrated as shown in Figure (2), pad is depicted before (dotted line) and after (solid line) tilting it through an angle (δ).

\[
h_{\text{total}} = h_{\text{oil}} + h_{r.d}
\]

(1)

Where: \( h_{r.d} \) is the oil film thickness due to radial displacement.

This pad is located at an angular position (\( \theta_p \)) relative to a stationary coordinate system with origin \((O_b),(O_c)\) represents the angle to the line-of-centers which connects \((O_b)\) to \((O_j)\). In the TPJB case defines (\( \theta \)) with respect to \((O_b)\) and \( h_{oil} \) stationary coordinate system, with the counter-clockwise direction being positive. the tilting pad journal bearing (TPJB) film thickness expression,[ 6].

\[
h_{\text{oil}} = O_b N - O_b J
\]

(2)

\( O_b N = R_e - (R_c - R) \cos \theta - \theta_p) - \delta (R_e + t_e) \sin \theta - \theta_p) \)

\( O_b J = r - e \cos(\theta_e - \theta) \)

(3)

\( h_1 = C_p - C_p PF \cos(\theta - \theta_p) \)

\( h_2 = (r + t_p) \delta \sin(\theta - \theta_p) + e \cos(\theta_e - \theta) \)

(4)

\( h_{oil} = h_1 + h_2 \)

(5)

**Equilibrium Journal Position**

The pressure distribution must first be obtained by numerically solving the Reynolds’ equation (dynamically loaded equation). This pressure must then be used to balance the forces on the journal and the moments on each pad in the TPJB so that the equilibrium position of the journal can be determined.

**Pressure Distribution**

Reynolds’ equation (6) can be solved over the domain of each pad surface in the TPJB to give the pressure distribution. A typical meshed pad surface with essential pressure boundary conditions.

The finite element method will be chosen for simplicity. In this method, basis functions are used to approximate the exact solution to a differential equation, [7].

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial P}{\partial z} \right) = 6u \frac{\partial h}{\partial x} + 12 \frac{\partial^2 h}{\partial t^2}
\]
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\[ \frac{\partial h}{\partial t} = X_s r \theta_s + X_e r \theta_e \]

\[ \sum F_x = \sum_{i=1}^{N_{pads}} \int_{x_i}^{x_{i+1}} (P \cdot \cos \theta) dx \cdot dz = 0 \quad \ldots \quad (10a) \]

\[ \sum F_y = \sum_{i=1}^{N_{pads}} \int_{x_i}^{x_{i+1}} (P \cdot \sin \theta) dx \cdot dz - W_p = 0 \quad \ldots \quad (10b) \]

Parameters Controlling the Elastic Deformation and Dynamic Coefficients

(i) Pivot Offset (\( \alpha \))

\[ \alpha = \frac{\theta_p}{\beta} \quad \ldots \quad (11) \]

Where:

- \( \theta_p \) : is the pivot angle measured from the pad leading edge.
- \( \beta \) : is the pad extent angle.

When the pivot offset (\( \alpha \)) is equal 0.5 the pads are centrally pivoted. Offset pivot bearings with (\( \alpha \)) large than 0.5 are used in rotating machinery which will not run in reverse. The offset pivot ensures a converging wedge for all conditions.[7].

(ii) Bearing Preload Factor (PF)

Preload is often used to adjust bearing coefficients in order to obtain specific rotor response characteristics. The preload can vary from 0 to 1, with
each pad having different radii and hence different preload [7].

\[ PF = \frac{C_p - C_r}{C_p} \quad \ldots \quad (12) \]

(iii) External Loading Condition

The direction of loading also affects the static and dynamic force characteristics of a tilting pad bearing. The load can either be directed towards on a pad (LOP), or directed between the pads (LBP). The (LOP) configuration is chosen for lightly loaded bearings operating at high speed cases, whereas the (LBP) configuration is chosen for heavily loaded bearings [8].

(vi). Pivot Flexibility

The contact between a bearing pad and the supporting pivot occurs through a small contact area which can be easily susceptible to wear. Most tilting pad pivots show non-linear stiffness with a strong dependency on the applied load and must be evaluated by Hertzian contact models, [2].

\[ d_{\text{pivot}} = \frac{F_{\text{pivot}}}{K_{\text{pivot}}} \quad \ldots \quad (13) \]

Where: \( K_{\text{pivot}} \) = pivot stiffness,
\( d_{\text{pivot}} \) = pivot deflection
\( F_{\text{pivot}} \) = component of the applied load passing through the pivot

Thermal Distribution over Pad

Equation (4) can now be solved by specifying an inlet temperature at the leading edge of each tilted pad. The inlet temperature \( T_{in} \) at one of the pads is taken to be the sum of the lubricant supply temperature and the estimated average temperature rise \( \Delta T_{av} \) [9].

\[ H = 25.5 \, u_j^{0.7} \, \mu_e^{-0.2} (r \, \Delta p)^{-0.4} \quad \ldots \quad (16) \]

Elastic and Thermal Deformation of Pads

The deformation of elastic structures is described by means of the constitutive relation and equilibrium equation. The constitutive equation for a linearly elastic material is given as, [10].

\[ \sigma = C \, \varepsilon + \sigma_0 \quad \ldots \quad (17) \]

The equilibrium equation for a stationary elastic media is given by the following:

\[ \nabla \cdot \sigma + b = f \quad \ldots \quad (18) \]

Where:
\( \sigma \) = the stress tensor and strain tensor respectively.
\( \varepsilon \) = The residual stress tensor \( \sigma_0 \)

(typically due to thermal effects).
\( b, f \) = the body force vector and load vector respectively.

The pad deformation is composed as the linear...
superposition of elastic deflection due to the hydrodynamic pressure and thermal expansions due to the temperature gradients. The thermal distribution from solution to pad conduction equation is applied to obtain thermal expansion, while the oil pressure is integrated to obtain forces; these forces are resolved into components in the X and Y directions and applied on the pad surface to obtain mean deformations. Sommerfeld number is widely used as a characteristic number for journal – bearing equation (19) which relates the operating variables, viscosity, speed and bearing clearance , [8].

Evaluation of the Dynamic Coefficients

Tilting pad bearing stiffness and damping coefficients are obtained after bearing static equilibrium is achieved. The pivot and pad tilt degrees of freedom should be included in determining the dynamic force coefficients. The stiffness coefficients are given by perturbing the journal position (X and Y), pad tilt angle (δ) and pivot deformation equation (14). The changes in forces/ moments are obtained for these perturbations. The elastic deformation is represented as a change in the pad radius (Rp). The relationship for the stiffness coefficients is, [10].

\[ K_{ij}^{(eq)} = \frac{K_z \left( K_{ij} + K_z \right) - \omega^2 K_z C_{ij}^2}{\left( K_{ij} + K_z \right)^2 + \omega^2 C_{ij}^2} \]  
\[ C_{ij}^{(eq)} = \frac{K_z^2 C_{ij}}{\left( K_{ij} + K_z \right)^2 + \omega^2 C_{ij}^2} \]  
\[ K_z = m_p \omega^2 + K_{pivot} \]  

Results and Discussions

The results of this paper are illustrated below. Figure (3) shows how does the oil film thickness change with rotating angle (θ) for each pads surface, these variables values of oil film thickness depend on the position of the pad during the rotating operation, pad No.4 has maximum oil film thickness (hmax. =3.17 mm) and pad No.2 has minimum oil film thickness (hmin.=0.62 mm) these values are in the allowable range (by comparison these profile curves in Figure (3) with [12] for tilting 3-pads journal bearing, it is evident that both gave the same profile for oil film thickness but at different values).

Figure (4) shows the variations of generated pressure in the oil film.
thickness with rotating angle for each pad at load between pad and preload factor equal zero. From this figure it can be noted that, when there is an increases in the pivot offset (α) there is an increase in generated pressure. The position of maximum generated pressure over pad surface depends on the pivot offset (α) value.

Figures (5) and (6) shows the relationship between nondimensional Sommerfeld number and nondimensional dynamic bearing coefficients when the preload factor change from 0 to 0.5 values respectively for two types configurations load on pad and load between pads, when preload factor equal zero its more reasonable comparing with preload factor equal 0.5, because (Kxx) and (Cxx) are responsible about the stability of tilting pad bearing at high speed. The cross-coupled coefficients Kxy, Kyx, Cxy and Cyx are usually close to zero and negligible compared with the other linear coefficients, [7].

The Figures (7) and (8) illustrated the elastic deformation with thermal effect over surface of pad shell No.1 for load on pad configuration; at pivot offset are equal 0.6 and 0.5 respectively. It was found that the maximum elastic deformation in the pad surface occurs in the center of pivot for each figures are about 0.009125 mm and 0.008429 mm respectively. When comparison between these figures, figure (7) shows graded bedding of colors, that means the effect of elastic deformation is continuous for more area of pad surface.

Conclusions

From the results of this work the following conclusions can be obtained:

1- The maximum and minimum of the oil film thickness value depends on the type of bearing pivot and the angular position of pad.

2-When the pad preload factor is increase, the dynamic stiffness and damping coefficients will increase also; this result gives the best performance for bearing.

3-The bearing load carrying capacity increases when the pivot offset (α) increase, because when the pivot offset equal 0.5 this means the load carrying distributed uniformly over the pad surface.

4-When pivot offset increases that mean the elastic deformation also increases (because the elastic deformation depends on the value of generated pressure).
References


[9]. San Andrés, L., and Jackson, M., "Measurements of the Static


Table (1) Tilting Pads Bearing Details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pads</td>
<td>5</td>
</tr>
<tr>
<td>Radius of bearing</td>
<td>5.4035 cm</td>
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<tr>
<td>Radius of journal</td>
<td>5.35 cm</td>
</tr>
<tr>
<td>Pad thickness</td>
<td>1.21 cm</td>
</tr>
<tr>
<td>Angular dimension of pad</td>
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</tr>
<tr>
<td>Leading edge of each pads (LOP)</td>
<td>59°, 131°, 203°, 275°, 347°</td>
</tr>
<tr>
<td>Leading edge of each pads (LBP)</td>
<td>23°, 95°, 167°, 239°, 311°</td>
</tr>
<tr>
<td>Pad tilt angle</td>
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<tr>
<td>Fractional angular position of pivot</td>
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</tr>
<tr>
<td>Preload factor</td>
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<tr>
<td>Bearing load</td>
<td>4 kN to 4.5 kN</td>
</tr>
<tr>
<td>Lubricant viscosity</td>
<td>0.04 N/m²·sec.</td>
</tr>
<tr>
<td>Pad Configuration</td>
<td>LOP and LBP</td>
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<tr>
<td>Length diameter ratio</td>
<td>0.55</td>
</tr>
<tr>
<td>Journal speed</td>
<td>9000 r.p.m</td>
</tr>
</tbody>
</table>

(a) Load Between Pad (LBP) (b) Load on Pad (LOP)

Figure (1) Configurations for Tilting 5-Pad Bearings

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Figure (2) Tilting Pad Bearing with Cylindrical Pivot, [7].

Figure (3) Oil Film Thickness Variation with Angular Position over 5-Pad
The Dynamic Coefficients and Elastic Deformation with Thermal Effect For Cylindrical Pivot Tilting 5-Pad Bearing

Figure (4) Pivot Offset Effect on the Distribution Pressure at Load Between Pad (LBP) and PF =0

Figure (5) The Preload Factor Effect on The Bearing Dynamic Coefficients
The Dynamic Coefficients and Elastic Deformation with Thermal Effect For Cylindrical Pivot Tilting 5-Pad Bearing

Figure (6) The Pad Loading Position Effect on The Bearing Dynamic Coefficients

Figure (7) Elastic Deformation (mm) with Thermal Effect Over Pad No.1 (LOP, PF=0, $\alpha=0.6$)
Figure (8) Elastic Deformation (mm) with Thermal Effect Over Pad No.1 (LOP, PF=0, α=0.05)