An important consideration in the choice of a material is the way it behave when subjected to force. The mechanical properties of a material are a measure of the resistance it show to the application of the basic type of force:

1. Tensile force
2. Compressive force
3. Shear Force

**Stress**

**Normal stress:**
The application of an external force to a body cause internal resisting force within the body, whose resultant is equal in magnitude but opposite in direction to the applied force.

If the a bar subjected to a longitudinal axial force causing internal resisting force distributed continuously over the cross section of the bar as shown in the free diagram. If the applied force passes through the centered of the cross section of the bar, the resisting force will be distributed uniformly over the cross section for resisting loads of geometrically similar members. The uniform distribution of internal resisting can be express in the form of force per unit area as:

\[
\text{Stress (}\sigma\text{)} = \frac{F}{A}
\]

Where \( F \) – The applied force
\( A \)- cross sectional area

And if the cross sectional area is normal to the direction of the load, the stress called “Normal stress”

**Bearing stress:**
Bearing stress occurs when there is contact between two bodies. The external applied force is known as bearing and the pressure between the two bodies is known as a bearing stress. Bearing stress occurs between the post and plate, the plate and footing and between the footing and soil.

**Shearing stress:**
Shearing stresses occur when the force being resisted act in the plane of the reacting area.

\[
T = \frac{\text{Shearing force}}{\text{Area being sheared}}
\]

**H.W.:**
1. An aluminum bar having a cross sectional area of 160 mm\(^2\) carries the axial loads at the positions shown in Fig. below., compute the stress at each parts of aluminum
2. Determine the stress in each section shown in Fig below

Deformation

When an engineering material is subjected to forces, their atoms may be change their equilibrium positions. The total change in a dimension due to an applied force is known as deformation (Δ).

1. Elastic deformation:
   If the atom can resume their equilibrium positions when the imposed force are released, the deformation is termed elastic. Elastic deformation that is recoverable and indicates the relative resistance of a material. **Elasticity**: is the property of a material to return to its initial form and dimension after the deforming force is removed.

2. Plastic deformation:
   If an engineering undergoes deformation which exceed the elastic capacity (elastic deformation), the deformation is permanent and termed plastic. Plastic deformation is non recoverable and leaves the atoms permanently displaced from their original positions when the forces are released. The process of plastic deformation is shown below:
**Strain**

**Engineering strain:**
When a member is subjected to a tensile or compressive stress, it undergoes a deformation ($\Delta$). Tensile force causes an elongation of the body, while compressive causes a shortening of the dimension of the body in the direction of the force. The elongation (or shortening) per unit length is called strain ($\varepsilon$).

Average strain ($\varepsilon$) = $\Delta / L_0$, mm/mm dimensionless

Where $L_0$ – is the original length

The strain at any position is more correctly named as true strain, i.e. the ratio of the change in dimensions to the instantaneous dimensions.

$\varepsilon_{true} = \ln (\varepsilon + 1)$

**Shear strain:**
It is defined as the ratio of displacement ($X$) to the distance between the planes ($h$).

Shear strain = $X/h = \tan \Theta$
Modulus of elasticity (E)

Hook's law states that in elastic bodies stress is proportional to strain provided that the elastic limit is not exceeded.

\[ \Delta \sigma = E \epsilon \]

Where E is the constant of proportionality relating stress and strain. It is sometimes known as Young's modulus or more commonly the modulus of elasticity. E has the same units of stress.

\[ \text{Stress/strain} = E \]

\[ \frac{F}{A} = \left( \frac{\Delta L}{L} \right) * E \]

\[ \Delta L = \frac{F * L}{E * A} \]

Modulus of rigidity (G)

Where G is the constant of proportionality relating shear stress and shear strain. It is sometimes known as the modulus of rigidity. G has the same units of stress.

\[ T = G \text{ Shear strain} \]

Poisson's ratio (\( \mu \))

Because of the constancy of volume, when a material is deformed in one direction, there is a corresponding displacement or deformation in a direction perpendicular to it.

For example, consider the bar in Fig. below. If the axial load is applied, the elongates in the X-direction. The ratio of the strain in Y-direction to the strain in X-direction is termed {Poisson's ratio (\( \mu \))}. And expressed as:

\[ \mu = \frac{\epsilon_y}{\epsilon_x} \]

Where:

\( \mu \)- Poisson's ratio
\( \epsilon_y \)- Lateral strain
\( \epsilon_x \)- Direct strain

H.W.:

1. A steel bar 6m long is 5cm in diameter for 3m of it's length and 2.5cm in diameter for the second 3m as shown in Fig. below. The smallest diameter part of the bar is with 11 N/mm² tensile stress due to the applied load. Find the final diameters of the two parts after applied load. E of the steel is 200*10³ N/mm² and \( \mu \) of the steel is 0.28
**General expression for strain**

We can obtain a general expression for the strain on an element subjected to tensile force in three perpendicular directions as shown in Fig.:

**Case 1:** When the tensile stress effect in X- direction only:
- A – Direct strain $\varepsilon_x = + \frac{\delta x}{E}$
- B – Induced strain due to X- stress:
  1. Induced strain in X – direction due to X- stress = 0
  2. Induced strain in Y – direction due to X- stress ($\varepsilon_y = - \mu \varepsilon_x = - \mu \left(\frac{\delta x}{E}\right)$)
  3. Induced strain in Z – direction due to X- stress ($\varepsilon_z = - \mu \varepsilon_x = - \mu \left(\frac{\delta x}{E}\right)$)

**Case 2:** When the tensile stress effect in Y- direction only:
- A – Direct strain $\varepsilon_y = + \frac{\delta y}{E}$
- B – Induced strain due to Y- stress:
  1. Induced strain in X – direction due to Y- stress ($\varepsilon_x = - \mu \varepsilon_y = - \mu \left(\frac{\delta y}{E}\right)$)
  2. Induced strain in Y – direction due to Y- stress = 0
  3. Induced strain in Z – direction due to Y- stress ($\varepsilon_z = - \mu \varepsilon_y = - \mu \left(\frac{\delta y}{E}\right)$)

**Case 3:** When the tensile stress effect in Z- direction only:
- A – Direct strain $\varepsilon_z = + \frac{\delta z}{E}$
- B – Induced strain due to Y- stress:
  1. Induced strain in X – direction due to Z- stress ($\varepsilon_x = - \mu \varepsilon_z = - \mu \left(\frac{\delta z}{E}\right)$)
  2. Induced strain in Y – direction due to Z- stress ($\varepsilon_y = - \mu \varepsilon_z = - \mu \left(\frac{\delta z}{E}\right)$)
  3. Induced strain in Z – direction due to Z- stress = 0

Therefore, generalized Hook's law equations in tensions are:

$$
\varepsilon_x = + \left(\frac{\delta y}{E}\right) - \mu \left(\frac{\delta y}{E}\right) - \mu \left(\frac{\delta z}{E}\right)
$$

$$
\varepsilon_y = - \mu \left(\frac{\delta x}{E}\right) + \left(\frac{\delta y}{E}\right) - \mu \left(\frac{\delta z}{E}\right)
$$

$$
\varepsilon_z = - \mu \left(\frac{\delta x}{E}\right) - \mu \left(\frac{\delta y}{E}\right) + \left(\frac{\delta z}{E}\right)
$$

Generalized Hook's law equations in compression are:

$$
\varepsilon_x = - \left(\frac{\delta y}{E}\right) + \mu \left(\frac{\delta y}{E}\right) + \mu \left(\frac{\delta z}{E}\right)
$$

$$
\varepsilon_y = + \mu \left(\frac{\delta x}{E}\right) - \left(\frac{\delta y}{E}\right) + \mu \left(\frac{\delta z}{E}\right)
$$

$$
\varepsilon_z = + \mu \left(\frac{\delta x}{E}\right) + \mu \left(\frac{\delta y}{E}\right) - \left(\frac{\delta z}{E}\right)
$$

**Temperature stresses**

If a bar is not restrained in any way, an increase in temperature will cause an increase in it's dimensions, and decrease in temperature will cause a decrease in it's dimensions. It is usual to describe the dimensional change due to due to temperature changes in terms of the change in a linear dimensions. Thus, the change in length of a bar, $\Delta L$, is directly proportional to both the temperature change of the bar, $\Delta T$, and the original length of the bar, $L_0$: 
The constant of the proportionality is called the linear coefficient of expansion (\( \alpha \)) which is defined as change in length per unit length for a one degree change in temperature.

The final length: 
\[ L_f = L_0 + \Delta L \]

\[
L_f = L_0 \{ 1 + \alpha (\Delta T) \}
\]

\[ \Delta A = 2\alpha W_0 H_0 (\Delta T) \]

Thus the coefficient of expansion of area expansion can be taken as twice the coefficient of linear expansion.

\[ \Delta V = 3\alpha W_0 H_0 L_0 (\Delta T) \]

The coefficient of expansion of volume expansion can be taken as three times the coefficient of linear expansion.

**Thermal strain**

Besides stresses, changes in temperatures can cause deformation of materials. For homogenous isotropic materials, a changes in temperature of \( \Delta T \) degree causes uniform linear strain in every direction. Expressed as an equation, the thermal strain are:

\[ \varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha \Delta T \]

Where \( \alpha \) – the coefficient of linear of thermal expansion

The linear thermal strain for small strains is directly additive to linear strains due to stress. On this basis a typical modification of the equation:

\[ \varepsilon_x = + (\frac{\delta y}{E}) - \mu (\frac{\delta y}{E}) - \mu (\frac{\delta z}{E}) \]

To include thermal strain is:

\[ \varepsilon_x = + (\frac{\delta y}{E}) - \mu (\frac{\delta y}{E}) - \mu (\frac{\delta z}{E}) + \alpha \Delta T \]

**H.W.:**
1. Derive the generalized Hook's law equation for the body shown in fig.

2. A rod 2m long expands by 1mm when heated from 8 °C to 70 °C. What is the coefficient of linear expansion of the material from which the rod is made?