ENGGIEERING STATISTICS

(Lectures)

University of Technology,
Building and Construction Engineering Department
(Undergraduate study)

PROBABILITY THEORY

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A random variable refers to a measurement or observation that cannot be known in advance.

Roman letter is used to represent a random variable, the most common letter being $X$. A lower case $x$ is used to represent an observed value corresponding to the random variable $X$. So the notation $X = x$ means that the observed value of $X$ is $x$.

The set of all possible outcomes or values of $X$ we might observe is called the sample space.

The set of all possible outcomes of a random experiment is called the sample space of the experiment. The sample space is denoted as $S$.

**EXAMPLE 1:**
Consider an experiment in which you select a plastic pipe, and measure its thickness.

Sample space as simply the positive real line because a negative value for thickness cannot occur

$$S = R^+ = \{ x \mid x > 0 \}$$

If it is known that all connectors will be between 10 and 11 millimeters thick, the sample space could be

$$S = \{ x \mid 10 < x < 11 \}$$
If the objective of the analysis is to consider only whether a particular part is *low, medium, or high* for thickness, the sample space might be taken to be the set of three outcomes:

\[ S = \{ \text{low}, \text{medium}, \text{high} \} \]

If the objective of the analysis is to consider only whether or not a particular part conforms to the manufacturing specifications, the sample space might be simplified to the set of two outcomes,

\[ S = \{ \text{yes}, \text{no} \} \]

that indicate whether or not the part conforms.

**A discrete random variable** meaning that there are gaps between any value and the next possible value.

**A continuous random variable** meaning that for any two outcomes, any value between these two values is possible.

**EXAMPLE 2:**

If two connectors are selected and measured, the sample space is depending on the objective of the study.

If the objective of the analysis is to consider only whether or not the parts conform to the manufacturing specifications, either part may or may not conform. The sample space can be represented by the four outcomes:
\[ S = \{ yy, yn, ny, nn \} \]

If we are only interested in the number of conforming parts in the sample, we might summarize the sample space as

\[ S = \{ 0, 1, 2 \} \]

In random experiments in which items are selected from a batch, we will indicate whether or not a selected item is replaced before the next one is selected. For example, if the batch consists of three items \( \{a, b, c\} \) and our experiment is to select two items \textbf{without replacement}, the sample space can be represented as

\[ S_{\text{without}} = \{ ab, ac, ba, bc, ca, cb \} \]

\[ S_{\text{with}} = \{ aa, ab, ac, ba, bb, bc, ca, cb, cc \} \]

**Events:**

Often we are interested in a collection of related outcomes from a random experiment.

An \textbf{event} is a subset of the sample space of a random experiment.

Some of the basic set operations are summarized below in terms of events:

- The \textbf{union} of two events is the event that consists of all outcomes that are contained in either of the two events. We denote the union as \( E_1 \cup E_2 \).

- The \textbf{intersection} of two events is the event that consists of all outcomes that are contained in both of the two events. We denote the intersection as \( E_1 \cap E_2 \).

- The \textbf{complement} of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the component of the event \( E \) as \( \bar{E} \).
EXAMPLE 3:

Consider the sample space $S \{yy, yn, ny, nn\}$ in Example 2. Suppose that the set of all outcomes for which at least one part conforms is denoted as $E_1$. Then,

$$E_1 = \{yy, yn, ny\}$$

The event in which both parts do not conform, denoted as $E_2$, contains only the single outcome, $E_2\{nn\}$. Other examples of events are $E_3 = \emptyset$, the null set, and $E_4 = S$, the sample space. If $E_5 = \{yn, ny, nn\}$,

$$E_1 \cup E_5 = S \quad E_1 \cap E_5 = \{yn, ny\} \quad \hat{E}_1 = \{nn\}$$

EXAMPLE 4:

Measurements of the time needed to complete a chemical reaction might be modeled with the sample space $S = R^+$, the set of positive real numbers. Let

$$E_1 = \{x \mid 1 \leq x < 10\} \quad \text{and} \quad E_2 = \{x \mid 1 < x < 118\}$$

Then,

$$E_1 \cup E_2 = \{x \mid 1 \leq x < 118\} \quad \text{and} \quad E_1 \cap E_2 = \{x \mid 3 < x < 10\}$$

Also,

$$\hat{E}_1 = \{x \mid x \geq 10\} \quad \text{and} \quad \hat{E}_1 \cap E_2 = \{x \mid 10 \geq x < 118\}$$
EXAMPLE 5:

Samples of concrete surface are analyzed for abrasion resistance and impact strength. The results from 50 samples are summarized as follows:

<table>
<thead>
<tr>
<th>abrasion resistance</th>
<th>impact strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>40</td>
</tr>
<tr>
<td>Low</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

Let \( A \) denote the event that a sample has high impact strength, let \( B \) denote the event that a sample has high abrasion resistance.

Determine the number of samples in \( A \cap B \), \( A' \), and \( A \cup B \)

The event \( A \cap B \) consists of the 40 samples for which abrasion resistance and impact strength are high. The event \( A' \) consists of the 9 samples in which the impact strength is low. The event \( A \cup B \) consists of the 45 samples in which the abrasion resistance, impact strength, or both are high.
Venn diagrams are often used to describe relationships between events and sets.

Two events, denoted as $E_1$ and $E_2$, such that

$$E_1 \cap E_2 = \emptyset$$

are said to be **mutually exclusive**.

The two events in Fig. 1(b) are mutually exclusive, whereas the two events in Fig. 1(a) are not. Additional results involving events are summarized below. The definition of the complement of an event implies that

$$1 \in E \cup 2 \notin E$$

The distributive law for set operations implies that

<table>
<thead>
<tr>
<th>Set theory</th>
<th>Probability theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space, $S$</td>
<td>Sample space, sure event</td>
</tr>
<tr>
<td>Empty set, $\emptyset$</td>
<td>Impossible event</td>
</tr>
<tr>
<td>Elements $a, b, \ldots$</td>
<td>Sample points $a, b, \ldots$ (or simple events)</td>
</tr>
<tr>
<td>Sets $A, B, \ldots$</td>
<td>Events $A, B, \ldots$</td>
</tr>
<tr>
<td>$A$</td>
<td>Event $A$ occurs</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>Event $A$ does not occur</td>
</tr>
<tr>
<td>$A \cup B$</td>
<td>At least one of $A$ and $B$ occurs</td>
</tr>
<tr>
<td>$A \cap B$</td>
<td>Both $A$ and $B$ occur</td>
</tr>
<tr>
<td>$A \subset B$</td>
<td>$A$ is a subevent of $B$ (i.e. the occurrence of $A$ necessarily implies the occurrence of $B$)</td>
</tr>
<tr>
<td>$A \cap B = \emptyset$</td>
<td>$A$ and $B$ are mutually exclusive (i.e. they cannot occur simultaneously)</td>
</tr>
</tbody>
</table>

**Probability** is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur. “The chance of rain today is 30%” is a statement that quantifies our feeling about the possibility of rain.
A 0 probability indicates an outcome will not occur. A probability of 1 indicates an outcome will occur with certainty.

![Diagram of event E with 100 elements and probability P(E) = 0.30 calculated as 30(0.01)]

**Fig. 2:** Probability of the event E is the sum of the probabilities of the outcomes in E.

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For a discrete sample space, the *probability of an event E*, denoted as $P(E)$, equals the sum of the probabilities of the outcomes in $E$.

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**EXAMPLE 6:**

A random experiment can result in one of the outcomes \{a, b, c, d\} with probabilities 0.1, 0.3, 0.5, and 0.1, respectively. Let $A$ denote the event \{a, b\}, $B$ the event \{b, c, d\}, and $C$ the event \{d\}. Then,

\[
P(A) = 0.1 + 0.3 = 0.4 \\
P(B) = 0.3 + 0.5 + 0.1 = 0.9 \\
P(C) = 0.1
\]

Also: $P(A') = 0.6$, $P(B') = 0.1$, $P(C') = 0.9$

$P(A \cap B) = 0.3$
$P(A \cup B) = 1$
$P(A \cap C) = 0$
EXAMPLE 7:
A visual inspection of a defects location on concrete element manufacturing process resulted in the following table:

<table>
<thead>
<tr>
<th>Number of defects</th>
<th>Proportion of concrete element</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
</tr>
<tr>
<td>5 or more</td>
<td>0.1</td>
</tr>
</tbody>
</table>

a) If one element is selected randomly from this process to inspected, what is the probability that it contains no defects?

The event that there is no defect in the inspected concrete elements, denoted as $E_1$, can be considered to be comprised of the single outcome, $E_1 = \{0\}$.

Therefore, $P(E_1) = 0.4$

b) What is the probability that it contains 3 or more defects?

Let the event that it contains 3 or more defects, denoted as $E_2$

$P(E_2) = 0.1 + 0.05 + 0.1 = 0.25$

EXAMPLE 8:
Suppose that a batch contains six parts with part numbers $\{a, b, c, d, e, f\}$. Suppose that two parts are selected without replacement. Let $E$ denote the event that the part number of the first part selected is $a$. Then $E$ can be written as $E \{ab, ac, ad, ae, af\}$. The sample space can be counted. It has 30 outcomes. If each outcome is equally likely,

$P(E) = 5/30 = 1/6$
ADDITION RULES

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

EXAMPLE 8:

The defects such as those described in Example 7 were further classified as either in the “center” or at the “edge” of the concrete elements, and by the degree of damage. The following table shows the proportion of defects in each category. What is the probability that a defect was either at the edge or that it contains four or more defects?

<table>
<thead>
<tr>
<th>Location in Concrete Element Surface</th>
<th>Center</th>
<th>Edge</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>514</td>
<td>68</td>
<td>582</td>
</tr>
<tr>
<td>High</td>
<td>112</td>
<td>246</td>
<td>358</td>
</tr>
<tr>
<td>Total</td>
<td>626</td>
<td>314</td>
<td>940</td>
</tr>
</tbody>
</table>

Let \( E_1 \) denote the event that a defect contains four or more defects, and let \( E_2 \) denote the event that a defect is at the edge.

<table>
<thead>
<tr>
<th>Defects Classified by Location and Degree</th>
<th>Center</th>
<th>Edge</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.30</td>
<td>0.10</td>
<td>0.40</td>
</tr>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
<td>0.04</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.04</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>5 or more</td>
<td>0.07</td>
<td>0.03</td>
<td>0.10</td>
</tr>
<tr>
<td>Totals</td>
<td>0.72</td>
<td>0.28</td>
<td>1.00</td>
</tr>
</tbody>
</table>
The requested probability is $P (E_1 \cup E_2)$. Now, $P (E_1) = 0.15$ and $P (E_2) = 0.28$. Also, from the table above, $P (E_1 \cap E_2) = 0.04$

Therefore, $P (E_1 \cup E_2) = 0.15 + 0.28 - 0.04 = 0.39$

What is the probability that concrete surface contains less than two defects (denoted as $E_3$) or that it is both at the edge and contains more than four defects (denoted as $E_4$)?

The requested probability is $P (E_3 \cup E_4)$. Now $P (E_3) = 0.6$, and $P (E_4) = 0.03$. Also, $E_3$ and $E_4$ are mutually exclusive.

Therefore, $P (E_3 \cap E_4) = \emptyset$

and $P (E_3 \cup E_4) = 0.6 + 0.03 = 0.63$

for the case of three events:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
EXAMPLE 9:

Let $X$ denote the pH of a sample. Consider the event that $X$ is greater than 6.5 but less than or equal to 7.8. This probability is the sum of any collection of mutually exclusive events with union equal to the same range for $X$. One example is:

$$P(6.5 < X \leq 7.8) = P(6.5 < X \leq 7.0) + P(7.0 < X \leq 7.5) + P(7.5 < X \leq 7.8)$$

Another example is

$$P(6.5 < X \leq 7.8) = P(6.5 < X \leq 6.6) + P(6.6 < X \leq 7.1)$$
$$+ P(7.1 < X \leq 7.4) + P(7.4 < X \leq 7.8)$$

The best choice depends on the particular probabilities available.
(a) If a shaft is selected at random, what is the probability that the shaft conforms to surface finish requirements or to roundness requirements?
(b) What is the probability that the selected shaft conforms to surface finish requirements or to roundness requirements?
(c) What is the probability that the selected shaft either conforms to surface finish requirements or does not conform to roundness requirements?
(d) What is the probability that the selected shaft conforms to both surface finish and roundness requirements?

2-54. Cooking oil is produced in two main varieties: mono- and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

<table>
<thead>
<tr>
<th>type of oil</th>
<th>canola</th>
<th>corn</th>
</tr>
</thead>
<tbody>
<tr>
<td>type of</td>
<td>mono</td>
<td>13</td>
</tr>
<tr>
<td>unsaturation</td>
<td>poly</td>
<td>93</td>
</tr>
</tbody>
</table>

(a) If a bottle of oil is selected at random, what is the probability that it belongs to the polyunsaturated category?
(b) What is the probability that the chosen bottle is unsaturated canola oil?

2-55. A manufacturer of front lights for automobiles tests lamps under a high humidity, high temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 130 lamps:

<table>
<thead>
<tr>
<th>useful life</th>
<th>satisfactory</th>
<th>unsatisfactory</th>
</tr>
</thead>
<tbody>
<tr>
<td>intensity</td>
<td>117</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>satisfactory</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>unsatisfactory</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.
(b) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?
CONDITIONAL PROBABILITY

The conditional probability of an event $B$ given an event $A$, denoted as $P(B \mid A)$, is

$$
P(B \mid A) = \frac{P(A \cap B)}{P(A)}
$$

for $P(A) > 0$.

In a manufacturing process, 10% of the parts contain visible surface flaws and 25% of the parts with surface flaws are (functionally) defective parts. However, only 5% of parts without surface flaws are defective parts. The probability of a defective part depends on our knowledge of the presence or absence of a surface flaw.

Let $D$ denote the event that a part is defective and let $F$ denote the event that a part has a surface flaw.

Then, the probability of $D$ given, or assuming, that a part has a surface flaw as $P(D \mid F)$. This notation is read as the conditional probability of $D$ given $F$, and it is interpreted as the probability that a part is defective, given that the part has a surface flaw.
EXAMPLE 1:

Table 1 below provides an example of 400 parts classified by surface flaws and as (functionally) defective. For this table the conditional probabilities match those discussed previously in this section. For example, of the parts with surface flaws (40 parts) the number defective is 10.

<table>
<thead>
<tr>
<th></th>
<th>Surface Flaws</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes (event $D$)</td>
<td>No</td>
</tr>
<tr>
<td>Defective</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>No</td>
<td>30</td>
<td>342</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>360</td>
</tr>
</tbody>
</table>

Therefore,

$$P(D | F) = \frac{10}{40} = 0.25$$

and of the parts without surface flaws (360 parts) the number defective is 18. Therefore,

$$P(D | F') = \frac{18}{360} = 0.05$$

Figure 1: Tree diagram for parts classified

Therefore, $P(B | A)$ can be interpreted as the relative frequency of event $B$ among the trials that produce an outcome in event $A$. 
EXAMPLE 2:

Again consider the 400 parts in Table 1 above (example 1). From this table

\[
P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{10/400}{40/400} = \frac{10}{40}
\]

Note that in this example all four of the following probabilities are different:

\[
\begin{align*}
P(F) &= 40/400 \\
P(D) &= 28/400 \\
P(F|D) &= 10/28 \\
P(D|F) &= 10/40
\end{align*}
\]

Here, \( P(D) \) and \( P(D|F) \) are probabilities of the same event, but they are computed under two different states of knowledge.

Similarly, \( P(F) \) and \( P(F|D) \),

The tree diagram in Fig. 1 can also be used to display conditional probabilities.

\[
P(D|F) = \frac{10}{40} \quad \text{and} \quad P(D'|F) = \frac{30}{40}
\]

**Multiplication Rule (for counting techniques)**

If an operation can be described as a sequence of \( k \) steps, and

- if the number of ways of completing step 1 is \( n_1 \), and
- if the number of ways of completing step 2 is \( n_2 \) for each way of completing step 1, and
- if the number of ways of completing step 3 is \( n_3 \) for each way of completing step 2, and so forth,

then the total number of ways of completing the operation is

\[ n_1 \times n_2 \times \cdots \times n_k \]
Permutations

Another useful calculation is the number of ordered sequences of the elements of a set. Consider a set of elements, such as \( S \{a, b, c\} \). A permutation of the elements is an ordered sequence of the elements. For example, \( abc, acb, bac, bca, cab \), and \( cba \) are all of the permutations of the elements of \( S \).

The number of permutations of \( n \) different elements is \( n! \) where

\[
n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1
\]

In some situations, we are interested in the number of arrangements of only some of the elements of a set. The following result also follows from the multiplication rule.

The number of permutations of a subset of \( r \) elements selected from a set of \( n \) different elements is

\[
P_r^n = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}
\]

**EXAMPLE 3:**

A printed circuit board has eight different locations in which a component can be placed. If four different components are to be placed on the board, how many different designs are possible?

Each design consists of selecting a location from the eight locations for the first component, a location from the remaining seven for the second component, a location from the remaining six for the third component, and a location from the remaining five for the fourth component. Therefore,
Combinations

Another counting problem of interest is the number of subsets of \( r \) elements that can be selected from a set of \( n \) elements. Here, order is not important.

\[
P^8_5 = 8 \times 7 \times 6 \times 5 = \frac{8!}{4!} = 1680 \text{ different designs are possible.}
\]

**EXAMPLE 4:**

A printed circuit board has eight different locations in which a component can be placed. If five identical components are to be placed on the board, how many different designs are possible? Each design is a subset of the eight locations that are to contain the components. From the Equation above, the number of possible designs is

\[
\frac{8!}{5! \ 3!} = 56
\]

The following example uses the multiplication rule in combination with the above equation to answer a more difficult, but common, question.

**EXAMPLE 5:**

A bin of 50 manufactured parts contains three defective parts and 47 non-defective parts. A sample of six parts is selected from the 50 parts. Selected parts are not replaced. That is, each part can only be selected once and the sample is a subset of the 50 parts. How many different samples are there of size six that contain exactly two defective parts?
A subset containing exactly two defective parts can be formed by first choosing the two defective parts from the three defective parts.

\[
\binom{3}{2} = \frac{3!}{2!1!} = 3 \text{ different ways}
\]

Then, the second step is to select the remaining four parts from the 47 acceptable parts in the bin. The second step can be completed in

\[
\binom{47}{4} = \frac{47!}{4!43!} = 178,365 \text{ different ways}
\]

Therefore, from the multiplication rule, the number of subsets of size six that contain exactly two defective items is

\[3 \times 178,365 = 535,095\]

As an additional computation, the total number of different subsets of size six is found to be

\[
\binom{50}{6} = \frac{50!}{6!44!} = 15,890,700
\]

Therefore, the probability that a sample contains exactly two defective parts is

\[
\frac{535,095}{15,890,700} = 0.034
\]
S2-1. An order for a personal digital assistant can specify any one of five memory sizes, any one of three types of displays, any one of four sizes of a hard disk, and can either include or not include a pen tablet. How many different systems can be ordered?

S2-2. In a manufacturing operation, a part is produced by machining, polishing, and painting. If there are three machine tools, four polishing tools, and three painting tools, how many different routings (consisting of machining, following by polishing, and followed by painting) for a part are possible?

S2-3. New designs for a wastewater treatment tank have proposed three possible shapes, four possible sizes, three locations for input valves, and four locations for output valves. How many different product designs are possible?

S2-4. A manufacturing process consists of 10 operations that can be completed in any order. How many different production sequences are possible?

S2-5. A manufacturing operations consists of 10 operations. However, five machining operations must be completed before any of the remaining five assembly operations can begin. Within each set of five, operations can be completed in any order. How many different production sequences are possible?

S2-6. In a sheet metal operation, three notches and four bends are required. If the operations can be done in any order, how many different ways of completing the manufacturing are possible?

S2-7. A lot of 140 semiconductor chips is inspected by choosing a sample of five chips. Assume 10 of the chips do not conform to customer requirements.

(a) How many different samples are possible?

(b) How many samples of five contain exactly one nonconforming chip?

(c) How many samples of five contain at least one nonconforming chip?

S2-8. In the layout of a printed circuit board for an electronic product, there are 12 different locations that can accommodate chips.

(a) If five different types of chips are to be placed on the board, how many different layouts are possible?

(b) If the five chips that are placed on the board are of the same type, how many different layouts are possible?

S2-9. In the laboratory analysis of samples from a chemical process, five samples from the process are analyzed daily. In addition, a control sample is analyzed two times each day to check the calibration of the laboratory instruments.

(a) How many different sequences of process and control samples are possible each day? Assume that the five process samples are considered identical and that the two control samples are considered identical.

(b) How many different sequences of process and control samples are possible if we consider the five process samples to be different and the two control samples to be identical.

(c) For the same situation as part (b), how many sequences are possible if the first test of each day must be a control sample?

S2-10. In the design of an electromechanical product, seven different components are to be stacked into a cylindrical casing that holds 12 components in a manner that minimizes the impact of shocks. One end of the casing is designated as the bottom and the other end is the top.

(a) How many different designs are possible?

(b) If the seven components are all identical, how many different designs are possible?

(c) If the seven components consist of three of one type of component and four of another type, how many different designs are possible? (more difficult)

S2-11. The design of a communication system considered the following questions:

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?

(b) As in part (a), how many three-digit phone prefixes are possible that do not start with 0 or 1, but contain 0 or 1 as the middle digit?

(c) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

S2-12. A byte is a sequence of eight bits and each bit is either 0 or 1.

(a) How many different bytes are possible?

(b) If the first bit of a byte is a parity check, that is, the first byte is determined from the other seven bits, how many different bytes are possible?

S2-13. In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high viscosity material?

(b) What is the probability that at least one tank in the sample contains high viscosity material?

(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high viscosity material and exactly one tank in the sample contains material with high impurities?

S2-14. Plastic parts produced by an injection-molding operation are checked for conformance to specifications. Each tool contains 12 cavities in which parts are produced, and these parts fall into a conveyor when the press opens. An inspector chooses 3 parts from among the 12 at random. Two cavities are affected by a temperature malfunction that results in parts that do not conform to specifications.

(a) What is the probability that the inspector finds exactly one nonconforming part?

(b) What is the probability that the inspector finds at least one nonconforming part?

S2-15. A bin of 50 parts contains five that are defective. A sample of two is selected at random, without replacement.

(a) Determine the probability that both parts in the sample are defective by computing a conditional probability.

(b) Determine the answer to part (a) by using the subset approach that was described in this section.