CHAPTER 1
INTRODUCTION TO FOUNDATIONS

The soil beneath structures responsible for carrying the loads is the FOUNDATION. The general misconception is that the structural element which transmits the load to the soil (such as a footing) is the foundation. The figure below clarifies this point.

TYPES OF FOUNDATIONS

Foundations can be categorized into basically two types: Shallow and Deep.

Shallow Foundations:

These types of foundations are so called because they are placed at a shallow depth (relative to their dimensions) beneath the soil surface. Their depth may range from the top soil surface to about 3 times their breadth (about 6 meters). They include footings (spread and combined), and soil retaining structures (retaining walls, sheet piles, excavations and reinforced earth). There are several others of course.

Deep Foundations:

The most common of these types of foundations are piles. They are called deep because they are embedded very deep (relative to their dimensions) into the soil. Their depths may run over several 10s of meters. They are usually used when the top soil layer have low bearing capacity.
DESIGN CONSIDERATIONS
To perform satisfactorily, foundations must carry the loads (and moments) and have two main characteristics:

1. Be safe against overall shear failure (Bearing Capacity Failure).
2. Not undergo excessive displacement (Settlement).

These conditions will insure that the foundation i.e. the soil is safe and can carry the loads without major problems. Therefore, when designing foundations, these two characteristic must be satisfied.

In addition to satisfying the conditions for the foundation, the structural members (concrete, steel and/or wood) must be able to transfer the load to the soil without failing. In the case of concrete, two basic conditions must be satisfied:

1. No shear failure: This is satisfied by providing an adequate thickness of concrete.
2. No tension failure: This is satisfied by providing adequate steel reinforcement.

This course covers the analysis and design (geotechnical and concrete design) of the basic and most commonly used types of foundations including both shallow and deep foundations. Other types of foundations are covered in the follow-up course, "Foundation Engineering 2".

The following types of foundations will be covered in Foundation Engineering 1

FOOTINGS

Fig. 1 Spread Footings: (a) Square, (b) Rectangular, (c) Wall (Strip) and (d) Circular
Fig. 2 Rectangular Combined Footing

Fig. 8. Steel layout for a rectangular combined footing

Fig. 9. Trench and steel layout for a wall footing
EARTH RETAINING STRUCTURES

Fig. 3. Cantilever Retaining Wall

Fig. 4. Gravity Retaining Wall

Fig. 5. Counter fort Retaining Wall
DEEP FOUNDATIONS: PILES

Fig. 6. Various types of concrete piles

Fig. 7. Layout of Piles in Groups
### Table 1. Soil Types and Foundation Consideration

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Foundation Type</th>
<th>Reason(s) for use</th>
<th>Trouble Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAND</td>
<td>Footings</td>
<td>Easy to construct and economical</td>
<td>Bearing capacity may be a problem but in most cases it is sufficient.</td>
</tr>
<tr>
<td></td>
<td>Retaining Structures</td>
<td>Must be used since sand cannot support themselves</td>
<td>Excessive settlement in wet and loose deposits.</td>
</tr>
<tr>
<td></td>
<td>Deep Foundations (Piles)</td>
<td>Uses $\phi$ for friction resistance but low in bearing capacity</td>
<td>Confining pressure is usually low.</td>
</tr>
<tr>
<td>CLAY</td>
<td>Footings</td>
<td>Economic but may have problem with bearing capacity in saturated clays</td>
<td>Low bearing capacity.</td>
</tr>
<tr>
<td></td>
<td>Retaining Structures</td>
<td>Clays are self-supportive up to a certain height (critical). Must be used if height increases beyond the critical.</td>
<td>Generally low shear strength when wet.</td>
</tr>
<tr>
<td></td>
<td>Deep Foundations (Piles)</td>
<td>If bearing capacity is low, piles may be driven to rock. May change formation of clay.</td>
<td>High consolidation in soft clays.</td>
</tr>
</tbody>
</table>

### Table 2. Possible Solutions to some Problems in Foundations

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Nature of problem</th>
<th>Possible solution</th>
</tr>
</thead>
</table>
| SAND      | Settlement        | • Loose sands must be compacted  
• Lowering water table may result in sand densification. |
|           | Bearing Capacity  | • Compaction increases $c$ and $\phi$ and thus bearing capacity  
• Use deep foundations |
| Clay      | Consolidation     | • Lowering water table  
• Pre-loading  
• Drive pile to rock |
|           | Bearing Capacity  | • Compaction  
• Use deep foundations |
|           | Expansion or Swelling | • Treat or stabilize soil  
• Maintain constant water table  
• Alter soil nature (similar to stabilization)  
• Include swell pressure in design |
University of Technology

Building and Construction Engineering
Department

Roads and Bridges Section

FOUNDATION ENGINEERING COURSE

Subject:- Shallow foundation – Bearing capacity

Prepared by :- Prof. Dr. Kais Taha Shlash

Course designation : geo. 01
Bearing Capacity

Shallow (Depth \( D \) \( \leq \) Breadth \( B \))

Deep (\( D \gg B \))

Footing: It is an enlargement of the base of a column or wall for the purpose of transmitting the applied load to the subsoil at a pressure suited to the properties of soil.

Types of Shallow Foundations (\( D \leq B \))

1. Spread (isolated) Footing

   Spread

   Stepped

   Sloped

   used i. for high column load
   2. if no tensile reinforcement used

2. Combined Footing

   a. Rectangular
   b. Trapezoidal: used when space outside the structure is limited for a spread footing and the exterior column carries the largest load
   c. Strap or Cantilever may be used where the distance between columns is so great that a rectangular or trapezoidal footing becomes quite narrow, with resulting high bending moments, or where \( b \leq 4/3 \)

3. Mat (Raft) Foundation
4. Wall Footing (either continuous wall or many columns)

B. Deep Foundations (Depth \(D_f\) \(\gg\) Breadth \(B\))
1. Piles
2. Bridge Piers
3. Caissons

Design of Dimensions of Footings

1. Allowable Bearing Capacity \(q_{al}\): The maximum pressure which the soil can carry safely without risk of shear failure or excessive settlement. [units: \(\text{KN/m}^2\) or \(\text{kN/m}^2\) (kPa) or \(\text{psi}\)]

\[ q_{al} \geq \frac{Q}{A} \]

\[ A = \frac{L}{\text{Area}(G)} = \text{B.L} \]

The design can be of two approaches:

1. Uniform pressure intensity: This is done by locating the location of the resultant \(R\) at the same location of the center of gravity of the plan of footing (if possible).

2. Increasing pressure intensity: This is done by locating the location of the resultant \(R\) at the middle third of the plan of footing and calculating the pressures \(q_{max} < q_{al}\) and \(q_{min} \geq 0\).

\[ q_{max} = \frac{Q}{A} \left( 1 \pm \frac{6cL}{B} \pm \frac{6cB}{L} \right) \]

\[ \chi_c = \frac{\frac{1}{2} \left( B_1 + B_2 \right)}{B_1 + B_2} \]

\[ B_2 = \frac{\text{Area}(G)}{L} \]
Example 1: Design the following footing for
a) uniform pressure
b) resultant within middle third

Solution:

d) \( e = \frac{M}{Q} = \frac{80}{400} = 0.2 \ m \)

\( l = 2 \times (1.7 + 0.2) = 3 \ m \)

\( q_{all} = \frac{Q}{B \times L} \)

\( B = \frac{Q}{q_{all} L} = \frac{400}{100(3)} = 1.33 \ m \)

use 3m x 1.33m = 4m²

The location of the column is not the same as location of centroid.

b) \( q_{max} = \frac{Q}{B \times L} \left( 1 - \frac{e}{L} \right) \leq q_{all} \)

100 = \frac{400}{B(3)} \left( 1 - \frac{0.2}{3} \right)

\( B = 2.4 \ m \)

use 2.6m x 2.3m = 6m²

\( q_{min} = \frac{Q}{2.6(2.3)} \left( 1 - \frac{0.2}{2.6} \right) = 33.7 \text{ kPa} > 0 \text{ Ok} \)

The location of the column is at the centroid hence \( L \) is known (= 2.6m) but the pressure distribution must be calculated.
Example 2: Design a rectangular combined footing for a uniform soil pressure.

Solution
1. Find center of forces:
   \[ x = \frac{1000(3.3)}{1700} = 1.84 \text{ m from center of col. A} \]

2. Find the length of footing such that center of forces and centroid coincide.
   \[ l = 2(2.17 + 0.15) = 4.64 \text{ m} \quad \text{(use 4.65 m = l)} \]

3. Find the breadth of footing:
   \[ \frac{9_{all}}{9_{act}} = \frac{1900}{150} \]
   \[ 8 = 2.45 \text{ m} \]

Example 3: If in Example 2 the length of footing is 4 m, redesign the width of footing.

Solution
1. Find center of forces:
   \[ x = \frac{1000(3.3)}{1700} = 0.17 \text{ m from center of col. A} \]

2. Find the eccentricity:
   \[ e = x - \frac{L}{2} = (2.17 + 0.15) - 2.17 = 0.32 \text{ m} \]
   \[ e = 0 \text{ no uniform soil pressure} \]

3. Find width such that \( 9_{max} \leq 9_{all} \)
   \[ 9_{max} = \frac{V}{B L} \left( 1 + \frac{6e}{L} \right) = \frac{1200}{420 \times 4} \left( 1 + \frac{6(0.32)}{4} \right) = 150 \]
   \[ B = 4.19 \text{ m} \quad \text{(use} \ B = 4.2 \text{ m}) \]

4. Check minimum pressure:
   \[ 9_{min} = \frac{V}{B L} \left( 1 - \frac{6e}{L} \right) = \frac{1200}{420 \times 4} \left( 1 - \frac{6(0.32)}{4} \right) = 52.6 \text{ kPa} \]
   \[ \text{Area} = 4(4.2) = 16.8 \text{ m}^2 \]
Example 4: Redesign Example 3 for a triangular combined footing for a uniform soil pressure.

Solution:

1. Find center of forces:
   \[ x = \frac{1000 \times 2.1}{7500} = 2.17 \text{ m from center of total} \]
   \[ x = 2.17 + 0.13 = 2.3 \text{ m from edge} \]

2. For a uniform soil pressure, the centroid of the total mass must coincide with
   - Center of gravity
   \[ x_c = \frac{b_1}{a_1} \left( \frac{2b_1 + b_2}{a_1 + b_2} \right) \text{ from large end} \]
   \[ (4.2 \text{ m}) \frac{y}{2} = \frac{2b_1 + b_2}{a_1 + b_2} \]
   \[ (4.2 \text{ m}) = \frac{2 \times 5.66 + b_2}{5.66 + b_2} \]
   \[ b_2 = 9.29 \text{ m} \]

3. For a uniform pressure
   \[ P_{act} \geq P_{cr} = \frac{V}{A} \]
   \[ P_{act} = \frac{1.7 \text{ kN}}{(15)(14)} \]
   \[ b = 9.16 \text{ m} \]

4. Sketch to get:
   \[ b_1 = 5.66 \text{ - } b_2 = 5.66 - 4.18 = 1.48 \text{ m} \]
   \[ b_1 = 3.3 \text{ m} \]

5. Sketch to get:
   \[ b_2 = \sqrt{3} \times b_1 = \sqrt{3} \times 3.3 \text{ m} \]
   \[ A = \frac{Y}{L}, \quad \frac{15}{13} \times 15 = 11.33 \text{ m}^2 \]
   \[ b_2 = \frac{4}{L} \left( \frac{3}{2} x_c - 1 \right) \]
   \[ b_2 = 2(11.33) \left( \frac{3}{2} (2.68) - 1 \right) \]
   \[ b_2 = 14.7 \text{ m} \]
   \[ b_2 = \frac{L}{6} - \frac{b_2}{2} \]
   \[ b_2 = \frac{L}{6} - \frac{b_2}{2} \]

6. If \( x_c \geq \frac{L}{2} \) then you can use rectangular footing
   \[ x_c < \frac{L}{2} \text{ then triangular footing} \]
Behavior of Footing on Elastic/Plastic Material

1. In soils: strain is proportional to stress at low strains only or

\[ \varepsilon = \frac{\sigma}{E} \]

(i.e. the soil is elastic at low strains only)

2. As long as \( \tau_{\text{max}}\) (maximum shear stress) is less than shear strength (\( S \)) then the settlement is proportional to \( \tau_{\text{max}}\).

This occurs only for small values of \( \tau_{\text{max}}\).

Note: The largest value of \( \tau_{\text{max}}\) occurs along the center line at a depth roughly equals to \( \frac{B}{2} \) or \( 0.5B \)

A. First Yield: as \( \tau_{\text{max}}\) reaches a value such that \( \tau_{\text{max}} = S \) at some points, these points will yield. But these points are surrounded by adjacent points which can carry additional stress (i.e. \( \tau_{\text{max}} \) is still less than \( S \) at these points for the same value of \( \tau_{\text{max}}\))

B. Local Shear Failure: as \( \tau_{\text{max}}\) is increased the load-settlement curve steepens and the value of shear strength at the adjacent points is reached hence there will be a plastic zone under the loaded area.

C. General Shear Failure: for further increase in \( \tau_{\text{max}}\), the plastic zone spreads beyond the loaded area and the load-settlement curve steepens more.
Types of Shear Failure

1. General Shear Failure
   - Characterized by the existence of a well defined failure pattern which consists of a continuous slip surface from at least one edge of the footing to the ground surface.
   - Sudden and catastrophic and may be accompanied by tilting.
   - Observed in very dense sand or completely saturated clay.
   - No volume change ($\Delta V_i = 0$).
   - Very steep $\sigma - e$ curve (high modulus).
   - Mainly distortion.

2. Local Shear Failure
   - Characterized by a failure pattern which is clearly defined only immediately below the footing. Only after a considerable vertical displacement of the footing that the slip surface may appear at the ground surface.
   - No catastrophic collapse or tilting.
   - Observed in medium dense sand or clay with drainage.
   - Volume of bulges $< A. G$ ($\Delta V_i < \Delta V_k$).
   - Flat $\sigma - e$ curve (moderate modulus).
   - Distortion + consolidation.

3. Punching Failure
   - Characterized by a failure pattern that is not easily observed. Footing penetrates into the soil, under the load, without any bulging at the surface.
   - No visible collapse or tilting.
   - Observed in loose sand or soft clay.
   - Large volume change ($\Delta V_k$ is large).
   - Very flat $\sigma - e$ curve (very low modulus).

![Diagram of Stress-Settlement Curves](image)
Design Criteria

1. For any foundation there is some value of the applied pressure at which the settlement start to become very large and difficult to predict. This pressure is called the ultimate bearing capacity (q_u).

2. The actual pressure from the structure to the soil must not exceed allowable bearing capacity (q_all)

\[ q_{all} = \frac{q_{ult}}{F.S} \]

where F.S. = factor of safety

3. The settlement that occurs due to the applied pressure must be within permissible value

- No harmful effects on adjacent structures.

Terzaghi's Bearing Capacity Equations

Assumptions:

a. D ≤ B
b. Soil is plastic
c. Rough surface of footing
d. Neglect the shear resistance of soil above the horizontal plane
e. Replace the soil above foundation level with a surcharge \( \bar{q} = \gamma D_F \) placed at the foundation level
f. \( \beta = \phi \)
The application of load $Q$ tends to
1. push the wedge(s) into the ground
2. a lateral displacement of zones II and III. This disp.
   is resisted by shear stresses developed along the
   slip plane (ade) and (afg) and the weight of soil
   in the zones and $Vdp$.

From static equilibrium (see Table 4-1 pg 134 Bowles)

- Continuous footing
  \[ q_{ult} = C N_c + \bar{q} N_q + 0.5 \gamma B N_y \]
- Square footing
  \[ q_{ult} = 1.5 C N_c + \bar{q} N_q + 0.4 \gamma B N_y \]
- Circle footing
  \[ q_{ult} = 1.5 C N_c + \bar{q} N_q - 0.7 \gamma B N_y \]

where $q_{ult}$ = ultimate Bearing Capacity

$C$ = cohesion, $\phi$ = angle of internal friction of soil
$\bar{q}$ = Surcharge at foundation level = $\gamma D_1$
$\gamma$ = unit wt of the above soil
$D_1$ = Depth of footing
$B$ = Least width (or diameter) of footing

$N_q$, $N_c$, Terzaghi Bearing Capacity Factors

$N_q$, $N_c$ [see Table 4-2 pg 125 Bowles]

depends on the value of $\phi$

Soil parameters for local shear failure were
proposed by Terzaghi as

$C' = \frac{2}{3} C$, $\tan \phi' = \frac{2}{3} \tan \phi$
Conclusions
1. Pull increases with depth
2. Pull for cohesive soils (cohesionless) is directly dependent on footing size and $D_f$, but $D_f$ is more important
3. Pull for cohesive soils (cohesive) is independent on footing size. For $D_f = 5.0$, $N_c = 5.1$, $N_q = 1$, $N_p = 0$

Notes on Terzaghi B.C. equation
1. These equations are based on uncorrected assumptions that $p_o = 0$. It has been found that $p_o$ is closer to (45 \text{ kPa}). In other words, no adjustment for plane strain conditions has been considered by Terzaghi
2. Depth and shape factors are considered in general case
3. Don't include inclined load effects
4. Good results for sandy soils, not very reliable for clay soils

Example 6.4
A footing 2 x 2 m square is located at a depth of 3 m in silty clay sand, the shear strength parameter being $C = 30 \text{ kPa}$, $\phi = 35^\circ$. Determine the ultimate bearing capacity of the unit weight of the soil is $19 \text{ kN/m}^3$. Solve using Terzaghi B.C. eq. for square footing

$$Q_{ult} = 1.36 \cdot N_c + 0.43 \cdot N_q + 0.4 \cdot B \cdot N_p$$

For $\phi = 35^\circ$ and from Table 6.2 by Bowles

$$N_c = 5.2, N_q = 4.5, N_p = 42.2$$

$$\bar{q} = 1.0, \cdot 1.15 = 22.5 \text{ kN/m}^2 = 22.5 \text{ kPa}$$

$$Q_{ult} = 1.36 \cdot 5.2 \cdot (57.8) + 22.5 \cdot (4.5) + 0.4 \cdot (1.2) \cdot (42.2)$$

$$= 225.72 + 921.5 + 610.5$$

$$= 3778.2 \text{ kPa} \approx 3800 \text{ kPa}$$
C. Bearing Capacity

1. Gross pressure: is the total pressure on the surface of the foundation due to the weight of the foundation itself and the weight of the soil.

2. Net pressure: is the intensity of external loads on the base, excluding the dead weight of the foundation.

The bearing capacity equations \((q_{alt})\) are defined as:

\[
q_{alt} = \frac{Y D_f}{C_N + Y D_f (N_q - 1) + \frac{1}{2} \delta B N_b}
\]

Engineering, p. 311

a. The factor of safety \((F_s)\) with respect to a defined criterion of the net ultimate \(B = 9\), yields:

\[
F_s = \frac{q_{alt}}{Y D_f}
\]

b. The factor of safety of a design depends on:

(i) type of structure (permanent vs. temporary)

(ii) sensitivity of structure

(iii) extent of soil exploration

(iv) natural and seasonal conditions and assumptions made in the design

(v) extent of control during construction

It is recommended that factor of safety should lie between 2 and 4.
Meyerhof B.C. Equation

Assumptions:
1. $\beta = 45 + \phi$
2. No adjustment to $\phi$ for square or round footing

$\Phi = (1.1 - 0.1 \frac{B}{L}) \Phi_{TRAL}$

Equations

\[ q_{ult} = c N_c d_c + f' N_s q_y d_y + 0.5 \times R N_y S_{y} d_y \quad \text{[For Vertical Load]} \]

\[ q_{ult} = c N_c d_c i_c + f' N_s d_y i_y + 0.5 \times R N_y d_y i_y \quad \text{[For Inclined Load]} \]

where $N_c, N_y, N_s = \text{Meyerhof B.C. Factors (see Table 4-9)}$

$S_c, d_c, i_c = \text{shape, depth and inclination factors}$

[See Table 4-3 pg126]

Example 16: Resolve Ex. 1a using Meyerhof B.C. Equation

$\Phi = (1.1 - 0.1 \frac{B}{L}) \Phi = (1.1 - 0.1 \frac{25}{5}) 35 = 35$

For $\Phi = 35$ and from Table 4-4 $N_c = 46.1, N_y = 33.8, N_s = 37.1$

$S_c = 1 + 0.7 \left[ \tan^2 \left(45 + \frac{35}{2} \right) \right] \left[ \frac{2}{3} \right] = 1.738$

$S_y = 3.7 = 1 + 0.1 \left[ \tan^2 \left(45 + \frac{35}{2} \right) \right] \left[ \frac{2}{3} \right] = 1.389$

Shape Factors

Table 4.1

Shape Factors

$D_c = 1 + 0.7 \sqrt{kp} \left[ \frac{1.25}{2} \right] = 1.240$

Depth Factors

$D_y = D_c = 1 + 0.7 \sqrt{kp} \left[ \frac{1.25}{2} \right] = 1.240$

Table 4.3

shape Factors

\[ q_{ult} = c N_c d_c + f' N_s q_y d_y + 0.5 \times R N_y S_{y} d_y \]

\[ = 30(46.1)(1.24) + 1.25(33.8)(1.24) \]

\[ + 0.5(18.2)(33.1)(1.24) \]

\[ = 2980.5 + 1148.8 + 102.9 \]

\[ = 5133.2 \text{ kPa} \]


The General B.C. Equation (Hansen 1970)

\[ \Phi = 1.1 \frac{\varphi_{TERRA}}{\varphi_{TERRA}} \]

Assumptions:
1. \( \beta = 45 + \theta \)
2. \( \varphi_{TERRA} = 1.1 \)

Gives better results than Terszeghi B.C. equations.

\[ f_{u} = c \frac{N_{c}}{N_{l}} \left[ i \frac{g}{g_{i}} \left( 1 + \frac{d_{i}}{d} \right) \right] ^{2} \]

where \( N_{c}, N_{l}, N_{i} = \) Hansen B.C. Factors \( [\text{Table 4-4 Py 137}] \)

- \( S = \) shape factor
- \( d = \) depth factor
- \( i = \) inclination factor of load
- \( g = \) ground factor
- \( b = \) ground factor

Example 1c: Resolve Ex. 1b using General B.C. Eq.

\[ \Phi = 1.1 \frac{\varphi_{TERRA}}{\varphi_{TERRA}} \]

For \( \Phi = 35 \) : \( N_{c} = 44.1, N_{l} = 32.3, N_{i} = 33.9 \)

For \( \Phi = 40 \) : \( N_{c} = 75.7, N_{l} = 64.2, N_{i} = 79.5 \)

For \( \Phi = 38.5 \)

\[ N_{c} = \frac{N_{c} - N_{l}}{N_{i}} \Rightarrow N_{c} = 66.54 \]

Using Linear Interpolation

\[ N_{c} = \frac{38.5 - 35}{40 - 35} \cdot \frac{75.7 - 79.5}{44.1 - 64.2} \Rightarrow N_{c} = 69.93 \]

\[ f_{u} = c \frac{N_{c}}{N_{l}} \sqrt{1 + \frac{d}{d_{i}}} \left( 1 + \frac{d_{i}}{d} \right) \]

Factors:
- \( c = 1 \) since vertical force
- \( b = 1 \) since the adjacent ground is horizontal
- \( g_{i} = 1 \) since the base level is horizontal


\[ = 45.57 \times 2.53 \times 1.29 + 70.9 \times 2.5 = 779.4 \text{ kPa} > (f_{ult})_{\text{Meyerhofer}} > (f_{ult})_{\text{Terszeghi}} \]
Effect of Water Table on B.C

1. The influence of W.T can be accounted as follows:
   
1.1 No correction is needed for W.T at a depth equal to or greater than depth B below the footing base (Zone I).

2. If the W.T is at the base level then use \( Y_{sub} \) in the third term of B.C equation or you may use

\[
Y = Y_{sat} + W_y \quad \text{where}
\]

\[
W_y = \frac{Y_{sat}}{Y_{sat}} = \frac{Y_{sub}}{Y_{sat}} = \frac{20 - 10}{20} = 0.5 \quad \text{(Assuming } Y_{sat} = 20 \text{ kN/m}^2, Y_{sub} = 10 \text{ kN/m}^2)\]

3. If the W.T is in Zone II then use linear interpolation between cases 1 and 2 above i.e. \( W_y = 1 \) if \( b = B \)
   and \( W_y = 0.5 \) if \( b = 0 \) where \( b \) is distance from base to W.T.

   OR \( W_y = 0.5 + 0.5 \frac{b}{B} \)

4. If the W.T is at ground surface then use \( Y_{sub} \) in the second term in addition to using \( Y_{sub} \) in the third term

   OR \( W_y = 0.5 \quad \text{and} \quad W_y = 0.5 \)

5. If the W.T is in Zone III. then use linear interpolation between cases 2 and 4 above i.e. \( W_y = 1 \) if \( d = 0 \)
   and \( W_y = 0.5 \) if \( d = D_p \) where \( d \) is distance from base to W.T.

   OR \( W_y = 0.5 + (1 - \frac{d}{D_p}) \times 0.5 \)

\[
W_y = 1 - \left( \frac{d}{D_p} \right) \times 0.5
\]

Example 2: Resolve example 1a if \( Y_d = 18 \text{ kN/m}^2 \), \( e = 2.7 \) for the following cases:

1. If W.T at depth of 2m below foundation level
2. If W.T at depth of 1m
3. If W.T is at foundation level
4. If W.T is at depth of 0.5m below G.S
5. If W.T is at ground surface

Solution: Since \( Y_{sat} = \frac{G + \gamma \delta}{1 + e} \delta 

\text{but } Y_d = \frac{G}{1 + e} \delta \Rightarrow 18 = \frac{2.7}{1 + e} (10) \Rightarrow e = 0.5

\text{and } Y_{sat} = \frac{270 \times 0.5}{1 + 0.5} (10) = 21.3 \text{ kN/m}^2
Bearing Capacity

Case 1: W.T. will not affect $q_{ult}$ i.e. $W_x = 1, W_y = 1$

\[ q_{ult} = 22.54.2 + 931.5 + 210.56 = 3796.26 \text{ kPa} \]

Case 2: $b = 1 \text{ m}, W_y = 0.5 + 0.5(\frac{d}{2}) = 0.75, W_x = 1$

\[ q_{ult} = 22.54.2 + 931.5 + 0.4(21.33)(2)(42.4)(0.75) \]
\[ = 22.54.2 + 931.5 + 354.26 = 3708.7 \text{ kPa} \]

Case 3: $b = 0, W_y = 0.5, d = 0, W_x = 1$

\[ q_{ult} = 22.54.2 + 931.5 + 0.4(21.33)(2)(42.4)(0.5) \]
\[ = 22.54.2 + 931.5 + 352.37 = 3527.4 \text{ kPa} \]

Case 4: $W_y = 0.5, d = 1.25 - 0.5 = 0.75, W_x = 0.5 + (\frac{0.75 \times 0.5}{1.75}) = 0.7$

\[ q_{ult} = 22.54.2 + V_{sat} D_f N_y W_y + 0.4 V_{sat} D_f N_x W_x \]
\[ = 22.54.2 + (21.33)(1.25)(41.4)(0.7) + 0.4(21.33)(2)(42.4)(0.7) \]
\[ = 22.54.2 + 724.08 + 384.3 = 3542.6 \text{ kPa} \]

Or, you may calculate \( \bar{q} = 0.5(18) + 0.75(21.33 - 10) = 17.41 \text{ kPa} \)

And use \( V_{sat} \) in 3rd term without using \( W_x \) and \( W_y \)

i.e \( q_{ult} = 22.54.2 + 17.49(41.4) + 0.4(21.33)(2)(42.4) \)
\[ = 22.54.2 + 724.08 + 384.3 = 3542.6 \text{ kPa} \]

It is preferable to use the first approach.

Case 5: $W_y = 0.5, W_x = 0.5$

\[ q_{ult} = 22.54.2 + (21.33)(1.25)(41.4)(0.5) + 0.4(21.33)(2)(42.4)(0.5) \]
\[ = 22.54.2 + 551.9 + 361.7 = 3142.8 \text{ kPa} \]

Or, you may calculate \( \bar{q} = 1.25(21.33 - 10) = 14.18 \text{ kPa} \)

\[ q_{ult} = 22.54.2 + 14.16(41.4) + 384.3 = 3204.8 \text{ kPa} \]

Note: It can be noted that $q_{ult}$ is reduced from 3796 kPa to 3142.8 kPa due to the influence of W.T. i.e. it is reduced by $\frac{3796 - 3147}{3796} \times 100 = 17.1%$. 
**Table 4-1** Bearing-capacity equations by the several authors indicated

Terzaghi (see Table 4-2 for typical values and for \( K_p \) values)

**Footing type**

<table>
<thead>
<tr>
<th>Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous</td>
<td>( q_{un} = cN_c + \gamma N_s + 0.5 \gamma B N_s )</td>
</tr>
<tr>
<td>Square</td>
<td>( q_{un} = 1.5cN_c + \gamma N_s + 0.4 \gamma B N_s )</td>
</tr>
<tr>
<td>Round</td>
<td>( q_{un} = 1.3cN_c + \gamma N_s - 0.3 \gamma B N_s )</td>
</tr>
</tbody>
</table>

\[ N_s = \frac{a^2}{2 \cos(45 + \phi/2)} \]

\[ a = e^{0.75 - 0.75 \ln \phi} \]

\[ N_s = \frac{2 \cos \phi}{\cos^2 \phi} \]

Meyerhof (see Table 4-3 for shape, depth, and inclination factors)

**Vertical load:**

\[ q_{un} = cN_c d_i + \gamma N_s s \]

**Inclined load:**

\[ q_{un} = cN_c d_i + \gamma N_s s \cos \phi + 0.5 \gamma B N_s d_i \]

\[ N_s = \frac{e^{0.5 \tan(45 + \phi)}}{2} \]

\[ N_s = (N_s - 1) \cot \phi \]

\[ N_s = (N_s - 1) \tan(1.4 \phi) \]

Hansen (see Table 4-5 for shape, depth, and inclination factors)

**General:**

\[ q_{un} = cN_c + \gamma N_s + d_i g + 0.5 \gamma B N_s + d_i g + 0.5 \gamma B N_s \]

**when**

\( \phi = 0 \)

**use**

\[ q_{un} = 5.14(1 + s / d_i - \gamma \gamma B_i - g) \]

\[ N_s = \text{same as Meyerhof above} \]

\[ N_s = \text{same as Meyerhof above} \]

\[ N_s = 1.5(N_s - 1) \tan \phi \]

**Table 4-2** Bearing-capacity factors for the Terzaghi equations

Values of \( N_s \) for \( \phi \) of 30 and 48° are original Terzaghi values and used to back-calculate \( K_p \) for forward computations of \( N_s \) by author

<table>
<thead>
<tr>
<th>( \phi ) deg</th>
<th>( N_s )</th>
<th>( N_s )</th>
<th>( N_s )</th>
<th>( K_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.7</td>
<td>1.9</td>
<td>0.0</td>
<td>10.8</td>
</tr>
<tr>
<td>1</td>
<td>7.3</td>
<td>2.7</td>
<td>0.5</td>
<td>12.2</td>
</tr>
<tr>
<td>10</td>
<td>9.6</td>
<td>2.7</td>
<td>1.2</td>
<td>14.7</td>
</tr>
<tr>
<td>15</td>
<td>12.9</td>
<td>4.4</td>
<td>2.5</td>
<td>18.6</td>
</tr>
<tr>
<td>20</td>
<td>17.7</td>
<td>7.4</td>
<td>5.0</td>
<td>23.0</td>
</tr>
<tr>
<td>25</td>
<td>25.1</td>
<td>12.7</td>
<td>9.7</td>
<td>35.0</td>
</tr>
<tr>
<td>30</td>
<td>37.2</td>
<td>22.5</td>
<td>19.7</td>
<td>52.0</td>
</tr>
<tr>
<td>35</td>
<td>52.6</td>
<td>36.5</td>
<td>35.0</td>
<td>82.0</td>
</tr>
<tr>
<td>40</td>
<td>75.9</td>
<td>81.3</td>
<td>100.4</td>
<td>140.0</td>
</tr>
<tr>
<td>45</td>
<td>127.3</td>
<td>173.3</td>
<td>297.5</td>
<td>298.5</td>
</tr>
<tr>
<td>50</td>
<td>258.3</td>
<td>287.9</td>
<td>780.1</td>
<td>800.9</td>
</tr>
</tbody>
</table>

**Table 4-4** Bearing-capacity factors for the Meyerhof and Hansen bearing-capacity equations

Note that \( N_s \) and \( N_t \) are same for both equations

<table>
<thead>
<tr>
<th>( \phi ) deg</th>
<th>( N_s )</th>
<th>( N_t )</th>
<th>( N_t / N_s )</th>
<th>( 2 \tan \phi (1 - \sin \phi) )</th>
<th>( N_t / N_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.7</td>
<td>1.9</td>
<td>0.19</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>6.5</td>
<td>1.6</td>
<td>0.24</td>
<td>0.15</td>
<td>0.2</td>
</tr>
<tr>
<td>10</td>
<td>8.3</td>
<td>2.5</td>
<td>0.30</td>
<td>0.24</td>
<td>0.4</td>
</tr>
<tr>
<td>15</td>
<td>11.0</td>
<td>3.9</td>
<td>0.36</td>
<td>0.29</td>
<td>1.1</td>
</tr>
<tr>
<td>20</td>
<td>14.8</td>
<td>6.4</td>
<td>0.43</td>
<td>0.32</td>
<td>2.9</td>
</tr>
<tr>
<td>25</td>
<td>20.7</td>
<td>10.7</td>
<td>0.51</td>
<td>0.33</td>
<td>6.8</td>
</tr>
<tr>
<td>30</td>
<td>30.1</td>
<td>18.4</td>
<td>0.61</td>
<td>0.29</td>
<td>15.7</td>
</tr>
<tr>
<td>35</td>
<td>46.1</td>
<td>33.2</td>
<td>0.72</td>
<td>0.25</td>
<td>37.1</td>
</tr>
<tr>
<td>40</td>
<td>75.3</td>
<td>64.2</td>
<td>0.85</td>
<td>0.21</td>
<td>93.7</td>
</tr>
<tr>
<td>45</td>
<td>132.9</td>
<td>134.9</td>
<td>1.01</td>
<td>0.17</td>
<td>262.7</td>
</tr>
<tr>
<td>50</td>
<td>206.9</td>
<td>319.0</td>
<td>1.26</td>
<td>0.13</td>
<td>873.7</td>
</tr>
</tbody>
</table>

* \( N_{Mey} \) = Meyerhof value.
Table 4-5  
Shape, depth, inclination, and other factors for use in the Hansen bearing capacity equation in Table 4-1

<table>
<thead>
<tr>
<th>Shape factor</th>
<th>Depth factor</th>
<th>Inclination factor</th>
<th>Ground factor (see figure)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_e = 0.2B/L$</td>
<td>$d_e = 0.40/B$</td>
<td>$\nu = 0.5 - 0.5\sqrt{H/A}\psi_e$</td>
<td>$v' = \psi'/147^\circ$ for horizontal ground use $g' = 0.0$</td>
</tr>
<tr>
<td>$s_e = 1 + N_e B/N_e L$</td>
<td>$d_e = 0.4 \tan^{-1} \frac{D}{B}$</td>
<td>$i_e = i_e - (1 - i_e)(N_e - 1)$</td>
<td>$g' = 1 - \psi'/147^\circ$</td>
</tr>
<tr>
<td>$\nu = 1 + 0.4 \frac{D}{B}$</td>
<td>$\nu = 1 + 0.4 \frac{D}{E}$</td>
<td>$\nu = 0.5H \frac{1}{V + A_e \psi_e \cot \phi}$</td>
<td>$s_a = g_a = (1 - 0.5 \tan \psi)^2$</td>
</tr>
<tr>
<td>$s_e = 1 + (B/L) \tan \phi$</td>
<td>$d_e = 1 + 2 \tan \phi(1 - \sin \phi)^2 \frac{D}{B}$</td>
<td>$h_e = \frac{h_e}{147^\circ}$ for horizontal ground use $h' = 0.0$</td>
<td>$b_e = b_s = \exp \ 2\eta \tan \phi$</td>
</tr>
<tr>
<td>$s_e = 1 - 0.4B/L$</td>
<td>$d_e = 1.00$ for all $\phi$</td>
<td>$h_e = \frac{h_e}{147^\circ}$</td>
<td>$\eta$ radians for $b_e$</td>
</tr>
</tbody>
</table>

Table 4-5 cont'd

where $A_f$ = effective footing contact area
$L$ = effective footing length = $L_e - 2e$.
$W$ = effective footing width = $W_e - 2e$.
$D$ = depth of footing in ground.

$e$, $e_s$ = eccentricity of load with respect to center of footing area.
$c$ = cohesion of base soil.
$\phi$ = angle of internal friction of soil.
$H$, $V$ = load components parallel and perpendicular to footing, respectively.
$\tan \delta$ = coefficient of friction between footing and base soil (use $\delta = \phi$ for concrete poured on ground [Schultze and Horn (1967)].

$\eta$, $\psi$ = is shown in accompanying figure with positive directions shown.

Notes: Do not use shape factors in combination with inclination factors. Use $d_e$ and $i_e$ only in combination of $s_e$ with $d_e$, $g_e$, and $b_e$. When triaxial $\phi$ is used for plane-strain conditions, one may adjust to obtain: $\phi' = 1.1\phi_{true}$ (author's suggestion only for $\phi_{true} > 30^\circ$).

Limitations: $H \leq V \tan \delta + c_A$.

\[ i_e, i'_e > 0 \]
Kempthorn's Value for $N_e$ for Terzaghi equation only

1. $\left[ N_e \right]_{\text{vert}} = \left[ 1 + 0.2 \frac{F}{L} \right] \left[ N_e \right]_{\text{strip}}$

2. For depth $\frac{D}{B} < 2.5$
   
   $\left[ N_e \right]_{\text{strip}} = \left[ 1 + 0.2 \frac{D}{B} \right] \left[ N_e \right]_{\text{surface}}$

3. For depth $\frac{D}{B} > 2.5$
   
   $N_e = 1.5 \left[ N_e \right]_{\text{surface}}$

---

If $D_e = 8.4$ for a seepage with $B = 1.25 \text{ m}$, $L = 1.5 \text{ m}$, $D = 2 \text{ m}$

Find the undrained cohesion $C_u$ for another footing for:

$B = 1.25 \text{ m}$, $L = 1.5 \text{ m}$, $D = 2 \text{ m}$, $F = 0.3 \text{ kPa/m}$, $e = 0.5$

$V = 0.2 \text{ kPa}$

---

1. For footing
   
   $\left[ N_e \right]_{\text{vert}} = \left[ 1 + 0.2 \frac{D_e}{L} \right] \left[ N_e \right]_{\text{strip}}$

   $\frac{D_e}{L} = \frac{8.4}{1.5} = 5.6$

   $\left[ N_e \right]_{\text{strip}} = \frac{8.4}{1.5} = 5.6$

   $\left[ N_e \right]_{\text{surface}} = \frac{5.6}{1 + 0.2 \frac{5.6}{1.5}} = 5$

2. For footing
   
   $\left[ N_e \right]_{\text{strip}} = \left[ 1 + 0.2 \frac{D_e}{L} \right] \left[ N_e \right]_{\text{strip}}$

   $\frac{D_e}{L} = \frac{8.4}{1.5} = 5.6$

   $\left[ N_e \right]_{\text{vert}} = \frac{5.6}{1 + 0.2 \frac{5.6}{1.5}} = 5.6$

   $\left[ N_e \right]_{\text{strip}} = \frac{5.6}{1 + 0.2 \frac{5.6}{1.5}} = 5.6$

   $\left[ N_e \right]_{\text{surface}} = \frac{5.6}{1 + 0.2 \frac{5.6}{1.5}} = 5.6$

   $g_{ult} = C N_e = 8.16 C$

$F_{ult} = 36.1 \text{ kPa}$
Bearing Capacity

Meyerhof equation

Assumptions:

- \( P = V + S \)
- No adjustment to \( \phi \) for \( c = (1.1 - 0.1 \frac{b}{L}) \phi \)

Equation:

\[
Q = c + \phi N_q \frac{d_y}{g_y} dy = 0.5 \frac{y}{d_y} \frac{d_y}{g_y} dy
\]

\[
Q = c + \phi N_q \frac{d_y}{g_y} dy = 0.5 \frac{y}{d_y} \frac{d_y}{g_y} dy
\]

where \( N_q \) = Meyerhof Bearing factor

- \( c = \) shape, depth and inclination

See Table 4-2, pg 126

Example: Consider Ex. 1o using Meyerhof Bearing

\( \phi = 0.91 \) and \( B = \frac{b}{L} \)

\( \phi = (1.1 - 0.1 \frac{b}{L}) \phi = (1.1 - 0.1 \frac{2}{2}) = 0.85 \)

for \( \psi = \frac{y}{d_y} \)

from Table 4-4, \( N_q = 46 \), \( N_y = 3.7 \)

\( S_y = \tan\left(45 + \frac{\psi}{2}\right) \frac{d_y}{g_y} \left(\frac{\psi}{2}\right) = 1.732 \)

\( S_y = \tan\left(45 + \frac{\psi}{2}\right) \frac{d_y}{g_y} \left(\frac{\psi}{2}\right) = 1.732 \)

\( d_y = \sqrt{k_p} = \left[ \frac{1.25}{\psi} \right] = 1.218 \)

\( d_y = \sqrt{k_p} = \left[ \frac{1.25}{\psi} \right] = 1.218 \)

\( \phi_{tot} = N_{tot} + \phi N_y \frac{d_y}{g_y} \frac{d_y}{g_y} \)

\( \phi_{tot} = N_{tot} + \phi N_y \frac{d_y}{g_y} \frac{d_y}{g_y} \)

\( = 0.7 \times (1.732) (1.21) + 0.75 \times (2) (1.732) (1.21) \)

\( = 2.776 + 11.488 + 10.239 \)

\( = 21.523 \text{ kPa} \)
Eccentrically Loaded Footings

There are two approaches:

1. **Conventional Method**

   Replace eccentric load by a parallel concentric load of equal magnitude and a balancing moment for equilibrium.

   Find out the maximum stress and minimum stress under the footing using:
   \[ \sigma_{\text{max}} = \frac{Q}{A} \left( 1 \pm \frac{L}{L} \frac{L}{L} \right) \quad \text{[see Fig. 1 below]} \]

   \[ \sigma_{\text{max}} \text{ must be } \leq \text{Allowable} \quad \text{and} \quad \sigma_{\text{min}} > 0 \]

2. **Meyerhof Approach**

   In this method, the area of the footing is suitably reduced so that the load becomes concentric. The effective footing dimensions will become:
   \[ B' = B - 2c_y \quad L' = L - 2c_x \quad L' = B' L' \]

   Use \( B' \) in the third term in \( B \). Equation and in calculating shape factors and inclination factors only.

   Note: As a factor of safety, we may combine the two approaches.

---

**Diagram:**
Two-way eccentric loading in the footing.
Equivalent loading system adopted for the conventional method.
Bearing Capacity

Gross versus Net Soil Pressure [Bowles pg 211]

a. Gross (Total) pressure: is the actual pressure between the soil and the base of the foundation. This includes pressure due to the external loading of the foundation, the weight of the foundation itself and the weight of the soil above it.

b. Net pressure: is the intensity of external load at the base of footing (excluding the dead weight of the above soil).

c. The bearing capacity equations (9.44) are based on gross soils pressure which is everything above the foundation level. Settlement are caused only by net increases in soil pressure over the existing overburden pressure.

\[
(9)_{ult} = \frac{(9)_{ult} - \gamma D_s}{(9)_{allowable}} = \frac{(9)_{ult} - \gamma D_s}{(9)_{ult} - \gamma D_s} = \frac{\gamma D_s}{(9)_{ult} - \gamma D_s}
\]

**Factor of Safety** [Bowles pg 163 and pg 211]

The factor of Safety \( F_s \) with respect to shear failure is defined in terms of the net ultimate \( (9)_{ult} \) and

\[
F_s = \frac{(9)_{ult}}{(9)_{allowable}} = \frac{(9)_{ult} - \gamma D_s}{(9)_{ult} - \gamma D_s} = \frac{\gamma D_s}{(9)_{ult} - \gamma D_s}
\]

or \( (9)_{ult} = \frac{(9)_{allowable} \cdot (9)_{ult} - \gamma D_s}{F_s} \cdot \gamma D_s \)

The required factor of safety depends on

(i) Type of structure (permanent or temporary)
(ii) Sensitivity of structure
(iii) Extent of soil exploration
(iv) Nature of loading considered and assumption made in the design
(v) Extent of quality control during construction

It is recommended that factor of safety should lie between 2 and 4.
Example 4

Resolve Example 1b and check the adequacy of the footing if the load on the foundation base (including net of foundation) 

\( Q = 4,000 \text{ kN} \). The moment, \( M = 800 \text{ kN-m} \) and the \( F_s = 3 \).

Solution: There are two approaches

1. Conventional approach

   **Step 1:** Find allowable gross B.C.

   \[ q_{ult} = (q_{gross})_{ult} = 5153 \text{ kPa} \quad \text{from Example 1b} \]

   \[ (q_{gross})_{allowable} = \frac{(q_{ult})_{ult} - Y(R_f + 300)}{F_s} = \frac{5153 - 1.25(15) + (125)(18)}{3} \]

   **Step 2:** Find max and min pressure on soil

   \[ q_{max} = \frac{Q}{BL} \left( 1 + \frac{6D}{L} + \frac{6D^2}{L^2} \right) = \frac{4000}{(25)(2)} \left[ 1 + 0.5 \left( \frac{6(2)}{2} \right) \right] = 1600 \text{ kPa} < 1722 \]

   \[ q_{min} = \frac{Q}{BL} \left( 1 - \frac{6D}{L} - \frac{6D^2}{L^2} \right) = \frac{4000}{(25)(2)} \left[ 1 - 0.5 \left( \frac{2}{2} \right) \right] = 1000 > 0 \text{ kPa} \]

   The footing is O.K.

2. Meyerhof approach

   **Step 1:** Find \( q_{ult} \) for eccentric footing

   \[ B = B_1 - 2D_e = 2 - 2(0.7) = 1 \text{ m} \quad L_1 = 1 \text{ m} \quad c = 2 \quad o = 2 \text{ m} \]

   The shape factors are influenced only

   \[ s_c = 1 + 0.2 \frac{B_1}{L_1} = 1 + 0.2 \left[ \frac{20}{1} \right] = 1.59 \]

   \[ s_d = 1.0 \frac{B_1}{L_1} = 1.0 \left[ \frac{20}{1} \right] = 2.0 \]

   \[ q_{ult} = \left( 30 \right) \left( 42.1 \right) \left( 1.59 \right) \left( 1.24 \right) + \left( 125 \right) \left( 18 \right) \left( 33.3 \right) \left( 1.295 \right) \left( 1.12 \right) 

   + 0.5 \left( 18 \right) \left( 1.6 \right) \left( 33.1 \right) \left( 1.795 \right) \left( 1.12 \right) 

   = 2776.7 + 1086.7 + 774.8 = 4638.2 \text{ kPa} \quad \text{< 5152 kPa} \]

   **Step 2:** Find allowable gross B.C.

   \[ (q_{gross})_{allowable} = \frac{4638.2 - 1.25(15) + 1.25(18)}{3} \text{ kPa} \]

   **Step 3:** Find actual pressure on soil

   \[ q_{ult} = \frac{4000}{B_1L_1} = \frac{4000}{(25)(2)} = 1250 \text{ kPa} \quad \text{< 1544 kPa} \quad \text{O.K.} \]

   **Note:** Resolve using Terzaghi and General B.C.
Example 5
Design the following square footing using Terzaghi B.C equation (C=5:5v=2.5)

Solution:

Step 1: Find $Y_{sfu}$ and $Y_{fgy}$

$$Y_{sfu} = \frac{G + e}{1 + e} \quad Y_{fgy} = \frac{2.7 + 0.91}{1 + 0.81}$$

$$Y_{sfu} = \frac{3.6}{1.81} = 1.94 \text{ kN/m}^2$$

$$Y_{fgy} = \frac{3.7}{1.81} = 2.04 \text{ kN/m}^2$$

Step 2: Apply B.C equation

$$Q_{ult} = \frac{N_x}{3} + \frac{N_y}{2} w_T + 0.4 \gamma B N_z W_z$$

For $v=10$, $N_x = 9.6$, $N_y = 2.7$, $N_z = 12$ (From Table 4.2 Bold)

For W.T at Foundation level $w_T = 1$, $w_T = 0.5$

$$Q_{ult} = (9.6)(5.6) + (14.9)(0.5)(2.7)(1) + (0.4)(19.6) + (5.8)(0.5)$$

$$Q_{ult} = 512.1 + 465 = 977.1$$

Step 3: Find $(Q_{gross})_{allowable}$

$$(Q_{gross})_{allowable} = \frac{(Q_{gross})_{ult}}{1 - 0.4}$$

$$= \frac{977.1 + 4.65}{0.6}$$

$$= 212 + 1.86 B$$

Step 4: Find the actual pressure on soil

$$q = \frac{Q}{A} = \frac{700}{B^2}$$

Step 5: Equating $(Q_{gross})_{allowable} = q$ and find $B$

$$212 + 1.86 B = \frac{700}{B^2}$$

$$1.86 B^3 + 212 B^2 - 700 = 0$$

Solving by Trial and error

For $B = 2$ => $1.86 (2)^3 + 212 (2)^2 - 700 = 162.5 > 0$ $\checkmark$

For $B = 1.5$ => $1.86 (1.5)^3 + 212 (1.5)^2 - 700 = -216.7 < 0$ $\times$

$$B = 1.8 \Rightarrow 1.86 (1.8)^3 + 212 (1.8)^2 - 700 = -277 < 0$$ $\times$

H.W. 2: Redesign using Meyerhof and Hansen B.C equations.

H.W. 3: Redesign if an additional moment of M=50 kN.m in one direction is applied [use Terzaghi B.C. and both approaches].
Bearing Capacity

Layers on Layered Soils [See Bowles pg 149]

In the previous analysis, the soil is assumed to be homogeneous throughout the zone of shear ($\approx 0.5 \times \tan (45 + \varphi)$). There are two approaches to solve such problems:

1. Soils' Approach

When the soil strength does not vary more than 5% within the depth of shear zone then a weighted average of strength parameters ($C, \phi$) are computed:

$$ C_0 = C_h \frac{H_i}{H} \tan \phi_i + C_s \frac{H_s}{H} \tan \phi_s + \ldots + C_n \frac{H_n}{H} \tan \phi_n $$

These values are used in Eq. 6.3:

If there is greater variation in the soil strength, then you may use the minimum values.

2. Reddy and Srinivasan's Approach

Can be applied to two layers of cohesive soils:

Use Hansen's Eq.:

$$ Q_{cb} = C, N_s (1 - s + d') + fN_q $$

where $C$ = cohesion of first layer

$N_s = B.C$ factor to be obtained from Fig 4.7 (Bowles)

$\delta = $ thickness of 1st layer

$K = $ ratio of the vertical to horizontal shear strength of the 1st layer (for anisotropic soil $K = 1$). Usually take $K = 1$

$c_v$ = cohesion of 2nd layer

$\delta = $ thickness of 2nd layer

$Q_{cb}, Q, d'$, $f$ and $N_q$ are as defined in Hansen eq.

$$ Q_{cb} = \frac{C, N_s (1 - s + d') + fN_q}{\delta / K} $$

For $Q, d', f$ and $N_q$ are as defined in Hansen eq.
Table 4.7 Bearing-capacity factors \( N_c \) for footings on \((R, h = 0)\) or adjacent to a slope

Refer to Fig. 4.11(b). A slope angle \( \theta \) may be used when depth of area is \( h < 0.1 \) or when \( h > 0.5 \). Approximate values given show extent of change of bearing capacity and depth of embedment.

<table>
<thead>
<tr>
<th>( h / \theta )</th>
<th>( \theta ), degrees</th>
<th>( N_c ), values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1-0.5</td>
<td>0</td>
<td>1.0-1.2</td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>15</td>
<td>1.2-1.4</td>
</tr>
<tr>
<td>1.0-2.0</td>
<td>30</td>
<td>1.4-1.6</td>
</tr>
<tr>
<td>2.0-5.0</td>
<td>45</td>
<td>1.6-1.8</td>
</tr>
<tr>
<td>5.0-10.0</td>
<td>60</td>
<td>1.8-2.0</td>
</tr>
</tbody>
</table>

Figure 4.11(b) shows a bearing capacity for footings on a slope with different values of \( h / \theta \). The values of \( N_c \) correspond to the table.

Figure 4.11(c) shows the effect of embedment depth on bearing capacity.
Example 6

Check the adequacy of the footing use $F = 3$, $K = 1.2$

Solution:

Step 1 Find $Y_{sat}$ and $Y_{dry}$ for each soil

\[
Y_{sat} = \frac{C_{uv} + Y_{w}}{1 + e} \quad Y_{dry} = \frac{C_{uv} - \phi}{1 + e} \cdot Y_{w}
\]

<table>
<thead>
<tr>
<th>Soil</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{sat}$ (kN/m)</td>
<td>15.4</td>
<td>18.6</td>
<td>19.4</td>
</tr>
<tr>
<td>$C_{uv}$ (kN/m)</td>
<td>15</td>
<td>18.8</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Step 2a Apply Souk's approach

- Since the soils 2 and 3 are cohesive ($\phi = 0$) and $c$ values do not vary more than 50% ($\frac{80 - 60}{80} = 25%$) then we may use Souk's approach.

\[
C_{uv} = \frac{C_{1}H_{i} + C_{2}H_{i}}{1 + 1/n} \quad H_{i} = \text{depth of zone of shear} = H_{i}
\]

\[
H_{i} = 0.5 \times L \quad H_{i} = 0.5 (1.5) \times \tan (45^\circ - \phi/2) - 0.5 - 0.22
\]

\[
C_{uv} = 60 (0.5) + 80 (0.25) = 66.2 \text{ kPa}
\]

\[
\phi_{uv} = \tan^{-1} \frac{H_{i} \tan \phi_{uv} + H_{i} \tan \phi}{2H_{i}} = 0 \quad \text{since} \quad \phi_{uv} = 0
\]

Step 2b Apply general P.C

9ult = 5.14 $S_{u} \left( 1 + \frac{C_{uv} - c'}{c'} \right)$ kPa \[\text{for} \theta = 0, \text{Table 4.1} \]

where $S_{u} = \text{undrained shear strength} = c = 66.2$

\[
S_{u} = 0.2 \times 0.2 \times \frac{2}{3} = 0.15
\]

\[
d' = 0.4 \times 0.4 \times \frac{1}{1.5} = 0.032
\]

\[
c' = 0.5 - 0.5 \sqrt{-\frac{d'}{1-\sigma}} = 0.5 - 0.5 \sqrt{-0} = 0 \quad \text{for} \ H_{0} \text{, i.e. no inclined load}
\]

\[
\theta = \tan^{-1} \frac{H_{i} \tan \phi + H_{i} \tan \phi}{2H_{i}} = 0
\]

\[
\phi = Y_{sat} + Y_{sub} \times 15 (0.8) + (9.4 - 10) (0.4) = 15.76 \text{ kPa}
\]

\[
9ult = 5.14 (66.2) (1 + 0.15 + 0.32 - 0 - 0) = 507.2 \text{ kPa}
\]

Step 4a Find allowable gross B.C

\[
(9_{gross})_{\text{allowable}} = \frac{9_{ult} - 3}{F} = \frac{507.2 - 15.76}{162.48} = 3.32
\]
Bearing Capacity

\[ \text{Step } 5 \text{a: Find actual pressure on soil} \]

We of concrete is footing = \[ (0.8)(2)(0.4) + (0.2)(0.3)(0.2) \] 24.305 kN

We of soil above footing = \[ ((1.5)(2)(0.8) - (0.5)(0.2)(0.8)) \] 15.349 kN

Total load at the base of footing (Q_{gross}) = 300 + 30.5 + 24.9 = 355.4 kN

Gross pressure at the base of footing (Q_{gross}) = 355.4 \text{ kPa} < 162.4 \text{ kPa}

\[ \text{Step } 2 \text{b: Apply Reddy and Srinivasan's approach} \]

\[ Q_{ult} = C_i \times N_c \times (1 + d_i + c_i) + g \times N_g \]

where \( C_i \) is cohesion of upper soil = 60 kPa

\( N_c \) = B.C. Factor from Fig. 4.3

for \( K = 1.2 \) (given) and \( e_1 = \frac{80}{80} = 1 \) and \( d = \frac{0.5}{0.75} \)

\( N_c = 6.1 \) from Fig. 4.3 b

\( N_g = 1 \) from Table 4.4

\( S_c = 0.15, c_i = 0.32, g = 15 \text{ kPa} \) [calculated in step 1a]

\[ Q_{ult} = 60 \times (6.1) \times (1 + 0.15 + 0.32) + 15 \times 76 (1) \]

\[ = 538.62 + 15 \times 76 = 1537 \text{ kPa} \]

\[ (Q_{gross})_{allowable} = \frac{Q_{ult}}{S} = \frac{537.6}{3} = 179.3 \text{ kPa} \]

\[ \text{Step 4b: Find actual pressure on soil} \]

\[ Q_{gross} = 121.5 \text{ kPa} < 179.3 \text{ kPa} \]
Bearing Capacity

1. Find exit at E such that exit angle, $\psi$ = $\frac{\pi}{3}$
2. Compute reduced $N_e$ based on failure surface Line $AB$ in the standard case and failure surface Line $AC$ in case $N_e' = N_e - \frac{L_e}{A_e}$ (or use Table 4.4)
3. Compute reduced $N_q$ as $\frac{N_q}{A_s}$ = $\frac{N_q}{A_s}$ for $A_s > A_b$
4. No correction for $N_q$
5. Use $q_{ul}$ for $N_e'$ and $q_{ul} = \phi \frac{q}{q}$ for $N_e'$ and $q_{ul}$

Example 2

Find allowable $Q$

Solution

Step 1: Find $N_e'$ and $q_{ul}$

Value of $N_e'$ and $q_{ul}$ depends

For $p = 0.5$, $q_s = 0.5$

Table 4.4

<table>
<thead>
<tr>
<th>$q_{ul}$</th>
<th>$N_e'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td>6.40</td>
</tr>
<tr>
<td>0.80</td>
<td>7.60</td>
</tr>
<tr>
<td>0.90</td>
<td>8.80</td>
</tr>
</tbody>
</table>

By linear int. relation for $p = 0.70$

$Q_{ul} = 6.40 - \frac{13.91}{0.80}$

$N_q = 22$

No need to calculate $N_e'$ since $q_{ul}$ however it can be calculated in the same manner.
Step 3. Apply General BE equation

\[ P_{BE} = (N_1)\phi_{BE} \sigma_{BE} + \frac{1}{2} B N_1 \phi_{BE} \sigma_{BE}^2 + \frac{1}{3} B^2 N_1 \phi_{BE} \sigma_{BE}^3 \]

since \( P_{BE} \) therefore no need to calculate 1st term.

\[
\begin{align*}
\phi &= 1 + \frac{L}{a} \tan \alpha = 1 + \frac{1}{2} \tan 30^\circ = 1.18 \\
\phi &= 1 + 0.4 \frac{L}{a} = 1 + 0.4 \times 1 = 1.4 \\
\phi &= 1 & Al = 1 \\
N_1 &= 16.1 & N_8 &= 37.9 - N_1 \\
&= 26.38 \\
&= \frac{33.9 - 15.1 \times 26.38}{75 - 70} = 38.7 - \frac{N_1}{75 - 70}
\end{align*}
\]

\[ P_{BE} = 34.0 + (3) \left(0.11\right) \left(27\right) \left(1.65\right) \left(1\right) + \frac{1}{2} \left(0.11\right) \left(4\right) \left(26.38\right) \left(0.6\right) \]

\[ = 15.46 \text{ kip*ft} \]

\[ (Q_{net})_{all} = (Q_{net})_{all} \left(70\right)/3 = 15.7 - \frac{5.13}{0.11} \text{ kip*ft}^{-1} \]

\[ (Q_{net})_{all} = \text{Area} \times 0.517 = 80.7 \text{ kip} \]

**Bearing Capacity**

There are two approaches:

**Terzaghi and Rank Approach**: Use Fig. 7.5 (below) below

**First**

\[ P_{BE} = \phi_{BE} \sigma_{BE} \]

\[ (Q_{net})_{all} = \text{Area} \times 0.517 \]

**Second**

\[ P_{BE} = \frac{1}{2} B N_1 \phi_{BE} \sigma_{BE}^2 + \frac{1}{3} B^2 N_1 \phi_{BE} \sigma_{BE}^3 \]

\[ (Q_{net})_{all} = \frac{1}{2} B N_1 \phi_{BE} \sigma_{BE}^2 + \frac{1}{3} B^2 N_1 \phi_{BE} \sigma_{BE}^3 \]

\[ (Q_{net})_{all} = \text{Area} \times 0.517 \]

**Equation**

\[ P_{BE} = \frac{1}{2} B N_1 \phi_{BE} \sigma_{BE}^2 + \frac{1}{3} B^2 N_1 \phi_{BE} \sigma_{BE}^3 \]

\[ (Q_{net})_{all} = \frac{1}{2} B N_1 \phi_{BE} \sigma_{BE}^2 + \frac{1}{3} B^2 N_1 \phi_{BE} \sigma_{BE}^3 \]

\[ (Q_{net})_{all} = \text{Area} \times 0.517 \]
Bearing Capacity

Meyerhof (1956, 1974) approximate equation:

\[ q_a = \frac{N}{F_i} \quad \text{for } B \leq B_v \]
\[ q_a = \frac{N}{F_i} \left( \frac{B_v}{B} \right)^{1/3} \quad \text{for } B > B_v \]

where \( N_o \) average corrected \( N \) for overburden pressure
for 0.5 \( B \) below footing

\( q_a \) allowable bearing pressure for settlement 0.25 mm in kPa or ksf

<table>
<thead>
<tr>
<th>ksf units</th>
<th>( \frac{q_a}{F_i} )</th>
<th>( \frac{q_a}{B} )</th>
<th>( \frac{F_i}{B} )</th>
<th>( \frac{B_v}{B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.05</td>
<td>0.08</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

\( k_d = 1.4 \) for 0.33 \( B \) \( \leq 1.33 \)

For mat foundation \( q_a = \frac{N_o}{F_i} \)

\( (q_a) \text{ any settlement} : (q_a)_{25 \text{ mm}} \frac{\text{settl}}{25} = \)

Bearing Capacity from Cone Penetration Test

1. Schmertmann (1975)

\[ N_c = \frac{q_c}{80} \quad \text{for sand} \]
\[ q_c = \text{cone resistance (kPa)} \]

\[ N_c \approx \phi \cdot \rho_c \cdot N_o, \ q_c \rightarrow B = \text{B.C. equation} \]

2. Meyerhof (1965)

\[ q_a = \frac{q_c}{80} \quad \text{for } B \leq B_v \]
\[ q_a = \frac{q_c}{80} \left( \frac{B_v}{B} \right)^{1/3} \quad \text{for } B > B_v \]

see above table for values of \( F_i \) and \( F_o \)
University of Technology

Building and Construction Engineering
Department

Roads and Bridges Section

FOUNDATION ENGINEERING COURSE

Subject:- Shallow foundation - settlement

Prepared by :- Prof. Dr. Kais Taha Shlash

Course designation : geo. 01
Types of Settlement

1. Immediate (elastic) Settlement ($S_i$) is caused by elastic and plastic deformations of the soil particles themselves. Mostly occurs in sands or fine-grained soils with $5 < 90\%$ [0-2 days].

2. Primary Consolidation Settlement ($S_c$) is caused by rearrangement of the soil particles, leading to a reduction in voids in the soil. Mostly occurs in fully saturated silty clay [years].

3. Secondary Consolidation Settlement ($S_s$) occurs after the completion of the consolidation of the soil due to creep of soil.

\[ S_{tot} = S_i + S_c + S_s \]

Settlement Considerations

The following settlement must be calculated and checked for a given project:

1. Total Settlement ($S_t$ or $S_i$)
2. Differential Settlement ($S_i - S_t$)
3. Angular Distortion ($\frac{S_i - S_t}{L}$)

Contact Pressure

The actual or the theoretical predicted pressure at the base of the footing is known as the contact pressure. The contact pressure distribution depends on:

1. Nature of footing (flexible or rigid)
2. Type of soil (cohesionless or cohesive)
3. Depth of footing

In practice, it is generally assumed that the pressure distribution at the base of the footing is uniform for centroidal loading and linearly varying for eccentric loading.
Settlement

Stresses in a Soil Mass due to Footing Pressure

Several methods are currently used to estimate the increased pressure on an element of soil at some depth in the strata.

A: Approximate Method: (2:1 method)

\[ \Delta q = \frac{Q}{(L+2)(B+2)} \] at and depth z

B: Boussinesq and Newmark Charts

Assumption:
- The soil is elastic, homogeneous, semi-infinite, and isotropic and obey Hooke's Law.

Types of external loads:
1. Concentric point load (use the following equation)

\[ \Delta q = \frac{3G}{2\pi R^2} \left(1 + \frac{R^2}{L^2}\right) \]

2. Circular Loaded Area (use Fig 5.4 Lambe)
3. Square Loaded Area (use Fig 5.4 Bowles)
4. Continuous Loaded Area (use Fig 5.4 Bowles)
5. Rectangular Loaded Area (use Fig 8.6 Lambe)
6. Triangular Loaded Area (use Fig 8.8 Lambe)
7. Embankment Loaded Area (use Fig 7.16 Des)
8. Irregular Loaded Area (use Fig 5.3 Bowles)

For more details see Lambe or Bowles (ex. 5.1)
Fig. 8.6 (a) Chart for use in determining vertical stresses below corners of loaded rectangular surface areas on elastic, isotropic material. Chart gives $f(m, n)$. (b) At point $A$, $\Delta \sigma_x = \Delta \sigma_y = f(m, n)$. (From Newmark, 1942)
of dimension $B$ to scale at various $z$ values or ratios of footing dimension and using the influence chart to find pressure intensities at various points both beneath and outside the footing prism. The pressure isobars for a circular foundation are not given since these are simple to obtain by proper interpretation of the influence chart (or by direct use of Eq. (5-31)).

Lateral pressure on an element of soil in a stratum, $q_s$, may also be computed by a Boussinesq equation as

$$q_s = \frac{Q}{2\pi z^2} \left[ 3 \sin^2 \theta \cos^2 \theta - \frac{(1 - 2\mu) \cos^2 \theta}{1 + \cos \theta} \right]$$  (5-7)
Example 3.2 A 10-ft high embankment is to be constructed as shown in Fig. 3.17. If the unit weight of compacted soil is 120 lb/ft³, calculate the vertical stress due solely to the embankment at A, B, and C.

**Solution** \( q = \gamma H = 120 \times 10 = 1200 \text{ lb/ft}^2 \)

**Vertical stress at A**: Using the method of superposition and referring to Fig. 3.18a,

\[
\sigma_{zA} = \sigma_z(1) + \sigma_z(2)
\]

For the left-hand section, \( b/z = 5/10 = 0.5 \) and \( a/z = 10/10 = 1 \). From Fig. 3.16, \( I_1 = 0.396 \). For the right-hand section, \( b/z = 15/10 = 1.5 \) and \( a/z = 10/10 = 1 \). From Fig. 3.16, \( I_2 = 0.477 \). So,

\[
- (a) = (I_1 + I_2)q = (0.396 + 0.477)1200 = 1047.6 \text{ lb/ft}^2
\]

**Vertical stress at B**: Using Fig. 3.18b,

\[
\sigma_{zB} = \sigma_z(1) + \sigma_z(2) - \sigma_z(3)
\]

For the left-hand section, \( b/z = 0/10 = 0 \), \( a/z = 5/10 = 0.5 \). So, from Fig. 3.16, \( I_1 = 0.14 \). For the middle section, \( b/z = 25/10 = 2.5 \), \( a/z = 10/10 = 1 \). Hence, \( I_2 = 0.493 \). For the right-hand section, \( I_3 = 0.14 \) (same as the left-hand section). So,

\[
\sigma_{zB} = I_1(120 \times 5) + I_2(120 \times 10) - I_3(120 \times 5) = 0.493(1200)
\]

\[= 591.6 \text{ lb/ft}^2\]

**Vertical stress at C**: Referring to Fig. 3.18c,

\[
\sigma = \sigma_z(1) - \sigma_z(2)
\]

For the left-hand section, \( b/z = 40/10 = 4 \), \( a/z = 10/10 = 1 \). So \( I_1 = 0.498 \). For the right-hand section, \( b/z = 10/10 = 1 \), \( a/z = 10/10 = 1 \). So \( I_2 = 0.456 \). Hence,

\[
\sigma_{zC} = (I_1 - I_2)q = (0.498 - 0.456)1200 = 50.4 \text{ lb/ft}^2
\]
Immediate Settlement

From theory of elasticity

\[ S_i = \frac{qB}{E_I} \cdot I_w \]

where \( S_i \): immediate settlement
\( q \): intensity of contact pressure \((-\sigma_y)\) (in units of \( E_I \))
\( B \): least lateral dimension of footing (in units of \( S_i \))
\( I_w \): influence factor which depends on shape of footing and its rigidity (typical values are given in Table 5-4 Bowles)

\( E_I \): elastic properties of soil (typical values in Tables 2-6 and 2-7 Bowles)

This equation was given for the deflection of a footing on the ground surface of a semi-infinite half space. When the footing is at depth \( D \) you may use Fig 5-8 Bowles which was proposed by Fox (1948) for correction

\( (S_i)_{\text{final}} = S_i \cdot F_I \)

\( F_I \): a factor depends on \( D/B \), \( H/B \) and \( M \) (Fig 5-8)

Table 5-4 Influence factors \( E_1 \), \( E_2 \) for various-shaped members and for flexible and rigid footings

<table>
<thead>
<tr>
<th>Shape</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>1.00</td>
<td>0.94</td>
<td>(edge)</td>
</tr>
<tr>
<td>Square</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Rectangular</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width</th>
<th>( L )</th>
<th>( B )</th>
<th>( D )</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>3.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>4.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Lee (1996)

† Others have used the value 0.95 for the rigid footing influence factor for circular footings

Figure 5-8: Influence factor for footing at a depth \( D \). Use actual footing width and \( L \) for \( L/B \) ratio.
Consolidation Settlement

- Consolidation: is the gradual reduction in volume of a fully saturated soil of low permeability due to drainage of the excess pore water pressure.
- One Dimensional Consolidation: the simplest case in which a condition of zero lateral strain is implicit.
- Swelling: is the reverse of consolidation: The gradual process of the gradual increase in volume of soil under negative excess pore water pressure.
- Consolidation Settlement: is the vertical displacement of the surface corresponding to the volume change at any stage of the consolidation process.

- Consolidation Test (Oedometer): The consolidation test is intended to provide basic information for making settlement calculations including the time rate of the settlement. An undisturbed sample is carefully trimmed and fitted into a rigid ring. Porous stones are placed on the top and bottom of the sample. Vertical load (consolidation pressure) is applied in increments and the soil moisture is allowed to escape through the porous stones. The amount of compression of the sample at various time intervals is measured by means of a dial gauge. The results of this test are plotted in the form of $e$-$p$ or $e$-$\log p$, $p$ being the consolidation pressure and $e$ being the corresponding void ratio of the soil.
The compressibility of the clay can be represented by

\[ \text{The rate of volume change} \ (\text{mv}) \quad (\text{m}^3/\text{kN}) \]

\[ \text{mv} \quad \text{change in volume per unit volume per unit stress} \]

\[ \text{mv} = \frac{e}{2} \frac{d^2 e}{d \log p} = \frac{h}{2} \frac{d^2 e}{d \log p} \]

\[ \text{mv} \quad \text{is not constant but ranges depending on the stress range} \]

The Compression Index \( e_c \)

\[ e_c \quad \text{slope in the linear portion of the e-log p} \]

\[ e_c = \frac{d e}{d \log p} \]

The expansion part of the e-log p plot can be approximated to a straight line, the slope of which is referred to as the expansion index \( e_g \) (or recompression index or swelling index). Approximately \( e_g = 0.009 (\text{LL}-10) \) for M.C. clay.
Method To Obtain Field Virging Compression Curve

a) Normally Consolidated Soil

\[
\begin{align*}
\Delta H &= \frac{E}{1+e_0} \Delta \varepsilon, \\
\Delta \varepsilon &= \frac{E}{1+e_0} \Delta \sigma
\end{align*}
\]

b) Over consolidated Soil

\[
\begin{align*}
\Delta H &= \frac{E}{1+e_0} \Delta \varepsilon, \\
\Delta \varepsilon &= \frac{E}{1+e_0} \Delta \sigma
\end{align*}
\]

Calculation of \( C_v \): Dimensional Consolidation Settlement

1. \( \frac{\Delta H}{H_0} = \frac{E}{1+e_0} \Delta \varepsilon \)
2. \( C_v = \frac{E}{1+e_0} \Delta \varepsilon \log \frac{\sigma_{0}}{\sigma_{v}} \)

For N.C. (CLay)

\( \Delta H : S_c = \frac{C_v}{1+e_0} \Delta \varepsilon = \frac{E}{1+e_0} \Delta \sigma \)

For O.C. (CLay) when \( \sigma_v < \sigma_c \)

\( S_c = \frac{C_v}{1+e_0} \log \frac{\sigma_c}{\sigma_v} \)

since \( \Delta \sigma = C_v \log \frac{\sigma_c}{\sigma_v} \)

For O.C. (CLay) when \( \sigma_v > \sigma_c \)

\( \Delta \sigma = \Delta \varepsilon, + \Delta \sigma_v \)

\( S_c = \left[ \frac{C_v}{1+e_0} \log \frac{\sigma_c}{\sigma_v} \right. \)

\( \left. + \frac{C_v}{1+e_0} \log \frac{\sigma_v}{\sigma_c} \right] \)
The Rate of Consolidation Settlement

Rate of consolidation settlement depends on the rate at which excess pore water can get dissipated which in turn depends on the permeability of soil, number of drainage paths and lengths of drainage paths.

\[ t = \frac{S_v}{C_v} \cdot d \]

where \( t \) = time required to reach a certain percentage of consolidation

\[ U = \frac{S_v \text{ any time}}{S_v \text{ any settlement}} \]

The time factor \( U \) is a coefficient depending upon the percentage of consolidation \( U \) see Fig 27 and Table 13. The time factor should be determined from consolidation tests (Casagranie log method).

\[ C_v = \frac{k(1+c)\sqrt{v}}{0.435 C_u \gamma w} \]

\[ C_v = \text{coefficient of consolidation to be determined from consolidation tests (Casagranie log method)} \]

\[ U = \frac{S_v \text{ any time}}{S_v \text{ any settlement}} \]

\[ t = \frac{S_v}{C_v} \cdot d \]

Fig 27
Settlement at any time $t$

1. Find $S_{100}$. Settlement at 100% consolidation. You may use one of the equations in page 12.

2. Find $T = \frac{t}{t_1}$ [The units must be compatible such that $T$ is dimensionless.]

3. Find $U_{av}$ with from:
   a) From Fig 27.3 Lambe (see pg 12):
   b) $T = \frac{t}{t_1} \left( \frac{U_{av}}{100} \right)^2$ for $100 \leq U_{av} \leq 53$
   c) $T = 1.781 - 0.973 \log (100 - U_{av})$ for $U_{av} > 53$

4. $S_{av} = (U_{av}) (S_{100})$

Secondary Settlemet

$S_{sec} = C_k \frac{H}{t_0} \log \frac{t_0}{t_1} + \frac{C_h}{t_0} H \log \frac{t_0}{t_1}$

$C_k$: Coefficient of secondary compression. It is obtained from $e - \log t$ or from $t = t_{100} \times$ time for the end of 100% consolidation.

$e_{life}$: Time for $e = e_{max}$ of building (use only if $t_{life} > t_{100}$)

$D_e = e_{life} - e_{max}$

Allowable Settlement:

The allowable total differential settlement and angular distortion are summarized in Table 5-6 and 5-7 Bowes.
University of Technology

Building and Construction Engineering Department

Roads and Bridges Section

FOUNDATION ENGINEERING COURSE

Subject:- Shallow foundation – Structural design

Prepared by :- Prof. Dr. Kais Taha Shlash

Course designation : geo. 01

Under preparation