EXP. NO.6
FILTERS
Low –pass filter (integrator R.C. circuit)

OBJECT:
To steady the behavior and response of R.C. Circuit.

APPARTUS:
1- Signal function generator
2- Oscilloscope
3- Resisters, capacitors)
4- A.V.O. meter.

THEORY:
Consider the circuit shown in fig. (1)
If the output is taken off the capacitor, as shown in Fig. (1), it will respond as a low-pass filter.

At \( f = 0 \) Hz,
\[
X_C = \frac{1}{2\pi fC} = \infty \Omega
\]

and the open-circuit equivalent can be substituted for the capacitor, as shown in Fig. (2), resulting in \( V_o = V_i \).
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At very high frequencies, the reactance is:

\[ X_C = \frac{1}{2\pi fC} \approx 0 \Omega \]

and the short-circuit equivalent can be substituted for the capacitor, as shown in Fig. (3), resulting in \( V_o = \) zero V.

A plot of the magnitude of \( V_o \) versus frequency will result in the curve of Fig. (4).

For filters, a normalized plot is employed more often than the plot of \( V_o \) versus frequency of Fig. (4).
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Normalization is a process whereby a quantity such as voltage, current, or impedance is divided by a quantity of the same unit of measure to establish a dimensionless level of a specific value or range.

A normalized plot in the filter domain can be obtained by dividing the plotted quantity such as $V_o$ of Fig. (4) with the applied voltage $V_i$ for the frequency range of interest. Since the maximum value of $V_o$ for the low-pass filter of Fig. (1) is $V_i$, each level of $V_o$ in Fig. (4) is divided by the level of $V_i$. The result is the plot of $A_v = V_o/V_i$ of Fig. (5). Note that the maximum value is 1 and the cutoff frequency is defined at the 0.707 level.

At any intermediate frequency, the output voltage $V_o$ of Fig. (1) can be determined using the voltage divider rule:

$$V_o = \frac{X_C \angle -90^\circ V_i}{R - jX_C}$$

or

$$A_v = \frac{V_o}{V_i} = \frac{X_C \angle -90^\circ}{R - jX_C} = \left(\frac{X_C \angle -90^\circ}{\sqrt{R^2 + X_C^2}} \right) \left\{ \tan^{-1} \left(\frac{X_C}{R}\right) \right\}$$

and

$$A_v = \frac{V_o}{V_i} = \frac{X_C \angle -90^\circ}{\sqrt{R^2 + X_C^2}} \angle -90^\circ + \tan^{-1} \left(\frac{X_C}{R}\right)$$
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The magnitude of the ratio $V_o/V_i$ is therefore determined by

$$A_v = \frac{V_o}{V_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}}$$

and the phase angle is determined by

$$\theta = -90^\circ + \tan^{-1} \frac{X_C}{R} = -\tan^{-1} \frac{R}{X_C}$$

For the special frequency at which $X_C = R$, the magnitude becomes

$$A_v = \frac{V_o}{V_i} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

which defines the critical or cutoff frequency of Fig. (5). The frequency at which $X_C = R$ is determined by

$$\frac{1}{2\pi f_c C} = R$$

and

$$f_c = \frac{1}{2\pi R C}$$

The impact of Eq. (8) extends beyond its relative simplicity. For any low-pass filter, the application of any frequency less than $f_c$ will result in an output voltage $V_o$ that is at least 70.7% of the maximum. For any frequency above $f_c$, the output is less than 70.7% of the applied signal.

Solving for $V_o$ and substituting $V_i = V_i < 0^\circ$ gives
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\[ V_o = \left[ \frac{X_C}{\sqrt{R^2 + X_C^2}} \right] \angle \theta \]
\[ V_i = \left[ \frac{X_C}{\sqrt{R^2 + X_C^2}} \right] \angle \theta \]
\[ V_o \angle 0^\circ \]

and

\[ V_o = \frac{X_C V_i}{\sqrt{R^2 + X_C^2}} \angle \theta \]

The angle \( \Theta \) is, therefore, the angle by which \( V_o \) lag \( V_i \). This angle change from 0 to \( 90^\circ \), if the input voltage is sine wave with angle =0 then the output voltage become sine wave with angle =\( 90^\circ \) (i.e. cosine wave) \( V_{in} = A \sin(\omega t) \)

\[ V_o = B \sin(\omega t - 90^\circ) = -B \cos(\omega t) \]

For this reason, this circuit called integrator.

Since \( \Theta = -\tan^{-1}(R/XC) \) is always negative (except at \( f = 0 \) Hz), it is clear that \( V_o \) will always lag \( V_i \), leading to the label lagging network for the network of Fig. (1).

At high frequencies, \( XC \) is very small and \( R/XC \) is quite large, resulting in \( \Theta = -\tan^{-1}(R/XC) \) approaching \(-90^\circ\). At low frequencies, \( XC \) is quite large and \( R/XC \) is very small, resulting in \( \Theta \) approaching \( 0^\circ \). At low frequencies, \( XC \) is quite large and \( R/XC \) is very small, resulting in \( \Theta \) approaching \( 0^\circ \).

At \( XC = R \), or \( f = f_c \), \( -\tan^{-1}(R/XC) = -\tan^{-1}1 = -45^\circ \).

A plot of \( \theta \) versus frequency results in the phase plot of Fig. (6).
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fig.(6) Angle by which Vo lags Vi.

PROCEDURE

1-Connect the cct. Shown in fig .(7):

![Diagram of the circuit](image)

very the frequency and measure Vo for every setting of (f). Tabulate your result as in table (1)

<table>
<thead>
<tr>
<th>F(Hz)</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1K</th>
<th>1.5K</th>
<th>5K</th>
<th>10K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vo</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vo/Vin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table (1)

2- using the oscilloscope to measure the phase shift θ for each frequency setting

3- apply a sine-wave voltage at the input terminals of the cct. of fig. (7).with V_in=10V_{p.p.}

For the values of ( f , R, C ) as in the table (2). **Draw Vo & V_{in}**
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<table>
<thead>
<tr>
<th>f</th>
<th>R</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1KHZ</td>
<td>1KΩ</td>
<td>0.15µF</td>
</tr>
<tr>
<td>500HZ</td>
<td>10KΩ</td>
<td>0.1µF</td>
</tr>
<tr>
<td>50 HZ</td>
<td>1KΩ</td>
<td>0.001µF</td>
</tr>
</tbody>
</table>

Table (2)

REQUIREMENTS:
1- draw a graph between the gain \( A=V_o/V_{in} \) versus frequency, find \( f_c \) and
   Compare it with that obtained from equation (8).
2- draw a graph between\( (\theta) \) and \( (f) \), from the graph find \( f_c \) at \( \theta = 45 \) and compare it
   with that obtained from equation (8).

DISCUSSION:

a. Sketch the output voltage \( V_o \) versus frequency for the low-pass R-C
   filter of Fig. (8).

\[ R \quad 1\, kΩ \]
\[ + \quad - \]
\[ V_i = 20\, V \angle 0° \]
\[ C \quad 500\, pF \]
\[ V_o \]

Fig.(8)
b. Determine the voltage \( V_o \) at \( f = 100\, kHz \) and 1 MHz, and compare
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the results to the results obtained from the curve of part (a).
c. Sketch the normalized gain $A_v = \frac{V_o}{V_i}$. 