Numerical Modelling of Transient Flow In Long Oil Pipe Line System

Dr. Shaker H. Aljanabi*
Received on:30/12/2009
Accepted on:3/6/2010

Abstract
The problem of unsteady flow are frequently encountered in long oil pipelines without the provision of surge tank due to sudden closing or opening of valve and pump trip. The oil-hammer are analyzed and predicted by using the Eulerian approach (MOC) and the Lagrangian approach (WCM). The investigation was carried out for sufficiently long time to demonstrate changing of pressure head with time. Transient conditions arising in long oil pipeline with pumping station, valve and branches are studied in details for KRK pipeline by these two approaches. A Computer programs for all these components were developed. The effect of valve closure, line branch junction and pump shut-down are studied taking into account the effect of line friction on pressure wave. A numerical model "UNSTEADY_ FRIC_WH" using MOC and Barr's explicit friction factor has been presented for solution of the transient flow situation of water hammer. Assessment of friction factor at any section in this unsteady transient flow conditions clearly indicates the effectiveness of using variable friction factor in contrast to the steady state friction as in the available numerical models.

Keywords: Numerical modeling, Transient flow, Pipe line.
Numerical Modelling of Transient Flow In Long Oil Pipe Line System

Notations

A: pipe cross-sectional area (m$^2$)

a: wave speed (m/s)

D: diameter of conduit (pipe) (m)

E: Young modules of pipe material (Pa)

e: pipe wall thickness (m)

f: Darcy-Weisbetch friction factor

g: acceleration due to gravity (m/s$^2$)

H: piezometric head (m)

K: compression modules for fluid (Pa)

P: pressure (Pa)

Q: flow discharge (m$^3$/s)

Re: Reynolds number = (VD/$\nu$)

t: time (second)

V: velocity (m/s)

X: coordinate axis along conduit length

$\rho$: fluid density (kg/m$^3$)

$\psi$: (1-$\Phi^2$)

$\Phi$: the Poisson ratio of pipe material

$\theta$: pipe angle w.r.t the horizontal(degree)

c: pipe roughness size(m)

$\nu$: is the kinematics viscosity of the fluid (m$^2$/s)

Introduction

The analysis of transient in oil pipeline, some time called oil-hammer analysis or surge analysis, is rather complex because of pipe line friction losses are large compared to the instantaneous pressure change caused by sudden variation of flow velocity (Chaudhry 1987). The basic unsteady flow equations along pipe due to transient effect (closing valve, pump start up or shutdown….etc,) are non linear and hence analytical solution are not possible. Wiggert and Sundquist 1977 solved the pipe line transient fixed grids projecting the characteristic form outside the fundamental grid size. Watt 1980 have solved for rise of pressure by Method Of Characteristics (MOC) for only 1.2 seconds and the transient friction values have not been considered. Shimadg and Okashimo 1984 solved the second order equation of water hammer by series of solution method and a Newton-Raphson method. They calculate only maximum surge pressure with constant friction factor. Chaudhry and Hussaini 1985 solved the water hammer equations by McCormack technique. Samani and khayatzadeh 2004 also used this method to solve the system of equations related to the effect opening or closing a valve located in a pipeline. They solved the non-linear terms by McCormack predictor corrector, and the friction losses are estimated by applying a coefficient of pressure drop.

Many works on the analysis of transient flow focused mainly on specific type of flows such as liquid flows in pipelines .Although these methods are suitable for the types of...
the flow for which they were developed, they usually suffer limitations when applied to other types of flow. For pipeline applications, the MOC widely used (Boules et al 2004). The MOC is considered the most accurate of Eulerian methods in its representation of governing equations but require numerous steps or calculation to solve a typical transient pipe flow problem. This method has been summarized by other researchers (Larock et al 1999, Chaudhry 1987, Watters 1984, Streeter and Wylie 1967) and implemented in various computer programs for pipe system transient analysis (Axworthy et al, 1999; Karney and McInnis, 1990). Transient condition arising in long oil pipelines can be adequately analyzed and predicted by use of the MOC; attenuation, line packing, pyramiding and rarefaction can be completely taken into account. As compared with shorter piping systems, oil pipelines with several pumping stations, having units in series or in parallel, and with special speed controls or valving action, have somewhat different boundary conditions. Since the pumping heads are almost completely used to overcome fluid friction, careful attention to fluid properties is essential; also the complications arising from various batches of oil traversing the system at any one time must be examined (Rainer 2004).

Another method used to solve transient flow equations in the event-oriented system simulation environment is the Lagrange approach (Wood 2005). In this method, the pressure wave propagation process is driven by distribution system activities. The wave characteristics method (WCM) is an example of such approach (Wood et al 2005, Boules et al 2004) and was first described in the literature as the wave plan method (Wood et al 1966). The method tracks the movement and transformation of pressure wave as they propagate through-out the system and compute new conditions either at first time intervals or at times which a change actually occurs (variable time internal).

Part-1: Method of Characteristic

Governing Equations

The dynamics equations in transients-state flow in closed conduit are the equations for the conservation of mass and the momentum equation. These equations are a set of hyperbolic partial differential equations. Using the Reynolds transport theorem and assuming one-dimensional flow of elastic conduit with slightly compressible fluid, which stretch or contract with respect to time, these equations can be written as follow (Chaudhry 1987).

Continuity equation
\[
\frac{\partial p}{\partial t} + V \frac{\partial p}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} = 0 \quad \ldots (1.1)
\]

For thin -walled conduit Robert 1999, proposed following expression for the wave speed; \( a = \sqrt{\frac{K}{1+\psi \frac{D}{e K/E}}} \).

Momentum equation;
\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} + g \sin \theta + \frac{f V |V|}{2D} = 0 \quad \ldots (1.2)
\]
In most engineering applications, the convective acceleration term, \( V(\partial V/\partial x) \) and \((V\partial P/\partial x)\) are very small compared to the other term and can be neglected. Instead of the flow velocity, equations (1.1 and 1.2) is modified as:

\[
\frac{\partial H}{\partial t} + \left(\frac{a^2}{gA}\right)\frac{\partial Q}{\partial x} = 0
\]

\[
\frac{\partial Q}{\partial t} + gA\left(\frac{\partial H}{\partial x}\right) + \left(\frac{f}{2DA}\right)Q|Q| = 0
\]

\[
\text{(1.3)}
\]

\[
\text{….. (1.4)}
\]

**Characteristic Equation**

In time domain methodology, Eq. 1.3 and 1.4 are normally solved using the method of characteristics (see for example, Wylie and Streeter 1993). This involves finding moving coordinate system in which the equation may be written an ordinary rather than partial differential equations. Consider the relation that results when we multiplying Eq. 1.4 by \( \lambda \) and add to Eq.1.3 yields:

\[
(\partial Q/\partial t + \lambda a^2 \partial Q/\partial x) + \lambda gA (\partial H/\partial t + \partial H/\partial x) + f Q|Q|/2DA = 0
\]

\[
\text{….. (1.5)}
\]

If the coefficient of \( \partial Q/\partial x \) and \( \partial H/\partial x \) inside the bracket were identical, in other word if \( \lambda = \pm a \), then the expressions in bracket could be written as:

\[
\begin{align*}
\partial Q/\partial t = \pm a \partial Q/\partial x; \quad \text{and } \partial H/\partial t = \pm a \\
\partial H/\partial x
\end{align*}
\]

\[
\text{….. (1.6)}
\]

And these are the derivative \( dV/dt \) and \( dH/dt \) on \( dx/dt = \pm a \). These lines \( dx/dt = \pm a \), are the characteristics lines, and the solution of these equations is accomplished by the MOC, yielding in difference notations. Use these expressions Eq.1. 4 becomes,

\[
C^+; Q_p - Q_R + (gA/a) (H_p - H_R) + (f \Delta t/2DA) Q_R |Q_R| \quad \text{…….. (1.7a)}
\]

\[
C^-; Q_p - Q_S - (gA/a) (H_p - H_S) + (f \Delta t/2DA) Q_S |Q_S| \quad \text{…….. (1.7b)}
\]

The C+ equation is valid along a C+ characteristics line in the \( x- t \) plane; (Fig.1) given by \( dx/dt = a \). The C’ equation is valid only along C’ characteristics line \( dx/dt = -a \).

Equation 1.7a can be written as

\[
Q_p = C_p - C_a H_p \quad \text{….. (1.8a)}
\]

and eq. (1.7b) as:

\[
Q_p = C_p + C_a H_p \quad \text{….. (1.8b)}
\]

Equations (1.8a) and (1.8b) give;

\[
Q_p = 0.5 (C_p + C_n) \quad \text{…….. (1.8c)}
\]

In which

\[
\begin{align*}
C_p &= Q_R + (gA/a) H_R - (f \Delta t / 2DA) Q_R |Q_R| \\
C_n &= Q_S - (gA/a) H_S - (f \Delta t / 2DA) Q_S |Q_S| \\
C_a &= gA/a
\end{align*}
\]

Streeter and Wylie 1993 proposed an interpolation procedure for computing R and S for the known conditions M,O and N. Repeating this for all points at time \( t+\Delta t \) allow one to march forward in time.

\[
\begin{align*}
Q_R &= Q_o + G.a (Q_M - Q_o) \\
Q_S &= Q_o + G.a (Q_N - Q_o) \\
H_R &= H_o + G.a (H_M - H_o)
\end{align*}
\]

\[
\text{….. (1.10)}
\]
where $G = \Delta t/\Delta x$. The two characteristics equations for the relationships between changes in head can be found:

$$H_P = 0.5(H_R + H_S) + (a/g A)(Q_R - Q_S) - (a/gA^2)(f \Delta t/2D)(Q_R Q_R - Q_S Q_S)$$

(1.11)

$$Q_P = 0.5 (Q_R - Q_S) + (g A/a) (H_R - H_S) - (f \Delta t/2DA) (Q_R Q_R - Q_S Q_S).$$

(1.12)

The governing equations are solved with the assumption of friction factor at every time step by Bart's explicit friction factor which is a suitable combination of the Poiseuille equation and the Colebrook-White function to cover the full range of flow conditions from laminar to turbulent (Mimi Das Saikia 2006).

$$1/\sqrt{f} = -2 \log_{10} [5.02 \log_{10} \{Re /4.518 \log_{10} (Re/7)/[1+ (Re) 0.52/29(D+e)^{0.7}] + 1/3.7(D/e)].$$

(1.13)

Equations (1.7a) and (1.7b) are used together with boundary conditions. Where as equations (1.11) and (1.12) concern the internal points [Fig.1].

**Boundary Conditions**

For the boundary conditions at the upstream end of a C pipe, the C+ characteristics, equation (1.7a) is valid and provides one equation in two unknowns. Hence, only one external condition is needed, which may be any relation containing one or both of the unknown. For the downstream boundary condition equation 1.7b is valid. The same value of $\Delta t$ is used through the piping system, as this permits solution of boundary condition at common junction of the various pipes.

**A: Valve Positioned Within A pipe**

For steady-state condition the flow rate $Q_0 = Q_{\text{initial}}$ and $H_0 = H_{\text{initial}} = H_{\text{res}}$.

For transient condition, the discharge ($Q_0$) at valve at time $n$ can be calculated as follow (Jerry, Lescovich 1998).

For given gate valve, $C_V = 12800$ for $Q$ in gpm and $\Delta P$ in lb/in$^2$, for S.I Unit use $K_V = 0.865 C_V$ for $Q$ in m$^3$/hr and $\Delta P$ in bar, or $K_V = 14.42 C_V$ for in l/min and $\Delta P$ in bar.

For water,

$$Q = C_V \sqrt{\Delta P/s} \quad \ldots \ldots \ (1.14)$$

For another liquids expressed $\Delta P$ as follows

$$\Delta P_1/\Delta P_2 = (\mu_1/\rho_2)^{1/4} \rho_1/\rho_2. \quad (1.15)$$

Where; $\mu$ is the viscosity and subscript 1 refer to liquid and 2 for water.

The flow rate at each opening ratio ($Y/D$) or area ratio AR is:

$$Q = e^{x/\tau} . Q_0 \quad \ldots \ldots \ (1.16)$$

$$x = \ln (\tau) . Y/D \ldots \ldots \ (1.17)$$

and,

$$Q/Q_0 = \tau \sqrt{\Delta H/\Delta} \quad \ldots \ldots \ (1.18)$$

Where $\tau = C_V/C_{V0}$ and $Y/D$ is the percent caudal obtained from a typical valve performance curve provided by the manufacturer.
B: Branching Junction

For the branching junction shown in figure 3, the following equations can be written:

1. Continuity equation
\[ Q_{P_{i+1,n+1}} = Q_{P_{i+1,1}} + Q_{P_{i+2,1}} \quad \ldots \ldots (1.19) \]

2. Characteristic equations
\[ Q_{P_{i+1,n+1}} = C_{P_i} - C_{a_i} H_{P_{i+1,1}} \quad \ldots \ldots (1.20) \]
\[ Q_{P_{i+1,1}} = C_{a_{i+1}} + C_{a_{i+1}} H_{P_{i+1,1}} \quad (1.21) \]
\[ Q_{P_{i+2,1}} = C_{a_{i+2}} + C_{a_{i+2}} H_{P_{i+2,1}} \quad (1.22) \]

3. Equation for total head
\[ H_{P_{i,n+1}} = H_{P_{i+1,1}} = H_{P_{i+2,1}} \quad \ldots \ldots (1.23) \]

The junction is neglected, and it is assumed that the velocity heads in all conduits are equal. Simultaneous solution of Eqs. (1.19) through (1.23) yields;
\[ H_{P_{i+1,1}} = (C_{P_i} - C_{a_{i+1}} - C_{a_{i+2}})/(C_{a_i} + C_{a_{i+1}} + C_{a_{i+2}}) \quad \ldots \ldots (1.24) \]

Now \( H_{P_{i+1,1}} \) and \( H_{P_{i+2,1}} \) can be determined from Eq.(1.24) and \( Q_{P_{j,n+1}} \) and \( Q_{P_{j,1}} \) from equ.(1.20) through equ. (1.22).

C: Centrifugal Pump at Upstream End

Transient pump starts when the start-up lasts for a shorter period than the time acceleration of fluid column in the pipe. A rough estimate for the time acceleration is the reflection time \( T_f = 2L/a \). General expression is that the pipeline attains its steady state flow in less one or two time \( T_f \).

Transient in pump shut-down are normally much more complicated and critical than at pump start-up. When pump shut-down the rotation speed reduce rapidly. The most important is the ratio between duration of shut-down and the reflection time of pipeline (M. Nicla,ny).

Calculation of flow rate \( Q \) and pressure head \( H_p \) in a boundary node placed at the beginning of the conduit (fig. 4) fed by the pump, are performed based on equation (1.7b),
\[ Q (t+\Delta t) = Q (t). N (t +\Delta t)/N (t) \quad (1.25) \]
being similarity law of pump characteristic at rotational speed change. Decreases rotational speed of pump units for short time interval \( \Delta t \) is described by an exponential curve (T. Larsen 2006) in the following form;
\[ N (t+\Delta t) = N (t). e^{Gr/N . \Delta t} \quad \ldots \ldots (1.26) \]
\[ N (t+\Delta t) : \text{rotational speed at the end of time interval } \Delta t \]
\[ N (t) : \text{rotational speed at the beginning of time interval } t, \]
\[ Gr \approx dN/dt = 900.P/\pi^2.I.N \quad \ldots \ldots (1.27) \]

Where \( P \): pump power consumption expressing as;
\[ P=\rho g Q H_t / \eta \quad \ldots \ldots (1.28) \]
\( H_t \): change in total head, \( \eta \): pump efficiency
\( I \): inertia moment of pump at rotational speed, its value supplied by manufacturer. Thorley (1999) presented an empirical equation, for centrifugal pump and motor if information is not available:

Centrifugal pumps
\[ I_p = 0.03768 [P/N]^{0.9556} \quad \ldots \ldots (1.29) \]

Motors
\[ I_m = 0.0043 [P/N]^{1.48} \quad \ldots \ldots (1.30) \]

Then, \( I = I_p + I_m \)

A rough estimate of duration of shut-down period can be found from
\[ \Delta t = K.E / P = 0.5 (I0 \eta/2 \rho g Q H) \quad \ldots \ldots (1.31) \]
Numerical Modelling of Transient Flow in Long Oil Pipe Line System

\[ \Delta H_3 = \Delta H_1 + F_1 (Q_3 - Q_1) \] \hspace{1cm} (2.2)

\[ \Delta H_4 = \Delta H_2 + F_2 (Q_4 - Q_2) \] \hspace{1cm} (2.3)

Where, \( F_1 = \frac{a}{g} A_1 \) \hspace{0.5cm} \( F_2 = \frac{a_3}{g} A_2 \)

The head \( H \), after wave action is;

\[ H_3 = H_1 + \Delta H_1 + \Delta H_3 \] \hspace{1cm} (2.4)

\[ H_4 = H_2 + \Delta H_2 + \Delta H_4 \] \hspace{1cm} (2.5)

The coefficient of characteristic equation \( A(t), B(t) \) and \( C(t) \) may vary with time and these represent the values at time of wave action. Equations (2-2) to (2-5) can be solved to obtain quadratic relationship for \( Q_0 \). This is;

\[ C(t) Q_0^2 + B(t) Q_0 - (F_1 + F_2) + b = 0 \] \hspace{1cm} (2.6)

\[ b = H_1 + 2\Delta H_1 + H_2 - 2\Delta H_2 + (F_1 + F_2)Q_i \] \hspace{1cm} (2.7)

Pipe Junction Analysis

A pressure wave of magnitude \( \Delta H \) impinging in one of junction legs, is transmitted to each of the adjoining legs. The magnitude of the transmitted wave \( \Delta H_T = T_i \Delta H \) where the transmission coefficient, \( T_i \), is given by

\[ T_i = \frac{2}{\sum (1/F_j)} \] \hspace{1cm} (2.8)

Where the summation refer to all legs connecting at the junction. A reflection back in leg \( i \) occurs magnitudes \( R_i \Delta H \) where (\( R_i \) is the reflection coefficient = \( T_i - 1 \)) (Wood et al 2005).

Consider a pressure wave of magnitude \( \Delta H \) approaching a three pipe junction with initial flow rates...
Q₁, Q₂, and Q₃ and an initial pressure head of H₁ (Fig. 6). The wave action at the junction result in a reflected wave of magnitude \( R \Delta H \) in the pipe 1 and a transmitted wave of magnitude \( T \Delta H \) down each of the remaining two pipes 2 and 3. The new flow rates as a resultant of the wave action are \( Q'_1, Q'_2 \) and \( Q'_3 \) and the resultant pressure head at the junction is \( H₂ \).

Based on the basic transient flow relationship an expression of the transmitted and reflected wave can be written;

\[
\Delta H (1-R_1) = F_1 (Q'_1 - Q_1) \quad (2.9)
\]

\[
\Delta HT_1 = F_2 (Q'_2 - Q_2) \quad \ldots \quad (2.10)
\]

\[
\Delta HT_1 = F_3 (Q'_3 - Q_3) \quad \ldots \quad (2.11)
\]

\[
(Q'_1 - Q_1) = (Q'_2 - Q_2) + (Q'_3 - Q_3)
\quad \ldots \quad (2.12)
\]

**Active Element (Pumps) Analysis**

The characteristic \( A(t), B(t) \) and \( C(t) \) in equation 2.1 may depend on various conditions including pump speed, pump head and other operating conditions and are explicitly determined before calculation for the effect of wave action are made. A simple expression that is useful for pump startup or shutdown is:

\[
\Delta H = A_R a^2 + B_R a|Q| + C_R Q |Q| \quad \ldots \quad (2.13)
\]

Where \( A_R, B_R \) and \( C_R \) are the coefficients of a quadratic curve that represents normal pump operating at full (rated) speed and \( a \) is the speed ratio (\( a = N/N_R \)). For variable speed operation during a startup or shutdown, \( N/N_R \) represent the ratio of rotational speed at any time during the transitional period, \( N \), to the rated speed of rotation, \( N_R \).

**Results And Discussion:**

The computational accuracy and performance of hydraulic transient modeling approach has not been comprehensively compared using object testing applied to system. Certainly, well-known numerical challenges in solving the transient flow equations exist, for example avoiding numerical dispersion and attenuation and eliminating unnecessary distortion of either the physical pipe system or its boundaries.

Both Eulerian and Lagrangian solution schemes are commonly used to approximate the solution of governing equations. The MOC requires numerous steps or calculations to solve a typical transient-pipe flow problem. Lagrangian methods, update the hydraulic state of the system at fixed or variable time interval at times when a change actually occurs. Each approach assumes that a steady-state hydraulic equilibrium solution is available that gives initial flow and pressure distribution throughout the system. Both the MOC and the WCM obtain solutions at interval time of \( \Delta t \) at all junctions and components. However the MOC also requires solution at all interior points for each time step. This requirement basically handles the effect of pipe wall frictions. The WCM handles these effects by using the pressure wave characteristics. Results for pressure head and flow variation are calculated for each time step at all components and junctions in the pipe system.
A scientific Computer program (WHOP) (Water Hammer Oil Pipeline) was designed by Quick Basic Language and applied on Pentium Four with (Ram 512 MB). The (WHOP) calculated the friction factor, head \( H \), and flow rate \( Q \) variation with variable time.

**Method of Characteristic (MOC)**

The following information are available;

\[
K_{oil} = 1.5 \text{ GPa}; \quad E_m = 210 \text{ GPa}; \quad \Psi = 1 - \Phi = 0.91; \quad \Phi = 0.3; \quad D = 0.672 \text{ m}; \quad A = 0.456 \text{ m}^2; \quad e = 0.0111125 \text{ m}; \quad \varepsilon = 10^{-4} \text{ m}; \quad \rho_{oil} = 836.6 \text{ Kg/m}^3; \quad \nu = 5.1 \times 10^6 \text{ m}^2/\text{s}^2; \quad L = 19312 \text{ m}; \quad Q = 0.34 \text{ m}^3/\text{s}; \quad H_{suc} = 153.3 \text{ m}; \quad H_{Disc} = 413 \text{ m}.
\]

Calculations were carried out using the data, for a complete pump shut-down occurring over a time of 35 seconds. The most interesting object of this simulated system, by modeled the pipeline into \( N \) (node = 13) equal reaches. A computational time period of \( \Delta t = 1.458 \text{ sec} \) is necessary. The solution to problem in oil transients are began with steady-state condition at \( t = 0 \), so that \( H \) and \( Q \) along the pipe are known initial values of each communication section.

The transient heads and flow rates at pump shutdown at each node along pipeline, for variable time intervals (Fig. 8&9) show that there is a dramatically change transient pressure and rate, depending on time and location. Also shows that \( H \& Q \) just at downstream of pump are less affected by transient flow.

The variation of transient head and flow rate for valve closure are shown in figures (10—13). These figures indicated that \( H\&Q \) upstream valve varies along pipeline with time interval at all grid points. Only at nodes near to the pump the values does not affected by valve closing. The transient pressure head for pump shutdown and valve closure is drawn in fig. (14), it is to be noted that there is a large effect of transient time on the maximum pressure. The careful of designing the valve and pumps must be considered in order to decrease the hydraulic problems. The effect of pipe friction on transient \( H\&Q \) along pipeline are shown in figs. (15-18) at pump trip and valve closure. It is clearly indicated the highly variations of results if applying the frictionless solution of characteristic equations.

Figure (19) shows the head variation for transient flow at suction and discharge side of pump after the pressure wave act on its (case C3A). The transient head up and downstream valve after wave action (case B2) are shown in figs. (20). It is clear from the figures that the maximal increase of pressure head have been serious restricted to the pump trip and valve admissible for the pipeline.

Figures (21) & (22) compare the transient results obtained using the MOC and the WCM solution approach at the pump and valve, respectively. The two methods produced results that are virtually indistinguishable (identical).

**Wave Characteristic Method (WCM)**

The following cases are studied in this approach:( table 1).

- **A. pipeline friction:** Case A1: Wave act from left to right. Case A2: Wave act from right to left.
- **B. Gate Valve**
Case B1: Wave act from left to right side after valve closing at variable area ratio (the wave not reached the valve).

Case B2: Wave reflected and transmitted at the valve at variable area ratio.

C. The pump: Case C1: Wave act from suction side only. Case C2: Wave act from discharge side only.

Case C3A: Change of pump speed without wave action. Case C3B: Change of pump speed with wave acts from suction side. Case C4: Waves act from both sides with changing of pump speed.

D: Line Junction: Case D1: Acting of pressure wave in each leg separately. Case D1A: Act on leg 1, D1B: Act on leg 2, D1C: Act on leg 3.

Case D2: Wave act simultaneously on legs. Case D3: Wave act on two pipes (1 and 2) at a period of times (wave act from leg 3 only).

The ability of the MOC to accurately model pipe friction in a system using just one calculation was substantiated by the virtually identical result obtained for cases studied. This accuracy held true even though pipe friction has a significant effect on a solution. The excellent agreement between the MOC and WCM solutions for this (figs.27&28) confirms that the computed effect of wall friction is similar for the two methods. The MOC presented in this paper, applied to estimate unsteady flow, enables consideration of different boundary conditions and analysis of transient states in hydraulic system.

Conclusions

The special problem arising in the analysis of transients in oil pipelines are readily handled on the digital computer by use the method of characteristic solution of the basic equations of water hammer. Since the equation go back to fundamentals, and include fluid friction, such phenomena as line packing, attenuation, pyramiding, and rarefaction are automatically taken into account through proper handling of the boundary conditions.

Both the MOC and WCM methods are capable of accuracy solving for transient pressure and flows in oil pipe system, including the effect of pipe friction. It is very important to mention here the possibilities to build a complete system by using these methods and different numerical experiment could be done to achieve the best performance and system optimization.

Any transient analysis is subjected to inaccuracies because of incomplete information regarding the piping system, its components and degree of skeletonization. Properly developed and calibrated models for transient analysis greatly improve the ability of oil utilities to determine adequate surge protection, strengthen the integrity of the system.

References

### Table 1 (WCM results).

<table>
<thead>
<tr>
<th>Case</th>
<th>Qi</th>
<th>ΔH1</th>
<th>H1</th>
<th>ΔH2</th>
<th>H2</th>
<th>Qo</th>
<th>ΔH3</th>
<th>H3</th>
<th>ΔH4</th>
<th>H4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>.34</td>
<td>37.3</td>
<td>413</td>
<td>0</td>
<td>375.7</td>
<td>.436</td>
<td>12.5</td>
<td>463</td>
<td>24.76</td>
<td>400.5</td>
</tr>
<tr>
<td>A2</td>
<td>.34</td>
<td>0</td>
<td>413</td>
<td>37.3</td>
<td>375.7</td>
<td>.231</td>
<td>27.544</td>
<td>440.5</td>
<td>9.756</td>
<td>₪2.776</td>
</tr>
<tr>
<td>B1/AR</td>
<td>.09944</td>
<td>37.3</td>
<td>0</td>
<td>375.6</td>
<td>0</td>
<td>376.88</td>
<td>.09861</td>
<td>1.278</td>
<td>376.87</td>
<td>1.278</td>
</tr>
<tr>
<td>0.9</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09861</td>
<td>1.293</td>
<td>376.89</td>
<td>-1.293</td>
<td>375.55</td>
</tr>
<tr>
<td>0.8</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09841</td>
<td>1.6</td>
<td>377.1</td>
<td>-1.6</td>
<td>375.25</td>
</tr>
<tr>
<td>0.7</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09821</td>
<td>1.884</td>
<td>377.5</td>
<td>-1.884</td>
<td>374.96</td>
</tr>
<tr>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09791</td>
<td>2.37</td>
<td>378</td>
<td>-2.37</td>
<td>374.4</td>
</tr>
<tr>
<td>0.5</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09751</td>
<td>3.05</td>
<td>378.7</td>
<td>-3.05</td>
<td>373.8</td>
</tr>
<tr>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09661</td>
<td>4.33</td>
<td>379.93</td>
<td>-4.33</td>
<td>372.5</td>
</tr>
<tr>
<td>0.2</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.09601</td>
<td>5.25</td>
<td>381</td>
<td>-5.25</td>
<td>371.6</td>
</tr>
<tr>
<td>0.1</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.07531</td>
<td>37.115</td>
<td>412.7</td>
<td>-37.115</td>
<td>339.8</td>
</tr>
<tr>
<td>B2/AR</td>
<td>.09944</td>
<td>37.3</td>
<td>375.6</td>
<td>0</td>
<td>376.88</td>
<td>.1225</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.12251</td>
<td>1.28</td>
<td>414.7</td>
<td>35.5</td>
<td>412.3</td>
</tr>
<tr>
<td>0.8</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.12231</td>
<td>2.11</td>
<td>415</td>
<td>35.2</td>
<td>412</td>
</tr>
<tr>
<td>0.7</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.12211</td>
<td>2.57</td>
<td>415.5</td>
<td>34.73</td>
<td>411.3</td>
</tr>
<tr>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.12151</td>
<td>3.34</td>
<td>416.24</td>
<td>33.96</td>
<td>410.7</td>
</tr>
<tr>
<td>0.5</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.12081</td>
<td>4.42</td>
<td>417.3</td>
<td>32.88</td>
<td>409.63</td>
</tr>
<tr>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.11951</td>
<td>6.42</td>
<td>419.3</td>
<td>-6.42</td>
<td>407.63</td>
</tr>
<tr>
<td>0.2</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.11061</td>
<td>20.12</td>
<td>433</td>
<td>-20.12</td>
<td>412.94</td>
</tr>
<tr>
<td>0.1</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.08988</td>
<td>52.0</td>
<td>465</td>
<td>-52.0</td>
<td>413.08</td>
</tr>
<tr>
<td>C1</td>
<td>.34</td>
<td>51.00</td>
<td>153.3</td>
<td>0</td>
<td>413</td>
<td>.4941</td>
<td>10.1</td>
<td>214.4</td>
<td>40.84</td>
<td>453.84</td>
</tr>
<tr>
<td>C2</td>
<td>.34</td>
<td>0</td>
<td>153.3</td>
<td>37.3</td>
<td>413</td>
<td>.2041</td>
<td>37.09</td>
<td>190.37</td>
<td>22.0</td>
<td>450.2</td>
</tr>
<tr>
<td>C3A/AR</td>
<td>.8</td>
<td>.34</td>
<td>0</td>
<td>153.3</td>
<td>0</td>
<td>413</td>
<td>.1621</td>
<td>45.23</td>
<td>198.53</td>
<td>322</td>
</tr>
<tr>
<td>0.7</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.0871</td>
<td>63.933</td>
<td>217.33</td>
<td>-63.933</td>
<td>349</td>
</tr>
<tr>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.01941</td>
<td>81</td>
<td>234.5</td>
<td>81</td>
<td>332</td>
</tr>
<tr>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.00741</td>
<td>130.85</td>
<td>284.15</td>
<td>-130.85</td>
<td>282.15</td>
</tr>
<tr>
<td>0.2</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.17781</td>
<td>165.9</td>
<td>319.2</td>
<td>-165.9</td>
<td>297.1</td>
</tr>
<tr>
<td>C3B/AR</td>
<td>.8</td>
<td>.34</td>
<td>51.00</td>
<td>153.3</td>
<td>0</td>
<td>413</td>
<td>.33661</td>
<td>51.86</td>
<td>256.16</td>
<td>-0.86</td>
</tr>
<tr>
<td>0.7</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.2681</td>
<td>69.2</td>
<td>273.5</td>
<td>18.2</td>
<td>394.84</td>
</tr>
<tr>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.2071</td>
<td>84.556</td>
<td>288.856</td>
<td>-33.56</td>
<td>379.44</td>
</tr>
<tr>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.1151</td>
<td>107.77</td>
<td>312.07</td>
<td>-56.77</td>
<td>356.23</td>
</tr>
<tr>
<td>0.2</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.0971</td>
<td>112.31</td>
<td>316.61</td>
<td>-61.31</td>
<td>339.8</td>
</tr>
<tr>
<td>C3 C/AR</td>
<td>.8</td>
<td>.34</td>
<td>0</td>
<td>153.3</td>
<td>37.3</td>
<td>413</td>
<td>.04681</td>
<td>74.09</td>
<td>227.4</td>
<td>86.79</td>
</tr>
<tr>
<td>0.7</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.1131</td>
<td>114.173</td>
<td>267.7</td>
<td>-77.17</td>
<td>73.13</td>
</tr>
<tr>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.1251</td>
<td>117.5</td>
<td>270.8</td>
<td>80.2</td>
<td>370.10</td>
</tr>
<tr>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.2451</td>
<td>198.9</td>
<td>302.44</td>
<td>-111.54</td>
<td>338.76</td>
</tr>
<tr>
<td>C4/AR</td>
<td>1.0</td>
<td>.34</td>
<td>51.00</td>
<td>153.3</td>
<td>37.3</td>
<td>413</td>
<td>.3791</td>
<td>41.1</td>
<td>245.4</td>
<td>47.155</td>
</tr>
<tr>
<td>511.155</td>
<td>0.8</td>
<td>.34</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>.2111</td>
<td>83.6</td>
<td>287.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>455</td>
<td>0.6</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.0741</td>
<td>118.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>322.5</td>
<td>-29.81</td>
<td>420.5</td>
<td>0.4</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-.0581</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>151.806</td>
<td>356.1</td>
<td>-63.275</td>
<td>87.00</td>
<td>0.2</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.101</td>
<td>162.13</td>
<td>366.93</td>
<td>-74.14</td>
<td>376.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. WCM results for branching System

<table>
<thead>
<tr>
<th></th>
<th>Before Action</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta H_1 )</td>
<td>( H_1 )</td>
<td>( T_1 )</td>
<td>( R_1 )</td>
<td>( H_2 )</td>
<td>( \Delta H_4 )</td>
<td>( \Delta H_5 )</td>
<td>( \Delta H_6 )</td>
</tr>
<tr>
<td>D1A</td>
<td>375.5</td>
<td>375.5</td>
<td>0.93</td>
<td>.076</td>
<td>722.5</td>
<td>346.96</td>
<td>346.96</td>
<td>-28.5</td>
</tr>
<tr>
<td>D1B</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>D1c</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>D2</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>D3: // ( \Delta t ):</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>( H_2 )</td>
<td>375.5</td>
<td>376</td>
<td>390</td>
<td>401.5</td>
<td>409</td>
<td>413.5</td>
<td>416.53</td>
</tr>
</tbody>
</table>

Figure (1) Basic element of the grid for an internal node within a pipe.

Figure (2) Valve Positioned

Figure (3) Branching Junction
Figure (4). Grid element of boundary node next to the pump

Figure (5) Condition at component before and after wave action.

Figure (6) Effect of pipe junction on pressure wave
Figure (7) Case study of pipeline system.

Figure (8) Variation of head at each node for pump shutdown (MOC)

Figure (9) Variation of flow rate at each node for pump shut-down (MOC)

Figure (10) Variation of head at each node of valve closing (MOC).

Figure (11) Variation of head along pipe for valve closing (MOC).
Numerical Modelling of Transient Flow In Long Oil Pipe Line System

Figure (12) Variation of flow rate along pipe at valve closing (MOC).

Figure (13) Variation of flow rate at each node for valve closing (MOC).

Figure (14) Pressure head characteristics for transient condition.
Numerical Modelling of Transient Flow In Long Oil Pipe Line System

Figure (15) Effect of pipe friction on transient flow rate (MOC, case 1)

Figure (16) Effect of pipe friction on pressure transient (MOC, case 1)

Figure (17) Effect of pipe friction on transient flow rate (MOC, case 2)

Figure (18) Effect of pipe friction on pressure transient (MOC, case 2)
Numerical Modelling of Transient Flow In Long Oil Pipe Line System

Figure (19) Pressure head at suction and discharge side of pump after wave action.

Figure (20) Pressure variation upstream and downstream of valve after wave action (case B2).

Figure (21) Comparison of MOC and WCM for pump shut-down.

Figure (22) Comparison of MOC and WCM for valve closure.