University of Technology
Mechanical Engineering Department
Final Examination/Second Semester

Subject: Mathematics II
Division: 1st Class Power Plants
Examiners: Dr. Ahmed A. Imran

Year: 2015/2016
Exam Time: 3 Hrs.
Date: 24 / 05 / 2016

## Answer Five Questions only

**Q1:** Find the general solution of the **non-homogenous** linear system?

$$\begin{aligned} 4x_1 - 8x_2 + 11x_3 - 11x_4 + 6x_5 &= 4 \\ 2x_1 - 4x_2 + 9x_3 + 5x_4 - 4x_5 &= 2 \\ x_1 - 2x_2 + 3x_3 - 2x_4 + x_5 &= 1 \\ 3x_1 - 6x_2 + 8x_3 - 9x_4 + 5x_5 &= 3 \end{aligned}$$

(14 marks)

**Q2:** /A/ Find the Maclaurin series for $\sin^2 x$, and then find the value of $\lim_{x\to 0} \frac{sin^2x - x^2}{x^4}$ ?

(7 marks)

/B/ Given the data in table below, find the value of f(5.5) using **straight forward interpolation method**?

| $x_i$ | 2 | 4 | 7 |
|---|---|---|---|
| $y_i$ | 0.693 | 1.386 | 1.946 |

(7marks)

**Q3:** /A/ Find the roots of the equation $z^3 - 27\,i = 0$ ?

(7 marks)

/B/ Evaluat $\int \frac{1-e^{-x}}{x} dx$ to five terms only using **Maclaurin** series.

(7 marks)

**Q4:**/A/ Fitting the data by using **second order polynomial regression**, and then find the correlation coefficient (**r**) ?

| x | 2 | 3 | 4 | 5 | 7 | 10 |
|---|---|---|---|---|---|---|
| y | 5.2 | 7.8 | 10.7 | 13 | 19.3 | 27.5 |

(14 marks)

**Q5:** /A/ Find the Taylor series for $f(x) = x\cos(x)$ at $a = 2\pi$ ?

(7 marks)

/B/ Prove that $tan^{-1}(z) = \frac{1}{2i} ln\left(\frac{1+zi}{1-zi}\right)$ ?

(7 marks)

$Q_6$: /A/ Experimental data for force (f) and velocity (v). It is modeled by power equation ($f = av^b$) where(a & b) are parameters. Use **transformation with logarithm (base-10) to linearization** this equation, then use **linear regression** to find (a and b) ?

| f | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| v | 25 | 70 | 380 | 550 | 610 | 1120 | 830 | 1450 |

(10 marks)

/B/ Evaluate $\lim_{x\to -2} \frac{xe^x - 4 + 2e^x - 2x}{1 + x\sin \pi x + \frac{x}{2} + 2\sin \pi x}$ by using **L'Hospital's** rule. (4 marks)

*Good luck*