



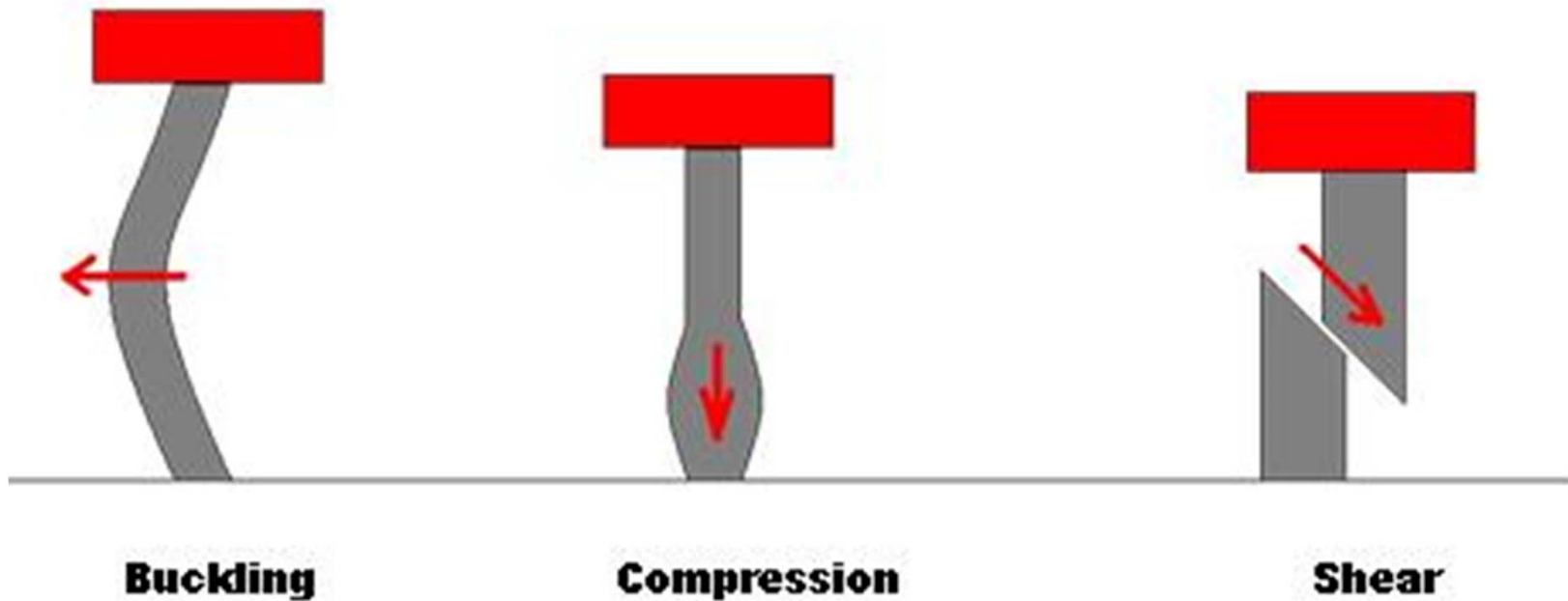
Mechanical Engineering Design II

Fourteenth & Fifteenth Lectures

Columns Design

Columns Design

A column is a long, slender member that carries an axial compressive load and that fails due to buckling rather than due to failure of the material of the column.



Failure due to buckling



Analysis of Columns

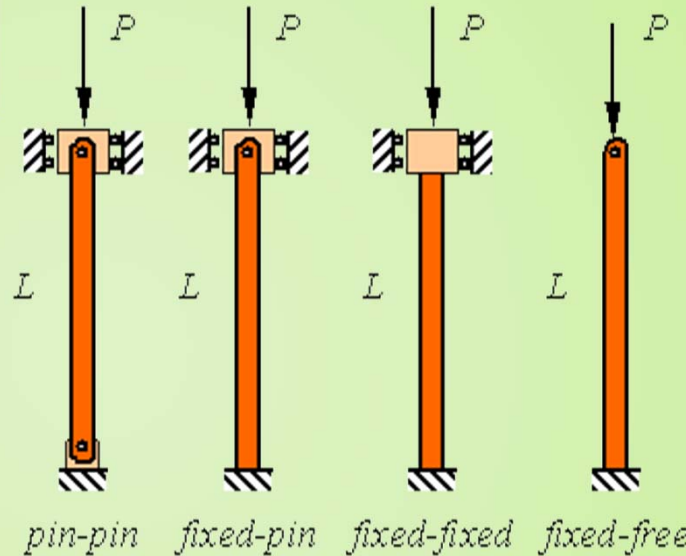
Properties of C.S

1. Cross Sectional Area (A)
2. Moment of Inertia (I)
3. Radius of gyration $r = \sqrt{\frac{I}{A}}$

Materials

1. Modulus of elasticity (E)
2. Yield strength (S_y)

Connections



Theoretical value	$K=1$	$K=0.7$	$K=0.5$	$K=2$
Practical value	$K=1$	$K=0.8$	$K=0.65$	$K=2.1$

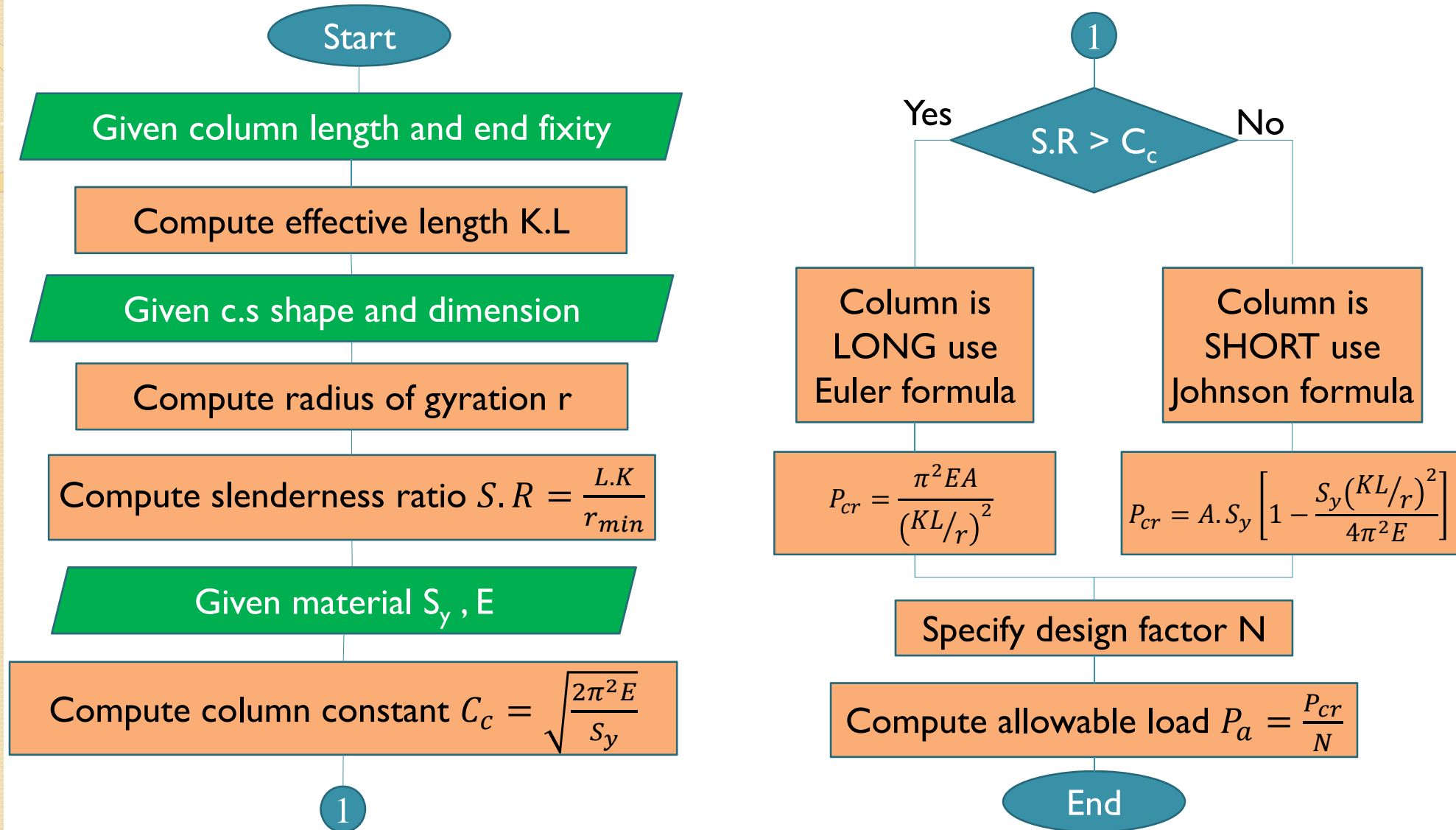
Column type

1. Effective length $L_e = K \cdot L$
2. Slenderness ratio $S.R = \frac{L_e}{r_{min}} = \frac{L \cdot K}{r_{min}}$
3. Column constant $C_c = \sqrt{\frac{2\pi^2 E}{S_y}}$
(Transition slenderness ratio)

If $S.R > C_c$ then column is LONG

If $S.R < C_c$ then column is SHORT

Flowchart for analyzing of straight centrally loaded column:



Example.I : A column has a solid circular cross section, 31.75 mm in diameter; it has a length of 1.3716 m and is pinned at both ends. If it is made from AISI 1020 cold-drawn steel, what would be a safe column loading?

Solution: use the flowchart above.

Results :

Step.1: $K = 1 \quad KL = 1 \times 1.3716 = 1.3716m$

Step.2: for solid round section $r = \frac{D}{4} = \frac{31.75}{4} = 7.9375mm$

Step.3: compute slenderness ratio $\frac{KL}{r} = \frac{1.3716}{7.9375} = 173$

Step.4: compute the column constant $C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2 (207Gpa)}{350Mpa}} = 108$

For material AISI 1020
cold-drawn steel:
E=207 Gpa
S_y=350 Mpa

Step.5: because $\frac{KL}{r} > C_c$ column is long , Eulers formula should be used :

$$A = \frac{\pi D^2}{4} = \frac{\pi (31.75)^2}{4} = 793.596 \text{ mm}^2$$

$$\text{The critical load is } P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 (207Gpa)(793.596 \text{ mm}^2)}{(173)^2} = 54265.6 \text{ N}$$

At this load the column just begin to buckle, now let N=3

$$\text{Allowable load } P_a = \frac{P_{cr}}{N} = \frac{54265.6}{3} = 18090.016 \text{ N (the safe load on the column)}$$

Example .2: Determine the critical load for:
 Mat.AISI 1040 hot rolled steel
 lower end is welded and upper was pinned

Solution: use the flowchart above.

Results :

Step.1: $K = 0.8$ $KL = 1 \times 1.3716 = 1.3716 \text{ mm}$

Step.2: for solid rectangular section $r = \frac{B}{\sqrt{12}}$

$$r = \frac{12}{\sqrt{12}} = 3.46 \text{ mm}$$

Step.3: compute slenderness ratio $\frac{KL}{r} = \frac{0.8 \times 280}{3.46} = 64.7$

Step.4: compute the column constant $C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2 (207 \text{ Gpa})}{290 \text{ Mpa}}} = 119$

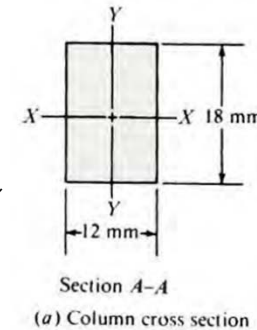
Step.5: because $\frac{KL}{r} < C_c$ column is short, J.B.Johnson formula should be used :

$$A = 12 \times 18 = 216 \text{ mm}^2$$

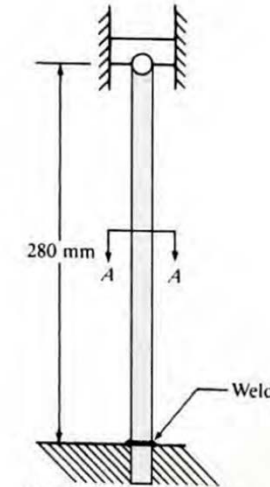
The critical load is $P_{cr} = A \cdot S_y \left[1 - \frac{S_y (KL/r)^2}{4\pi^2 E} \right] = 53.3 \text{ kN}$

At this load the column just begin to buckle, now let $N=3$

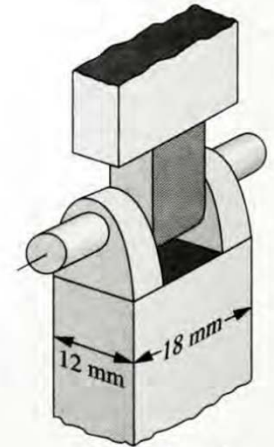
Allowable load $P_a = \frac{P_{cr}}{N} = \frac{53.3}{3} = 17.8 \text{ kN}$ (the safe load on the column)



(a) Column cross section



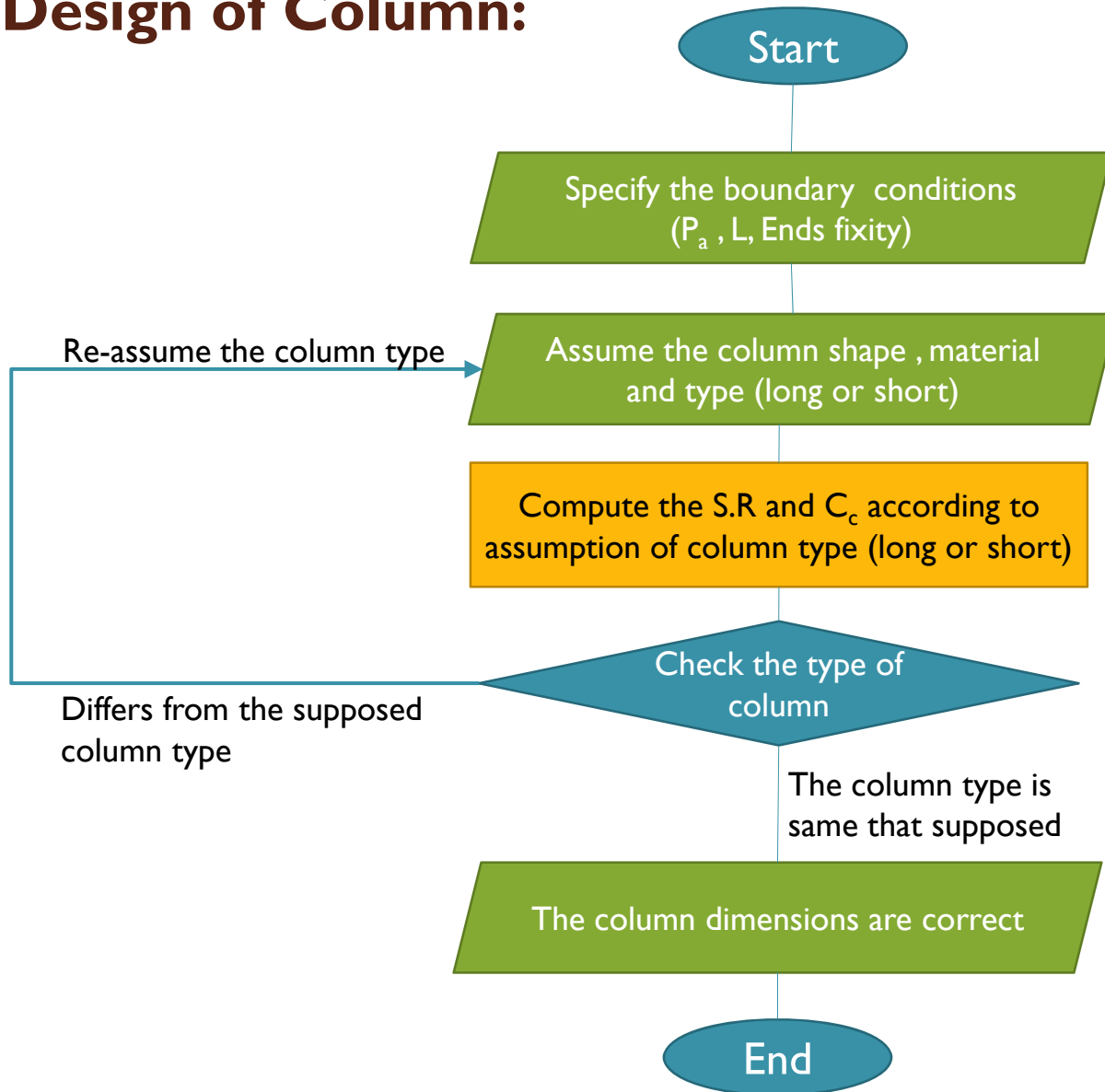
(b) Sketch of column installation



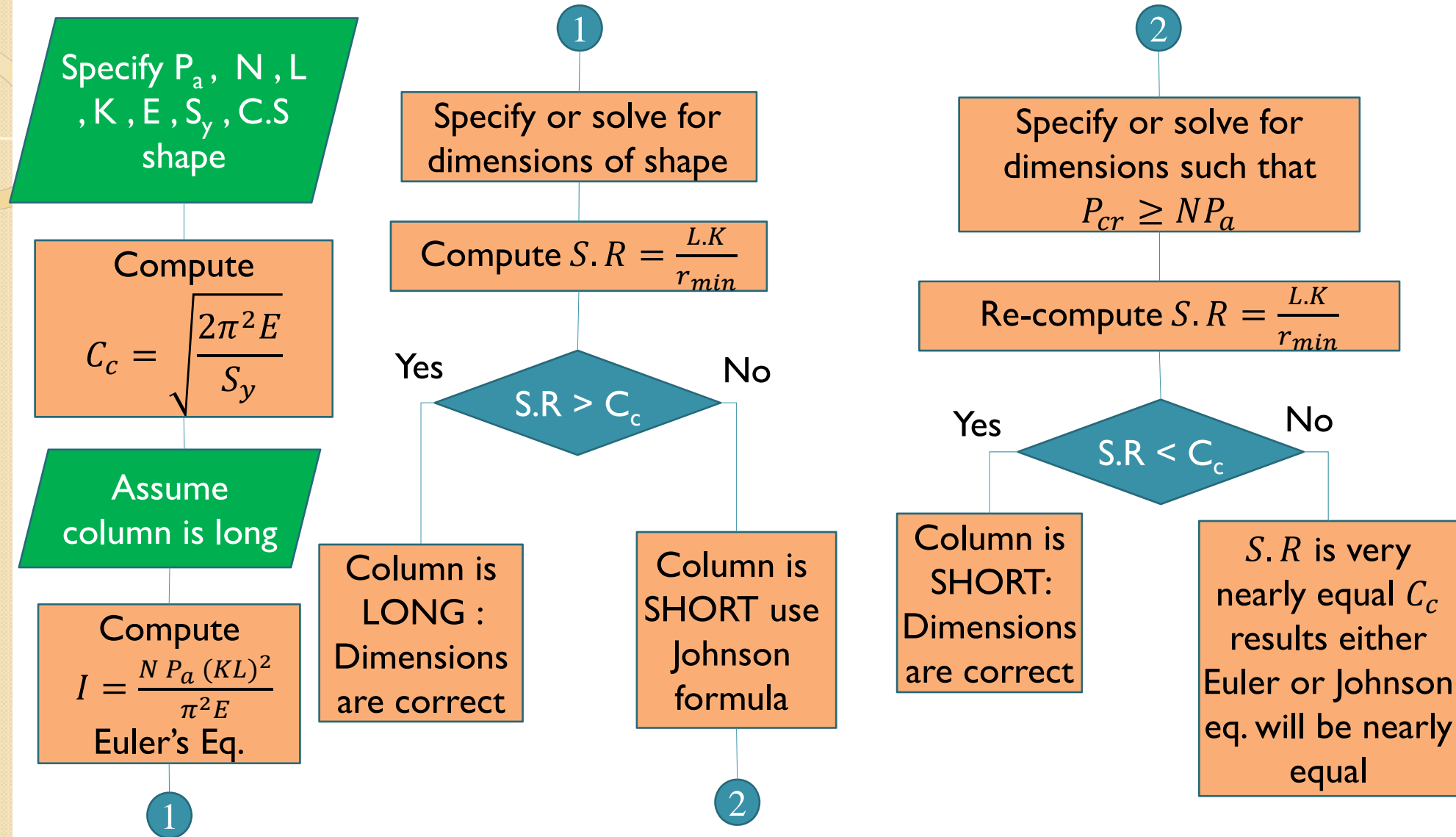
(c) Sketch of pinned connection

For material AISI 1040
 hot rolled steel:
 $E=207 \text{ Gpa}$
 $S_y=290 \text{ Mpa}$

The Design of Column:



Flowchart for designing of straight centrally loaded column:



Design assuming a long column (Euler's Eq.)

$$I = \frac{P_{cr} (KL)^2}{\pi^2 E} = \frac{N P_a (KL)^2}{\pi^2 E}$$

For solid circular section $I = \frac{\pi D^4}{64}$

$$\therefore D = \left[\frac{64 N P_a (KL)^2}{\pi^2 E} \right]^{1/4}$$

Design assuming a short column (Johnson's Eq.)

$$P_{cr} = A \cdot S_y \left[1 - \frac{S_y (KL/r)^2}{4\pi^2 E} \right]$$

For solid circular section $A = \frac{\pi D^2}{4}$, $r = \frac{D}{4}$

$$\therefore D = \left[\frac{4 N P_a}{\pi S_y} + \frac{4 S_y (KL)^2}{\pi^2 E} \right]^{1/2}$$

Example.3 : specify a suitable dia. of solid, round cross section for a machine link if it carry 43590.4 N of axial compressive load. $L=635\text{mm}$, ends will be pinned, $N=3$, Mat. AISI 1020 hot-rolled steel.

Analysis: * use the flowchart shown before.

* Assume the column is LONG.

$$\text{Results: } D = \left[\frac{64 N P_a (KL)^2}{\pi^2 E} \right]^{1/4} = \left[\frac{64 \times 3 \times 43590.4 (635\text{mm})^2}{\pi^2 (207 \times 10^9)} \right]^{1/4} = 26.924\text{mm}$$

$$\therefore r = \frac{D}{4} = 6.731\text{mm}, S.R = \frac{L.K}{r_{min}} = \frac{635 \times 1}{6.731} = 94.3 \text{ and } C_s = 138 \therefore S.R < C_s$$

Column is redesign as short column by using Johnson's eq.:

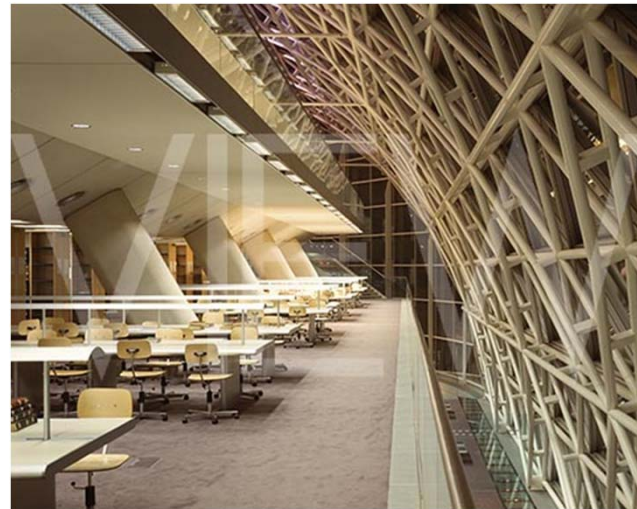
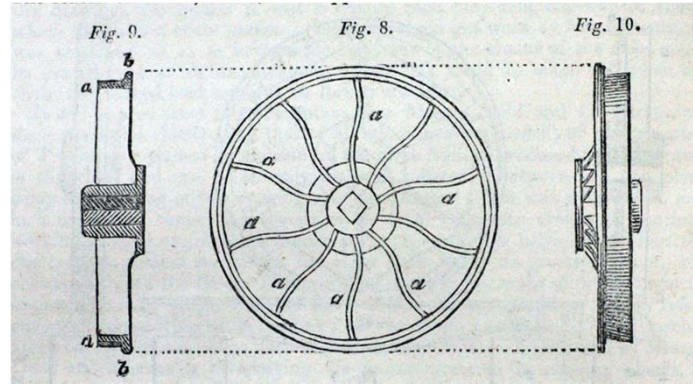
$$D = \left[\frac{4NP_a}{\pi S_y} + \frac{4 S_y (KL)^2}{\pi^2 E} \right]^{1/2} = \left[\frac{4(3)(43590.4)}{\pi (206.85\text{MPa})} + \frac{4 (206.85\text{MPa})(635\text{mm})^2}{\pi^2 (207 \times 10^9)} \right]^{1/2} = 31.242\text{mm}$$

$$\text{Checking the S.R again, we have : } S.R = \frac{L.K}{r_{min}} = \frac{635 \times 1}{31.242/4} = 81.3$$

Comments: this is still less than C_s , then our analyzing is acceptable .

Crooked Column:

The columns shown in figures below are crooked, bending occurs in addition to the column action.



The Euler and Johnson formulas assume that the column is straight and that the load acts in line with the centroid of the cross section of the column. So the crooked column has special formula:

$$P_a^2 - \frac{1}{N} \left[S_y A + \left(1 + \frac{ac}{r^2} \right) P_{cr} \right] P_a + \frac{S_y \cdot A \cdot P_{cr}}{N^2} = 0$$

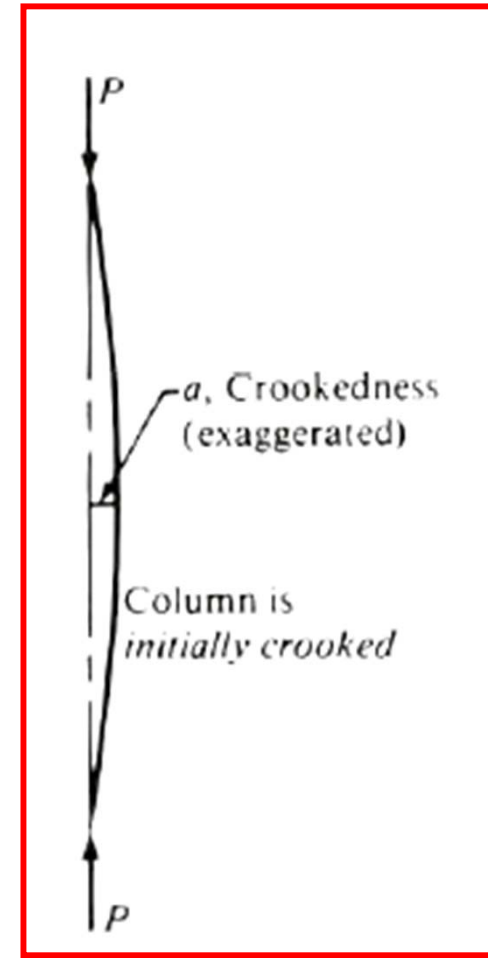
Where a = initial crookedness.

c = distance from the neutral axis of C.S about which bending occurs to its outer edge.

The equation can be written as :

$$P_a^2 + C_1 P_a + C_2 = 0 \quad \text{and} \quad P_a = 0.5 \left[-C_1 - \sqrt{C_1^2 - 4C_2} \right]$$

The smaller of the two possible solutions is selected.



Example.4 : A column has both ends pinned and has a length of 812.8mm. It has a circular cross section with a diameter of 19.05mm and an initial crookedness of 3.175mm. The material is AISI 1040 hot-rolled steel. Compute the allowable load for a design factor of 3.

Analysis: use equation above to evaluate C_1 and C_2 then find P_a

Results: $S_y = 289590 \text{ KPa}$

$$A = \frac{\pi D^2}{4} = 285.1784 \text{ mm}^2$$

$$r = \frac{D}{4} = 4.7752 \text{ mm} \quad \text{and} \quad c = \frac{D}{2} = 9.525 \text{ mm}$$

$$\frac{KL}{r} = 171 \quad \text{and} \quad P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 (207 \text{ GPa}) (285.1784 \text{ mm}^2)}{(171)^2} = 19909.25 \text{ N}$$

$$C_1 = -\frac{1}{N} \left[S_y A + \left(1 + \frac{ac}{r^2} \right) P_{cr} \right] = -42969 \quad C_2 = \frac{S_y \cdot A \cdot P_{cr}}{N^2} = 182.7 \times 10^6$$

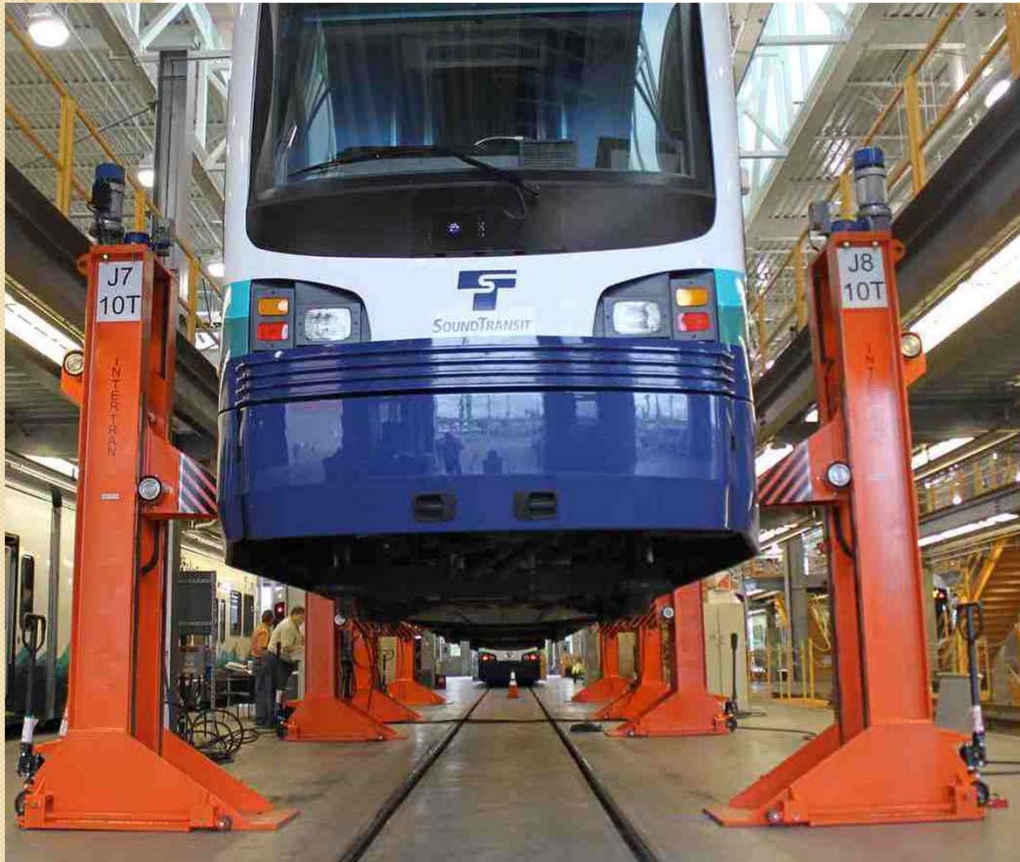
$$\therefore \text{The eq. is : } P_a^2 - 42969 P_a + 182.7 \times 10^6 = 0$$

$$\therefore P_a = 4784.7 \text{ N is the allowable load.}$$

Note: This solution process is most accurate for long column.

Eccentrically Loaded Columns:

An eccentric load is one that applied away from the centroidal axis of the c.s of column as shown in the following figures.

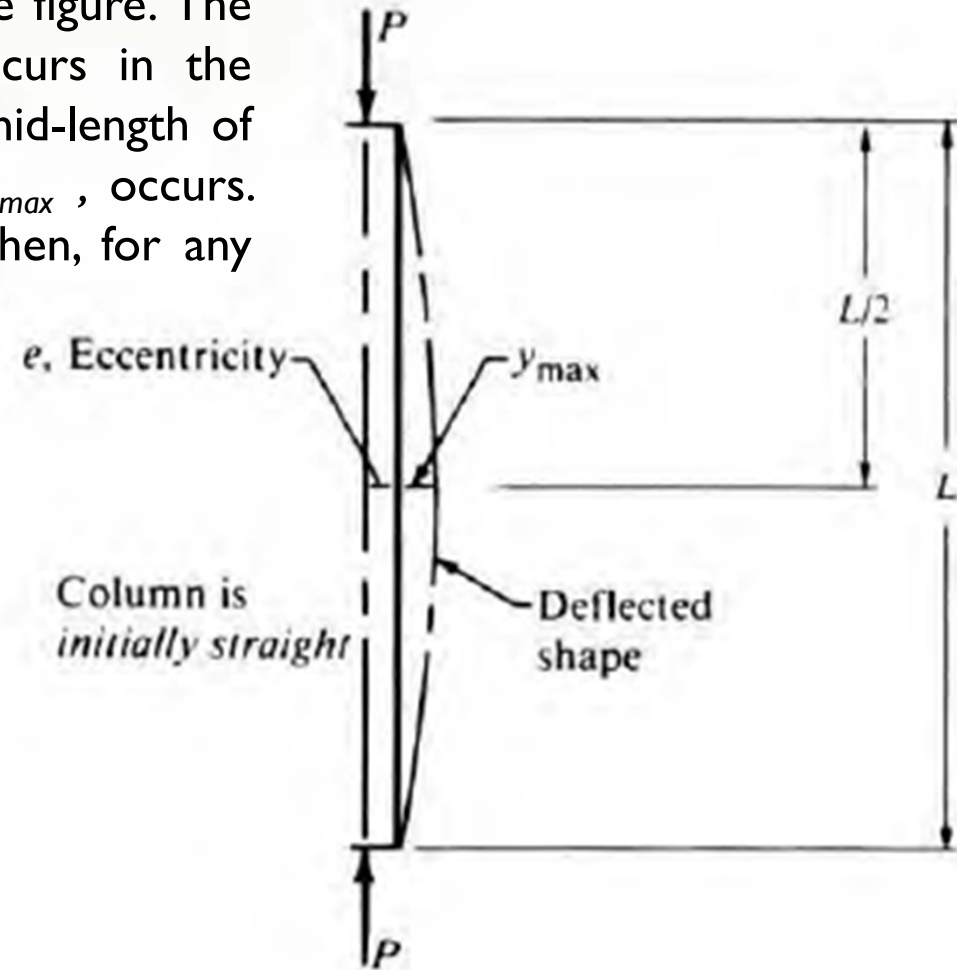


Such a load exerts bending in addition to the column action that results in the deflected shape shown in the figure. The maximum stress in the deflected column occurs in the outermost fibers of the cross section at the mid-length of the column where the maximum deflection, y_{max} , occurs. Let's denote the stress at this point as $\sigma_{L/2}$. Then, for any applied load, P ,

$$\sigma_{L/2} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \cdot \sec \left(\frac{KL}{2r} \sqrt{\frac{P}{AE}} \right) \right]$$

When max. stress is equal yield stress S_y

$$\therefore S_y = \frac{P_y}{A} \left[1 + \frac{ec}{r^2} \cdot \sec \left(\frac{KL}{2r} \sqrt{\frac{P_y}{AE}} \right) \right]$$



But $P_a = \frac{P_y}{N}$

$$\therefore \text{Required } S_y = \frac{NP_a}{A} \left[1 + \frac{ec}{r^2} \cdot \sec \left(\frac{KL}{2r} \sqrt{\frac{NP_a}{AE}} \right) \right]$$

* The above eq. cannot be solved for either A or P_a , therefore an iterative solution is required as will be demonstrated in the example.6 .

* Another critical factor may be amount of deflection of the axis of the column due to the eccentric load.

$$y_{max} = e \left[\sec \left(\frac{KL}{2r} \sqrt{\frac{P}{AE}} \right) - 1 \right]$$

Example.5 : For the column of Example.4, compute the maximum stress and deflection if a load of 4781.6N is applied with an eccentricity of 19.05mm. The column is initially straight.

Given: $e=19.05\text{mm}$, $D=19.05\text{mm}$, $L=812.8\text{mm}$, both ends pinned, $KL=812.8\text{mm}$
 $r=4.7752\text{mm}$, $c=D/2=9.525\text{mm}$, Mat.AISI 1040 hot-rolled steel

$$\text{Results: } \sigma_{L/2} = \frac{4781.6}{272.25} \left[1 + \frac{19.05 \times 9.525}{(4.7752)^2} \cdot \sec \left(\frac{812.8}{2(4.7752)} \sqrt{\frac{4781.6}{285.1784 \times 206.85 \times 10^9}} \right) \right]$$
$$= 202023.5 \text{ Kpa}$$

$$y_{max} = 19.05 \left[\sec \left(\frac{812.8}{2(4.7752)} \sqrt{\frac{4781.6}{285.1784 \times 206.85 \times 10^6}} \right) - 1 \right]$$
$$= 7.4022\text{mm}$$

Example.6 : The stress in the column found in Example.5 seems high for the AISI 1040 hot-rolled steel. Redesign the column to achieve a design factor of at least 3.

Results: * S_y for AISI 1040 HR to be 289590 Kpa

* If we choose to retain same material. The c.s of the column must increased to decrease stress.

* The objective is to find suitable value for A , c , and r such that $P_a=4781.6N$ $N=3$ & $L_e=8128mm$ & $e=19.05mm$ and $S < S_y$.

* The original design $D=19.05mm$, Let us try $D=25.4mm$ then:

$$A = \frac{\pi D^2}{4} = 506.482 mm^2, r = \frac{D}{4} = 6.35 mm, r^2 = 40.325 mm^2 \text{ \& } c = \frac{D}{2} = 12.7 mm$$

$$S = \frac{3 \times 4781.6}{506.45} \left[1 + \frac{19.05 \times 12.7}{40.325} \cdot \sec \left(\frac{812.8}{2(6.35)} \sqrt{\frac{3 \times 4781.6}{506.482 \times 207 \times 10^9}} \right) \right]$$

$$= 260217.3 \text{ Kpa} < \text{Required value for } S_y \text{ then we have satisfactory results}$$

$$y_{max} = 19.05 \left[\sec \left(\frac{812.8}{2(6.35)} \sqrt{\frac{4781.6}{506.482 \times 207 \times 10^9}} \right) - 1 \right] = 1.9304 \text{ mm}$$

\therefore dia assumed is satisfactory

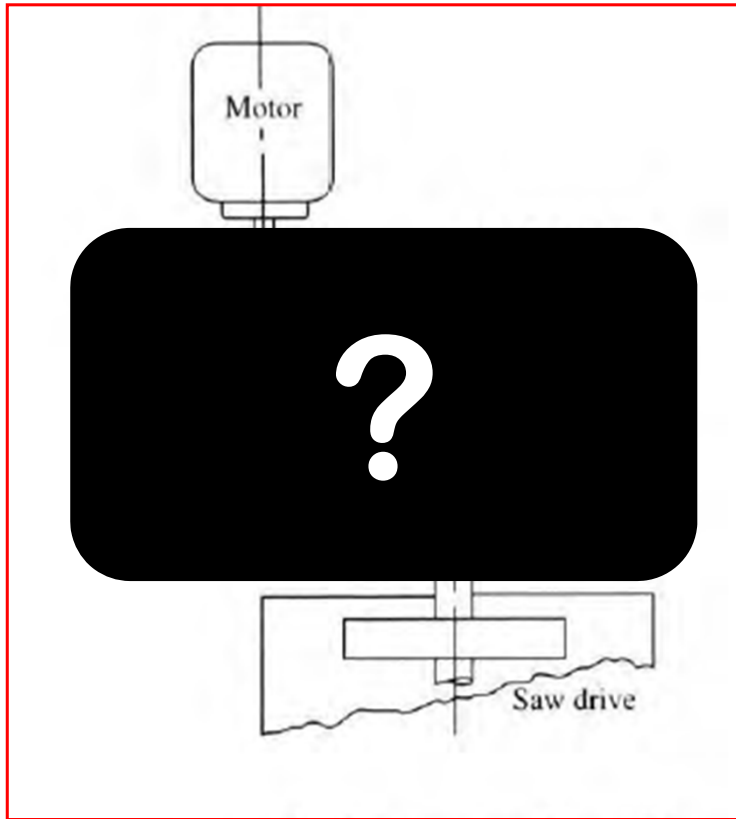


Mechanical Engineering Design II

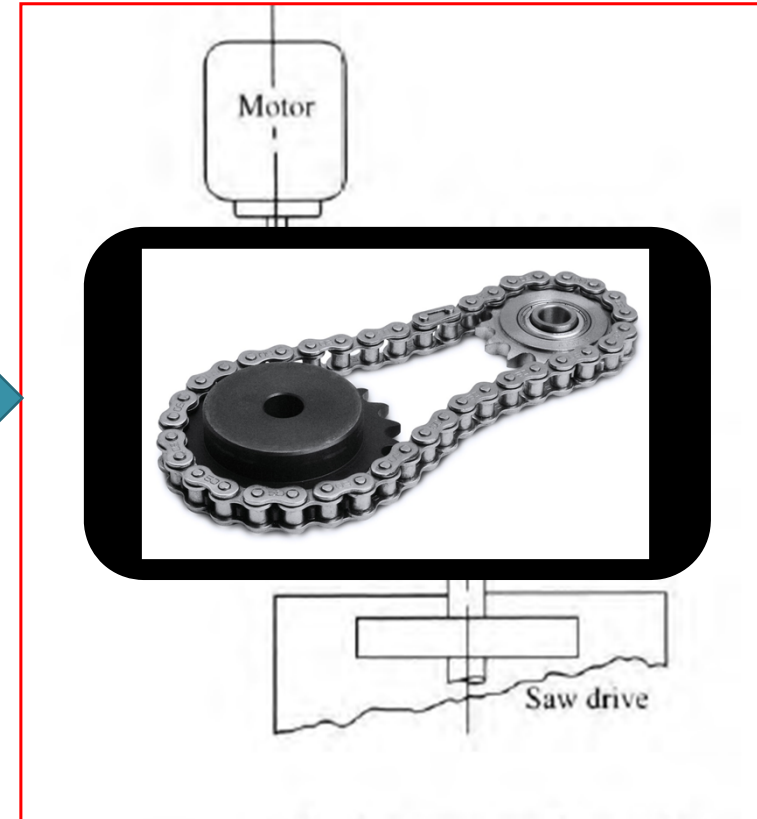
Sixteenth Lecture

Design of Chain Drive

Power Transmission Problem



Proposed solution (Chain Drive)



Transmitted Power is Known
Input and output range speed is Known

Flowchart for designing a chain drive:

Transmitted Power , Input and Output range speed

Specify a service factor from Table 7-8 page.290 (306Pdf)

TABLE 7-8 Service factors for chain drives

Load type	Type of driver		
	Hydraulic drive	Electric motor or turbine	Internal combustion engine with mechanical drive
Smooth (agitators; fans; light, uniformly loaded conveyors)	1.0	1.0	1.2
Moderate shock (machine tools, cranes, heavy conveyors, food mixers and grinders)	1.2	1.3	1.4
Heavy shock (punch presses, hammer mills, reciprocating conveyors, rolling mill drive)	1.4	1.5	1.7

1

COMPUTE THE DESIGN POWER

Design Power = service factor x transmitted power

COMPUTE THE DESIRED SPEED RATIO

Ratio = Input Speed / Middle Output Speed

Specify the standard chain size from Table 7-4 page.284 (300 Pdf)

TABLE 7-4 Roller chain sizes

Chain number	Pitch (in)	Roller diameter	Roller width	Link plate thickness	Average tensile strength (lb)
25	1/4	None	—	0.030	925
35	3/8	None	—	0.050	2100
41	1/2	0.306	0.250	0.050	2000
40	1/2	0.312	0.312	0.060	3700
50	5/8	0.400	0.375	0.080	6100
60	3/4	0.469	0.500	0.094	8500
80	1	0.626	0.625	0.125	14 500
100	1 1/4	0.750	0.750	0.156	24 000
120	1 1/2	0.875	1.000	0.187	34 000
140	1 3/4	1.000	1.000	0.219	46 000
160	2	1.125	1.250	0.250	58 000
180	2 1/4	1.406	1.406	0.281	80 000
200	2 1/2	1.562	1.500	0.312	95 000
240	3	1.875	1.875	0.375	130 000

2

Select the chain pitch from Tables 7-5, 7-6, and 7-7 page(287-289) (Pdf 303-305)

TABLE 7-5 Horsepower ratings—single strand roller chain no. 40

No. of teeth	0.500 inch pitch				Rotational speed of small sprocket, rev/min																				
	10	25	50	100	180	200	300	500	700	900	1000	1200	1400	1600	1800	2100	2500	3000	3500	4000	5000	6000	7000	8000	9000
11	0.06	0.14	0.27	0.52	0.91	1.00	1.48	2.42	3.34	4.25	4.70	5.60	6.49	5.57	4.66	3.70	2.85	2.17	1.72	1.41	1.01	0.77	0.61	0.50	0.00
12	0.06	0.15	0.29	0.56	0.99	1.09	1.61	2.64	3.64	4.64	5.13	6.11	7.09	6.34	5.31	4.22	3.25	2.47	1.96	1.60	1.15	0.87	0.69	0.57	0.00
13	0.07	0.16	0.31	0.61	1.07	1.19	1.75	2.86	3.95	5.02	5.56	6.62	7.68	7.15	5.99	4.76	3.66	2.79	2.21	1.81	1.29	0.98	0.78	0.00	
14	0.07	0.17	0.34	0.66	1.15	1.28	1.88	3.08	4.25	5.41	5.98	7.13	8.27	7.99	6.70	5.31	4.09	3.11	2.47	2.02	1.45	1.10	0.87	0.00	
15	0.08	0.19	0.36	0.70	1.24	1.37	2.02	3.30	4.55	5.80	6.41	7.64	8.86	8.86	7.43	5.89	4.54	3.45	2.74	2.24	1.60	1.22	0.97	0.00	
16	0.08	0.20	0.39	0.75	1.32	1.46	2.15	3.52	4.86	6.18	6.84	8.15	9.45	9.76	8.18	6.49	5.00	3.80	3.02	2.47	1.77	1.34	0.00		
17	0.09	0.21	0.41	0.80	1.40	1.55	2.29	3.74	5.16	6.57	7.27	8.66	10.04	10.69	8.96	7.11	5.48	4.17	3.31	2.71	1.94	1.47	0.00		
18	0.09	0.22	0.43	0.84	1.48	1.64	2.42	3.96	5.46	6.95	7.69	9.17	10.63	11.65	9.76	7.75	5.97	4.54	3.60	2.95	2.11	1.60	0.00		
19	0.10	0.24	0.46	0.89	1.57	1.73	2.56	4.18	5.77	7.34	8.12	9.66	11.22	12.64	10.59	8.40	6.47	4.92	3.91	3.20	2.29	0.09	0.00		
20	0.10	0.25	0.48	0.94	1.65	1.82	2.69	4.39	6.07	7.73	8.55	10.18	11.81	13.42	11.44	9.07	6.99	5.31	4.22	3.45	2.47	0.00			
21	0.11	0.26	0.51	0.98	1.73	1.91	2.83	4.61	6.37	8.11	8.98	10.69	12.40	14.10	12.30	9.76	7.52	5.72	4.54	3.71	2.65	0.00			
22	0.11	0.27	0.53	1.03	1.81	2.01	2.96	4.83	6.68	8.50	9.40	11.20	12.99	14.77	13.19	10.47	8.06	6.13	4.87	3.98	2.85	0.00			
23	0.12	0.28	0.56	1.08	1.90	2.10	3.10	5.05	6.98	8.89	9.83	11.71	13.58	15.44	14.10	11.19	8.62	6.55	5.20	4.26	3.05	0.00			
24	0.12	0.30	0.58	1.12	1.98	2.19	3.23	5.27	7.28	9.27	10.26	12.22	14.17	16.11	15.03	11.93	9.18	6.99	5.54	4.54	0.87	0.00			
25	0.13	0.31	0.60	1.17	2.06	2.28	3.36	5.49	7.59	9.66	10.69	12.73	14.76	16.78	15.98	12.68	9.76	7.43	5.89	4.82	0.00				
26	0.13	0.32	0.63	1.22	2.14	2.37	3.50	5.71	7.89	10.04	11.11	13.24	15.35	17.45	16.95	13.45	10.36	7.88	6.25	5.12	0.00				
28	0.14	0.35	0.67	1.31	2.31	2.55	3.77	6.15	8.50	10.82	11.97	14.26	16.53	18.79	18.94	15.03	11.57	8.80	6.99	5.72	0.00				
30	0.15	0.37	0.72	1.41	2.47	2.74	4.04	6.59	9.11	11.59	12.82	15.28	17.71	20.14	21.01	16.67	12.84	9.76	7.75	6.34	0.00				
32	0.16	0.40	0.77	1.50	2.64	2.92	4.31	7.03	9.71	12.38	13.68	16.30	18.89	21.48	23.14	18.37	14.14	10.76	8.54	1.41					
35	0.18	0.43	0.84	1.64	2.88	3.19	4.71	7.69	10.62	13.52	14.96	17.82	20.67	23.49	26.30	21.01	16.17	12.30	9.76	0.00					
40	0.21	0.50	0.96	1.87	3.30	3.65	5.38	8.79	12.14	15.45	17.10	20.37	23.62	26.85	30.06	25.67	19.76	15.03	0.00						
45	0.23	0.56	1.08	2.11	3.71	4.10	6.08	9.89	13.66	17.39	19.24	22.92	26.57	30.20	33.82	30.63	23.58	5.53	0.00						
	Type A				Type B										Type C										

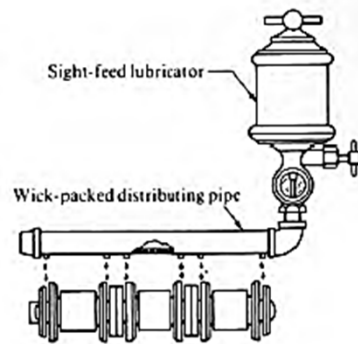
TABLE 7-6 Horsepower ratings—single strand roller chain no. 60

No. of teeth	0.750 inch pitch				Rotational speed of small sprocket, rev/min																					
	10	25	50	100	120	200	300	400	500	600	800	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500	5000	5500	6000	
11	0.19	0.46	0.89	1.72	2.05	3.35	4.95	6.52	8.08	9.63	12.69	15.58	11.85	9.41	7.70	6.45	5.51	3.94	3.00	2.38	1.95	1.63	1.39	1.21	0.00	
12	0.21	0.50	0.97	1.88	2.24	3.66	5.40	7.12	8.82	10.51	13.85	17.15	13.51	10.72	8.77	7.35	6.28	4.49	3.42	2.71	2.22	1.86	1.59	1.38	0.00	
13	0.22	0.54	1.05	2.04	2.43	3.96	5.85	7.71	9.55	11.38	15.00	18.58	15.23	12.08	9.89	8.29	7.08	5.06	3.85	3.06	2.50	2.10	1.79	0.00		
14	0.24	0.58	1.13	2.19	2.61	4.27	6.30	8.30	10.29	12.26	16.15	20.01	17.02	13.51	11.05	9.26	7.91	5.66	4.31	3.42	2.80	2.34	0.41	0.00		
15	0.26	0.62	1.21	2.35	2.80	4.57	6.75	8.90	11.02	13.13	17.31	21.44	18.87	14.98	12.26	10.27	8.77	6.28	4.77	3.79	3.10	2.60	0.00			
16	0.27	0.66	1.29	2.51	2.99	4.88	7.20	9.49	11.76	14.01	18.46	22.87	20.79	16.50	13.51	11.32	9.66	6.91	5.26	4.17	3.42	1.78	0.00			
17	0.29	0.70	1.37	2.66	3.17	5.18	7.65	10.08	12.49	14.88	19.62	24.30	22.77	18.07	14.79	12.40	10.58	7.57	5.76	4.57	3.74	0.00				
18	0.31	0.75	1.45	2.82	3.36	5.49	8.10	10.68	13.23	15.76	20.77	25.73	24.81	19.69	16.11	13.51	11.53	8.25	6.28	4.98	4.08	0.00				
19	0.33	0.79	1.53	2.98	3.55	5.79	8.55	11.27	13.96	16.63	21.92	27.16	26.91	21.35	17.48	14.65	12.50	8.95	6.81	5.40	0.20	0.00				
20	0.34	0.83	1.61	3.13	3.73	6.10	9.00	11.86	14.70	17.51	23.08	28.59	29.06	23.06	18.87	15.82	13.51	9.66	7.35	5.83	0.00					
21	0.36	0.87	1.69	3.29	3.92	6.40	9.45	12.46	15.43	18.38	24.23	30.02	31.26	24.81	20.31	17.02	14.53	10.40	7.91	6.28	0.00					
22	0.38	0.91	1.77	3.45	4.11	6.71	9.90	13.05	16.17	19.26	25.39	31.45	33.52	26.60	21.77	18.25	15.58	11.15	8.48	0.00						
23	0.40	0.95	1.85	3.61	4.29	7.01	10.35	13.64	16.90	20.13	26.54	32.88	35.84	28.44	23.28	19.51	16.66	11.92	9.07	0.00						
24	0.41	0.99	1.93	3.76	4.48	7.32	10.80	14.24	17.64	21.01	27.69	34.31	38.20	30.31	24.81	20.79	17.75	12.70	9.66	0.00						
25	0.43	1.04	2.01	3.92	4.67	7.62	11.25	14.83	18.37	21.89	28.85	35.74	40.61	32.23	26.38	22.11	18.87	13.51	10.27	0.00						
26	0.45	1.08	2.09	4.08	4.85	7.93	11.70	15.42	19.11	22.76	30.00	37.17	43.07	34.18	27.98	23.44	20.02	14.32	10.90	0.00						
28	0.48	1.16	2.26	4.39	5.23	8.54	12.60	16.61	20.58	24.51	32.31	40.03	47.68	38.20	31.26	26.20	22.37	16.01	0.00							
30	0.52	1.24	2.42	4.70	5.60	9.15	13.50	17.79	22.05	26.26	34.62	42.89	51.09	42.36	34.67	29.06	24.81	17.75	0.00							
32	0.55	1.33	2.58	5.02	5.98	9.76	14.40	18.98	23.52	28.01	36.92	45.75	54.50	46.67	38.20	32.01	27.33	19.56	0.00							
35	0.60	1.45	2.82	5.49	6.54	10.67	15.75	20.76	25.72	30.64	40.39	50.03	59.60	53.38	43.69	36.62	31.26	1.35	0.00							
40	0.69	1.66	3.22	6.27	7.47	12.20	18.00	23.73	29.39	35.02	46.16	57.18	68.12	65.22	53.38	44.74	38.20	0.00								
45	0.77	1.86	3.63	7.05	8.40	13.72	20.25	26.69	33.07	38.39	51.92	64.33	76.63	77.83	63.70	53.38	12.45	0.00								
	Type A				Type B								Type C													

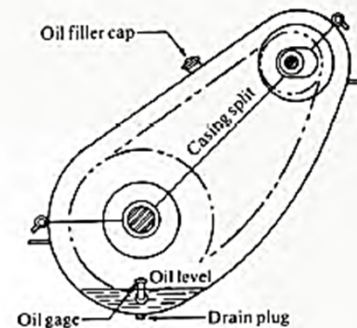
TABLE 7-7 Horsepower ratings—single strand roller chain no. 80

No. of teeth	1.000 inch pitch				Rotational speed of small sprocket, rev/min																					
	10	25	50	75	88	100	200	300	400	500	600	700	800	900	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500	
11	0.44	1.06	2.07	3.05	3.56	4.03	7.83	11.56	15.23	18.87	22.48	26.07	27.41	22.97	19.61	14.92	11.84	9.69	8.12	6.83	4.96	3.77	3.00	2.45	0.00	
12	0.48	1.16	2.26	3.33	3.88	4.39	8.54	12.61	16.82	20.59	24.53	28.44	31.23	26.17	22.35	17.00	13.49	11.04	9.25	7.90	5.65	4.30	3.41	2.79	0.00	
13	0.52	1.26	2.45	3.61	4.21	4.76	9.26	13.66	18.00	22.31	26.57	30.81	35.02	29.51	25.20	19.17	15.21	12.45	10.43	8.91	6.37	4.85	3.85	3.15		
14	0.56	1.35	2.63	3.89	4.53	5.12	9.97	14.71	19.39	24.02	28.62	33.18	37.72	32.98	28.16	21.42	17.00	13.91	11.66	9.96	7.12	5.42	4.30	3.52		
15	0.60	1.45	2.82	4.16	4.86	5.49	10.68	15.76	20.77	25.74	30.66	35.55	40.41	36.58	31.23	23.76	18.85	15.43	12.93	11.04	7.90	6.01	4.77	0.00		
16	0.64	1.55	3.01	4.44	5.18	5.86	11.39	16.81	22.16	27.45	32.70	37.92	43.11	40.30	34.41	26.17	20.77	17.00	14.25	12.16	8.70	6.62	5.25	0.00		
17	0.68	1.64	3.20	4.72	5.50	6.22	12.10	17.86	23.54	29.17	34.75	40.29	45.80	44.13	37.68	28.66	22.75	18.62	15.60	13.32	9.53	7.25	0.00			
18	0.72	1.74	3.39	5.00	5.83	6.59	12.81	18.91	24.93	30.88	36.79	42.66	48.49	48.08	41.05	31.23	24.78	20.29	17.00	14.51	10.39	7.90	0.00			
19	0.76	1.84	3.57	5.28	6.15	6.95	13.53	19.96	26.31	32.60	38.84	45.03	51.19	52.15	44.52	33.87	26.88	22.00	18.44	15.74	11.26	0.36	0.00			
20	0.80	1.93	3.76	5.55	6.47	7.32	14.24	21.01	27.70	34.32	40.88	47.40	53.88	56.32	48.08	36.58	29.03	23.76	19.91	17.00	12.16	0.00				
21	0.84	2.03	3.95	5.83	6.80	7.69	14.95	22.07	29.08	36.03	42.92	49.77	56.58	60.59	51.73	39.36	31.23	25.56	21.42	18.29	13.09	0.00				
22	0.88	2.13	4.14	6.11	7.12	8.05	15.66	23.12	30.47	37.75	44.97	52.14	59.27	64.97	55.47	42.20	33.49	27.41	22.97	19.61	14.03					
23	0.92	2.22	4.33	6.39	7.45	8.42	16.37	24.17	31.85	39.46	47.01	54.51	61.97	69.38	59.30	45.11	35.80	29.30	24.55	20.97	15.00					
24	0.96	2.32	4.52	6.66	7.77	8.78	17.09	25.22	33.24	41.18	49.06	56.88	64.66	72.40	63.21	48.08	38.16	31.23	26.17	22.35	15.99					
25	1.00	2.42	4.70	6.94	8.09	9.15	17.80	26.27	34.62	42.89	51.10	59.25	67.35	75.42	67.20	51.12	40.57	33.20	27.83	23.76	8.16					
26	1.04	2.51	4.89	7.22	8.42	9.52	18.51	27.32	36.01	44.61	53.14	61.62	70.05	78.43	71.27	54.22	43.02	36.22	29.51	25.20	0.00					
28	1.12	2.71	5.27	7.77	9.06	10.25	19.93	29.42	38.78	48.04	57.23	66.36	75.44	84.47	79.65	60.59	48.08	39.36	32.98	28.16	0.00					
30	1.20	2.90	5.64	8.33	9.71	10.98	21.36	31.52	41.55	51.47	61.32	71.10	80.82	90.50	88.33	67.20	53.33	43.65	36.58	31.23						
32	1.28	3.09	6.02	8.89	10.36	11.71	22.78	33.62	44.32	54.91	65.41	75.84	86.21	96.53	97.31	74.03	58.75	48.08	40.30	5.65						
35	1.40	3.38	6.58	9.72	11.33	12.81	24.92	36.78	48.47	60.05	71.54	82.95	94.29	105.58	111.31	84.68	67.20	55.00	28.15	0.00						
40	1.61	3.87	7.53	11.11	12.95	14.64	28.48	42.03	55.40	68.63	81.76	94.80	107.77	120.67	133.51	103.46	82.10	40.16	0.00							
45	1.81	4.35	8.47	12.49	14.57	16.47	32.04	47.28	62.32	77.21	91.98	106.65	121.24	135.75	150.20	123.45	72.28	0.00								
Type A					Type B										Type C											

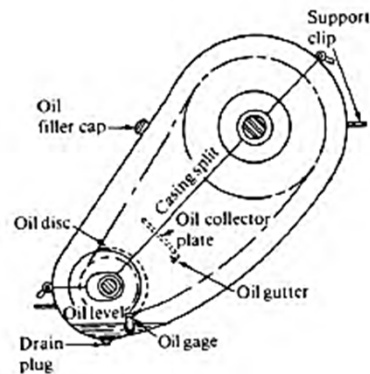
Select the lubrication type at the rotational speed of smaller sprocket from Tables 7-5, 7-6, and 7-7 page(287-289) (Pdf 303-305)



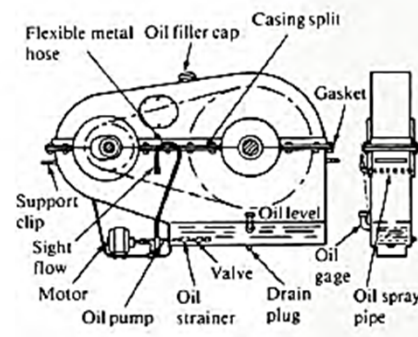
(a) Drip feed lubrication (Type A)



(b) Shallow bath lubrication (Type B)



(c) Disc or slinger lubrication (Type B)



(d) Oil stream lubrication (Type C)

COMPUTE THE REQUIRED NUMBER OF TEETH ON THE LARGE SPROCKET:

$$N_2 = N_1 \times \text{Ratio}$$

COMPUTE THE ACTUAL EXPECTED OUTPUT SPEED:

$$n_2 = n_1 (N_1/N_2)$$

COMPUTE THE PITCH DIAMETERS OF THE SPROCKETS

$$D_1 = \frac{P_1}{\sin(180/N_1)} \quad \text{and} \quad D_2 = \frac{P_2}{\sin(180/N_2)}$$

ASSUME THE CENTER DISTANCE BETWEEN SPROCKETS

$$C \cong 30 \text{ to } 50 \text{ (pitches)}$$

COMPUTE THE REQUIRED CHAIN LENGTH IN PITCHES

$$L = 2C + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C}$$

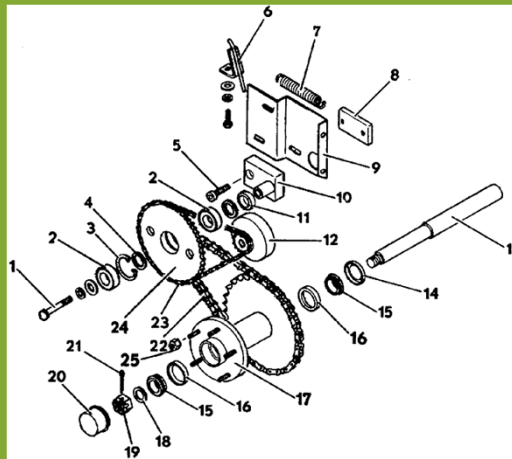
SPECIFY AN INTEGRAL & EVEN NUMBER OF PITCHES FOR THE CHAIN LENGTH, AND COMPUTE THE ACTUAL THEORETICAL CENTER DISTANCE

$$C = \frac{1}{4} \left[L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{N_2 + N_1}{2} \right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}} \right]$$

COMPUTE THE ANGLE OF WRAP OF THE CHAIN FOR EACH SPROCKET

$$\theta_1 = 180^\circ - 2 \sin^{-1} \left[\frac{D_2 - D_1}{2C} \right] \text{ (must be larger than } 120^\circ \text{)}$$

$$\theta_2 = 180^\circ + 2 \sin^{-1} \left[\frac{D_2 - D_1}{2C} \right]$$



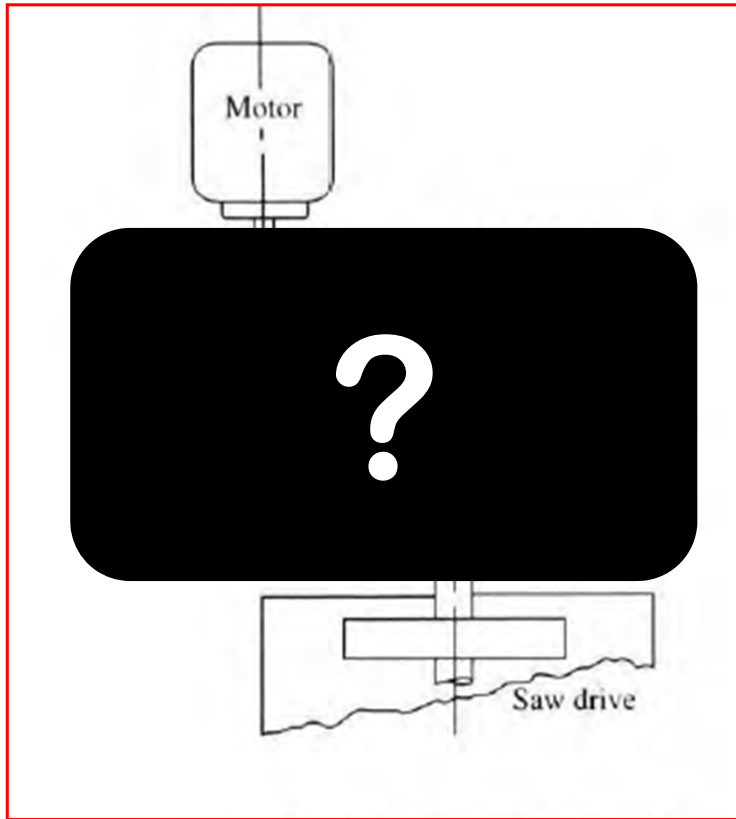


Mechanical Engineering Design II

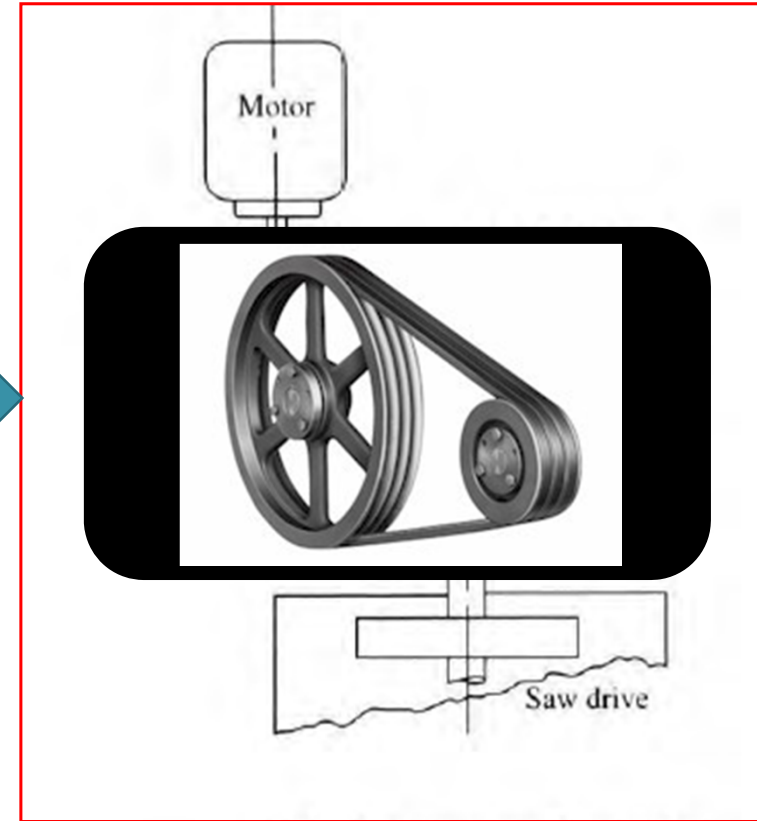
Seventeenth Lecture

Design of Belt Drive

Power Transmission Problem



Proposed solution (Belt Drive)



Transmitted Power is Known
Input and output range speed is Known

Types of Belt Drives



(a) **Wrapped construction**



(b) **Die cut, cog type**



(c) **Synchronous belt**



(d) **Poly-rib belt**

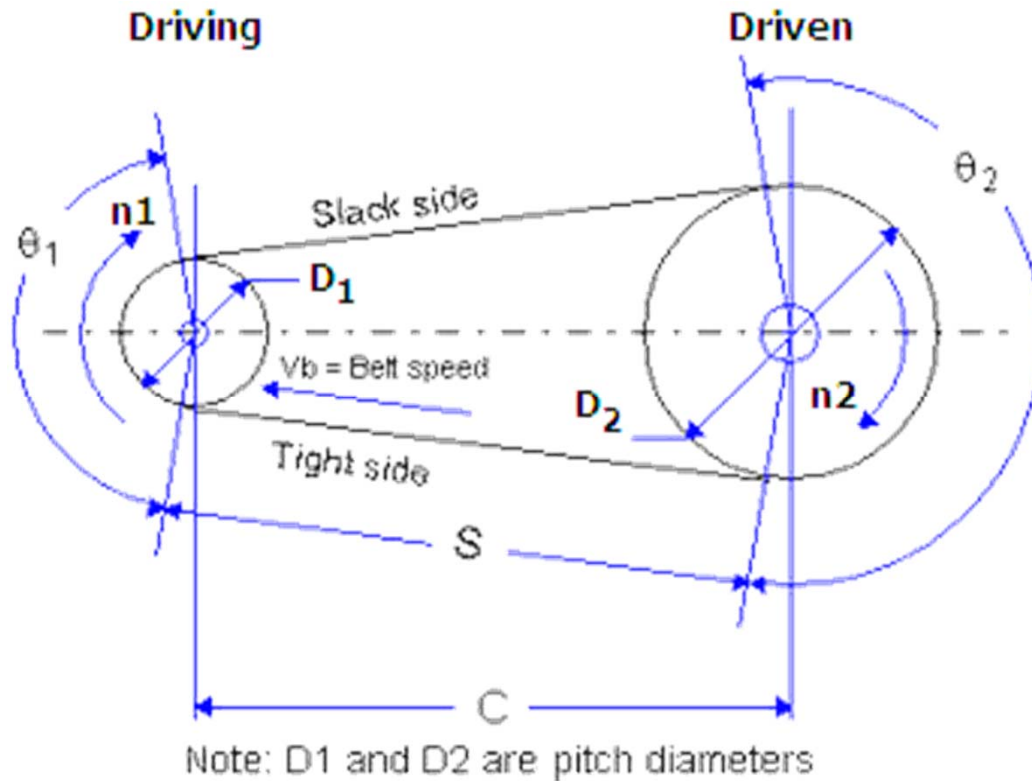


(e) **Vee-band**

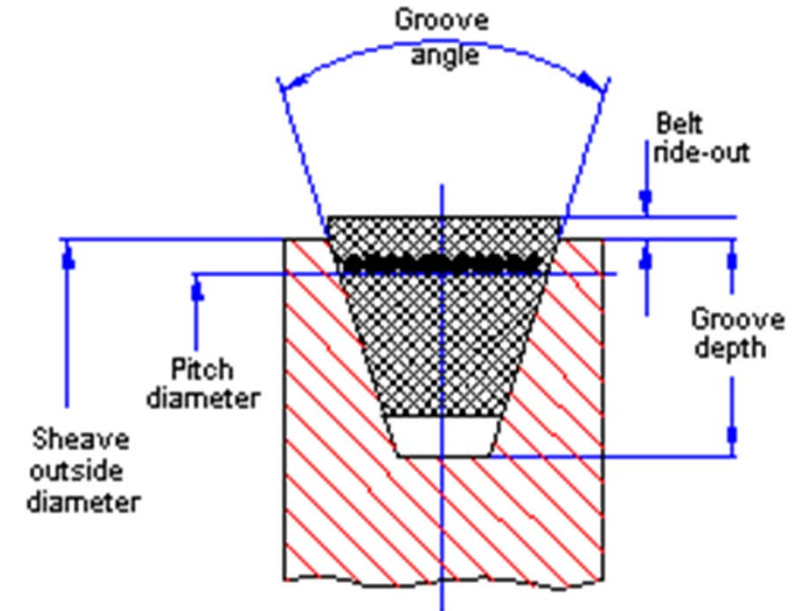


(f) **Double angle V-belt**

Basic Belt Drive Geometry



Typical belt section and groove geometry



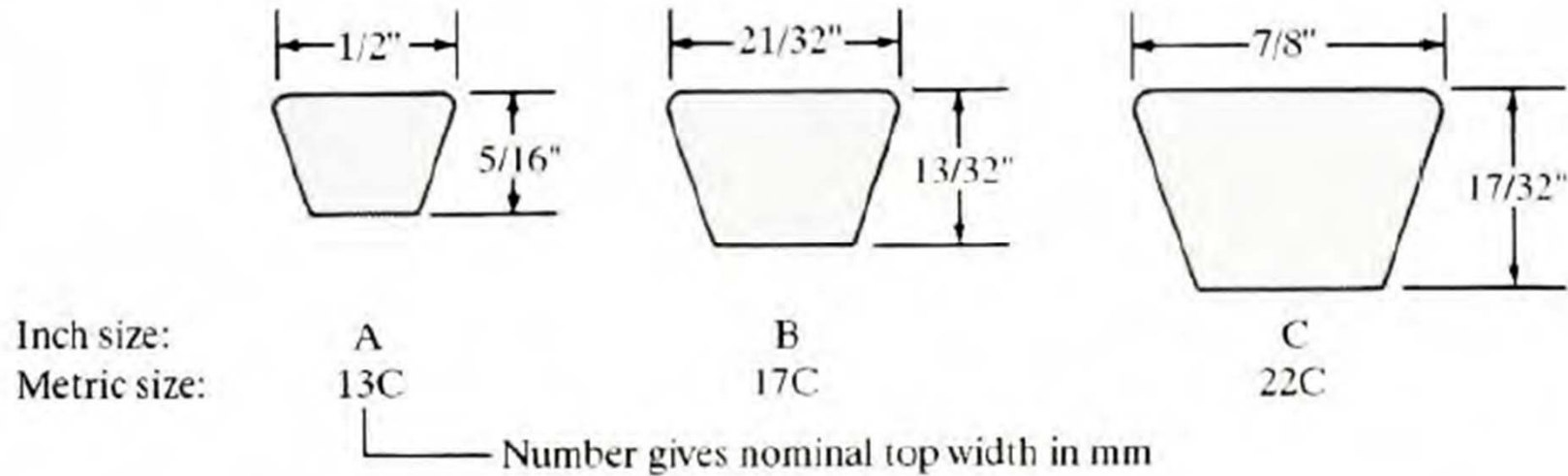
θ_1, θ_2 are the angles of wrap for small and big sheaves respectively

n_1, n_2 are the angular velocity for small and big sheaves respectively

C the center distance

S the span distance

Standard Belt Cross Sections



SAE Standard J636: V-belts and pulleys

SAE Standard J637: Automotive V-belt drives

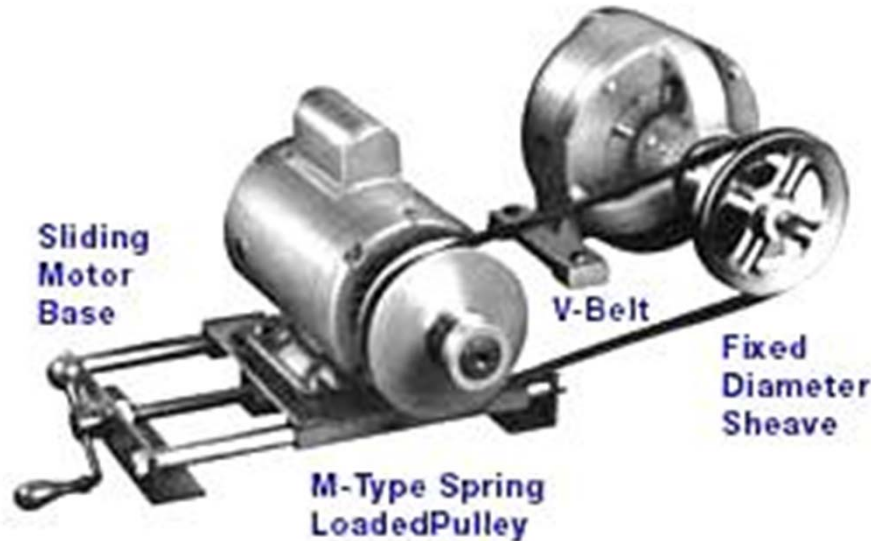
SAE Standard J1278: SI (metric) synchronous belts and pulleys

SAE Standard J1313: Automotive synchronous belt drives

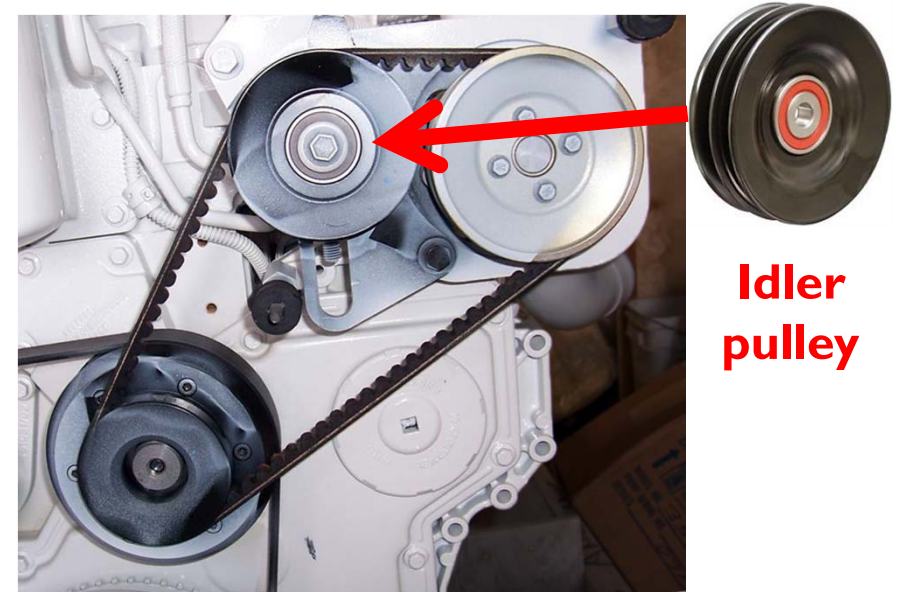
SAE Standard J1459: V-ribbed belts and pulleys

Design Conditions for V-belt

- The center distance must be adjustable in both directions, **or** if it was fixed, idler pulleys should be used.



or



- The nominal range of center distance should be: $D_2 < C < 3(D_2 + D_1)$
- The angle of wrap on smaller sheave (θ_1) should be $> 120^\circ$
- Most commercially available sheaves are cast iron, which should be limited to **(1981 m/min = 33 m/sec)**
- Consider an alternative type of drive, such as a gear type or chain, **if the belt speed is less than (304.8 m/min = 5 m/sec)**

V-Belt Drive Design

The rated power of the driving motor or other prime mover

Type of driver and driven load

The center distance

The size of the driving and driven sheaves

Speed of the smaller sheave



Designer

The service factor

The power rating for one belt

The belt length

The correction factor for belt length

The correction factor for the angle of wrap on the smaller sheave

The number of belts

The initial tension on the belt

Flowchart for designing a Belt drive:

Transmitted Power , Input and Output speed, Type of driver and driven load

Specify a service factor from Table 7-1 page.274 (290 Pdf)

TABLE 7-1 V-belt service factors

Driven machine type	Driver type					
	AC motors: Normal torque ^a DC motors: Shunt-wound Engines: Multiple-cylinder			AC motors: High torque ^b DC motors: Series-wound, compound-wound Engines: 4-cylinder or less		
	<6 h per day	6–15 h per day	>15 h per day	<6 h per day	6–15 h per day	>15 h per day
Agitators, blowers, fans, centrifugal pumps, light conveyors	1.0	1.1	1.2	1.1	1.2	1.3
Generators, machine tools, mixers, gravel conveyors	1.1	1.2	1.3	1.2	1.3	1.4
Bucket elevators, textile machines, hammer mills, heavy conveyors	1.2	1.3	1.4	1.4	1.5	1.6
Crushers, ball mills, hoists, rubber extruders	1.3	1.4	1.5	1.5	1.6	1.8
Any machine that can choke	2.0	2.0	2.0	2.0	2.0	2.0

^aSynchronous, split-phase, three-phase with starting torque or breakdown torque less than 175% of full-load torque.

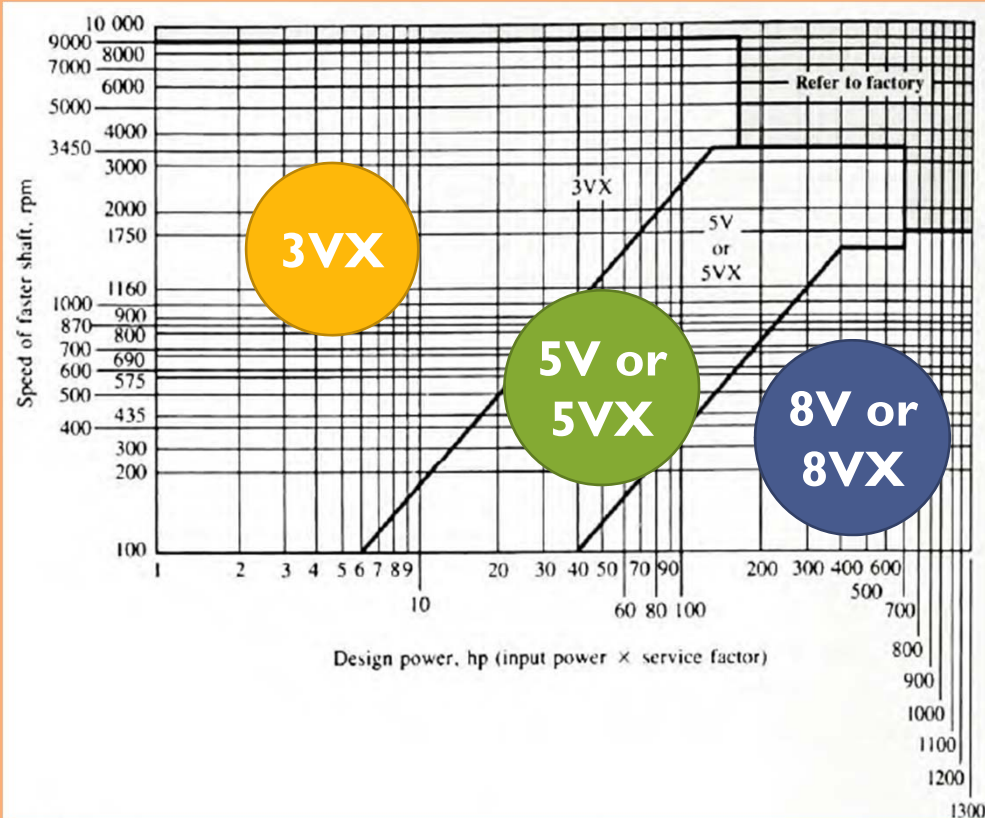
^bSingle-phase, three-phase with starting torque or breakdown torque greater than 175% of full-load torque.

1

COMPUTE THE DESIGN POWER

Design Power = service factor x transmitted power

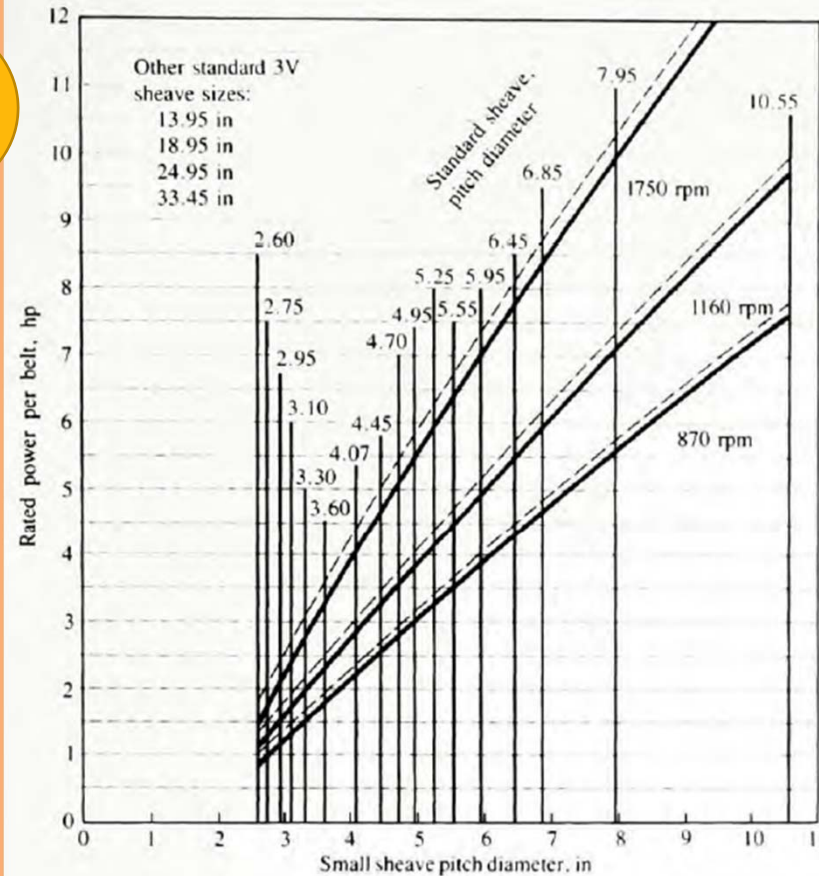
Specify the standard belt size from Figure 7-9 page.284 (300 Pdf)



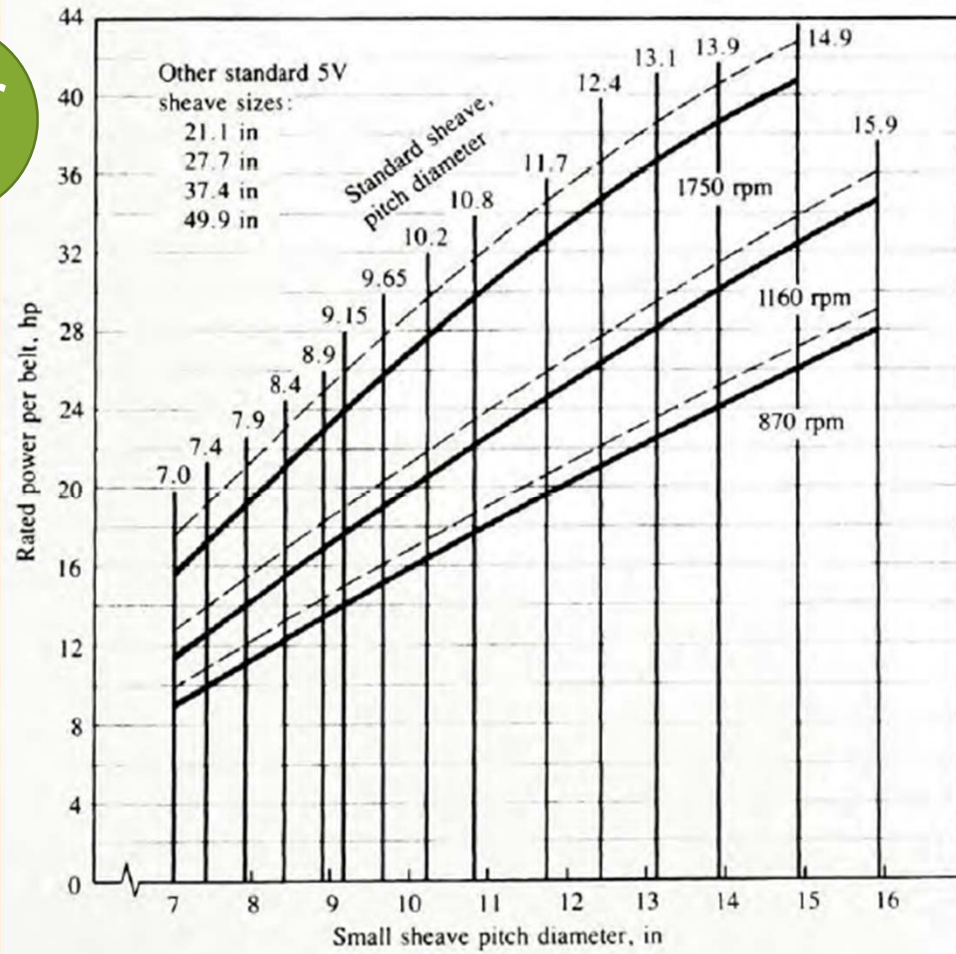
2

**State the power rating for one belt
from Figures (7-10, 7-11, and 7-12) page(275-276) (Pdf 291-292)**

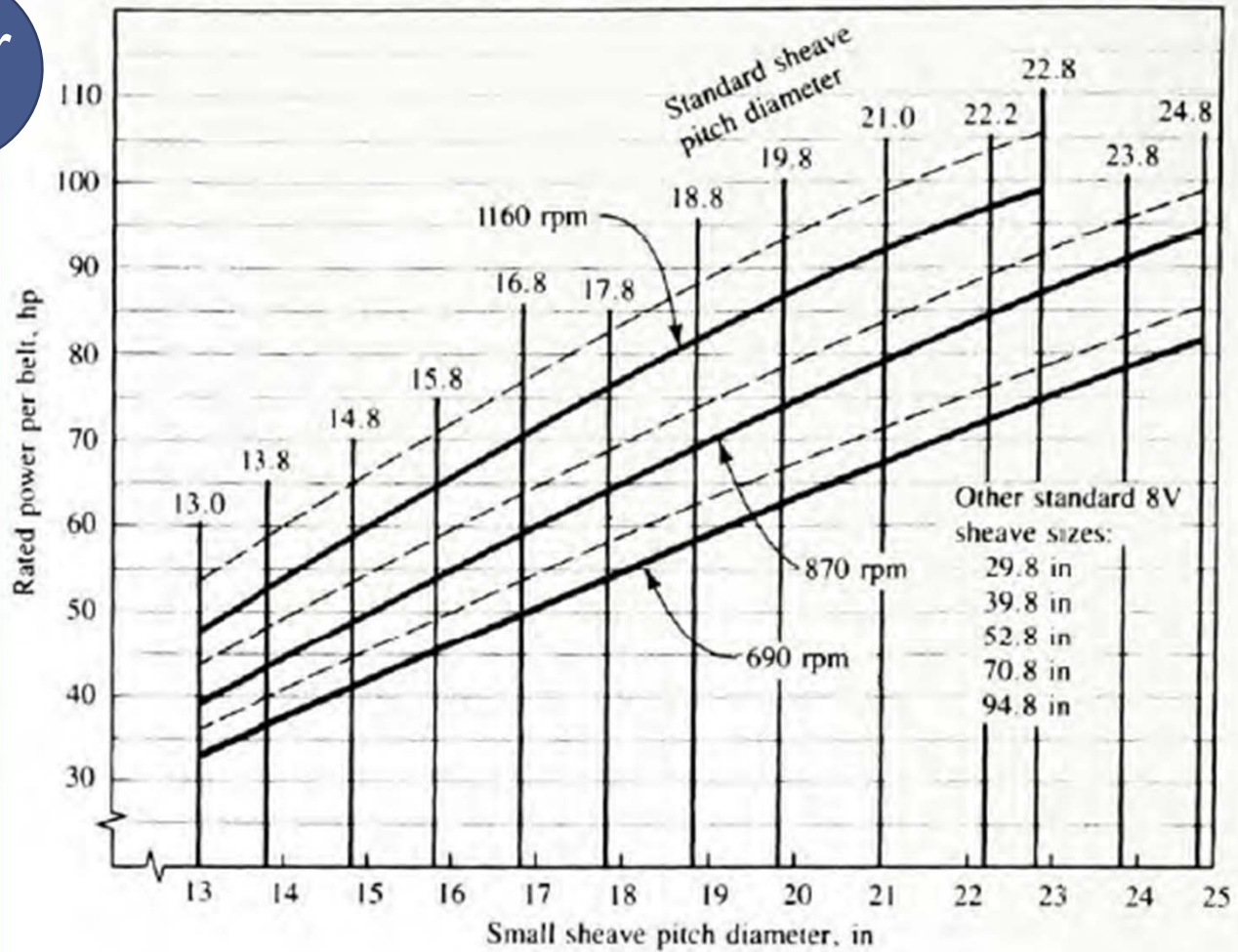
3VX



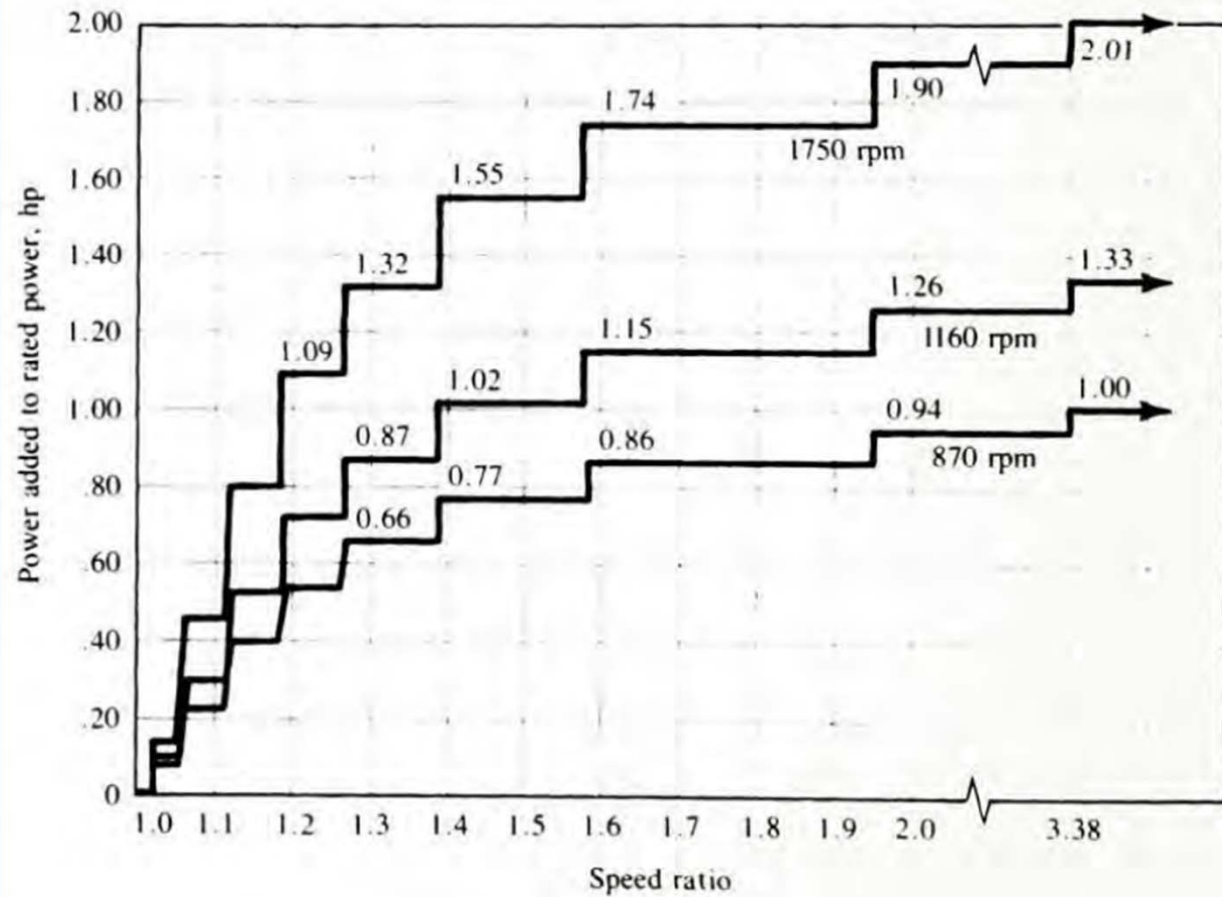
5V or
5VX



8V or
8VX



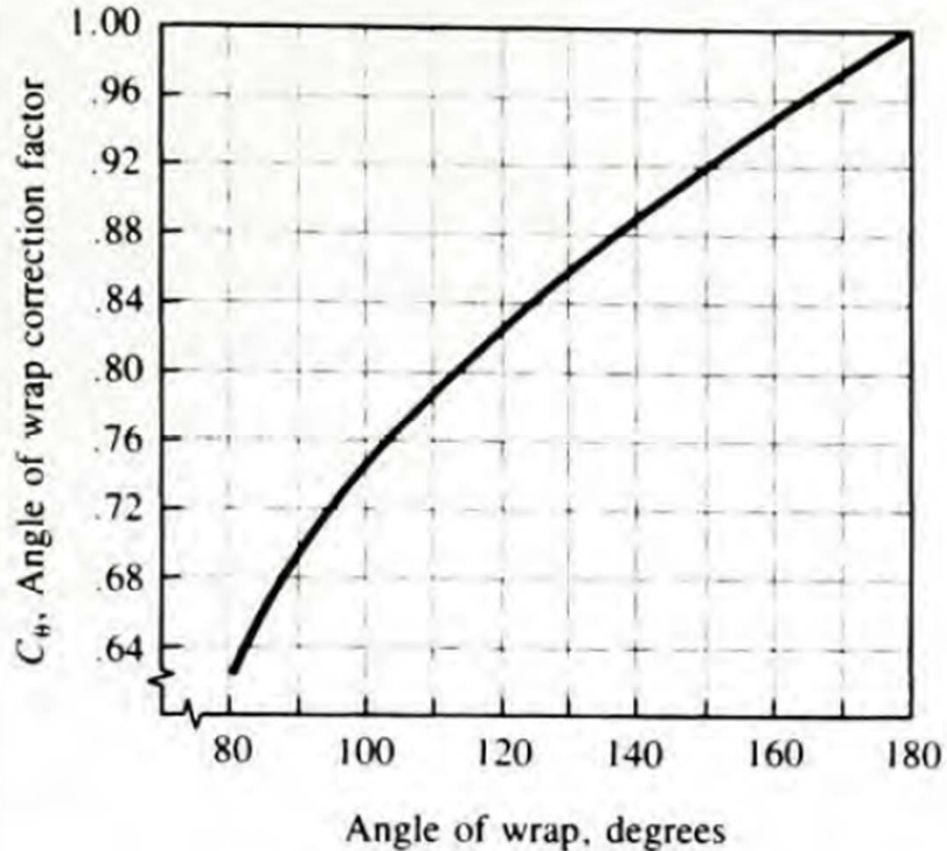
State the power added to rated power from Figure (7-13) page(276) (Pdf 292)



6

Compute the angle of wrap: $\theta_1 = 180^\circ - 2 \sin^{-1} \left[\frac{D_2 - D_1}{2C} \right]$

State the correction factor of wrap angle C_θ from figure (7-14) page (277) (293 pdf):



7

Specify a trial center distance: $D_2 < C < 3(D_2 + D_1)$

Compute the required belt length: $L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 + D_1)^2}{4C}$

Select the nearest standard belt length from Table (7-2) page (277) (293 pdf)

TABLE 7-2 Standard belt lengths for 3V, 5V, and 8V belts (in)

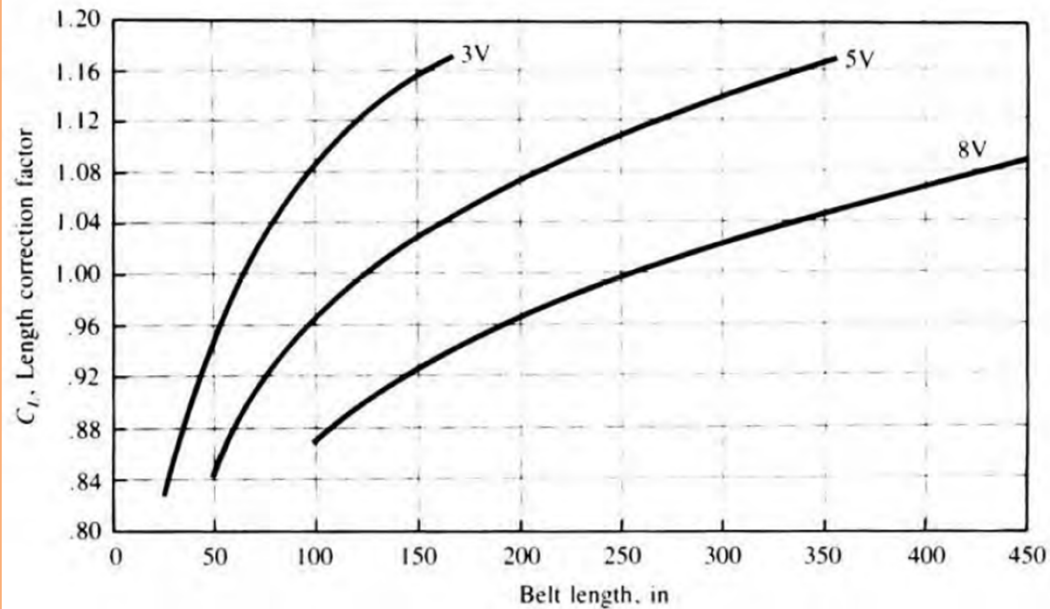
3V only	3V and 5V	3V, 5V, and 8V	5V and 8V	8V only
25	50	100	150	375
26.5	53	106	160	400
28	56	112	170	425
30	60	118	180	450
31.5	63	125	190	475
33.5	67	132	200	500
35.5	71	140	212	
37.5	75		224	
40	80		236	
42.5	85		250	
45	90		265	
47.5	95		280	
			300	
			315	
			335	
			355	
165				

Re-calculate the center distance and angle of wrap:

$$C = \frac{B + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16} \text{ where } B = 4L - 6.28(D_2 + D_1)$$

$$\theta_1 = 180^\circ - 2 \sin^{-1} \left[\frac{D_2 - D_1}{2C} \right]$$

State the length correction factor from Figure (7-15) page (277) (293 pdf)



Compute the corrected rated power per belt and the number of belts required to carry the design power:

$$\text{Corrected rated power} = C_{\theta} C_L P$$

where

$$P = (\text{actual rated power}) = \boxed{\begin{array}{c} \text{rated power} \\ (\text{fig. 7 - 10, 11 \& 12}) \end{array}} + \boxed{\begin{array}{c} \text{added power} \\ (\text{fig. 7 - 13}) \end{array}}$$

$$\text{Number of belts} = \frac{\text{Design Power}}{\text{Corrected Rated Power}}$$