

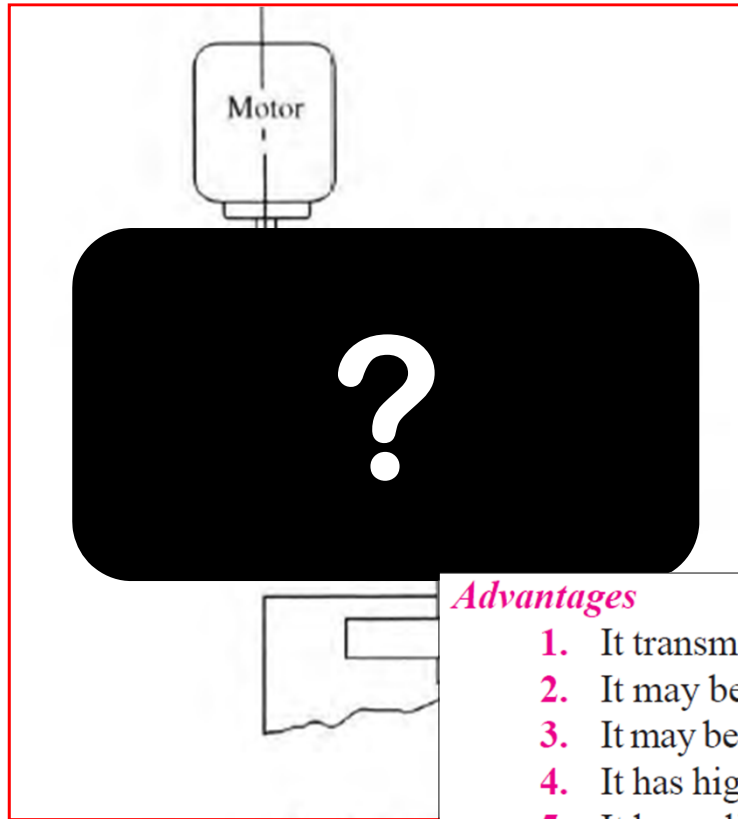


Mechanical Engineering Design II

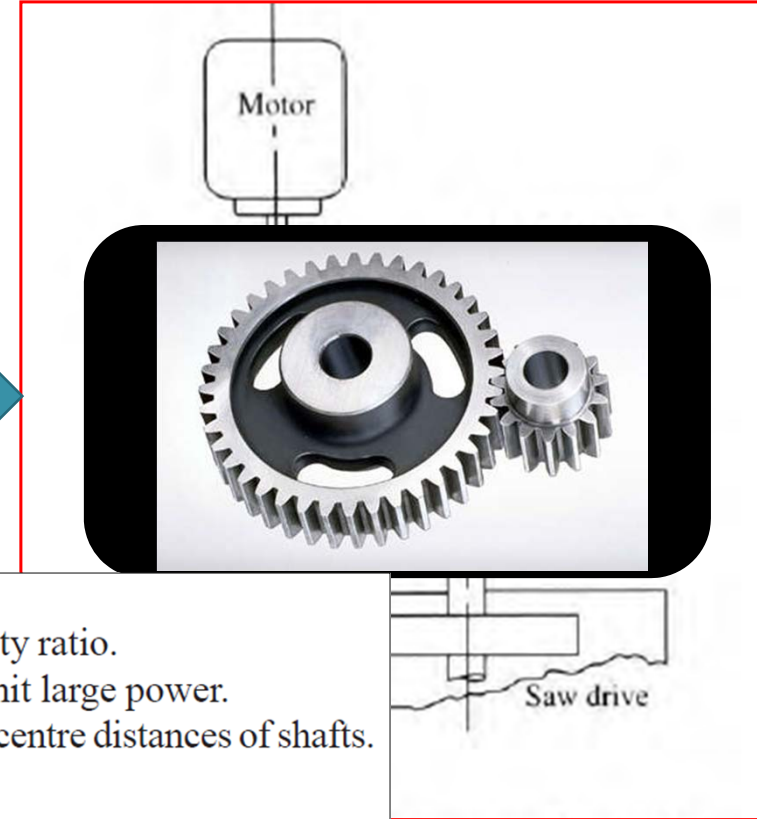
Eighteenth & Nineteenth Lectures

Design of Spur Gear

Power Transmission Problem



Proposed solution (Spur Gear)



Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It may be used for small centre distances of shafts.
4. It has high efficiency.
5. It has reliable service.
6. It has compact layout.

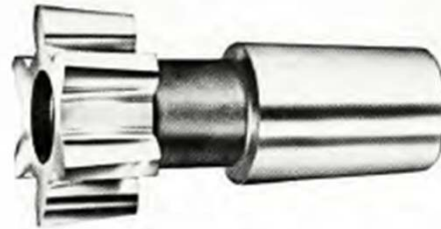
Disadvantages

1. Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.

Gear Manufacturing



(a) Form milling cutter



(b) Spur gear shaper cutter

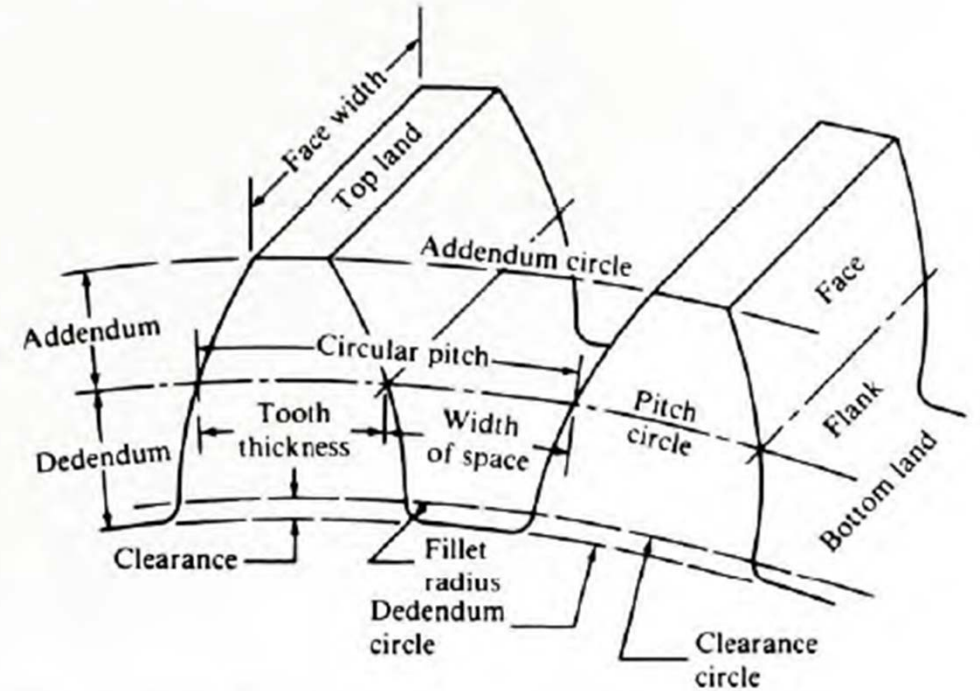
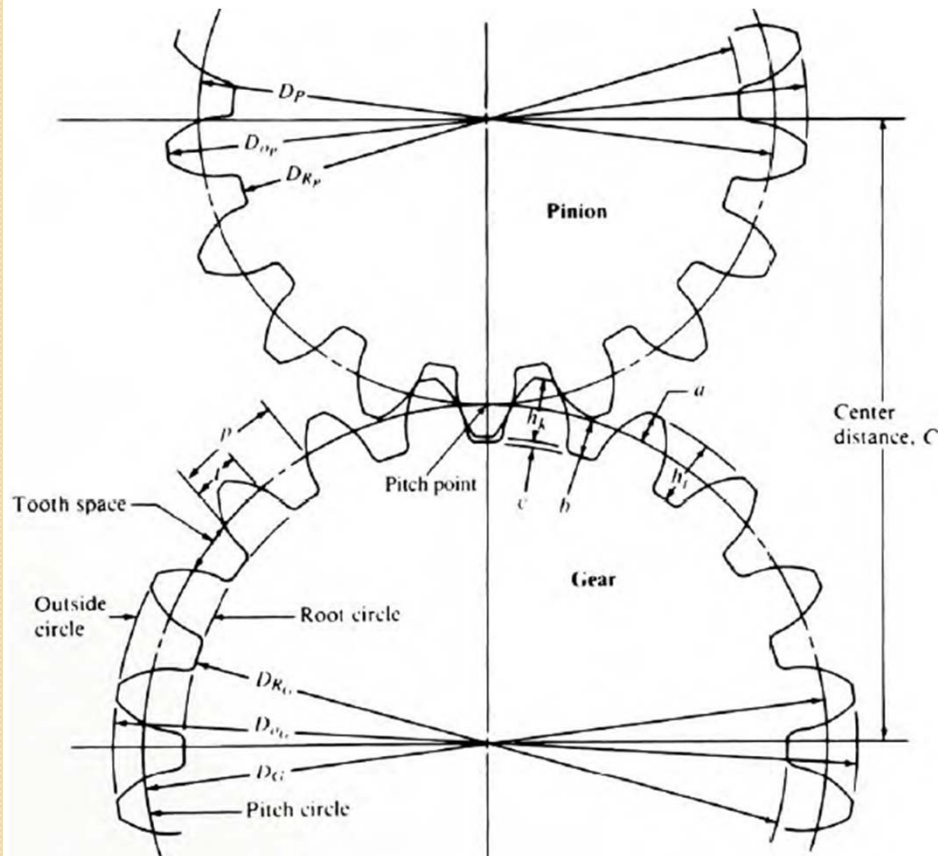


(c) Hob for small pitch gears having large teeth

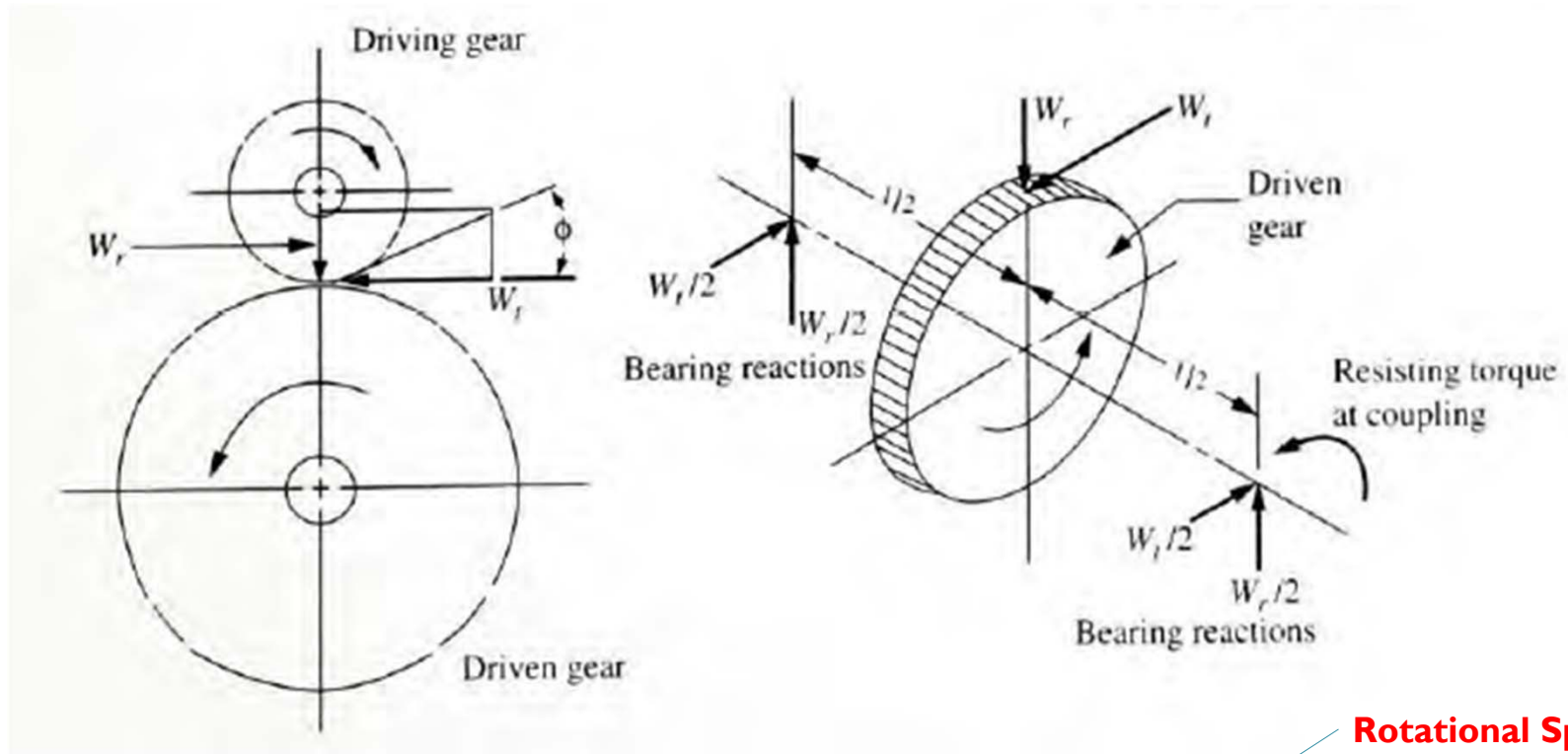


(d) Hob for high pitch gears having small teeth

Basic Spur Gear Geometry



Kinematics of Spur Gear



Torque (N.m)

$$T = W_t(R) = W_t(D/2) = P/n$$

Rotational Speed (rad/sec)

Tangential Force (N)

$$W_t = \frac{2P}{Dn}$$

Transmitted Power (watt)

Pitch Diameter (m)

Modes of Gear Tooth Failure

1. Bending failure.

Root failure

2. Pitting.

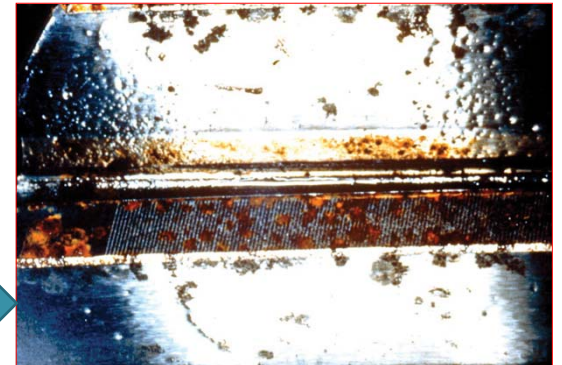
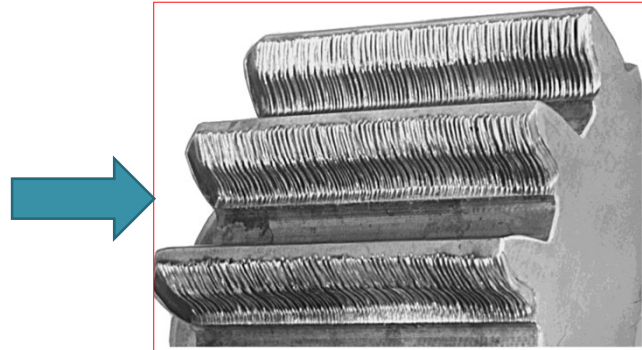
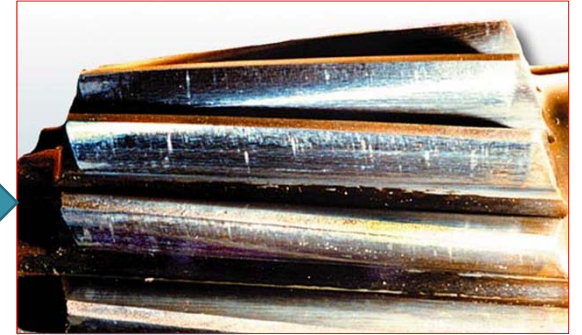
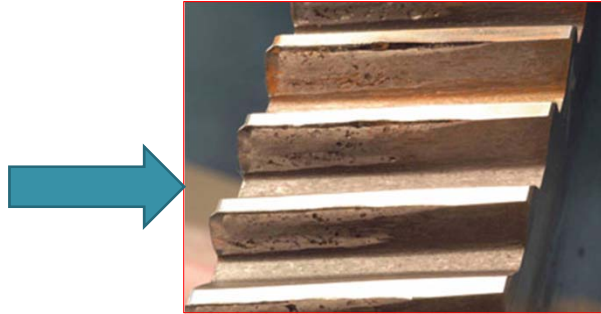
3. Scoring.

4. Abrasive wear.

5. Corrosive wear.

Surface failure

Dangerous Bar



Spur Gear Design

The power to be transmitted

Type of driver and driven load

The speed of the driving gear

The center distance

The speed of the driven gear or the velocity ratio

Other information related to problem specification



Designer

The gear teeth should not fail under static loading or dynamic loading during normal running conditions.

The gear teeth should have wear characteristics so that their life is satisfactory.

The use of space and material should be economical.

The alignment of the gears and deflections of the shafts must be considered.

The lubrication of the gears must be satisfactory.

Flowchart for spur gear designing process:

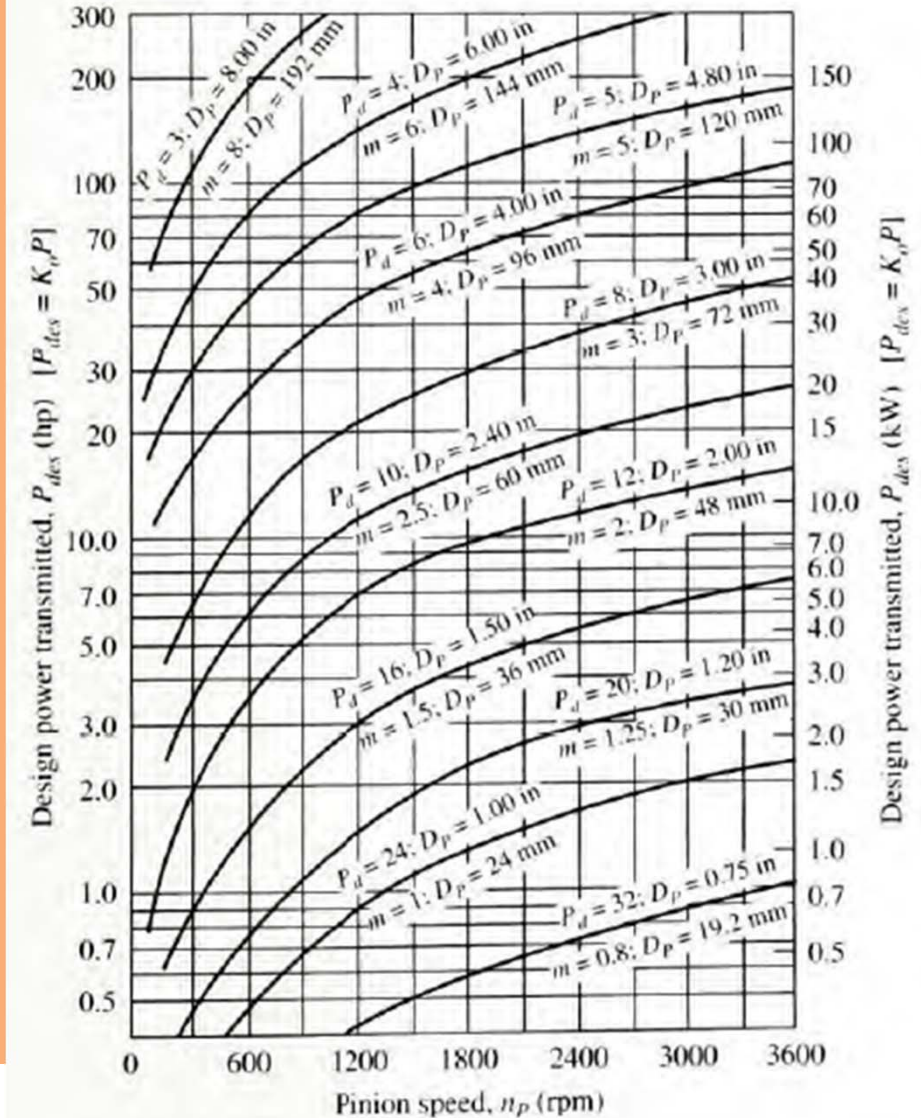
Transmitted Power , Input and Output speed, Center distance, Type of driver and driven load

Choose the over load factor (K_o) from Table (9-5) page(389) (405pdf)

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Compute the design power
Design Power = K_o x transmitted power

Find the trial value for
Diametral pitch (P_d) or Module (m)
and Pitch Diameter (D_p) from
Figure 9-27 page.409 (425 Pdf)



Specify the no. of teeth for Pinion N_P (from 17 to 20)

Compute the nominal velocity ratio $VR = \frac{n_P}{n_G}$

Compute the approximate no. of teeth for Gear

$$N_G = N_P \times VR$$

Compute the actual velocity ratio $VR = \frac{N_G}{N_P}$

Compute the actual output velocity $n_G = n_P \frac{N_P}{N_G}$

**Compute the pitch diameters $D_p = \frac{N_p}{P_d}$, $D_G = \frac{N_G}{P_d}$, center distance $c = \frac{N_P + N_G}{2P_d}$,
 pitch line speed $v_t = \frac{\pi D_p n_P}{60}$ and tangential force, $W_t = \frac{2P}{D_P n_P}$**

Specify the face width within the following recommended range for general machine drive gears:

$$8/P_d < F < 16/P \quad \text{Nominal value of } F = 12/P$$

Specify the quality number Q_v , from Table (9-2) page (378) (394 pdf)

Application	Quality number	Application	Quality number
Cement mixer drum drive	3-5	Small power drill	7-9
Cement kiln	5-6	Clothes washing machine	8-10
Steel mill drives	5-6	Printing press	9-11
Grain harvester	5-7	Computing mechanism	10-11
Cranes	5-7	Automotive transmission	10-11
Punch press	5-7	Radar antenna drive	10-12
Mining conveyor	5-7	Marine propulsion drive	10-12
Paper-box-making machine	6-8	Aircraft engine drive	10-13
Gas meter mechanism	7-9	Gyroscope	12-14

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0-800	6-8	0-4
800-2000	8-10	4-11
2000-4000	10-12	11-22
Over 4000	12-14	Over 22

4

Analyzing of gear tooth failure mode

Root (Bending) Failure Mode

Bending Stress Number

$$S_t = \frac{W_t P_d}{F J} K_o K_s K_m K_B K_v < S_{at} \frac{Y_N}{K_R (S.F)}$$

Surface (Pitting, Scoring,...) Failure Mode

Contact Stress Number

$$S_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}} < S_{ac} \frac{Z_N C_H}{K_R (S.F)}$$

Find the values of factors
 $(J, I, K_s, K_m, K_B, K_v, C_p, Y_N, Z_N, C_H, K_R, S.F)$ as in the following steps

5

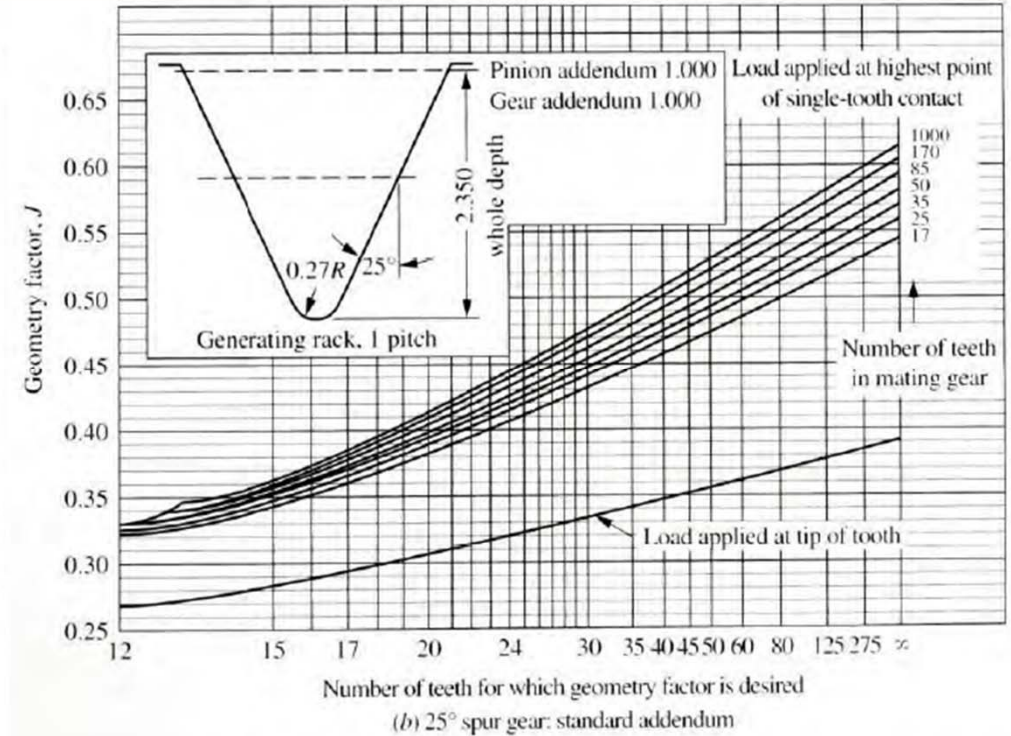
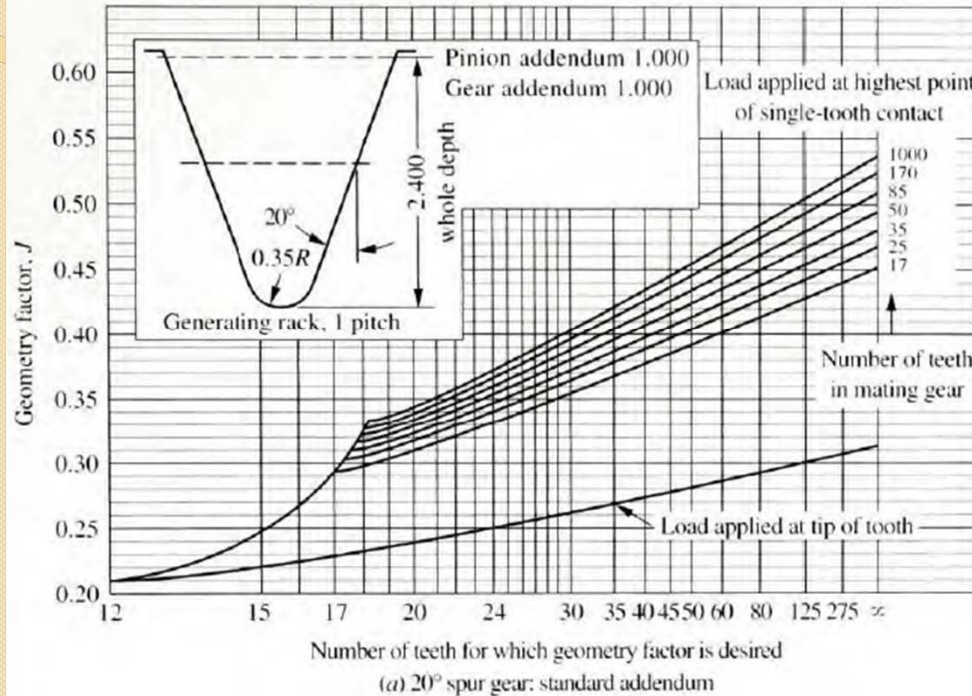
Specify the type of material for the gears to find the Elastic Coefficient C_p from Table (9-9) page(400) (Pdf 416)

Pinion material	Modulus of elasticity, E_p , lb/in ² (MPa)	Gear material and modulus of elasticity, E_G , lb/in ² (MPa)					
		Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Mall. iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nod. iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

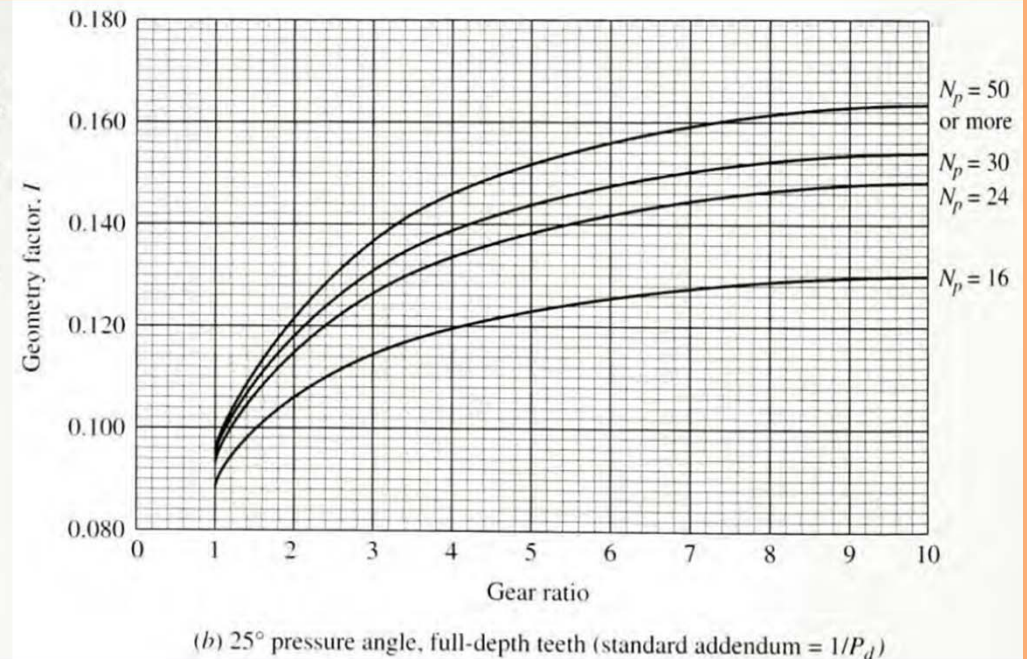
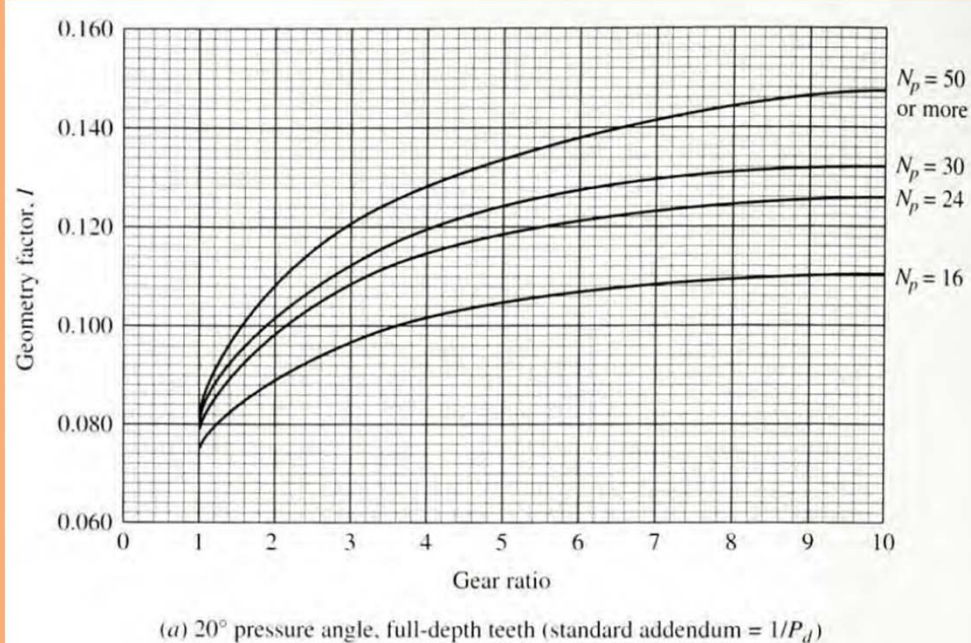
Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are (lb/in²)^{0.5} or (MPa)^{0.5}.

Specify the bending geometry factor (J) from figure (9-17) page (387) (403pdf):



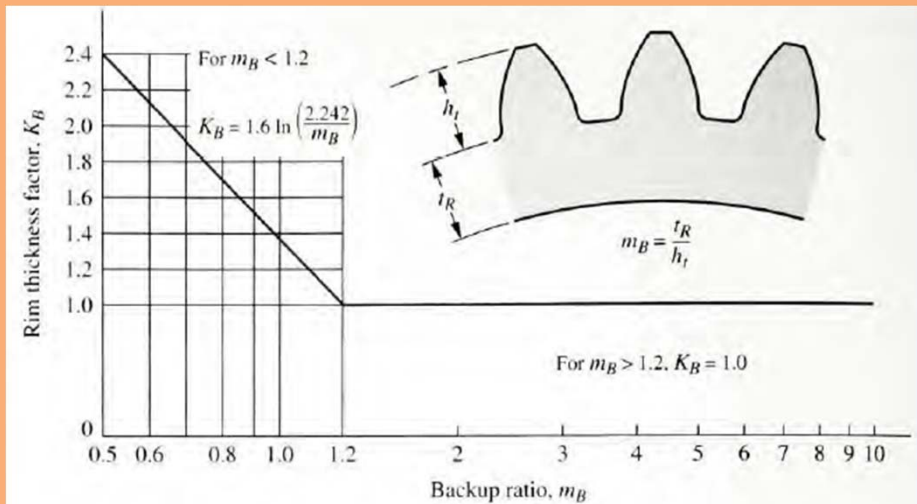
Specify the pitting geometry factor (I) from figure (9-23) page (402) (418 pdf):



**Specify the size factor (K_S) from Figure (9-6)
page (389) (293 pdf)**

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

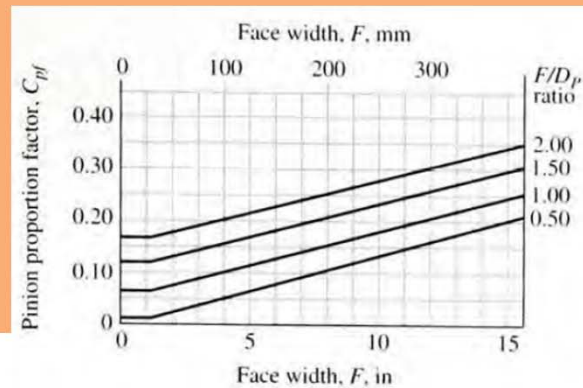
**Specify the rim thickness factor (K_B) from
Figure (9-20) page (392) (408 pdf)**



Determine the load distribution factor (K_m) : $K_m = 1.0 + C_{pf} + C_{ma}$

Where C_{pf} = pinion proportion factor from figure (9-18) page(391) (407 pdf)

C_{ma} = mesh alignment factor from figure (9-19) page(391) (407 pdf)



D_p = Pinion diameter

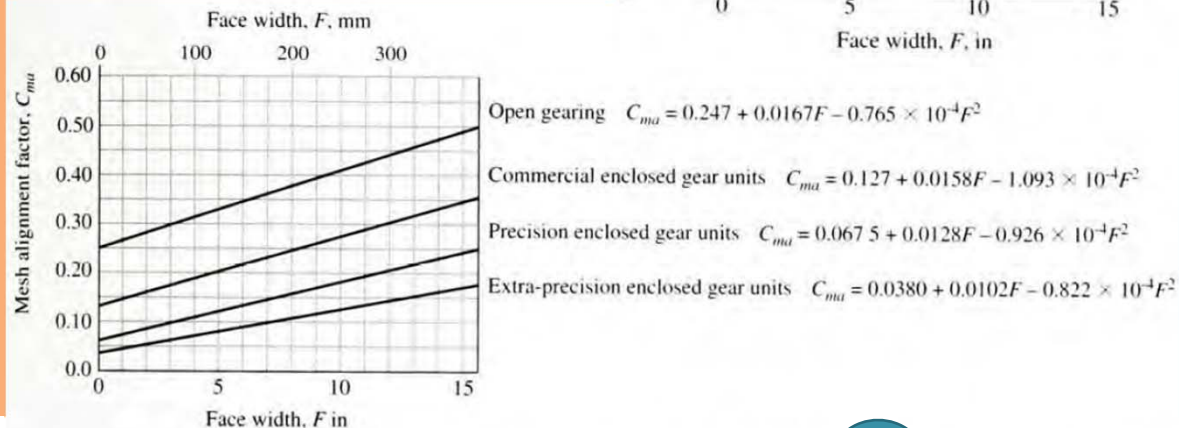
For $F/D_p < 0.50$, use curve for $F/D_p = 0.50$

When $F \leq 1.0$ in. ($F \leq 25$ mm)

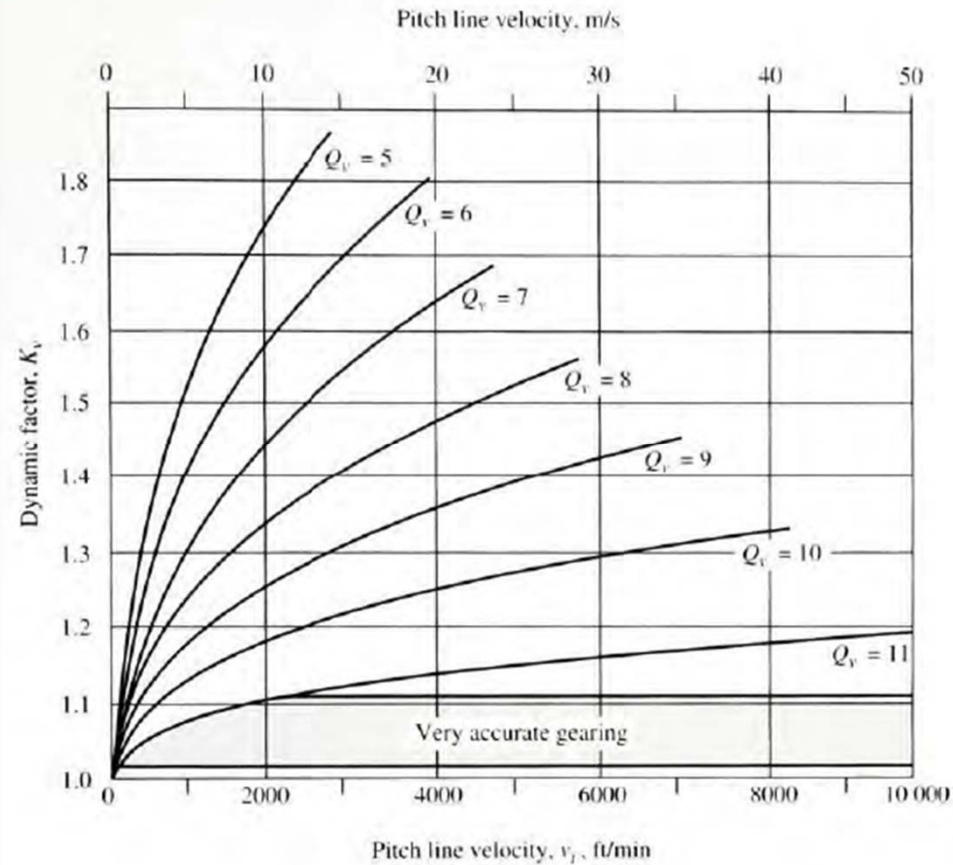
$$C_{pf} = \frac{F}{10D_p} - 0.025$$

When $1.0 \leq F < 15$,

$$C_{pf} = \frac{F}{10D_p} - 0.0375 + 0.0125F$$

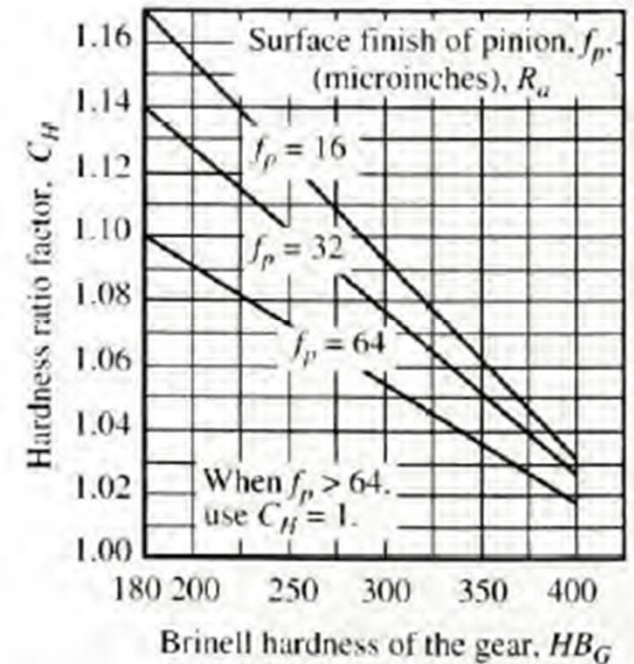
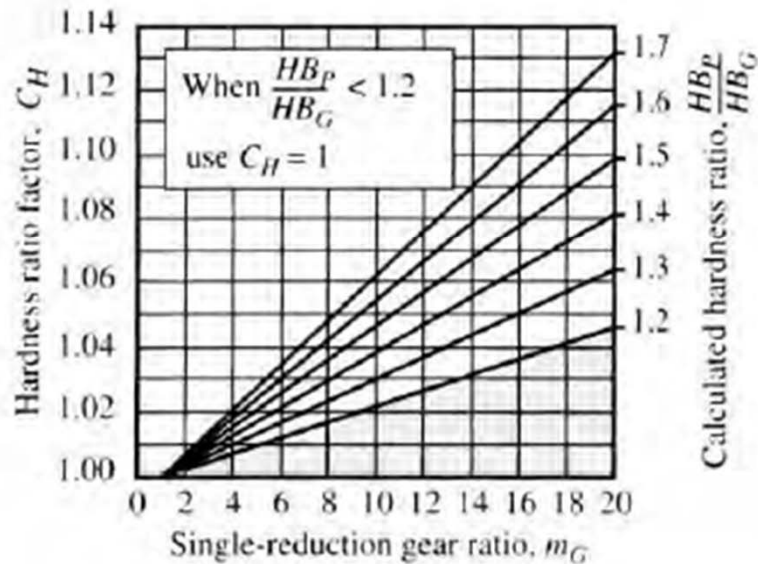


Specify the dynamic factor (K_v) from figure (9-21) page (393) (409 pdf):



Specify the safety factor (S.F) typically from 1 to 1.5

Specify the hardness ratio factor from Figure (9-25 & 26) page (404) (420 pdf)



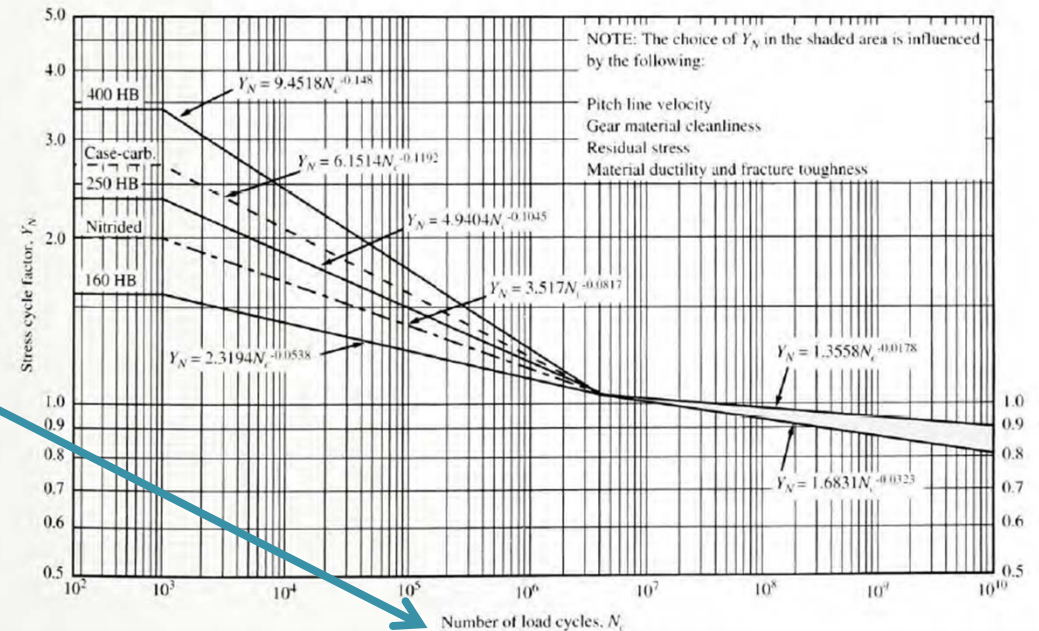
Specify the reliability factor (K_R) from Table (9-8) page (396) (412 pdf):

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

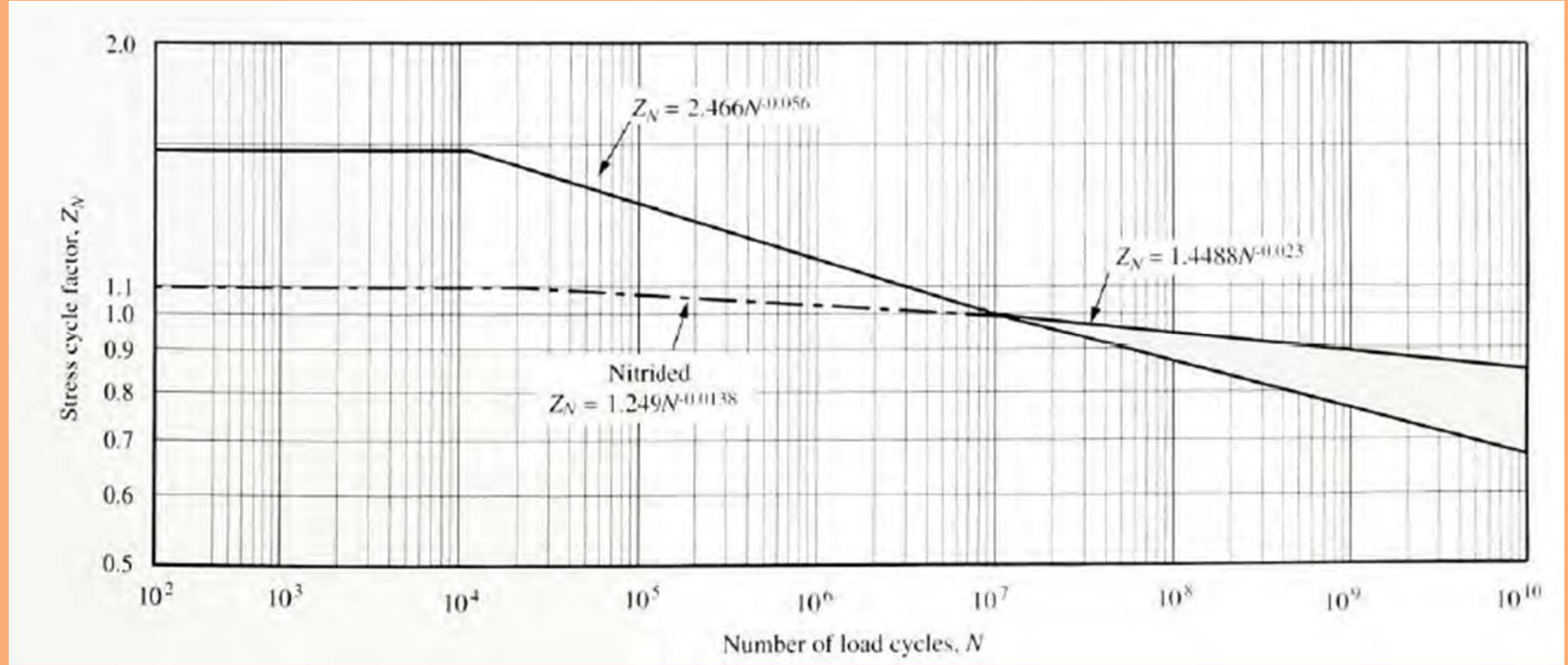
Specify the stress cycle life (Y_N) from Figure (9-8) page (395) (411 pdf):

TABLE 9-7 Recommended design life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000



Specify the pitting resistance stress cycle factor (Z_N) from figure (9-24) page (403) (419 pdf):



Choose material for pinion and gear or (S_{ac} , S_{at}) from figures [(9-10) page (379) (395pdf) , (9-11) page (380) (396pdf)] with tables [(9-3) page(381) (397pdf) , (9-4) page (385) (401pdf)] and see also Appendix 3 to 5 [p(A-6) to (A-11)].

Check if the selected material satisfy the following design conditions:

$$S_t \frac{K_R(S.F)}{Y_N} < S_{at}$$

$$S_c \frac{K_R(S.F)}{Z_N C_H} < S_{ac}$$

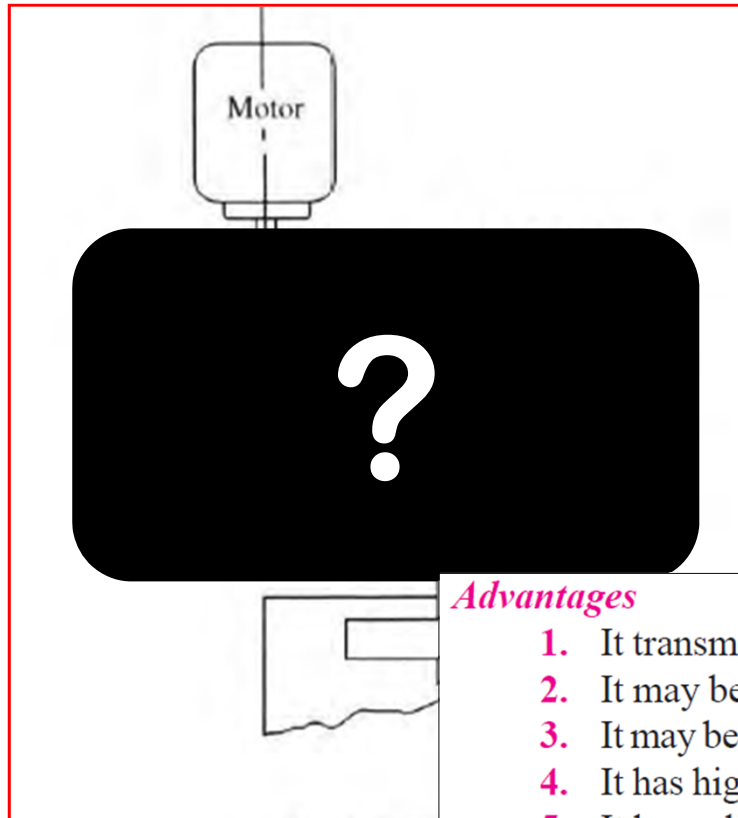


Mechanical Engineering Design II

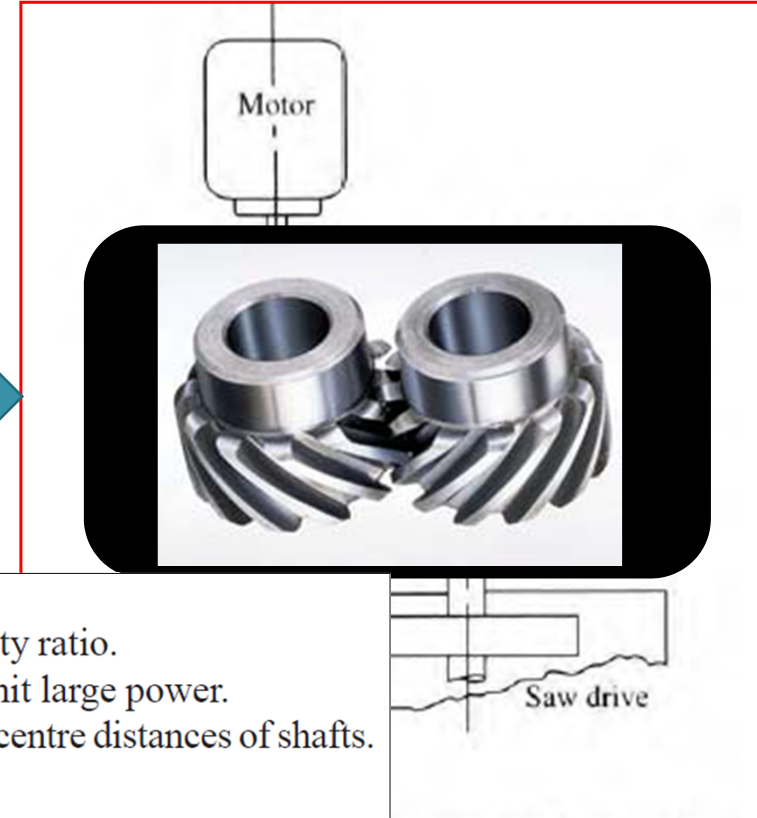
Twentieth Lecture

Design of Helical Gear

Power Transmission Problem



Proposed solution (Helical Gear)



Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It may be used for small centre distances of shafts.
4. It has high efficiency.
5. It has reliable service.
6. It has compact layout.

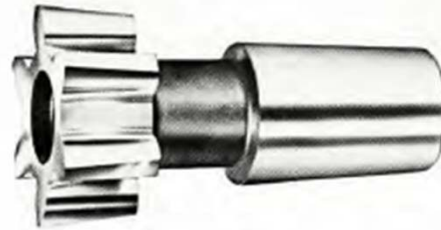
Disadvantages

1. Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.

Gear Manufacturing



(a) Form milling cutter



(b) Spur gear shaper cutter

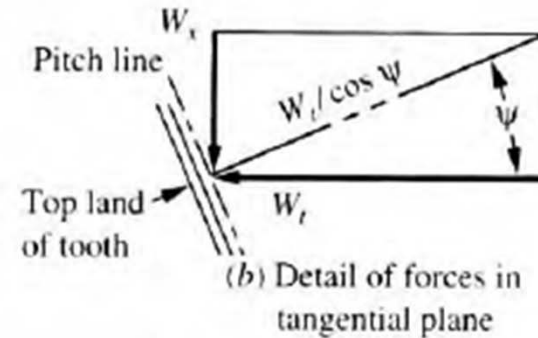
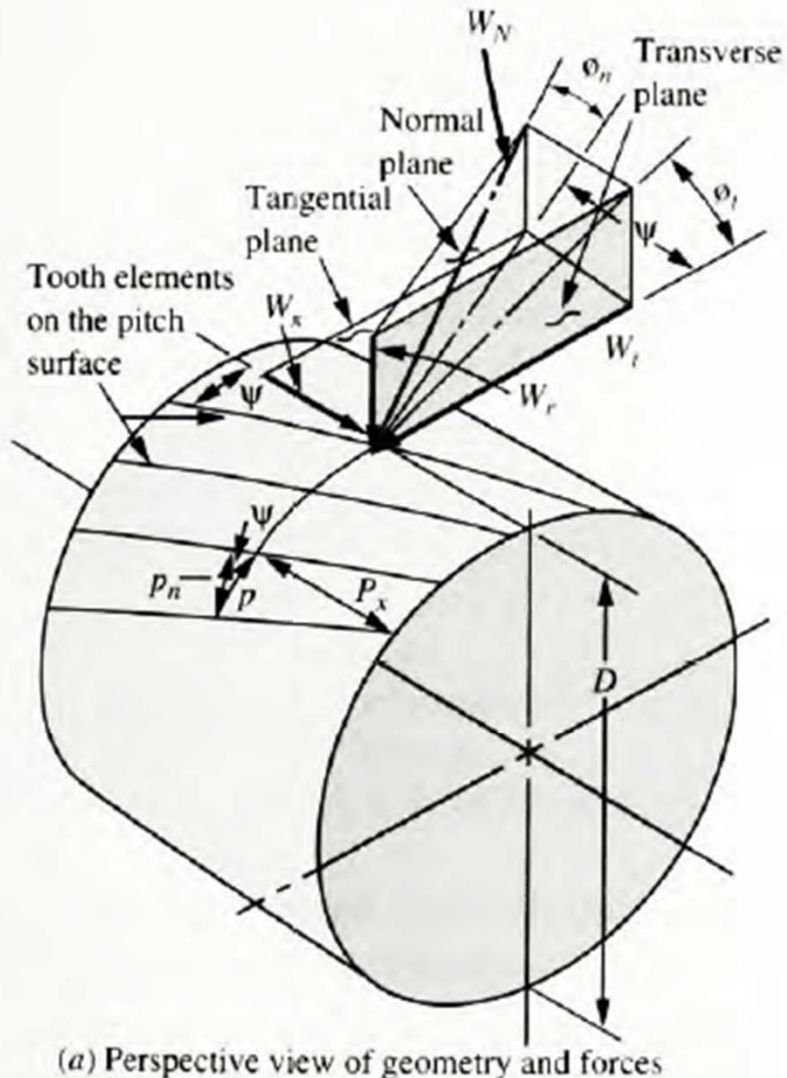


(c) Hob for small pitch gears having large teeth

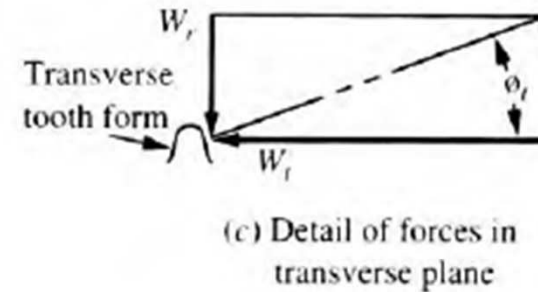


(d) Hob for high pitch gears having small teeth

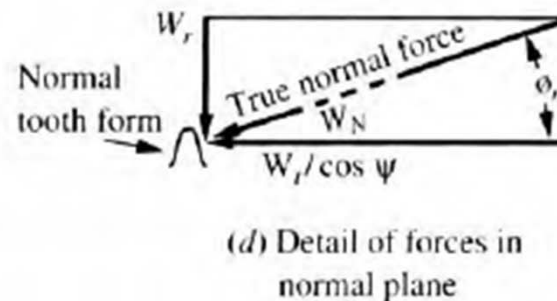
Basic Helical Gear Geometry and force Kinematics



$$\begin{aligned}\psi &= \text{Helix angle} \\ \tan \psi &= W_x / W_t \\ W_x &= W_t \tan \psi\end{aligned}$$



$$\begin{aligned}\phi_t &= \text{Transverse} \\ &\text{pressure angle} \\ \tan \phi_t &= W_r / W_t \\ W_r &= W_t \tan \phi_t\end{aligned}$$



$$\begin{aligned}\phi_n &= \text{Normal} \\ &\text{pressure angle} \\ \tan \phi_n &= \frac{W_r}{W_t / \cos \psi} \\ W_r &= \frac{W_t \tan \phi_n}{\cos \psi}\end{aligned}$$

Modes of Gear Tooth Failure

1. Bending failure.

Root failure

2. Pitting.

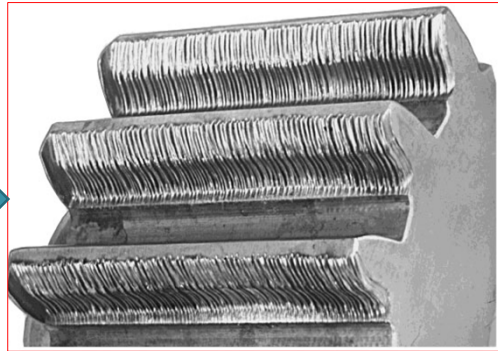
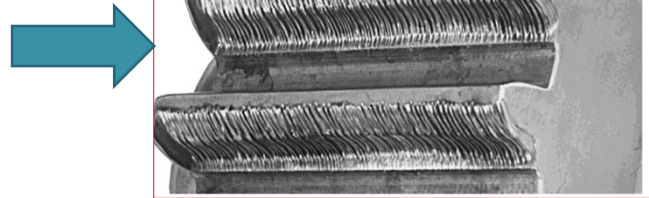
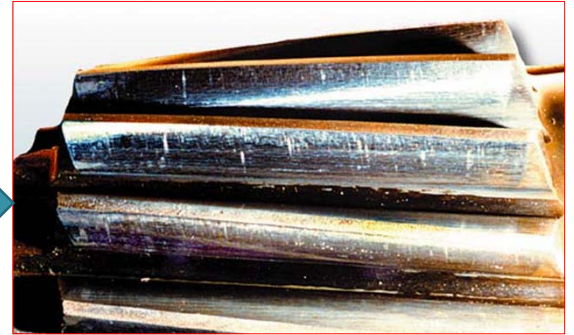
Surface failure

3. Scoring.

4. Abrasive wear.

5. Corrosive wear.

Dangerous Bar



Helical Gear Design

The power to be transmitted

Type of driver and driven load

The speed of the driving gear

The center distance

The speed of the driven gear or the velocity ratio

Other information related to problem specification



Designer

The gear teeth should not fail under static loading or dynamic loading during normal running conditions.

The gear teeth should have wear characteristics so that their life is satisfactory.

The use of space and material should be economical.

The alignment of the gears and deflections of the shafts must be considered.

The lubrication of the gears must be satisfactory.

Flowchart for spur gear designing process:

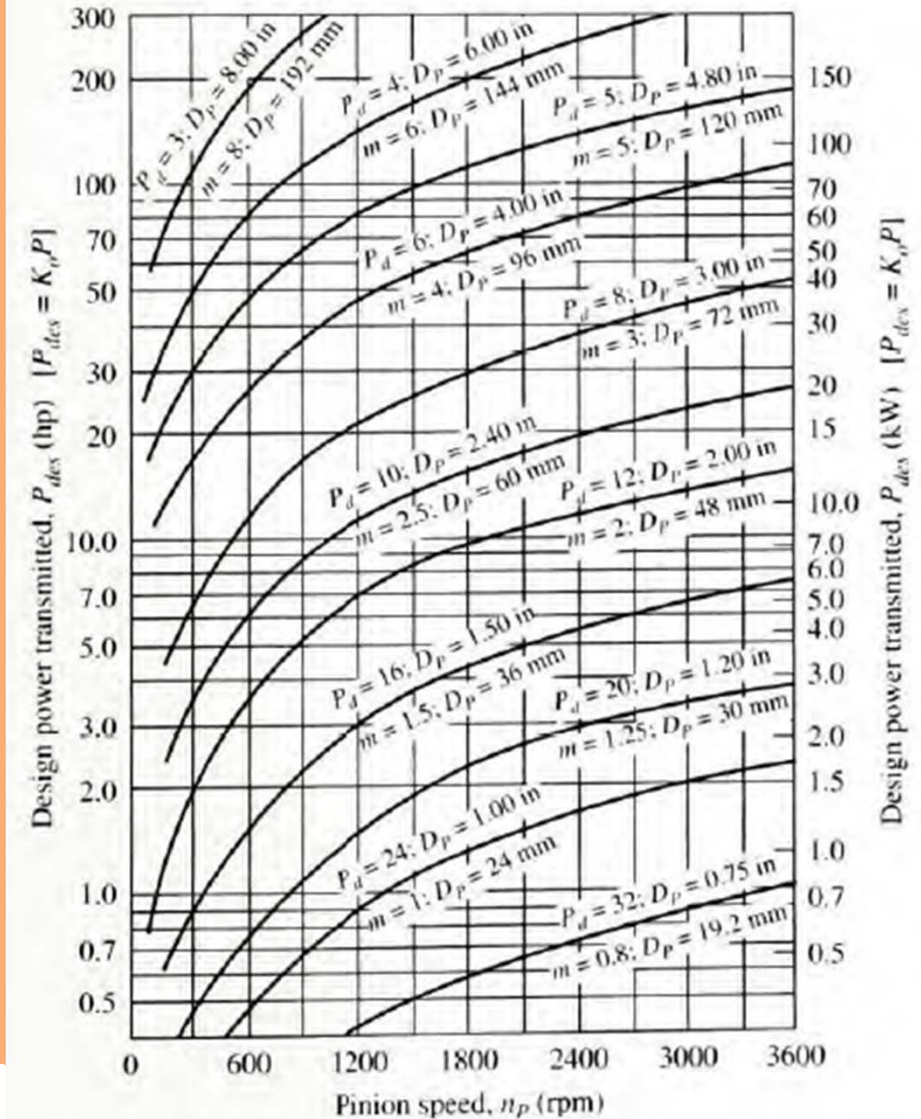
Transmitted Power , Input and Output speed, Center distance, Type of driver and driven load

Choose the over load factor (K_o) from Table (9-5) page(389) (405pdf)

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Compute the design power
Design Power= K_o x transmitted power

Find the trial value for
Diametral pitch (P_{dn}) or Module (m)
and Pitch Diameter (D_p) from
Figure 9-27 page.409 (425 Pdf)



2

Specify the no. of teeth for Pinion N_P (from 17 to 20)

Compute the nominal velocity ratio $VR = \frac{n_P}{n_G}$

Compute the approximate no. of teeth for Gear $N_G = N_P \times VR$

Compute the actual velocity ratio $VR = \frac{N_G}{N_P}$

Compute the actual output velocity $n_G = n_P \frac{N_P}{N_G}$

Compute the pitch diameters $D_p = \frac{N_p}{P_d}$, $D_G = \frac{N_G}{P_d}$, center distance $c = \frac{N_P + N_G}{2P_d}$, pitch line speed $v_t = \frac{\pi D_p n_P}{60}$, axial pitch $P_x = \frac{\pi}{P_d \tan \psi}$, $P_d = P_{dn} \cos \psi$ and tangential force, $W_t = \frac{60P}{\pi D_p n_p}$

3

Compute the face width : $F = 2 P_x$

Specify the quality number Q_v , from Table (9-2) page (378) (394 pdf)

Application	Quality number	Application	Quality number
Cement mixer drum drive	3-5	Small power drill	7-9
Cement kiln	5-6	Clothes washing machine	8-10
Steel mill drives	5-6	Printing press	9-11
Grain harvester	5-7	Computing mechanism	10-11
Cranes	5-7	Automotive transmission	10-11
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Mining conveyor	5-7	Marine propulsion drive	10-12
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Gas meter mechanism	7-9	Gyroscope	12-14

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0-800	6-8	0-4
800-2000	8-10	4-11
2000-4000	10-12	11-22
Over 4000	12-14	Over 22

4

Analyzing of gear tooth failure mode

Root (Bending) Failure Mode

Bending Stress Number

$$S_t = \frac{W_t P_d}{F J} K_o K_s K_m K_B K_v < S_{at} \frac{Y_N}{K_R (S.F)}$$

Surface (Pitting, Scoring,...) Failure Mode

Contact Stress Number

$$S_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}} < S_{ac} \frac{Z_N C_H}{K_R (S.F)}$$

**Find the values of factors
($J, I, K_s, K_m, K_B, K_v, C_p, Y_N, Z_N, C_H, K_R, S.F$) as in
the following steps**

5

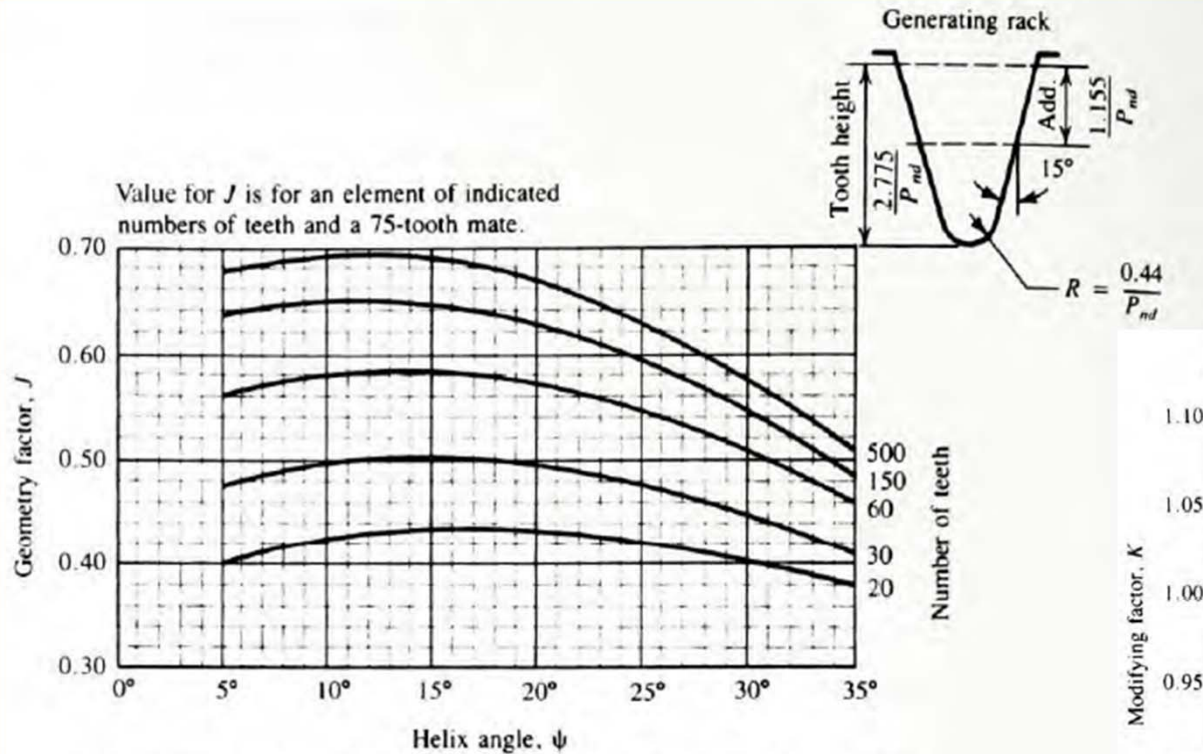
Specify the type of material for the gears to find the Elastic Coefficient C_p from Table (9-9) page(400) (Pdf 416)

Pinion material	Modulus of elasticity, E_p , lb/in ² (MPa)	Gear material and modulus of elasticity, E_G , lb/in ² (MPa)					
		Steel	Malleable iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
		30×10^6 (2×10^5)	25×10^6 (1.7×10^5)	24×10^6 (1.7×10^5)	22×10^6 (1.5×10^5)	17.5×10^6 (1.2×10^5)	16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Mall. iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nod. iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are (lb/in²)^{0.5} or (MPa)^{0.5}.

Specify the bending geometry factor (J) for 15° normal pressure angle from figure (10-5) page (456) (472pdf):

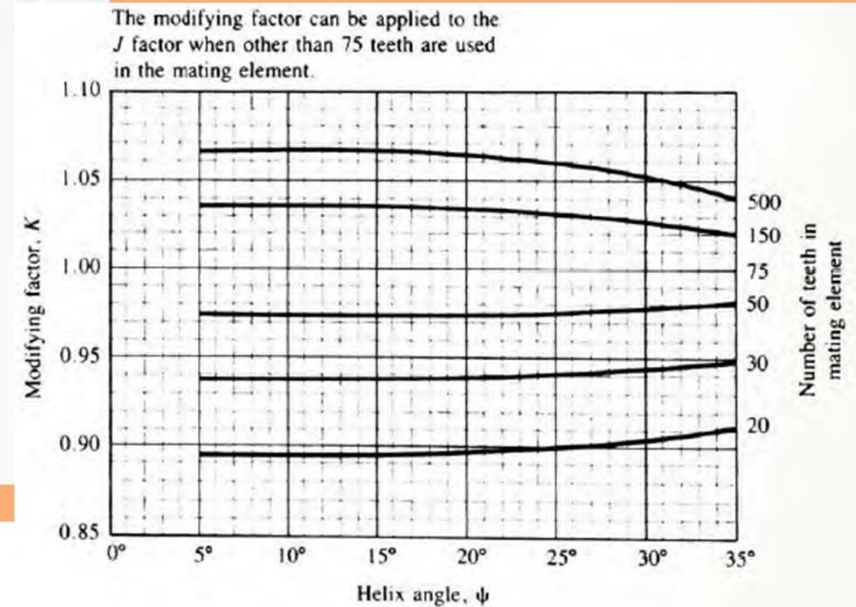


(a) Geometry factor (J) for 15° normal pressure angle and indicated addendum

$$J = \frac{Y}{K_t}$$

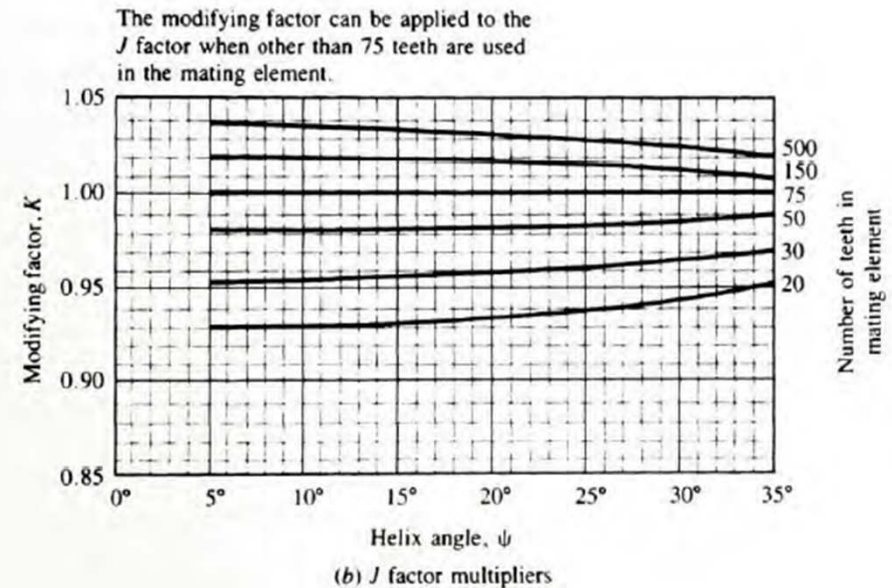
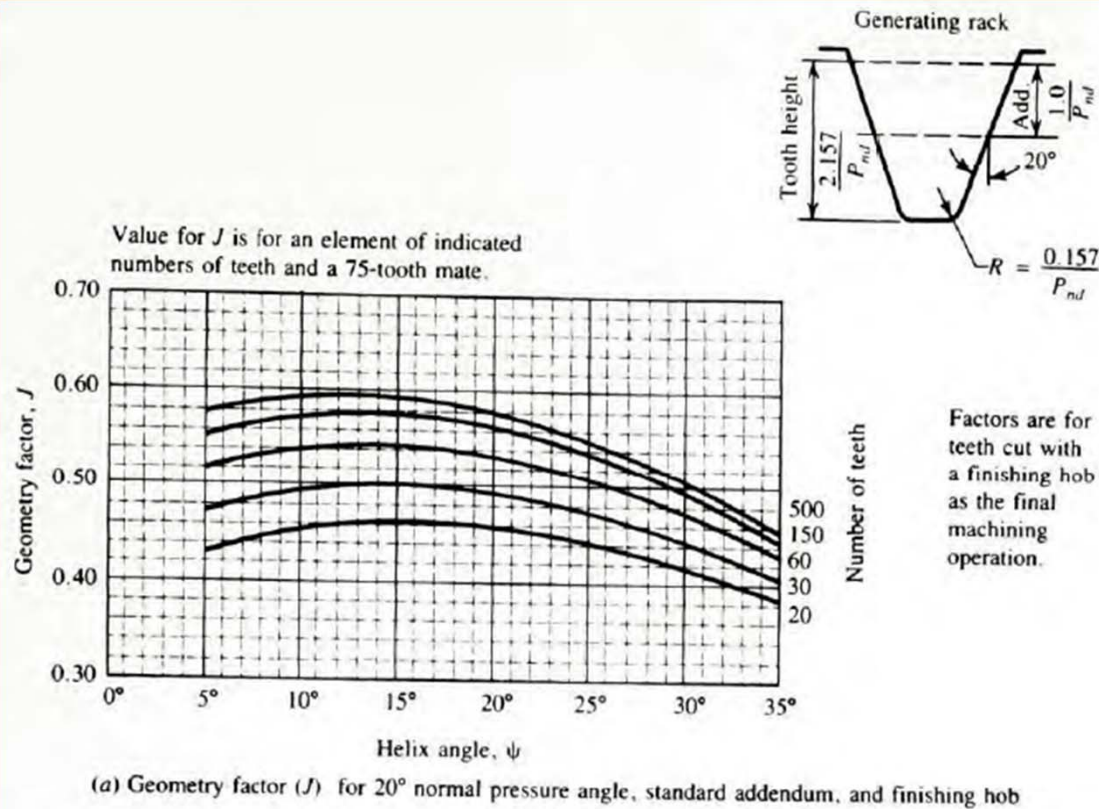
$Y = \text{Lewis factor}$

$K_t = \text{stress concentration factor}$

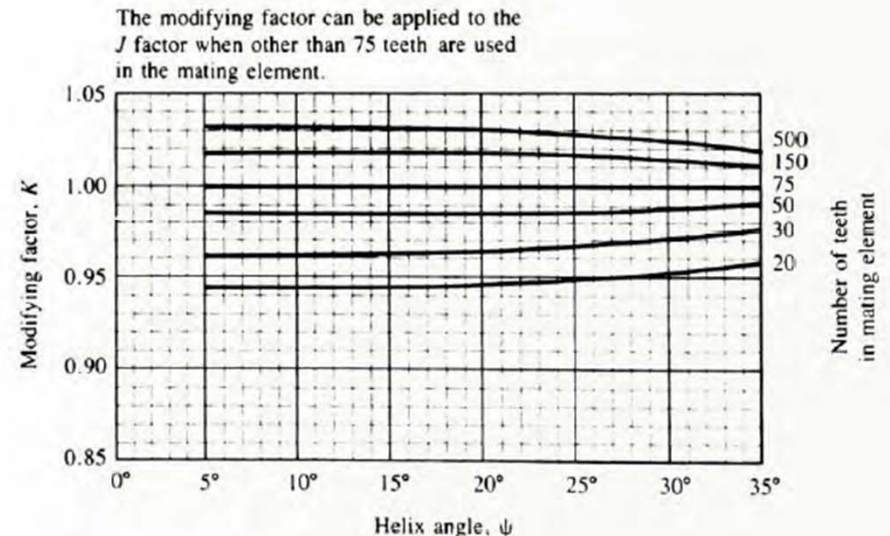
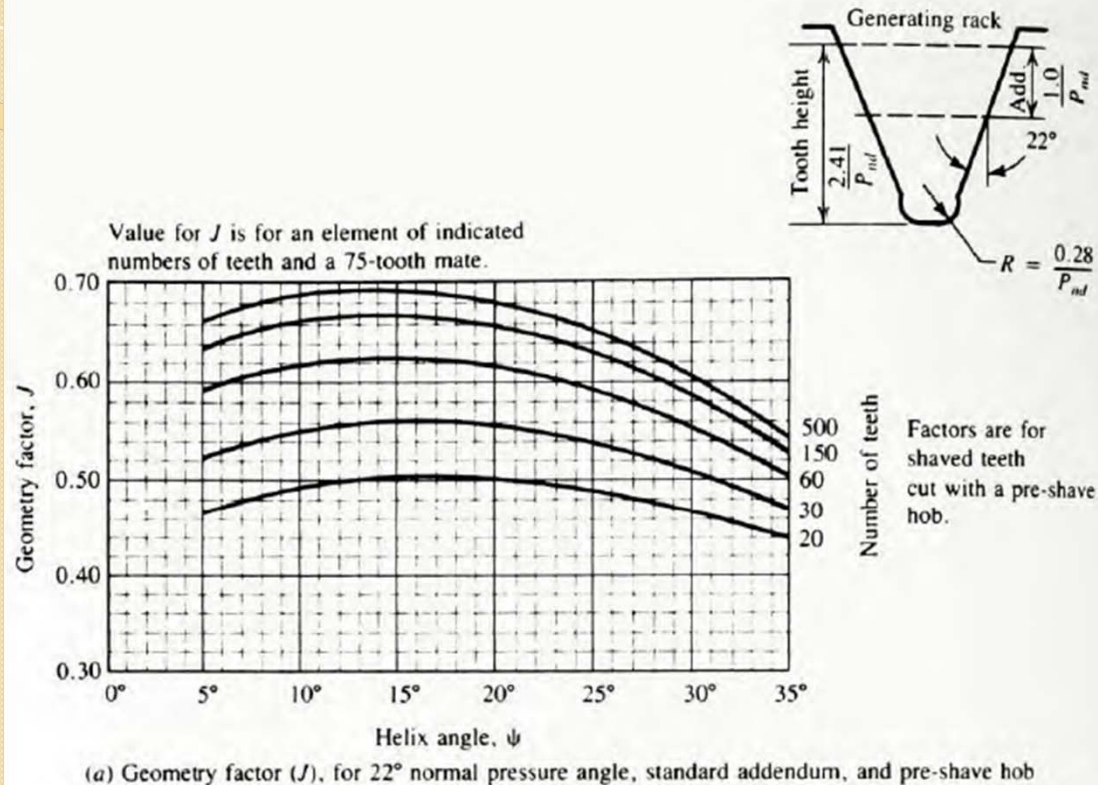


(b) J factor multipliers

Specify the bending geometry factor (J) for 20° normal pressure angle from figure (10-6) page (457) (473pdf):



Specify the bending geometry factor (J) for 22° normal pressure angle from figure (10-7) page (458) (474pdf):



Specify the pitting geometry factor (I) with 20° normal pressure angle from Table(10-1) page (459) (475pdf):

A. Helix angle $\psi = 15.0^\circ$

Gear teeth	Pinion teeth				
	17	21	26	35	55
17	0.124				
21	0.139	0.128			
26	0.154	0.143	0.132		
35	0.175	0.165	0.154	0.137	
55	0.204	0.196	0.187	0.171	0.143
135	0.244	0.241	0.237	0.229	0.209

B. Helix angle $\psi = 25.0^\circ$

Gear teeth	Pinion teeth					
	14	17	21	26	35	55
14	0.123					
17	0.137	0.126				
21	0.152	0.142	0.130			
26	0.167	0.157	0.146	0.134		
35	0.187	0.178	0.168	0.156	0.138	
55	0.213	0.207	0.199	0.189	0.173	0.144
135	0.248	0.247	0.244	0.239	0.230	0.210

Specify the pitting geometry factor (I) with 20° normal pressure angle from Table(10-2) page (460) (476pdf):

A. Helix angle $\psi = 15.0^\circ$

Gear teeth	Pinion teeth					
	14	17	21	26	35	55
14	0.130					
17	0.144	0.133				
21	0.160	0.149	0.137			
26	0.175	0.165	0.153	0.140		
35	0.195	0.186	0.175	0.163	0.143	
55	0.222	0.215	0.206	0.195	0.178	0.148
135	0.257	0.255	0.251	0.246	0.236	0.214

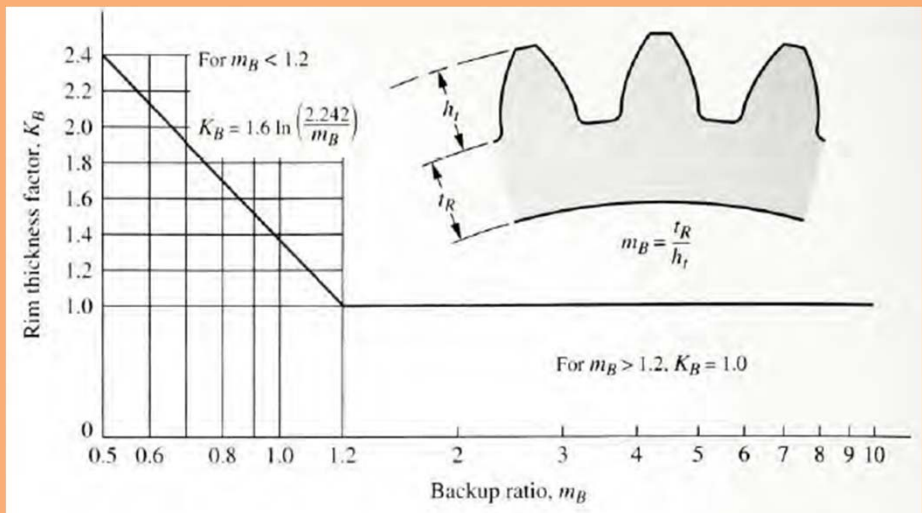
B. Helix angle $\psi = 25.0^\circ$

Gear teeth	Pinion teeth						
	12	14	17	21	26	35	55
12	0.129						
14	0.141	0.132					
17	0.155	0.146	0.135				
21	0.170	0.162	0.151	0.138			
26	0.185	0.177	0.166	0.154	0.141		
35	0.203	0.197	0.188	0.176	0.163	0.144	
55	0.227	0.223	0.216	0.207	0.196	0.178	0.148
135	0.259	0.258	0.255	0.251	0.246	0.235	0.213

Specify the size factor (K_s) from Table(9-6) page (389) (293 pdf)

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

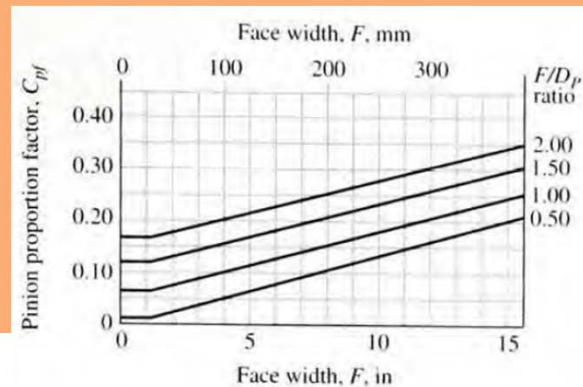
Specify the rim thickness factor (K_B) from Figure (9-20) page (392) (408 pdf)



Determine the load distribution factor (K_m) : $K_m = 1.0 + C_{pf} + C_{ma}$

Where C_{pf} = pinion proportion factor from figure (9-18) page(391) (407 pdf)

C_{ma} = mesh alignment factor from figure (9-19) page(391) (407 pdf)



D_p = Pinion diameter

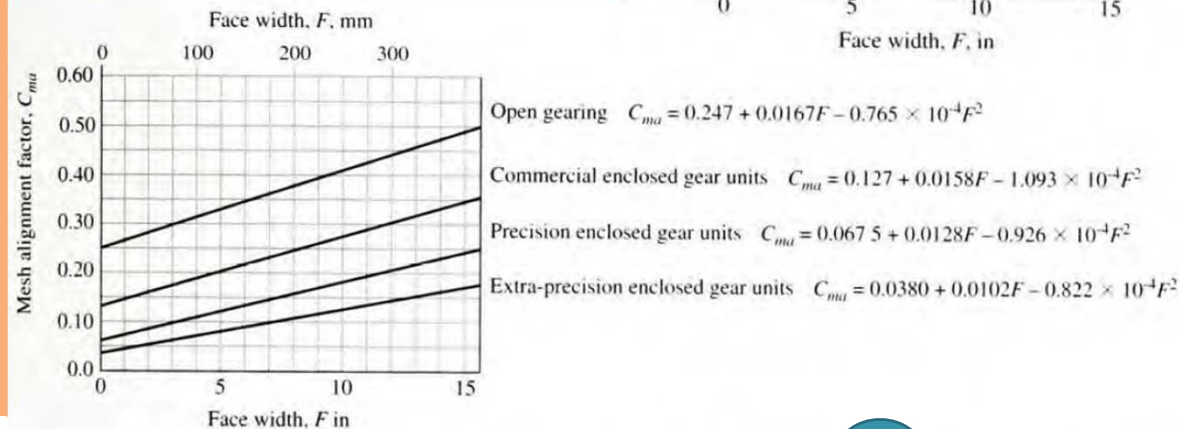
For $F/D_p < 0.50$, use curve for $F/D_p = 0.50$

When $F \leq 1.0$ in. ($F \leq 25$ mm)

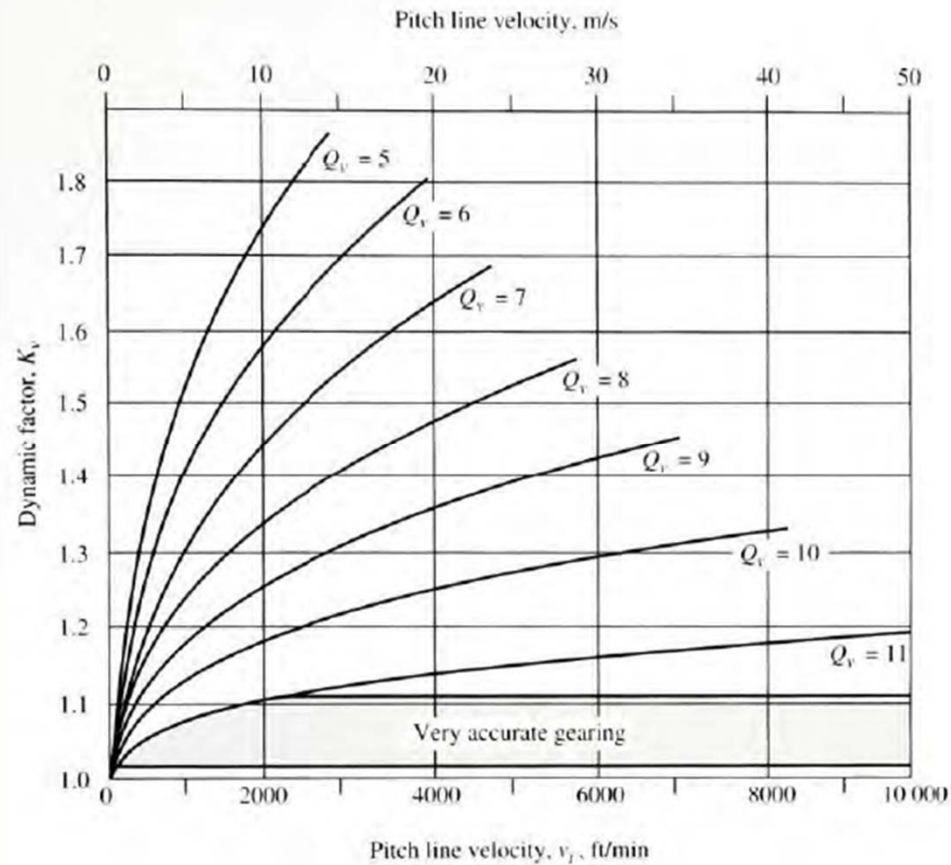
$$C_{pf} = \frac{F}{10D_p} - 0.025$$

When $1.0 \leq F < 15$,

$$C_{pf} = \frac{F}{10D_p} - 0.0375 + 0.0125F$$

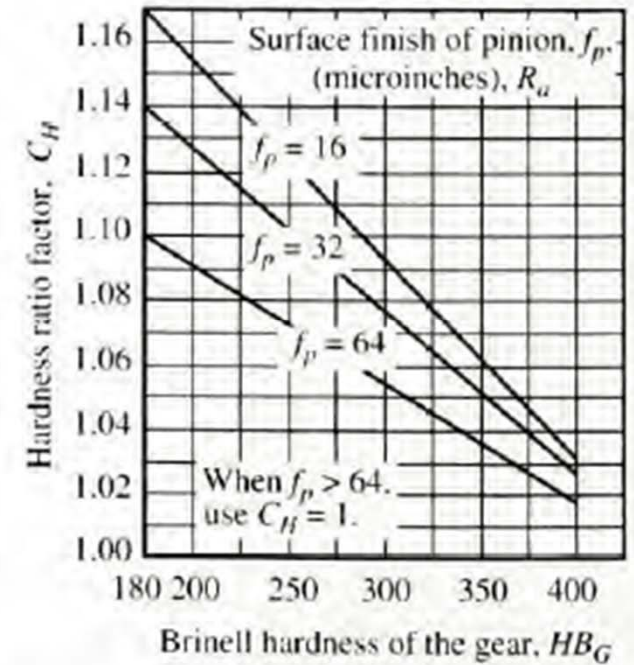
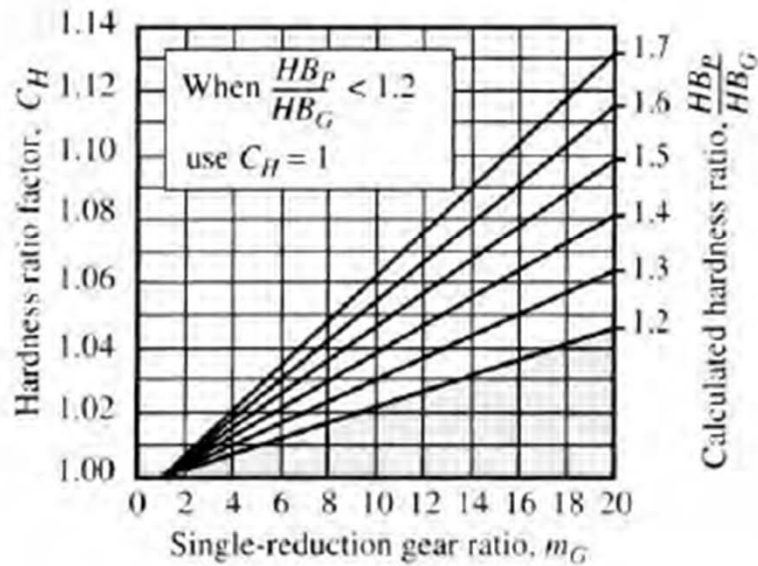


Specify the dynamic factor (K_v) from figure (9-21) page (393) (409 pdf):



Specify the safety factor (S.F) typically from 1 to 1.5

Specify the hardness ratio factor (C_H) from Figure (9-25 & 26) page (404) (420 pdf)



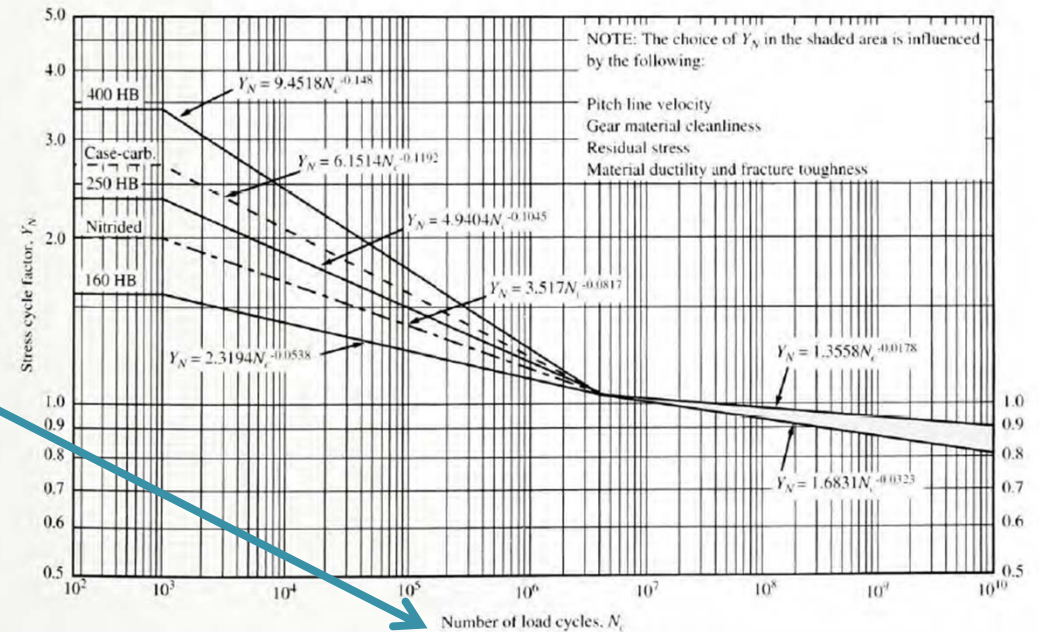
Specify the reliability factor (K_R) from Table (9-8) page (396) (412 pdf):

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

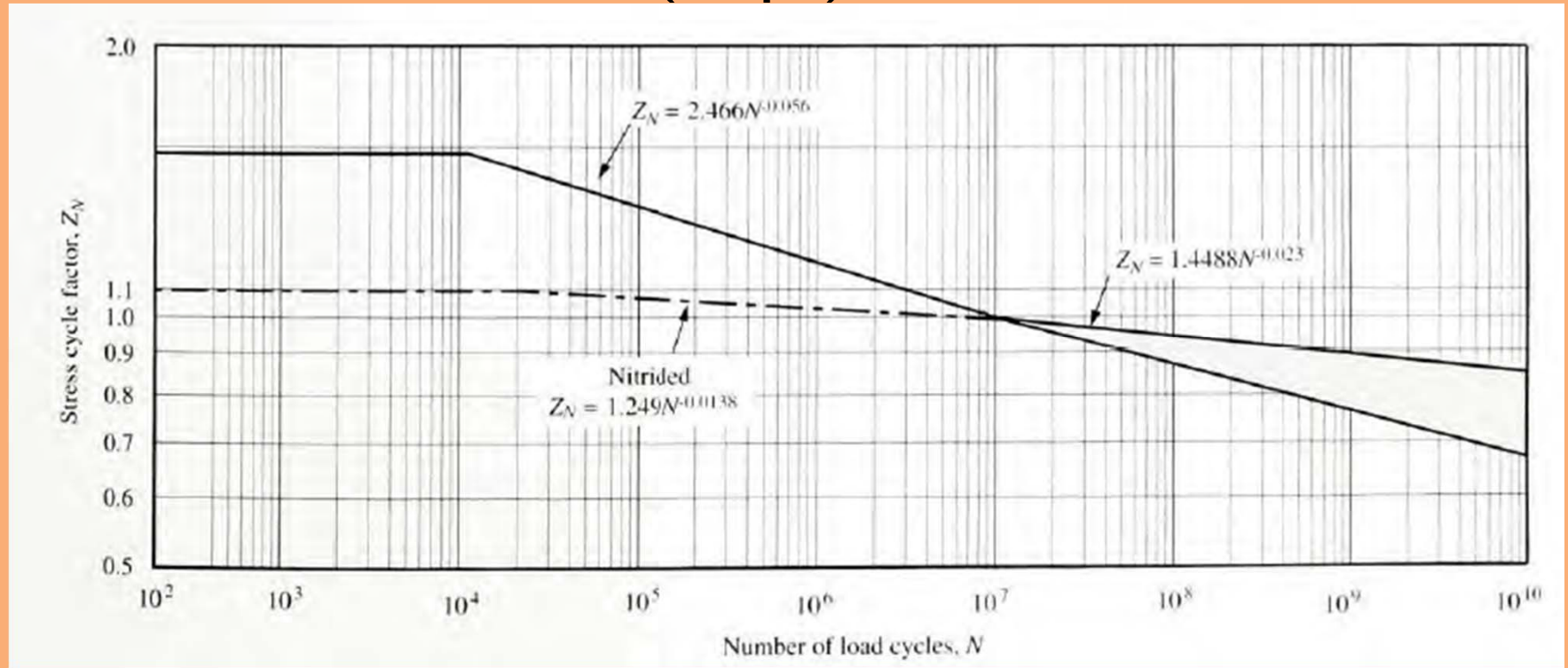
Specify the stress cycle life (Y_N) from Figure (9-8) page (395) (411 pdf):

TABLE 9-7 Recommended design life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000



Specify the pitting resistance stress cycle factor (Z_N) from figure (9-24) page (403) (419 pdf):



Choose material for pinion and gear or (S_{ac} , S_{at}) from figures [(9-10) page (379) (395pdf) , (9-11) page (380) (396pdf)] with tables [(9-3) page(381) (397pdf) , (9-4) page (385) (401pdf)] and see also Appendix 3 to 5 [p(A-6) to (A-11)].

Check if the selected material satisfy the following design conditions:

$$S_t \frac{K_R(S.F)}{Y_N} < S_{at}$$
$$S_c \frac{K_R(S.F)}{Z_N C_H} < S_{ac}$$

Example (10-2) p.461(477pdf):

A pair of helical gears for a milling machine drive is to transmit 48.47 kW (65 hp) with a pinion speed of 3450 rpm and a gear speed of 1100 rpm. The power is from an electric motor. Design the gears.

Solution:

Given data:

$P = \text{transmitted power} = 48.47 \text{ kW (65 hp)}$

$\text{Power source} = \text{electric motor}, \text{ driven machine} = \text{milling machine}$

$n_p = 3450 \text{ rpm}, n_g = 1100 \text{ rpm}$

Initial assumptions:

$$P_{d_n} = 12, N_p = 24, \psi = 15^\circ, \phi = 20^\circ, Q = 8$$

Basic dimensions computations:

$$P_d = P_{dn} \cos \psi = 12 \cos(15^\circ) = 11.59 \text{ mm}$$

$$P_x = \frac{\pi}{P_d \tan \psi} = \frac{\pi}{11.59 \tan(15^\circ)} = 25.7 \text{ mm}$$

$$\phi_t = \tan^{-1}(\tan \phi_n / \cos \psi) = \tan^{-1}[\tan(20^\circ) / \cos(15^\circ)] = 20.65^\circ$$

$$D_p = N_p / P_d = 24 / 11.59 = 2.07 \text{ in} = 52.6 \text{ mm}$$

$$F = 2P_x = 2(25.7) = 51.41 \text{ mm} = 2.25 \text{ in}$$

Gear kinematics computations:

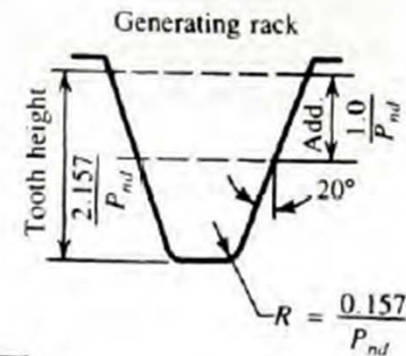
$$v_t = \frac{\pi D_p n_p}{60} = \frac{\pi(52.6 \times 10^{-3})(3450)}{60} = 9.5 \frac{\text{m}}{\text{s}}$$

$$W_t = \frac{60P}{\pi D_p n_p} = \frac{60(48.47 \times 10^3)}{\pi(52.6 \times 10^{-3})(3450)} = 5.1 \text{ kN} = 1146 \text{ lb}$$

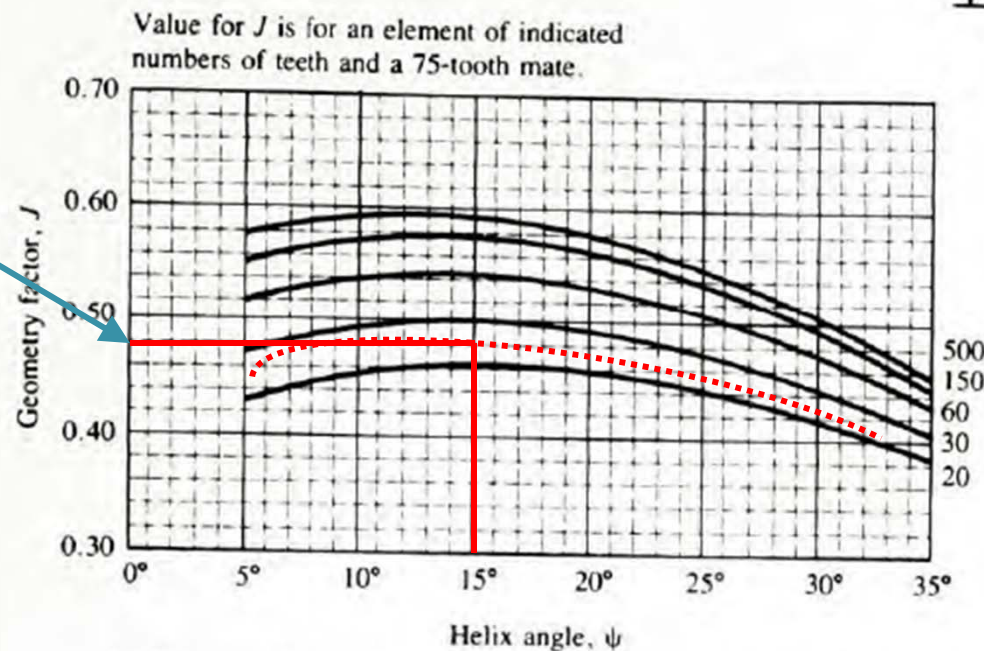
$$VR = N_G / N_p = n_p / n_G = 3450 / 1100 = 3.14 ,$$

$$N_G = N_p(VR) = 24(3.14) = 75.3 \cong 75 \text{ teeth}$$

J_p from $N_p=24$ and $N_G=75$ (figure 10-6):



$J_p = 0.48$

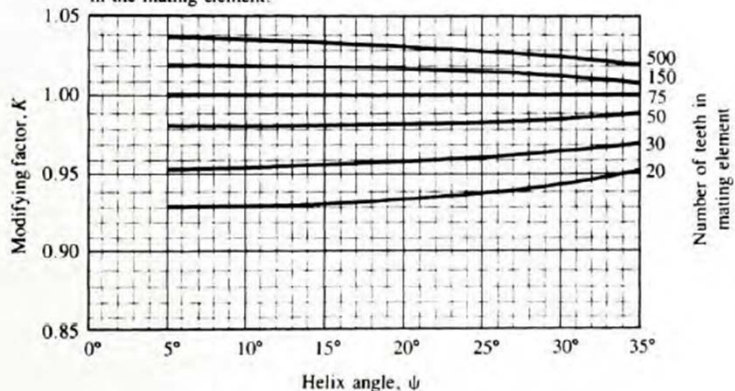


Number of teeth

Factors are for teeth cut with a finishing hob as the final machining operation.

(a) Geometry factor (J) for 20° normal pressure angle, standard addendum, and finishing hob

The modifying factor can be applied to the J factor when other than 75 teeth are used in the mating element.



(b) J factor multipliers

K_o from power source (electric motor) and driven machine (milling machine)(table 9-5):

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

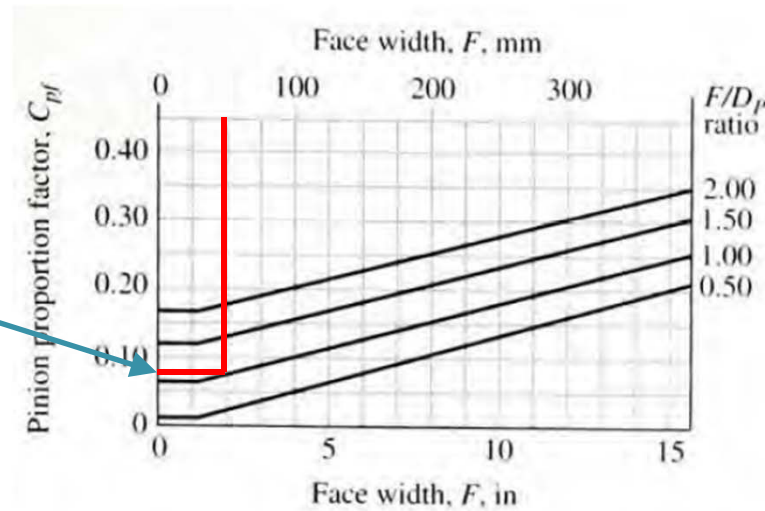
K_s from $P_d = 11.59 > 5$ (table 9-6):

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

K_m from $F = 51.41 \text{ mm}$ & $D_p = 52.6 \text{ mm}$, $F/D_p = 0.977$ (figure 9-18 & 19):

$$K_m = 1.0 + C_{pf} + C_{ma} \\ = 1 + 0.09 + 0.17 = 1.26$$

$$C_{pf} = 0.09$$



D_p = Pinion diameter

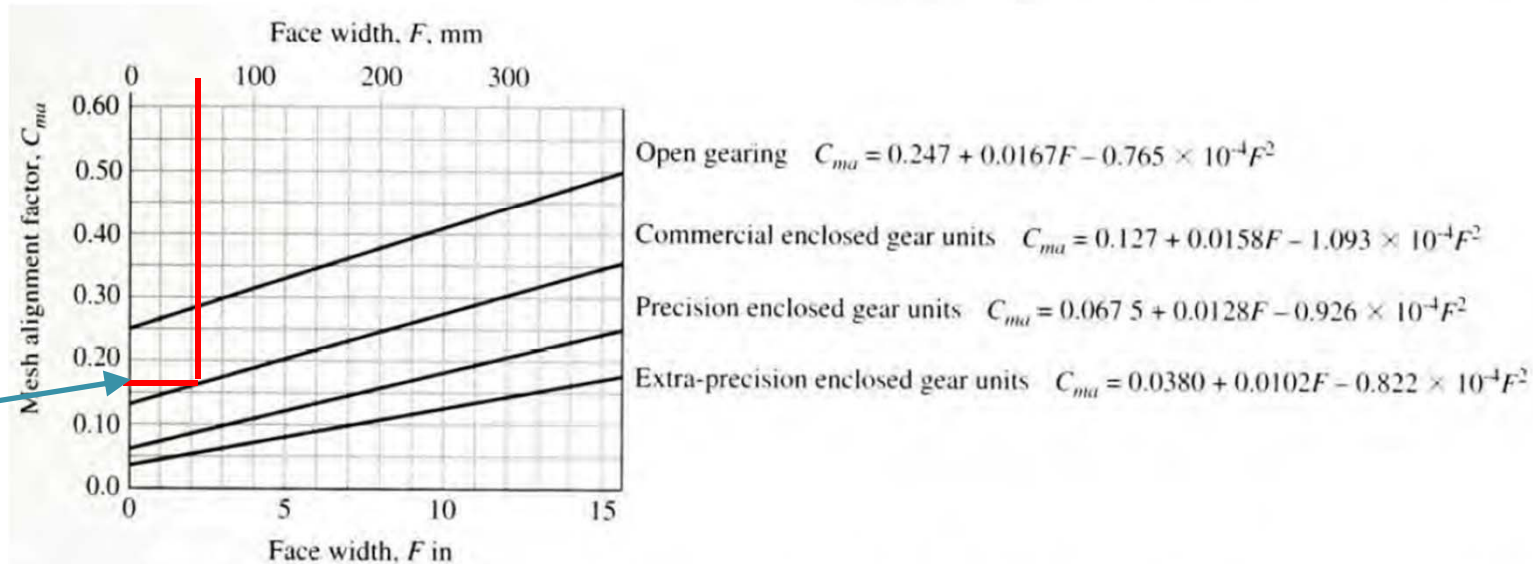
For $F/D_p < 0.50$, use curve for $F/D_p = 0.50$

When $F \leq 1.0 \text{ in.}$ ($F \leq 25 \text{ mm}$)

$$C_{pf} = \frac{F}{10D_p} - 0.025$$

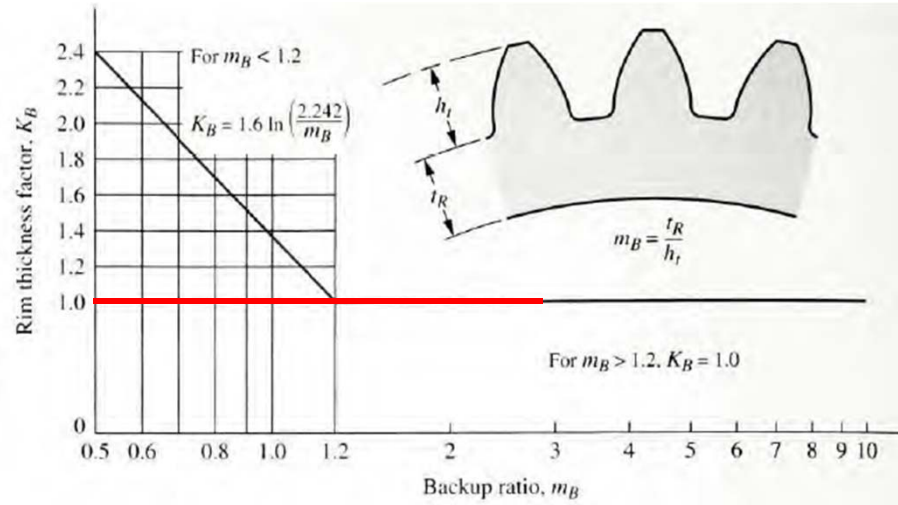
When $1.0 \leq F < 15$,

$$C_{pf} = \frac{F}{10D_p} - 0.0375 + 0.0125F$$

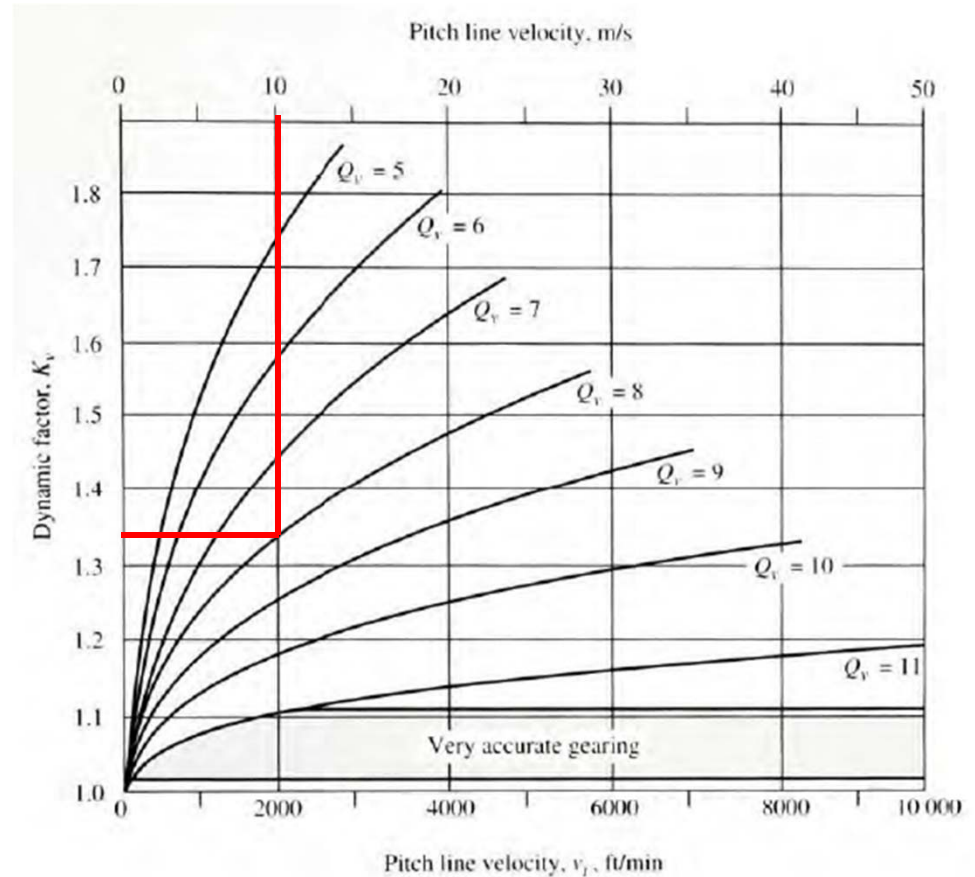


$$C_{mf} = 0.17$$

$K_B = 1$ from (solid gear) (figure 9-20):



$K_v = 1.35$
from $Q_v = 8$ & $v_t = 9.5 \text{ m/s}$ (figure 9-21):



**Design for reliability of 0.999
(less than one failure in 1000): $K_R=1.25$**

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

No unusual conditions seem to exist in this application beyond those already considered in the various K factors. Therefore we use a service factor S.F of 1.0

Design life: Let's design for 10 000 h of life as suggested in Table 9-7 for multipurpose gearing. Then, using Equation (9-18), we can compute the number of cycles of loading. For the pinion rotating at 3450 rpm with one cycle of loading per revolution,

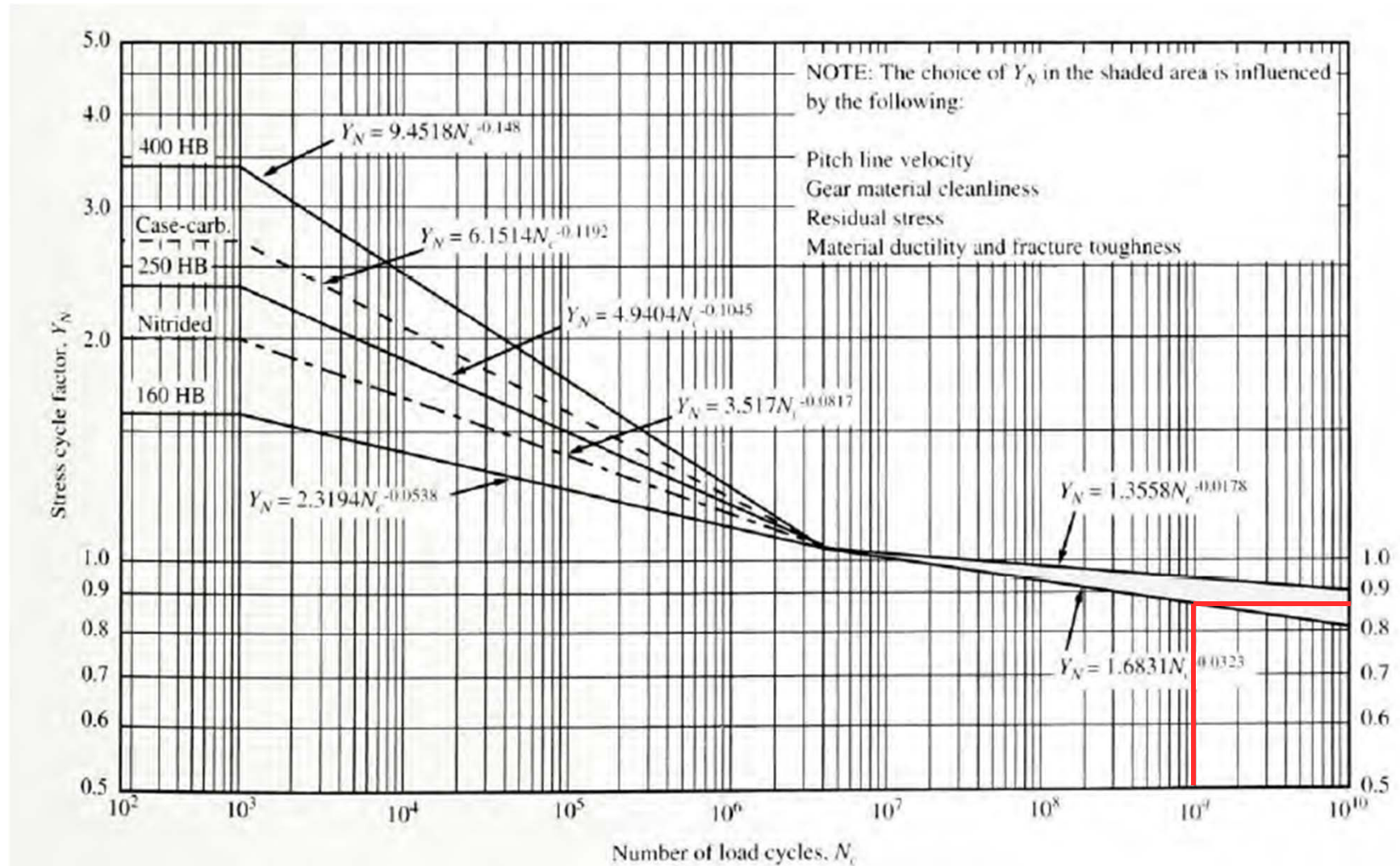
$$\begin{aligned}
 N_{c_p} &= (60)(L)(n_p)(q) \\
 &= (60)(10000)(3450)(1) = 2.1 \times 10^9 \text{ cycles}
 \end{aligned}$$

(q) = number of load applications per revolution

TABLE 9-7 Recommended design life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000

Specify the stress cycle life (Y_N) from Figure (9-8) page (395) (411 pdf):
 $Y_N=0.85$



The bending stress in the pinion can now be computed:

$$S_{tp} = \frac{W_t P_d}{F J_p} K_o K_s K_m K_B K_v \left(\frac{K_R \times S.F}{Y_N} \right)$$

$$= \frac{(1146)(11.59)}{(2.25)(0.48)} (1.5)(1)(1.26)(1)(1.35) \left(\frac{1.25 \times 1}{0.85} \right) = 46.145 \text{ ksi} = 318.16 \text{ MPa}$$

Specify the type of material for the gears
to find the Elastic Coefficient C_p from
Table (9-9) page(400) (Pdf 416)

Pinion material	Gear material and modulus of elasticity, E_G , lb/in ² (MPa)						
	Modulus of elasticity, E_p , lb/in ² (MPa)	Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Mall. iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nod. iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are (lb/in²)^{0.5} or (MPa)^{0.5}.

Specify the pitting geometry factor (I) with 20° normal pressure angle from Table(I0-I) page (459) (475pdf):

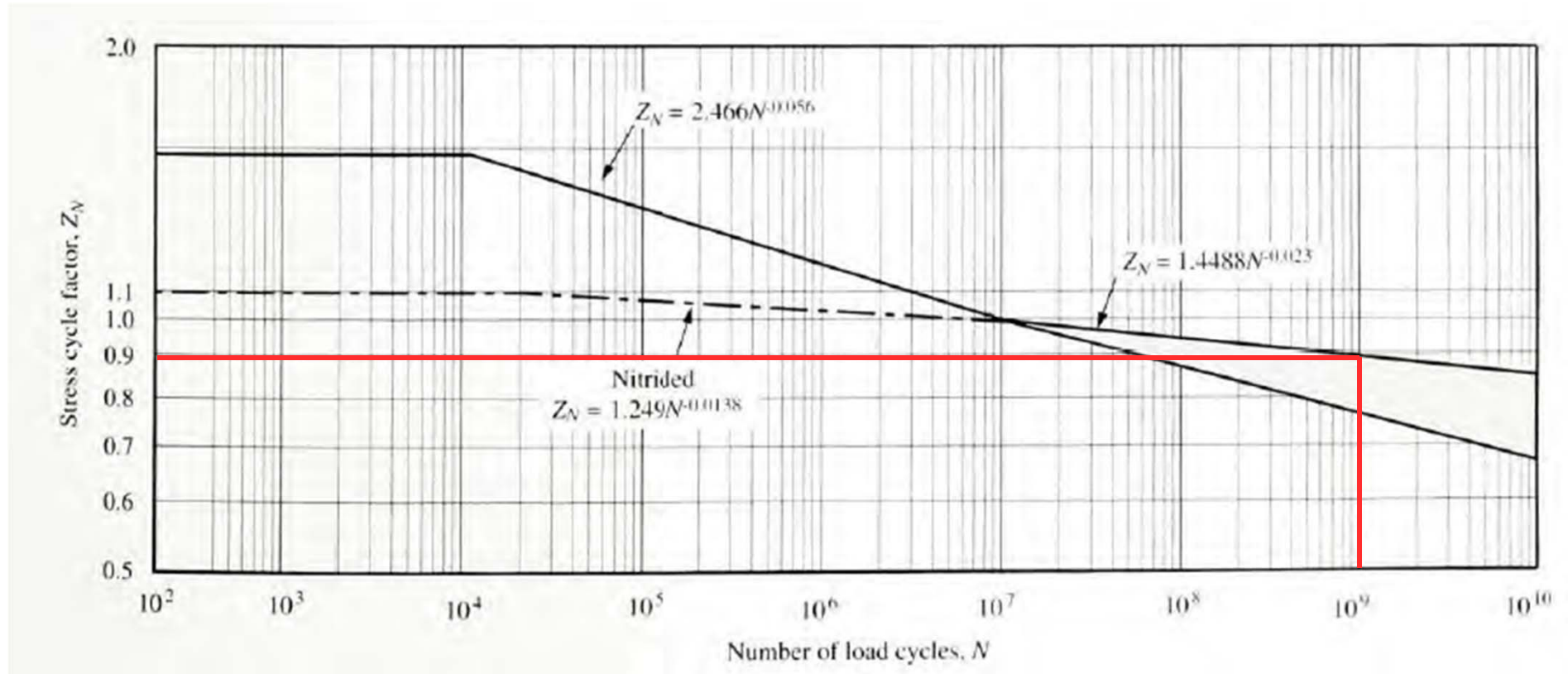
A. Helix angle $\psi = 15.0^\circ$

Gear teeth	Pinion teeth					
	17	21	24	26	35	55
17	0.124					
21	0.139	0.128				
26	0.154	0.143		0.132		
35	0.175	0.165		0.154	0.137	
55	0.204	0.196		0.187	0.171	0.143
135	0.244	0.241	0.202	0.237	0.229	0.209

B. Helix angle $\psi = 25.0^\circ$

Gear teeth	Pinion teeth					
	14	17	21	26	35	55
14	0.123					
17	0.137	0.126				
21	0.152	0.142	0.130			
26	0.167	0.157	0.146	0.134		
35	0.187	0.178	0.168	0.156	0.138	
55	0.213	0.207	0.199	0.189	0.173	0.144
135	0.248	0.247	0.244	0.239	0.230	0.210

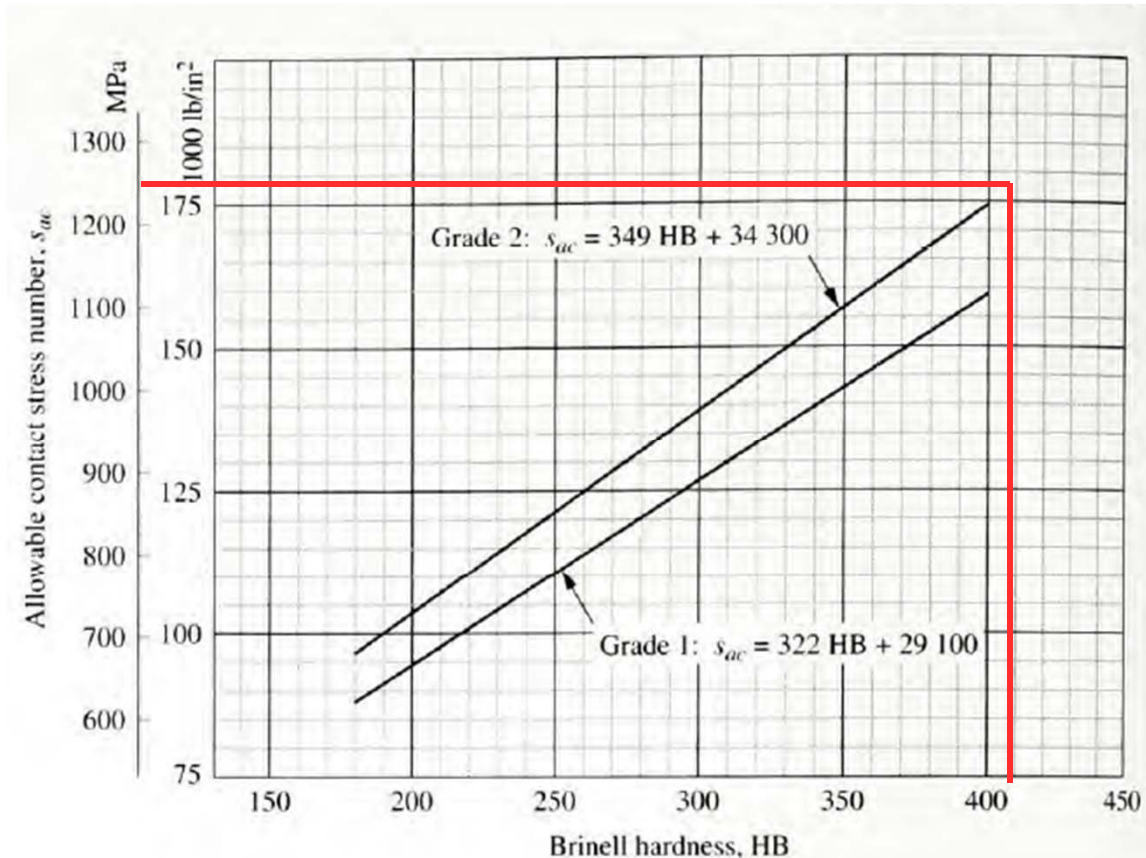
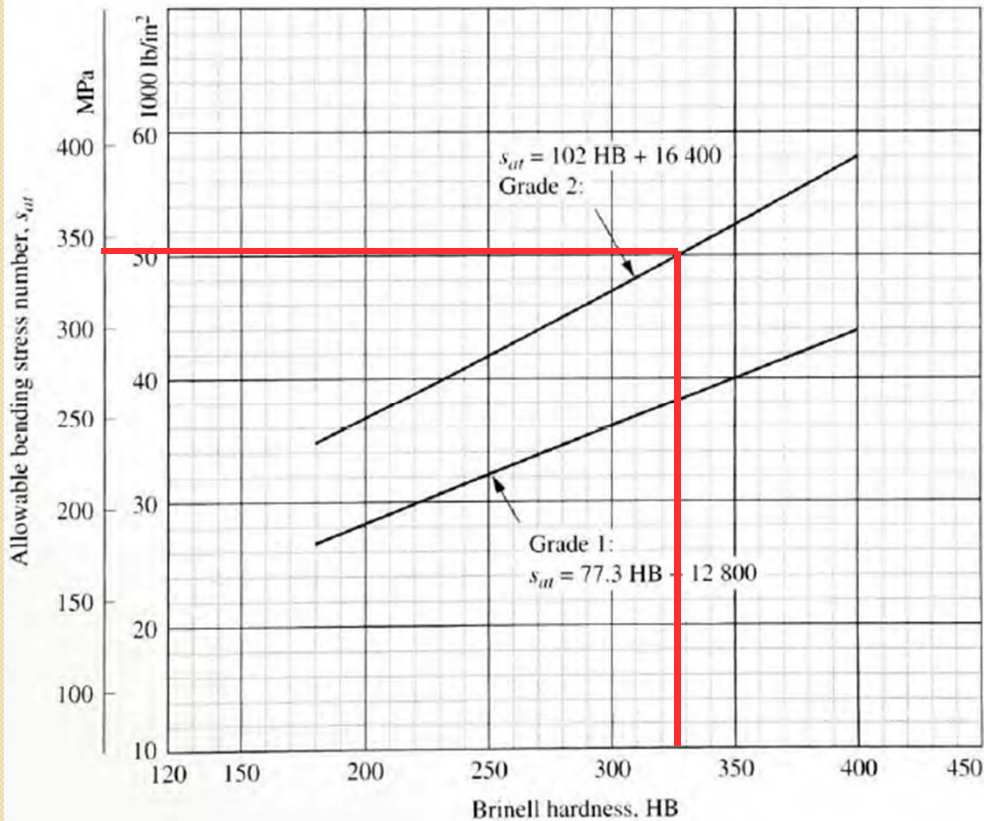
Specify the pitting resistance stress cycle factor (Z_N) from figure (9-24) page (403) (419 pdf): $Z_N=0.89$



for $\frac{HB_P}{HB_G} = 1$ use $C_H = 1$

The pitting stress number in the pinion can now be computed:

$$\begin{aligned}
 S_c &= C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}} \left(\frac{K_R (S.F.)}{Z_N C_H} \right) \\
 &= (2300) \sqrt{\frac{(1146)(1.5)(1)(1.26)(1.35)}{(2.25)(2.071)(0.202)}} \left(\frac{1.25 \times 1}{0.89 \times 1} \right) \\
 &= 180 \text{ ksi} = 1241 \text{ MPa}
 \end{aligned}$$



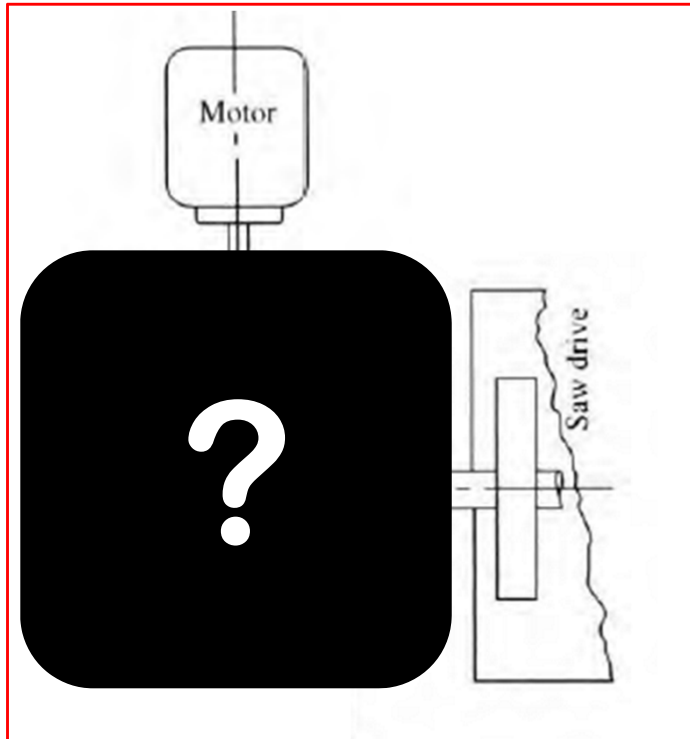


Mechanical Engineering Design II

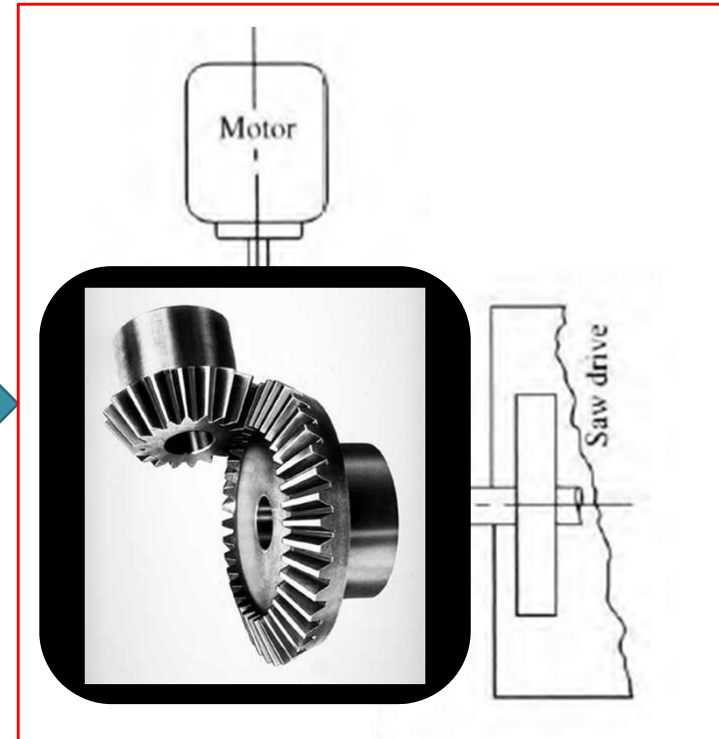
Twenty-one Lecture

Design of Bevel Gear

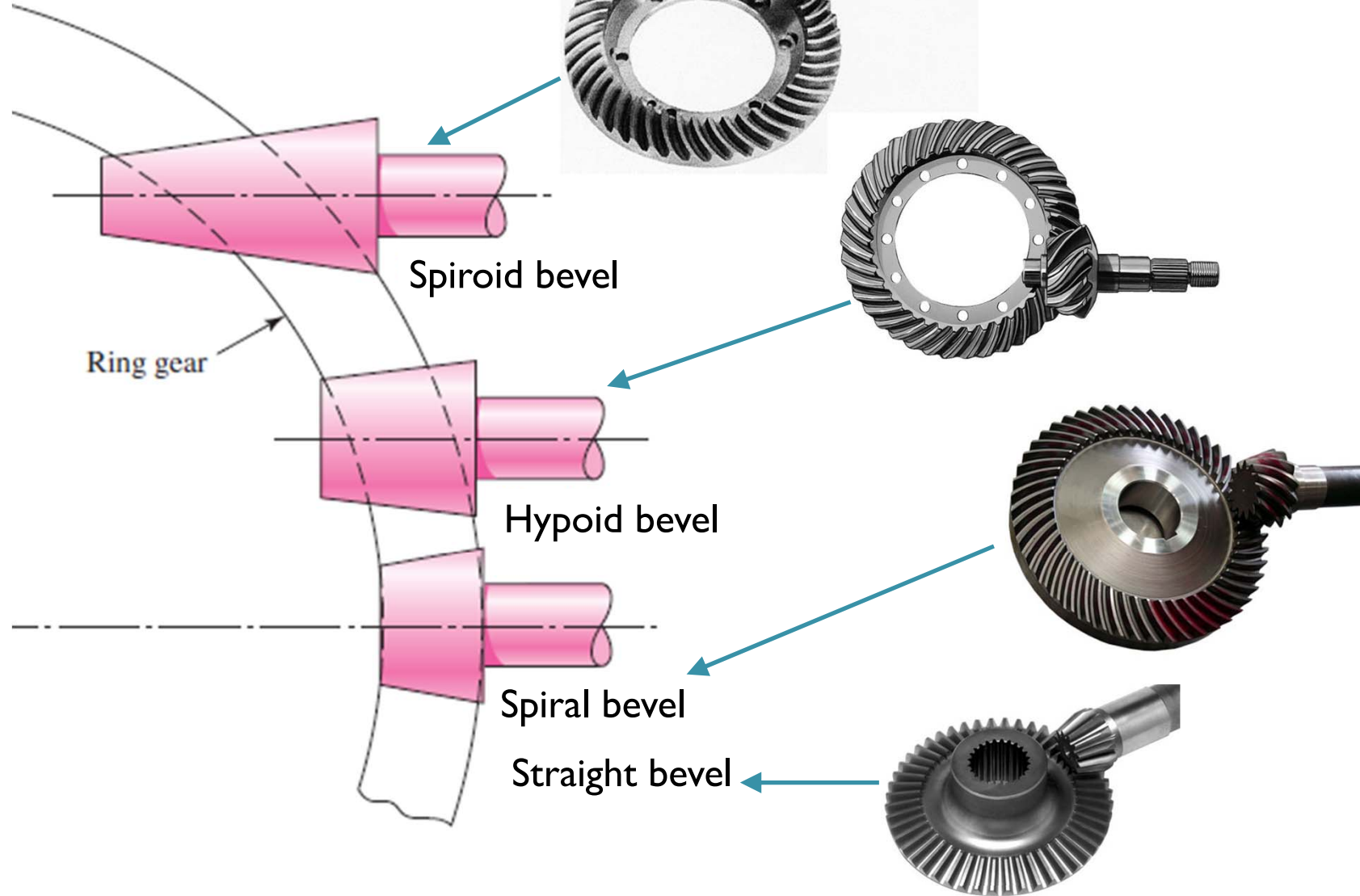
Power Transmission Problem



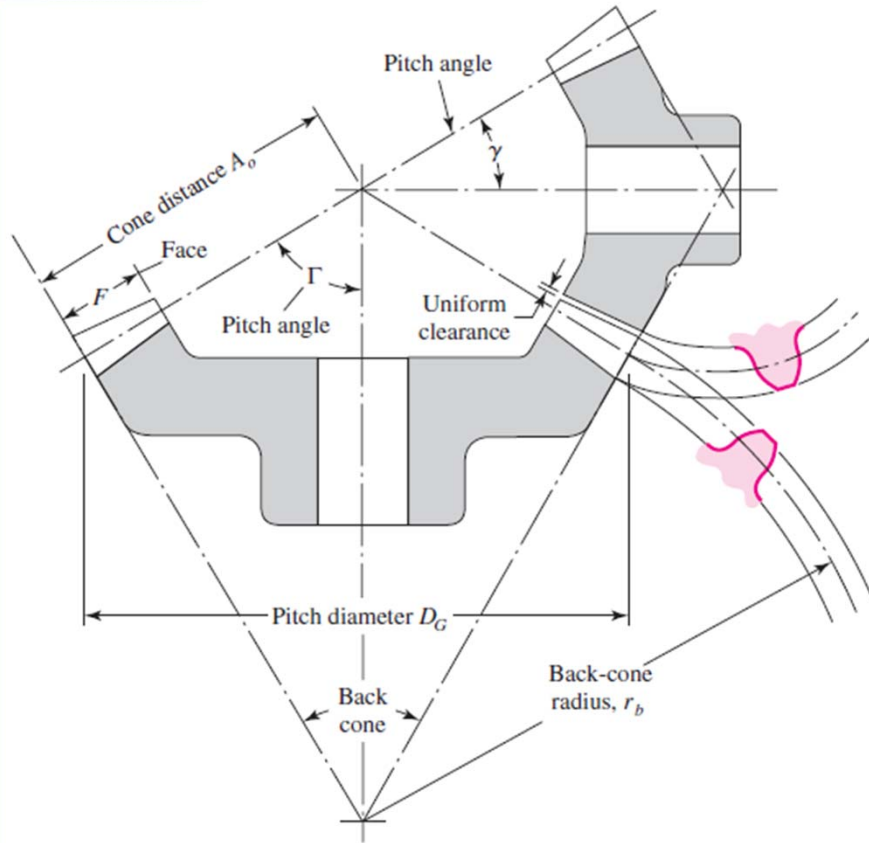
Proposed solution (Bevel Gear)



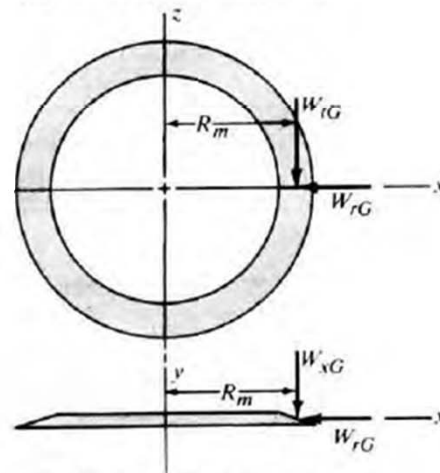
Types of Bevel Gears



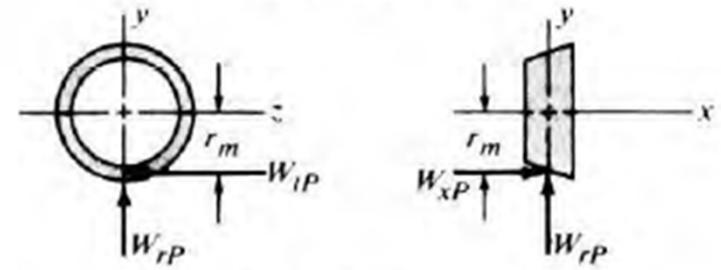
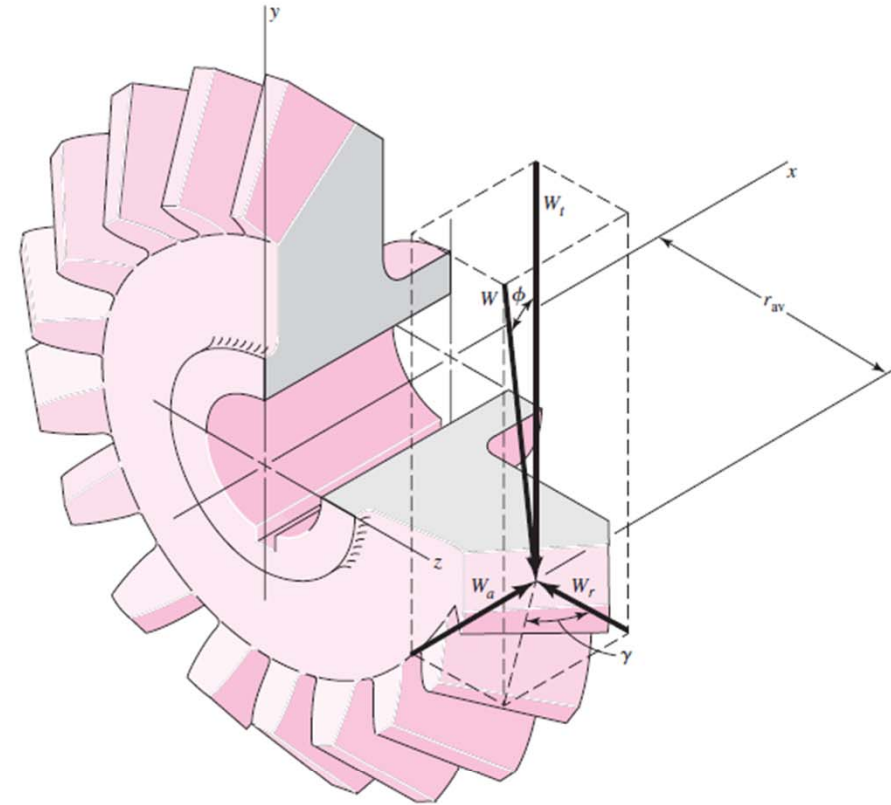
Basic bevel Gear Geometry and force Kinematics



Notes: Shaded area is pitch cone surface.
Considering magnitudes:
 $W_{tP} = W_{tG}$
 $W_{xP} = W_{rG}$
 $W_{rP} = W_{xG}$



(c) Free-body diagram: gear



(b) Free-body diagram: pinion

Modes of Gear Tooth Failure

1. Bending failure.

Root failure

2. Pitting.

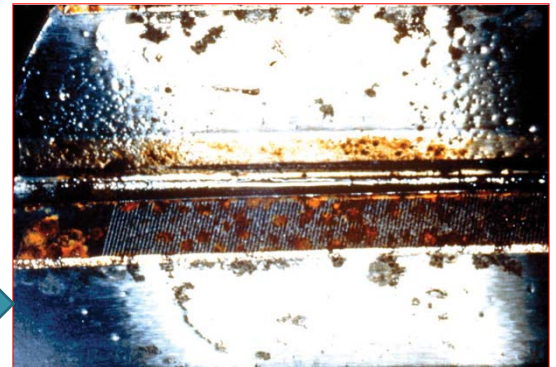
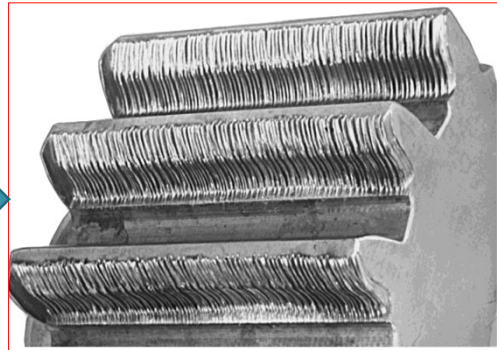
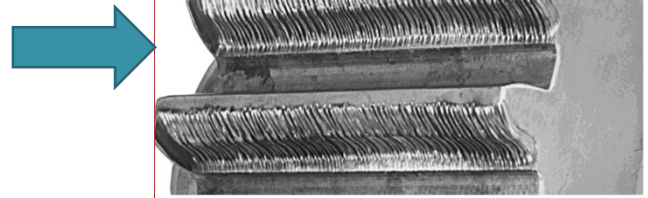
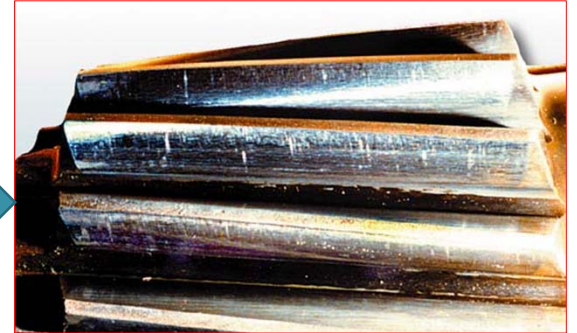
3. Scoring.

4. Abrasive wear.

5. Corrosive wear.

Surface failure

Dangerous Bar



Bevel Gear Design

The power to be transmitted

Type of driver and driven load

The speed of the driving gear

The center distance

The speed of the driven gear or the velocity ratio

Other information related to problem specification



Designer

The gear teeth should not fail under static loading or dynamic loading during normal running conditions.

The gear teeth should have wear characteristics so that their life is satisfactory.

The use of space and material should be economical.

The alignment of the gears and deflections of the shafts must be considered.

The lubrication of the gears must be satisfactory.

Flowchart for bevel gear designing process:

Transmitted Power , Input and Output speed, Center distance, Type of driver and driven load

Choose the over load factor (K_o) from Table (9-5) page(389) (405pdf)

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Compute the design power
Design Power= K_o x transmitted power

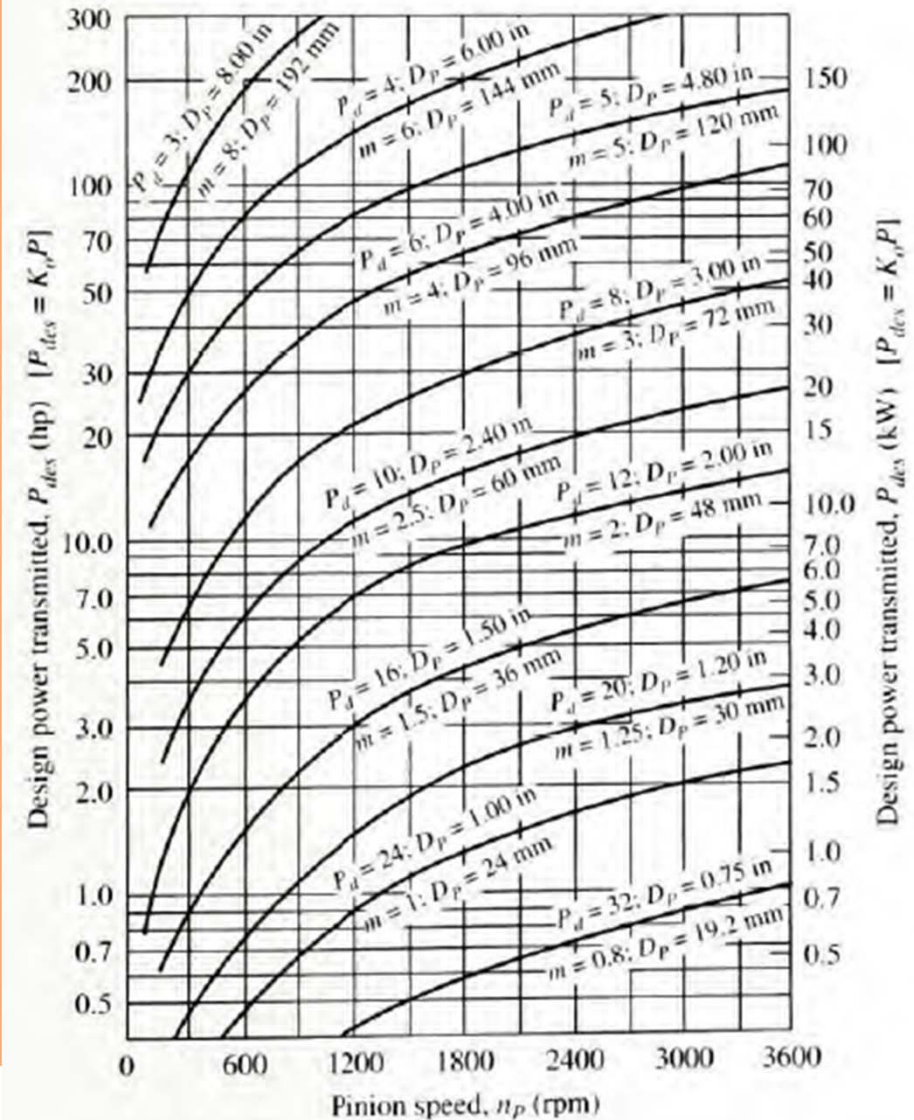
1

Find the trial value for
Diametral pitch (P_d) or Module (m)
 from Figure 9-27 page.409 (425 Pdf)

Standard_diametral_pitches_(teeth/in)

1.25
 1.5
 1.75
 2
 2.5
 3
 3.5
 4
 5
 6
 7
 8
 9
 10
 11
 12
 13
 14
 15
 16
 18
 20
 24
 32
 48
 64
 72
 80
 96
 120
 200

2



Specify the no. of teeth for Pinion N_P (from 17 to 20)

Compute the nominal velocity ratio $VR = \frac{n_P}{n_G}$

Compute the approximate no. of teeth for Gear $N_G = N_P \times VR$

Compute the actual velocity ratio $VR = \frac{N_G}{N_P}$

Compute the actual output velocity $n_G = n_P \frac{N_P}{N_G}$

Compute the pitch diameters $D_p = \frac{N_p}{P_d}$, $D_G = \frac{N_G}{P_d}$,
face width $F = \min(0.3A_o, 10/P_d)$, $A_o = \frac{D_p}{2 \sin \gamma} = \frac{D_G}{2 \sin \Gamma}$
pinion mean radius $r_m = D_p/2 - (F/2) \sin \gamma$ where $\gamma = \tan^{-1}(N_p/N_G)$,
gear mean radius $R_m = D_G/2 - (F/2) \sin \Gamma$ where $\Gamma = \tan^{-1}(N_G/N_P)$,
pitch line speed $v_t = \frac{2\pi r_m n_P}{60}$, and tangential force, $W_t = \frac{60P}{2\pi r_m n_p} = \frac{T}{r_m}$

3

Analyzing of gear tooth failure mode

Root (Bending) Failure Mode

Bending Stress Number

$$S_t = \frac{W_t P_d K_o K_s K_m}{F J} \frac{Y_N}{K_v} < S_{at} \frac{Y_N}{K_R (S.F)}$$

Surface (Pitting, Scoring,...) Failure Mode

Contact Stress Number

$$S_c = C_p C_b \sqrt{\frac{W_t}{F D_p I} \frac{C_o C_m}{C_v}} < S_{ac} \frac{Z_N C_H}{K_R (S.F)}$$

Find the values of factors
 $(J, I, K_s, K_m, K_v, C_p, C_b, C_o, C_m, C_v, Y_N, Z_N, C_H, K_R, S.F)$
as in the following steps

4

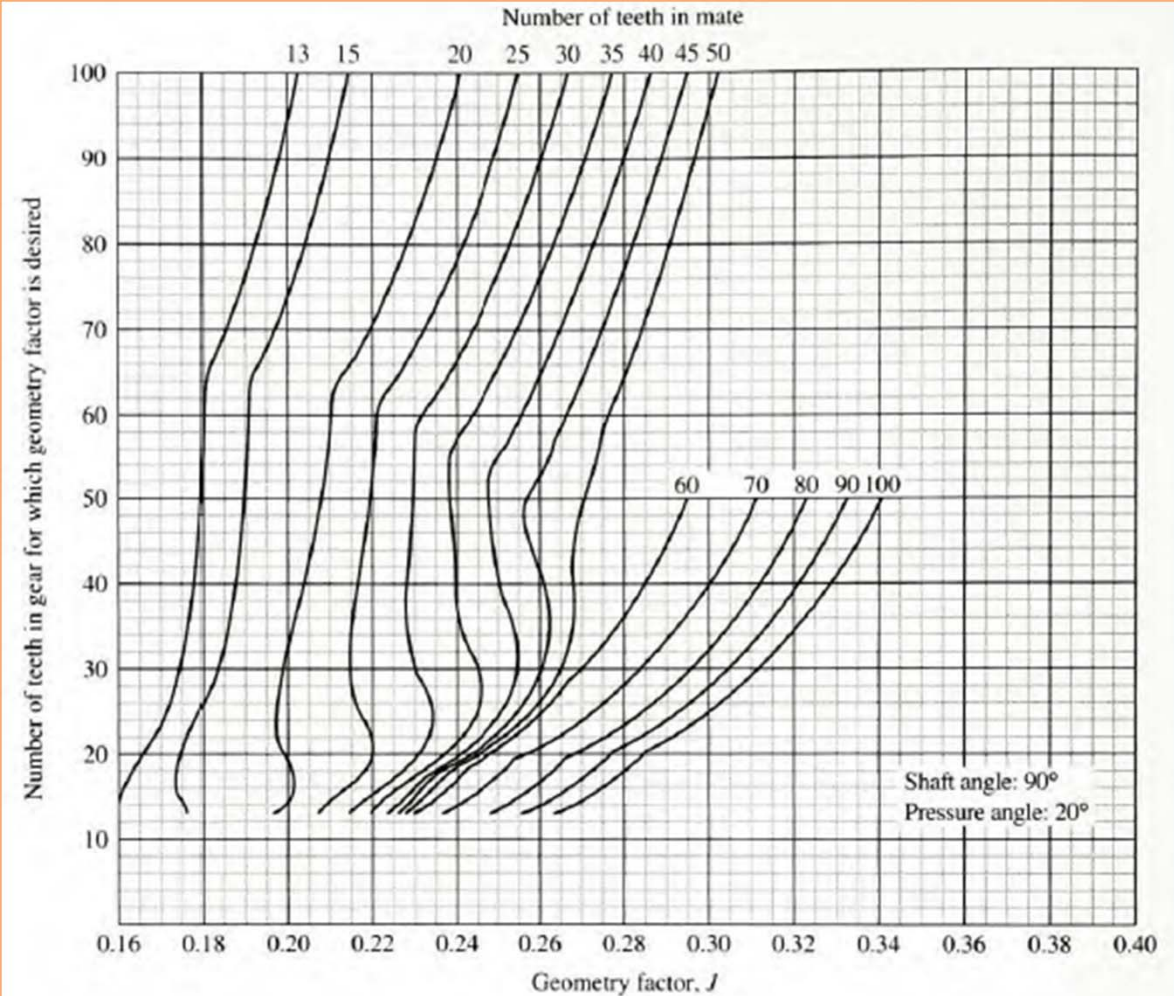
Specify the type of material for the gears to find the Elastic Coefficient C_p from Table (9-9) page(400) (Pdf 416)

Pinion material	Modulus of elasticity, E_p , lb/in ² (MPa)	Gear material and modulus of elasticity, E_G , lb/in ² (MPa)					
		Steel	Malleable iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
		30×10^6 (2×10^5)	25×10^6 (1.7×10^5)	24×10^6 (1.7×10^5)	22×10^6 (1.5×10^5)	17.5×10^6 (1.2×10^5)	16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Mall. iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nod. iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

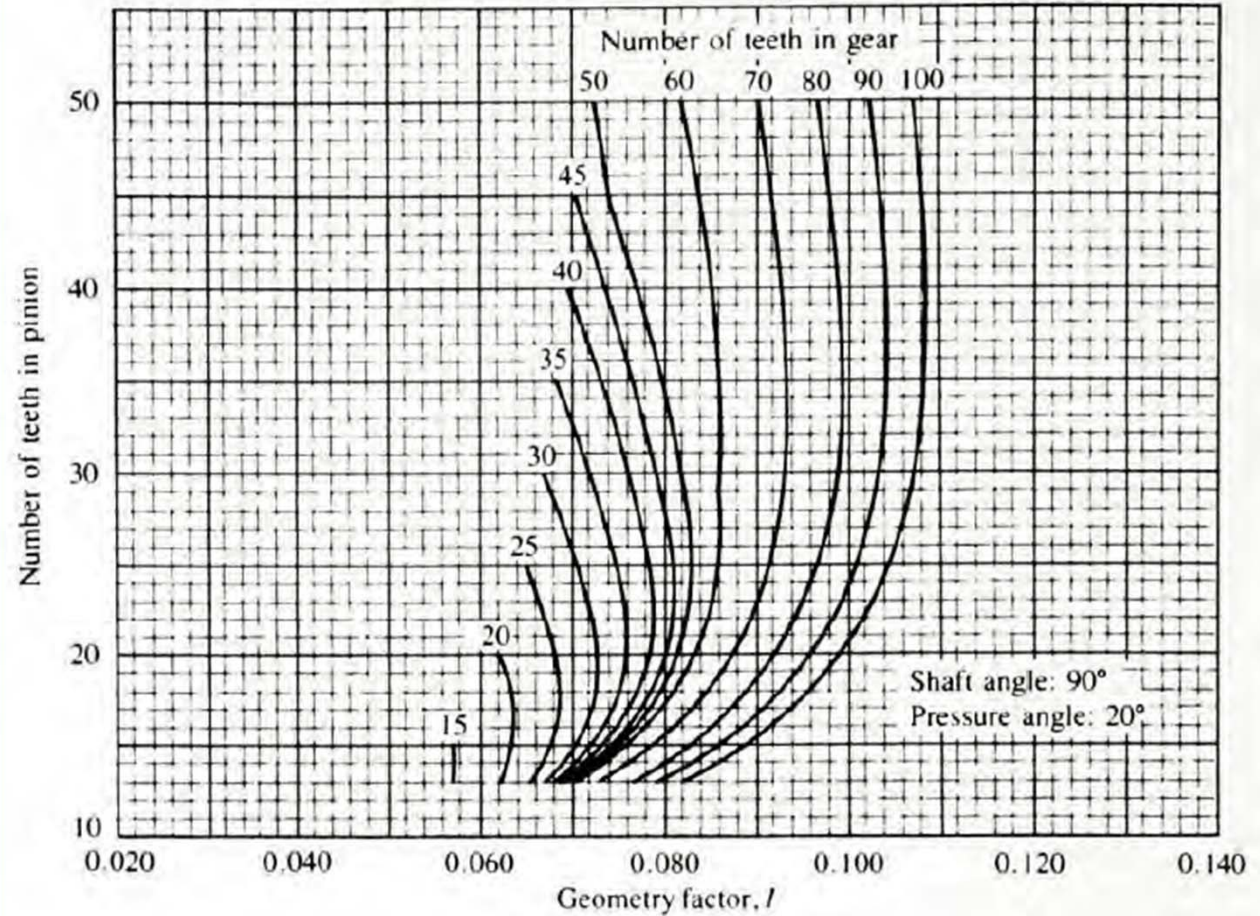
Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are (lb/in²)^{0.5} or (MPa)^{0.5}.

Specify the bending geometry factor (J) for 20° pressure angle and 90° shaft angle from figure (10-13) page (472) (488pdf):



Specify the pitting geometry factor (I) with 20° normal pressure angle and 90° shaft angle from Figure (10-14) page (474) (490pdf):



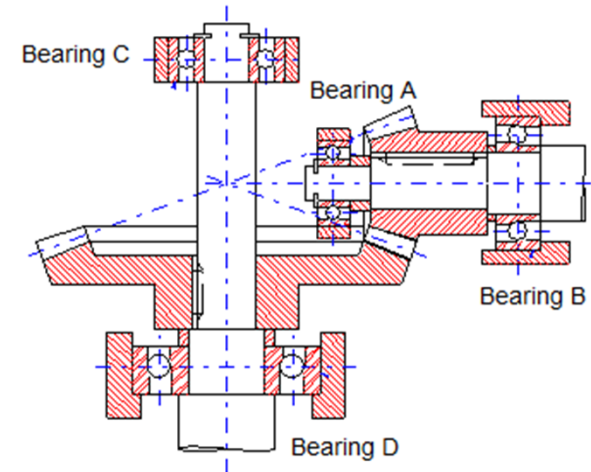
Specify the size factor (K_s) from Table(9-6) page (389) (293 pdf)

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

Specify the load distribution factor (K_m) from Table(10-3) page (471) (487 pdf)

Type of gearing	Both gears straddle-mounted	One gear straddle-mounted	Neither gear straddle-mounted
General commercial-quality	1.44	1.58	1.80
High-quality, commercial gearing	1.20	1.32	1.50

Straddle mounted gears



Specify the quality number Q_v , from Table (9-2) page (378) (394 pdf)

Application	Quality number	Application	Quality number
Cement mixer drum drive	3-5	Small power drill	7-9
Cement kiln	5-6	Clothes washing machine	8-10
Steel mill drives	5-6	Printing press	9-11
Grain harvester	5-7	Computing mechanism	10-11
Cranes	5-7	Automotive transmission	10-11
Punch press	5-7	Radar antenna drive	10-12
Mining conveyor	5-7	Marine propulsion drive	10-12
Paper-box-making machine	6-8	Aircraft engine drive	10-13
Gas meter mechanism	7-9	Gyroscope	12-14

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0-800	6-8	0-4
800-2000	8-10	4-11
2000-4000	10-12	11-22
Over 4000	12-14	Over 22

Choose material for pinion and gear or (S_{ac} , S_{at}) from figures [(9-10) page (379) (395pdf) , (9-11) page (380) (396pdf)] with tables [(9-3) page(381) (397pdf) , (9-4) page (385) (401pdf)] and see also Appendix 3 to 5 [p(A-6) to (A-11)].

Determine the dynamic factor (K_v) from the following equation:

$$K_v = \left[\frac{K_z}{K_z + \sqrt{v_t}} \right]^u$$

$$u = \frac{8}{(2)^{0.5Q}} - S_{at} \left[\frac{125}{E_P + E_G} \right] \text{ and } K_z = 85 - 10(u)$$

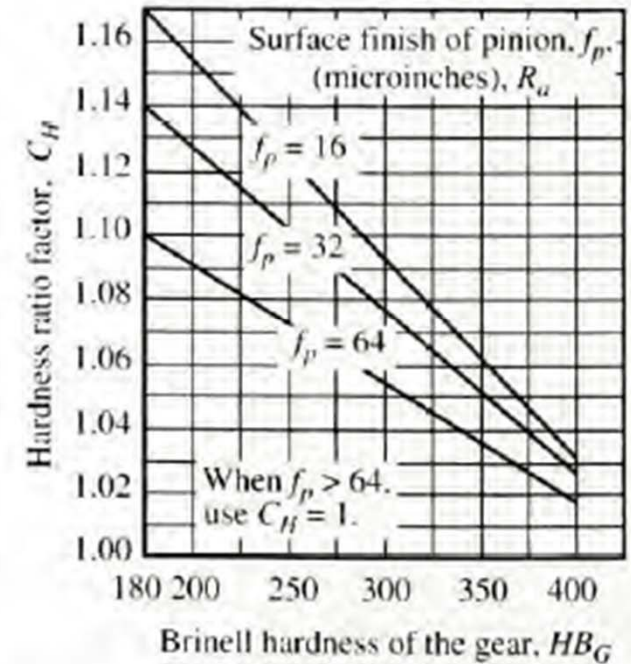
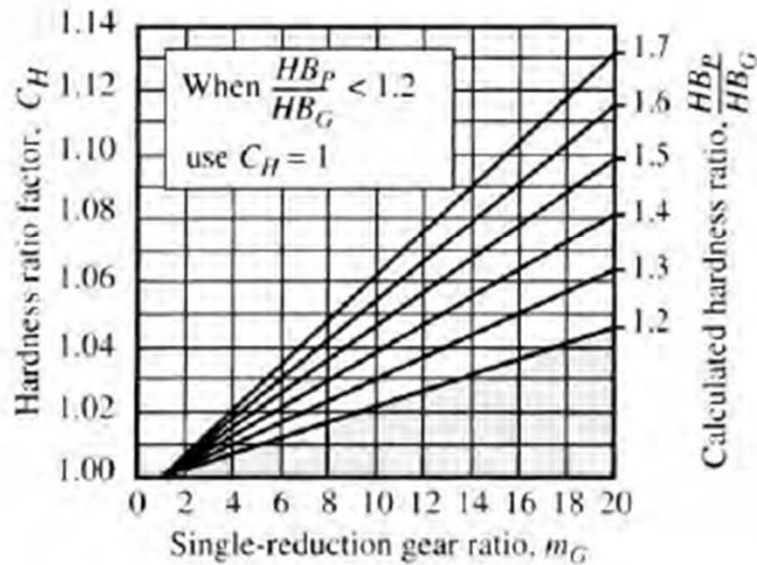
if $u = \text{negative value}$ then use $u = 0.0$

For checking K_v must be greater than ($K_{v_{min}} = \frac{2}{\pi} \tan^{-1}(v_t/333)$), if its not achieved then a higher quality number should be specified

Note : the calculation of inverse tangent must be in radians

Specify the safety factor (S.F) typically from 1 to 1.5

Specify the hardness ratio factor (C_H) from Figure (9-25 & 26) page (404) (420 pdf)



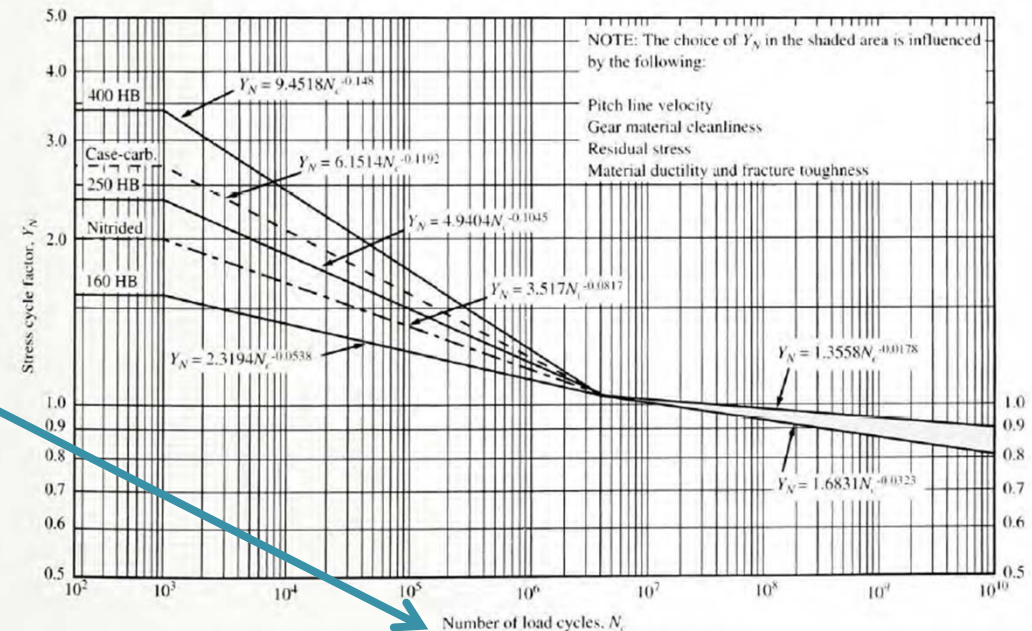
Specify the reliability factor (K_R) from Table (9-8) page (396) (412 pdf):

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

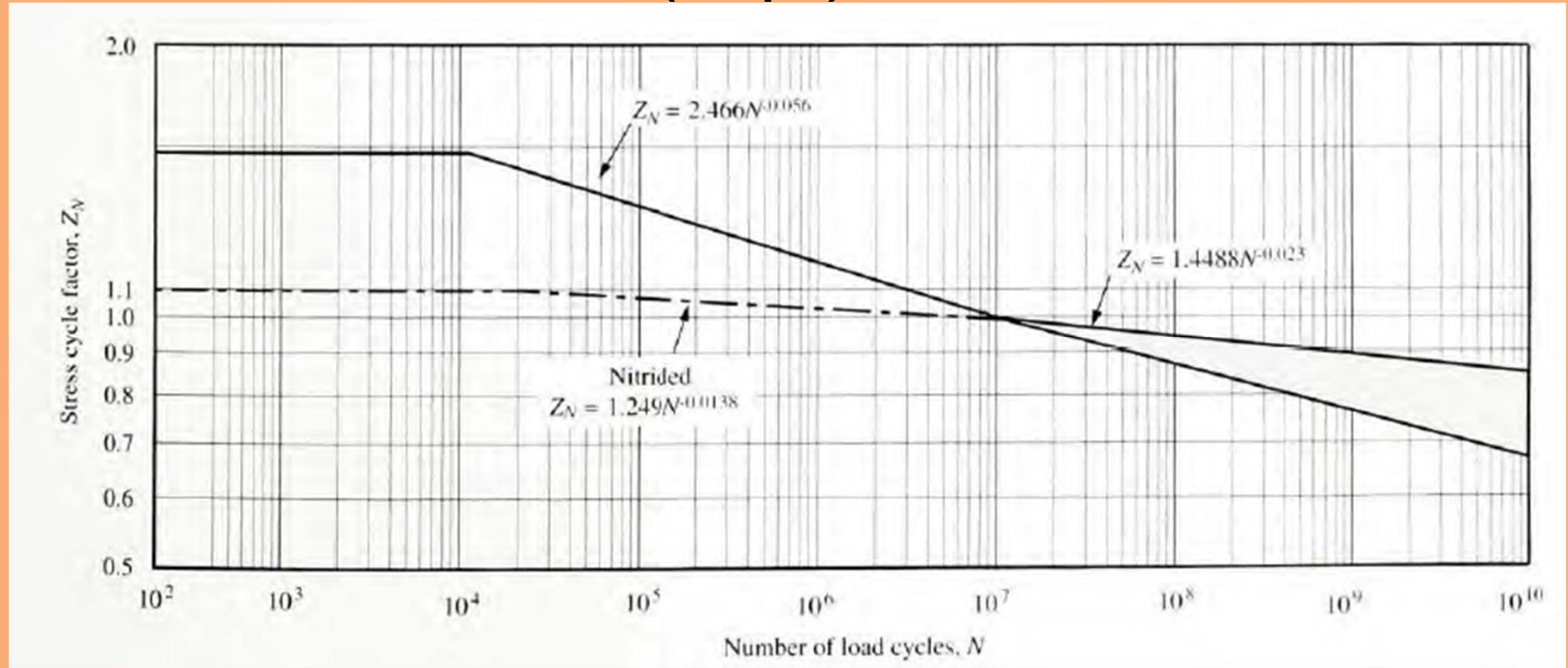
Specify the stress cycle life (Y_N) from Figure (9-8) page (395) (411 pdf):

TABLE 9-7 Recommended design life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000



Specify the pitting resistance stress cycle factor (Z_N) from figure (9-24) page (403) (419 pdf):



The factors C_o , C_v and C_m are the same as K_o , K_v and K_m

Using $C_b = 0.634$ allows the use of the same allowable contact stress as for spur and helical gear

Check if the selected material satisfy the following design conditions:

$$S_t \frac{K_R(S.F)}{Y_N} < S_{at}$$
$$S_c \frac{K_R(S.F)}{Z_N C_H} < S_{ac}$$