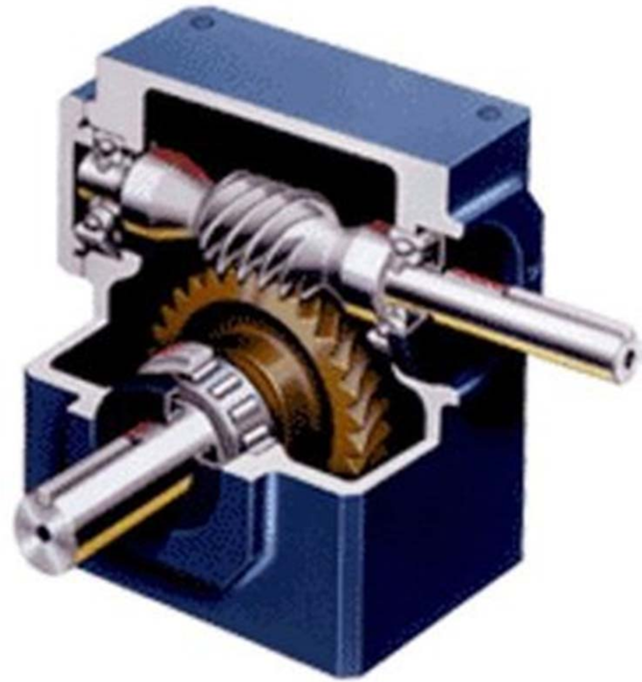
Mechanical Engineering Design II

Twenty-two Lecture

Design of Worm Gear

Power Transmission Problem

Proposed solution (Worm Gear)



Design Requirements

- ✓ high velocity ratios in a single step in a minimum of space
- ✓ non-intersecting shafts at right angles

Specifications of Worm Gears

Advantages:

(1) A large speed ratio;

(2) Silent and smooth operation;

(3) Small drive size

(4) Better load distribution;

(5) Self-locking action.

Disadvantages:

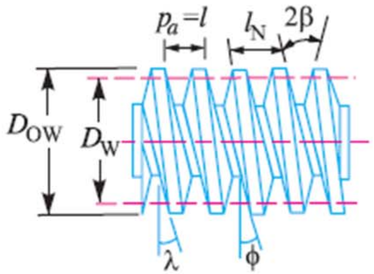
(1) Low efficiency;

(2) Expensive antifriction materials;

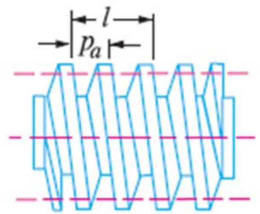
(3) Considerable sliding speed;

(4) Considerable heat generated.

Types of Worm Gears

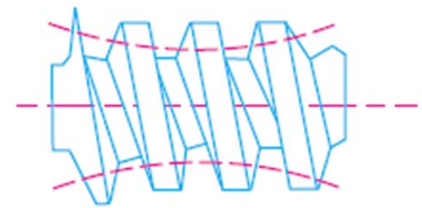


Single threaded.

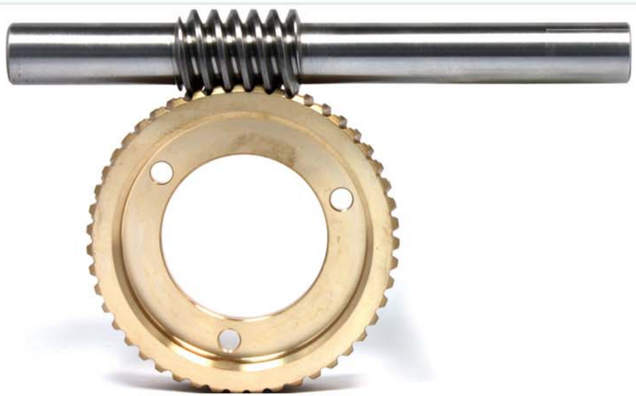


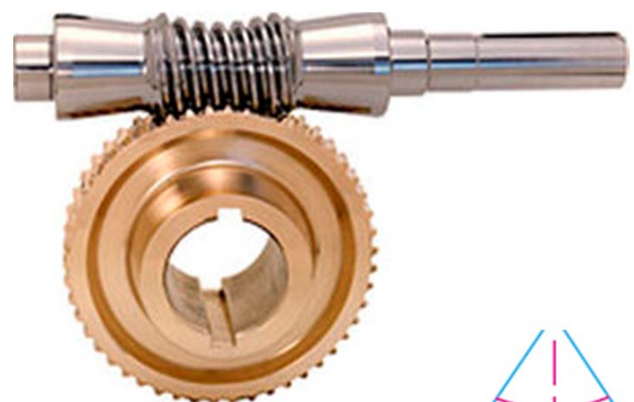
Double threaded.

(*a*) Cylindrical or straight worm.

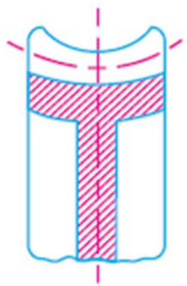


(*b*) Cone or double enveloping worm.

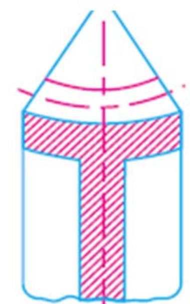




(*a*) Straight face.

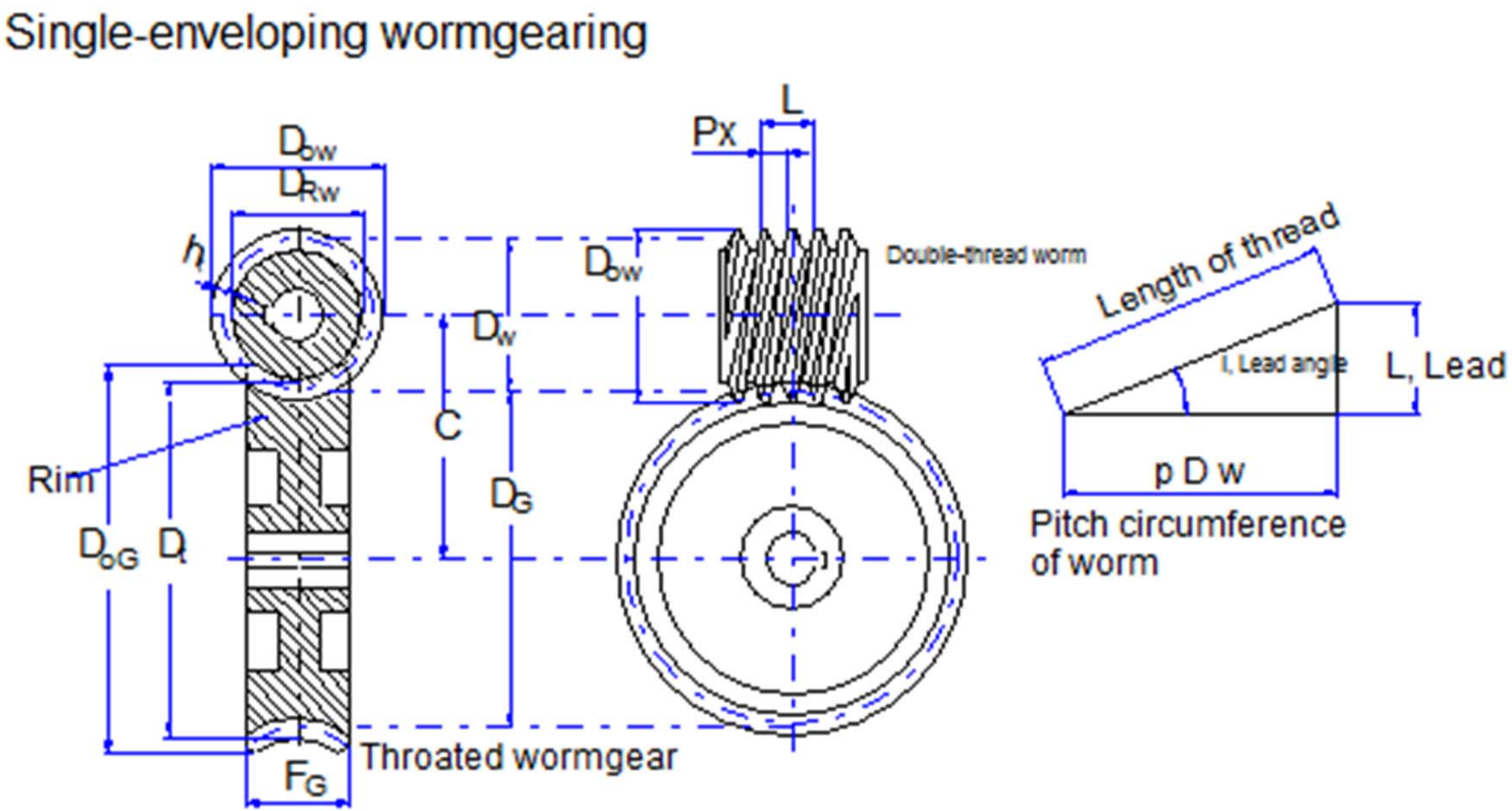


(*b*) Hobbed straight face.



(*c*) Concave face.

Basic Worm Gear Geometry



D_G = pitch diameter of the gear D_W = pitch diameter of the worm

N_G = number of teeth in the gear N_G = number of teeth in the worm

C = center distance L = lead $\lambda =$ lead angle P_x = axial pitch

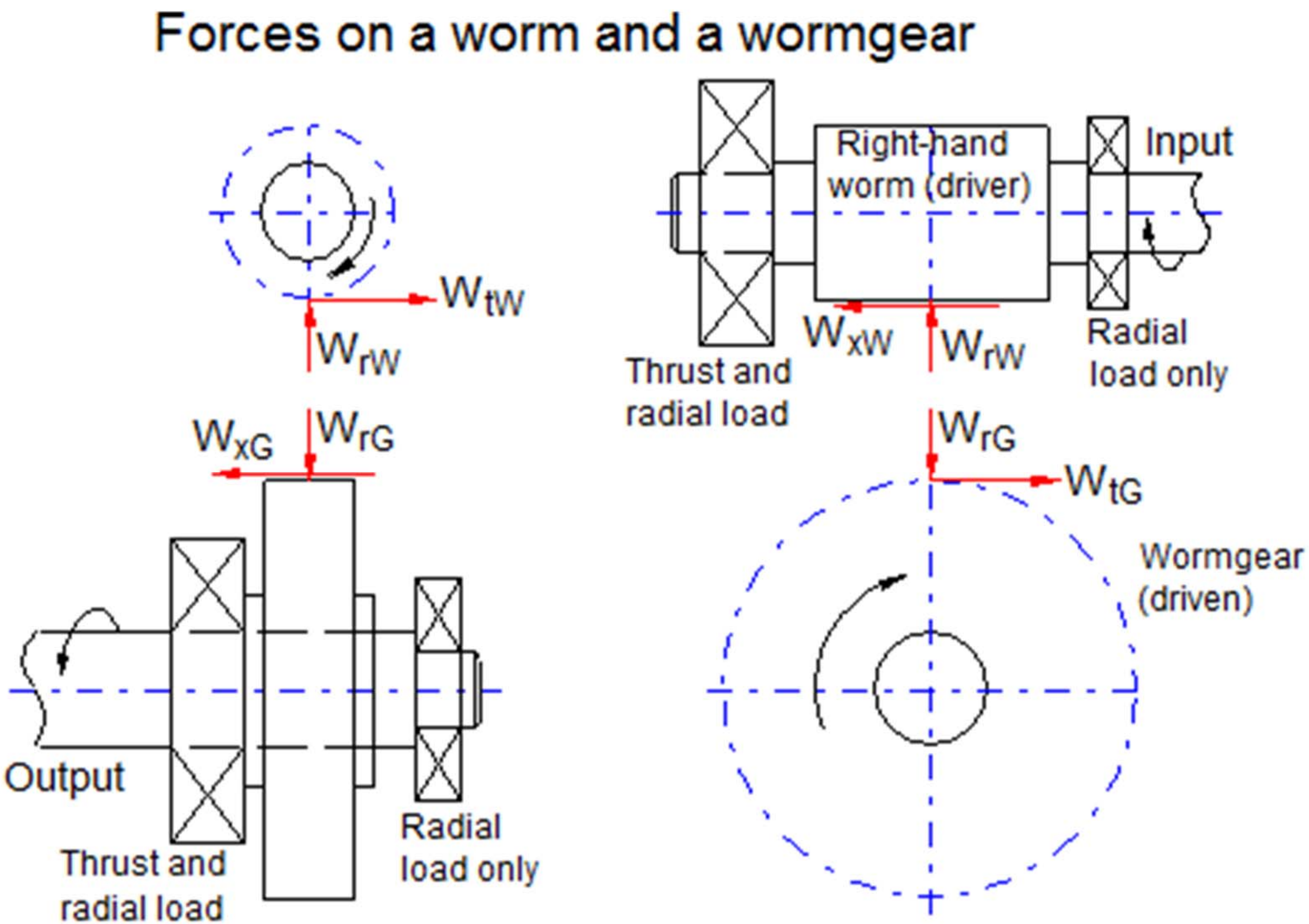
F_G = Face width of gear F_w = Face length of the worm

Forces on Worm Gear

$$W_{tG} = W_{xW}$$

$$W_{xG} = W_{tW}$$

$$W_{rG} = W_{rW}$$



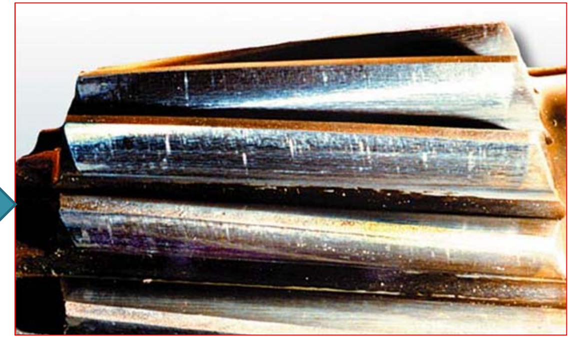
Modes of Gear Tooth Failure

1. Bending failure.

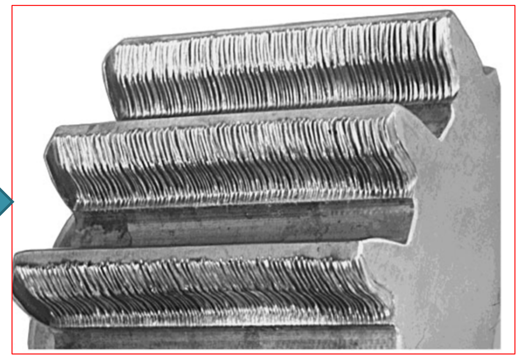
Root failure

2. Pitting.

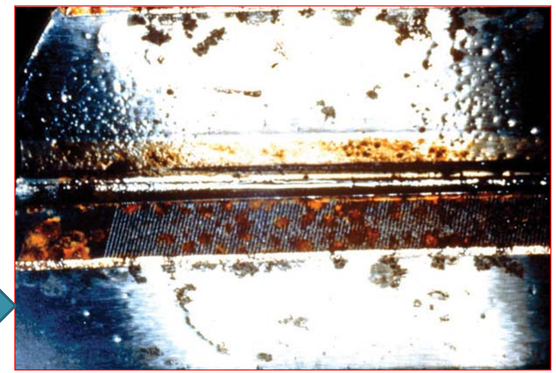
3. Scoring.



4. Abrasive wear.



5. Corrosive wear.



Surface failure

Dangerous Bar

Worm Gear Design

- The power to be transmitted
- Type of driver and driven load
- The speed of the driving gear
- The center distance
- The speed of the driven gear or the velocity ratio
- Other information related to problem specification

Designer

- The gear teeth should not fail under static loading or dynamic loading during normal running conditions.
- The gear teeth should have wear characteristics so that their life is satisfactory.
- The use of space and material should be economical.
- The alignment of the gears and deflections of the shafts must be considered.
- The lubrication of the gears must be satisfactory.

Flowchart for worm gear designing process:

Transmitted Power , Input and Output speed, Center distance, Type of driver and driven load

Specify the no. of threads for Worm N_W (from 2 to 8 or more)

Specify the diametral pitch P_d (3 , 4 , 5 , 6 , 8 , 10 , 12 , 16 , 24 , 32 , 48)

Specify the Pressure angle ϕ_n (14.5° , 20° , 25° , 30°)

Compute the nominal velocity ratio $VR = \frac{n_w}{n_G} = \frac{N_G}{N_P}$

Compute the circular pitch for Gear $p = \pi/P_d$ = axial pitch for worm P_x , normal circular pitch $p_n = p\cos\lambda$, and the face width of gear $F = 2p$

Compute the lead $L = N_W \times P_x$

Compute the pitch diameter of the worm within $\frac{C^{0.875}}{1.6} > D_W > \frac{C^{0.875}}{3}$

1

1

Compute the lead angle $\lambda = \tan^{-1}(L/\pi D_W)$

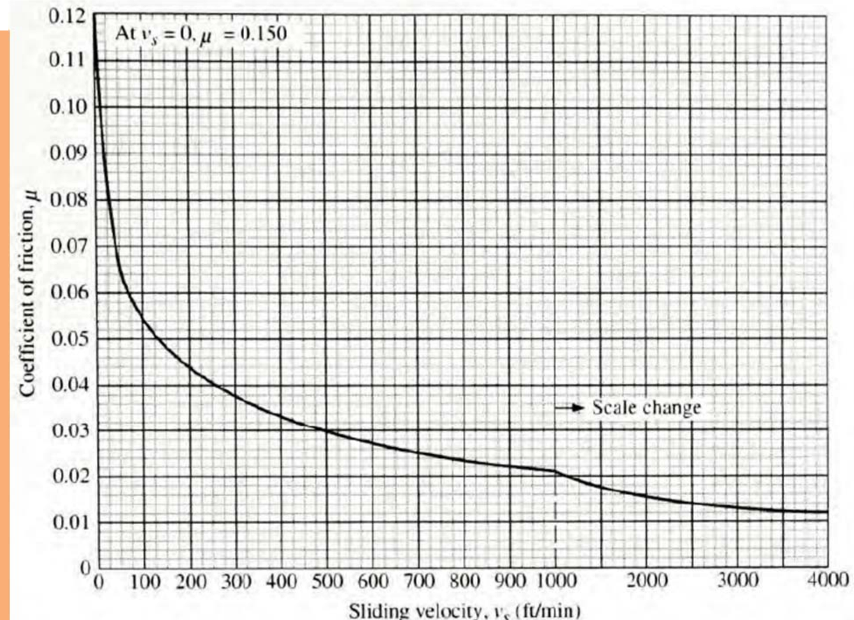
Compute the pitch diameter of gear $D_G = N_G/P_d$

Compute the pitch line speed of the worm and gear

$$v_{tw} = \frac{2\pi D_W n_w}{60} \quad , v_{tG} = \frac{2\pi D_G n_G}{60}$$

Compute the sliding velocity $v_s = v_{tG}/\sin\lambda = v_{tw}/\cos\lambda$

Find the coefficient of friction μ from figure (10-18) page(477) (493pdf)



choice of formula depends on the sliding velocity. *Note:* v_s must be in ft/min in the formulas; 1.0 ft/min = 0.0051 m/s.

2

Compute the output torque $T_o = \frac{P_o}{\omega_G} = W_{tG}(\frac{D_G}{2})$,

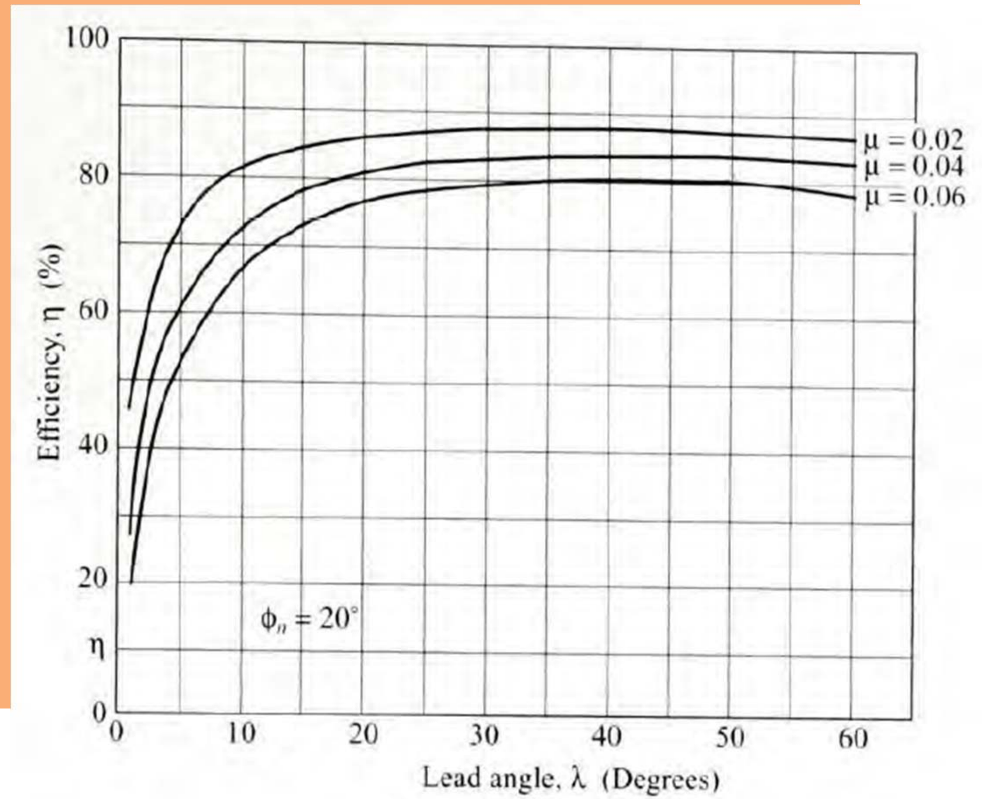
$$W_{xG} = W_{tG}\frac{\cos\phi_n \sin\lambda + \mu\cos\lambda}{\cos\phi_n \cos\lambda - \mu\sin\lambda} \quad , \quad W_{rG} = W_{tG}\frac{\sin\phi_n}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$$

Friction force $W_f = \frac{\mu\ W_{tG}}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$, **Power loss** $P_L = v_s W_f/60$,

Input Power $P_i = P_o + P_L$,

Efficiency $\eta = \frac{P_o}{P_i} = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n - \mu\tan\lambda}$

or from figure (10-19) page (479) (495pdf)



3

Analyzing of gear tooth failure mode

Root (Bending) Failure Mode

Bending Stress

$$\sigma = \frac{W_d}{yF_G p_n} < S_{at}$$

Surface (Pitting, Scoring,…) Failure Mode

Surface Durability (Rated Tangential Load)

$$W_{tR} = C_s D_G^{0.8} F_e C_m C_v > W_{tG}$$

Find the values of factors (y, K_v , C_s , C_m , C_v) as in the following steps

4

4

Specify the Lewis form factor for worm gear teeth from Table (10-4) page(482) (Pdf 498)

ϕ_n	y
$14\frac{1}{2}°$	0.100
20°	0.125
25°	0.150
30°	0.175

Compute the dynamic load factor $K_v = 1200/(1200 + v_{tG})$ (v_{tG} in ft/min)

Compute the dynamic load $W_d = W_{tG}/K_v$

5

Specify the material factor (C_s) from figure (10-20) page (483) (499pdf) or from the following equations:

Sand-Casting Bronzes:

$for\ D_G > 63.5mm$

$$C_s = 1189.636 - 476.545\log_{10}(D_G)$$

$for\ D_G < 63.5mm$

$$C_s = 1000$$

Static-Chill-Cast or Forged Bronzes:

$for\ D_G > 203.2mm$

$$C_s = 1411.651 - 455.825\log_{10}(D_G)$$

$for\ D_G < 203.2mm$

$$C_s = 1000$$

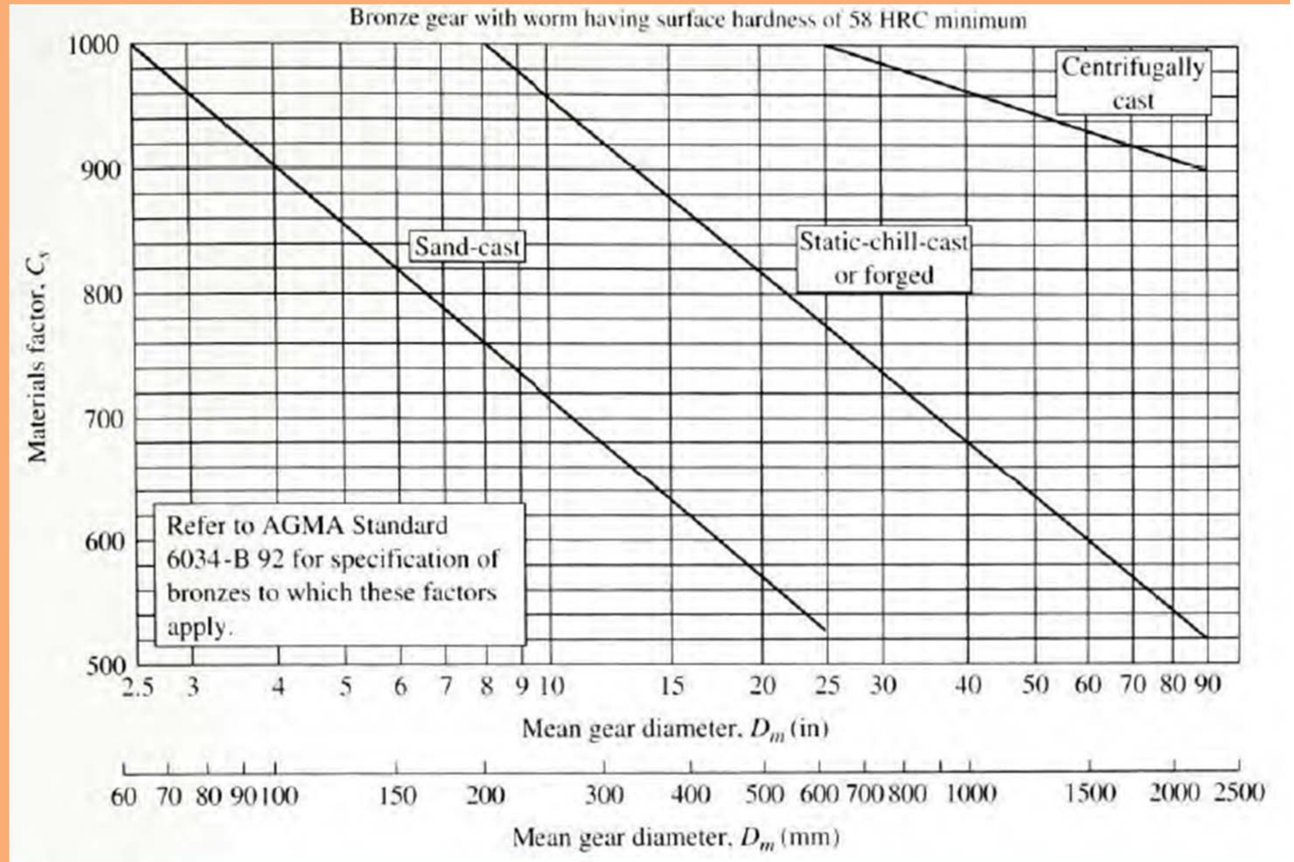
Centrifugally Cast Bronzes:

$for\ D_G > 635\ mm$

$$C_s = 1251.291 - 179.750\log_{10}(D_G)$$

$for\ D_G < 635\ mm$

$$C_s = 1000$$



Note: if standard addendum gears are used, $D_m = D_G$

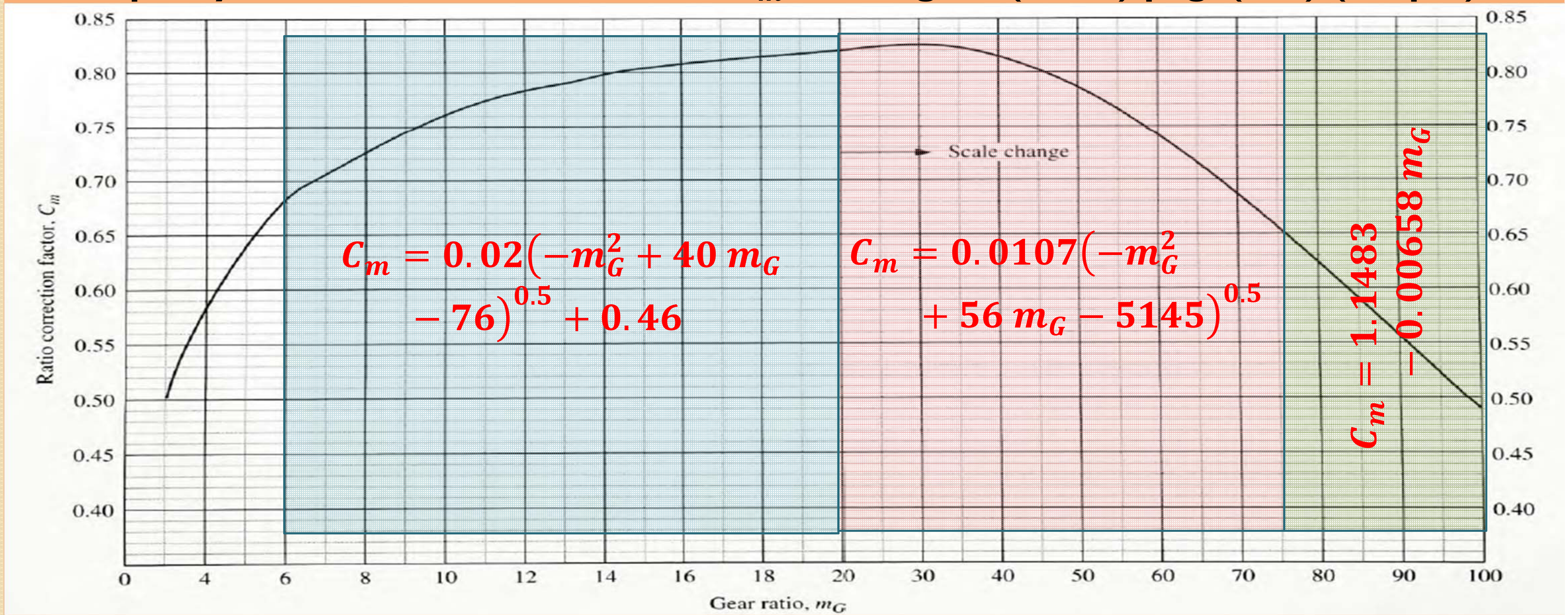
6

Compute the effective face width in inches

$if\ F_G < 0.667\ (D_w)\ then\ \ F_e = F_G$

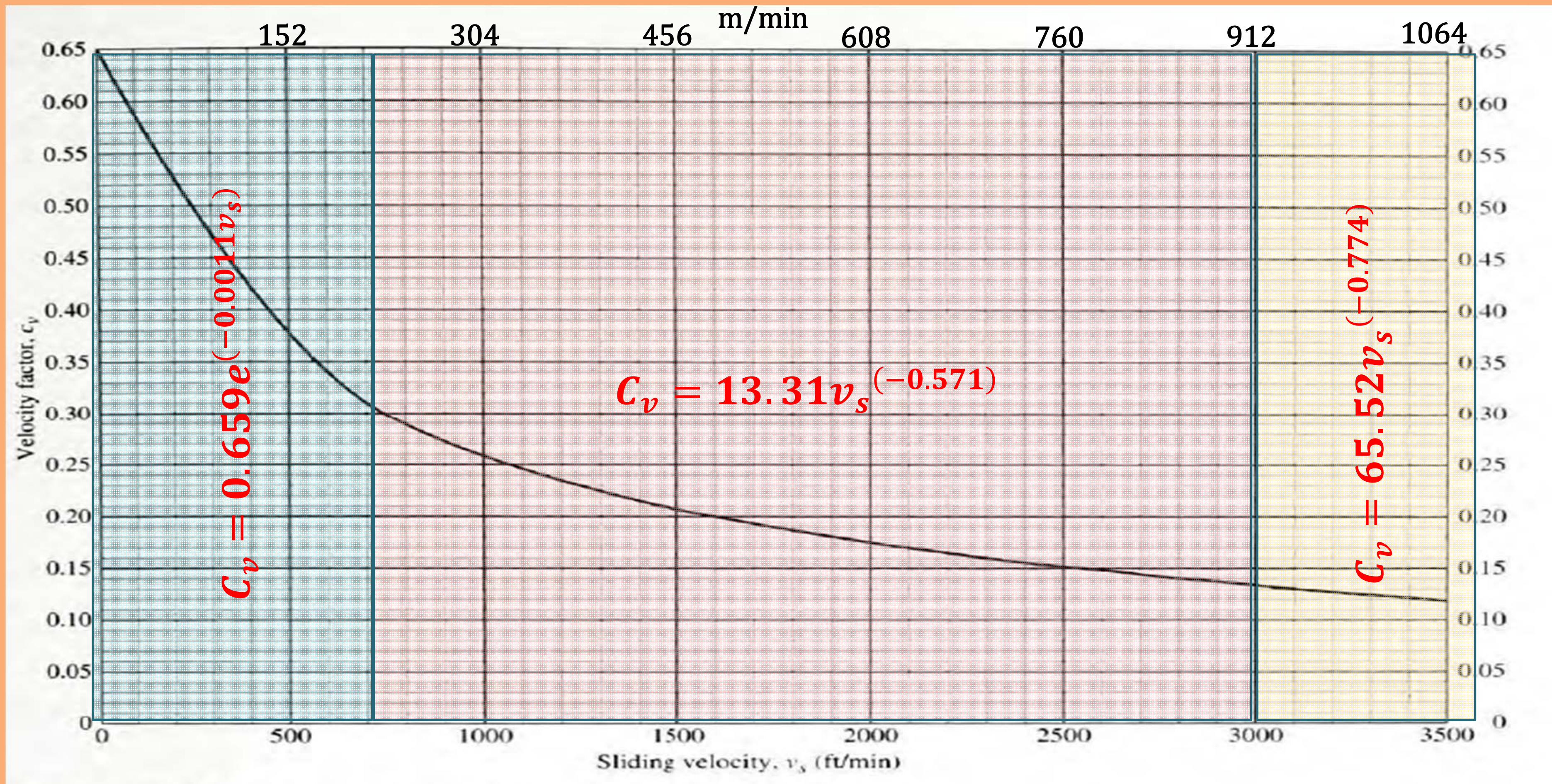
$else\ if\ F_G > 0.667\ (D_w)\ then\ \ F_e = 0.67\ (D_w)$

Specify the Ratio correction factor C_m from figure (10-21) page(484) (500pdf)



7

Specify the velocity factor (C_v) from figure(10-22) page (485) (501 pdf)



Check if the selected dimensions satisfy the following design conditions:

First condition

$\sigma < S_{at}$ ($\mathbf{117\ MPa}$ **for manganese gear bronze**

, $\mathbf{165.5\ MPa}$ **for phosphor gear bronze ,**

and for cast iron use approximately $\mathbf{0.35}\ \sigma_u$

Second condition

$$W_{tR} > W_{tG}$$

Third condition

$$maximum\ deflection\ of\ worm < 0.005\sqrt{P_x}$$

Example Problem 10–8 Is the wormgear set described in Example Problem 8–7 satisfactory with regard to strength and wear when operating under the conditions of Example Problem 10–7? The wormgear has a face width of 1.25 in.

Solution From previous problems and solutions,

$$W_{tG} = 962 \text{ lb} \qquad VR = m_G = 17.33$$
$$v_{tG} = 229 \text{ ft/min} \qquad v_s = 944 \text{ ft/min}$$
$$D_G = 8.667 \text{ in} \qquad D_W = 2.000 \text{ in}$$

Assume 58 HRC minimum for the steel worm. Assume that the bronze gear is sand-cast.

Stress

$$K_v = 1200/(1200 + v_{tG}) = 1200/(1200 + 229) = 0.84$$
$$W_d = W_{tG}/K_v = 962/0.84 = 1145 \text{ lb}$$
$$F = 1.25 \text{ in}$$
$$y = 0.125 \text{ (from Table 10–4)}$$
$$p_n = p \cos\lambda = (0.5236)\cos\ 14.04^\circ = 0.508 \text{ in}$$

Then

$$\sigma = \frac{W_d}{yFp_n} = \frac{1145}{(0.125)(1.25)(0.508)} = 14\,430 \text{ psi}$$

The guidelines in Section 10–12 indicate that this stress level would be adequate for either manganese or phosphor gear bronze.

Surface Durability: Use Equation (10–36):

$$W_{tR} = C_s D_G^{0.8} F_e C_m C_v \tag{10–36}$$

C Factors: The values for the C factors can be found from Figures 10–20, 10–21, and 10–22. We find

$$C_s = 740 \text{ for sand-cast bronze} \quad \text{and} \quad D_G = 8.667 \text{ in}$$
$$C_m = 0.184 \quad \text{for } m_G = 17.33$$
$$C_v = 0.265 \quad \text{for } v_s = 944 \text{ ft/min}$$

We can use $F_e = F = 1.25$ in if this value is not greater than 0.67 times the worm diameter. For $D_W = 2.000$ in,

$$0.67D_W = (0.67)(2.00 \text{ in}) = 1.333 \text{ in}$$

Therefore, use $F_e = 1.25$ in. Then the rated tangential load is

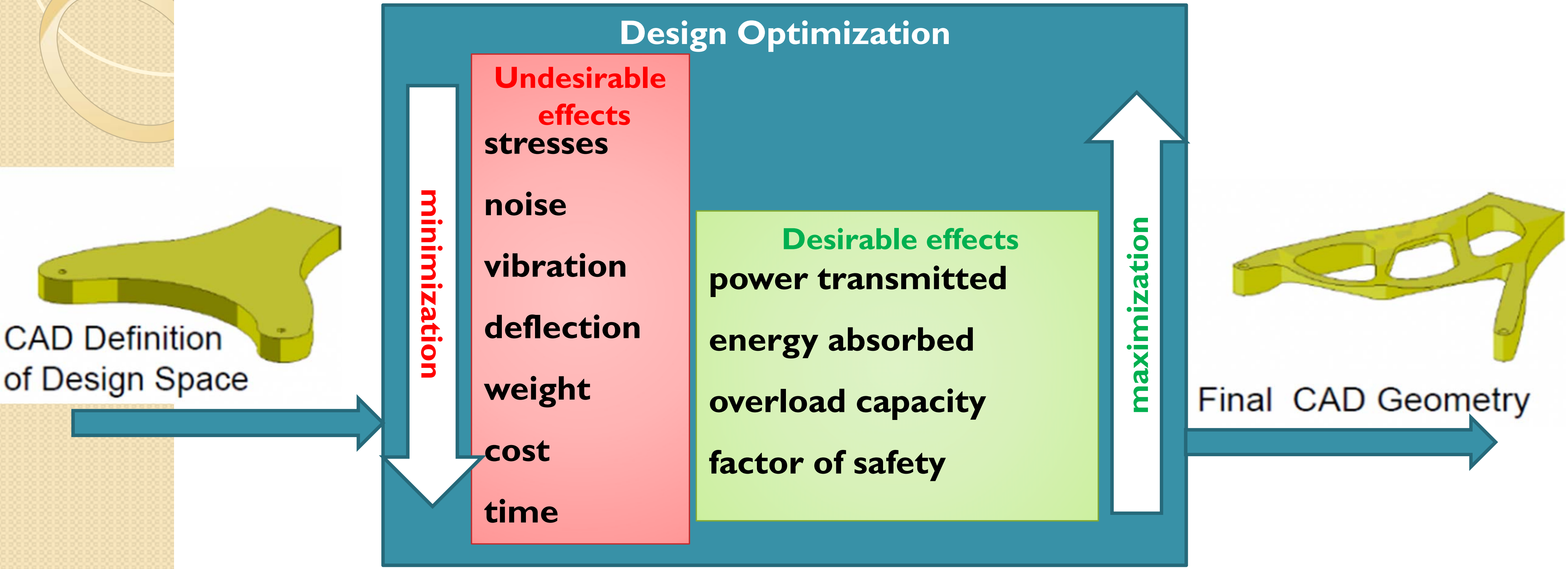
$$W_{tR} = (740)(8.667)^{0.8}(1.25)(0.814)(0.265) = 1123 \text{ lb}$$

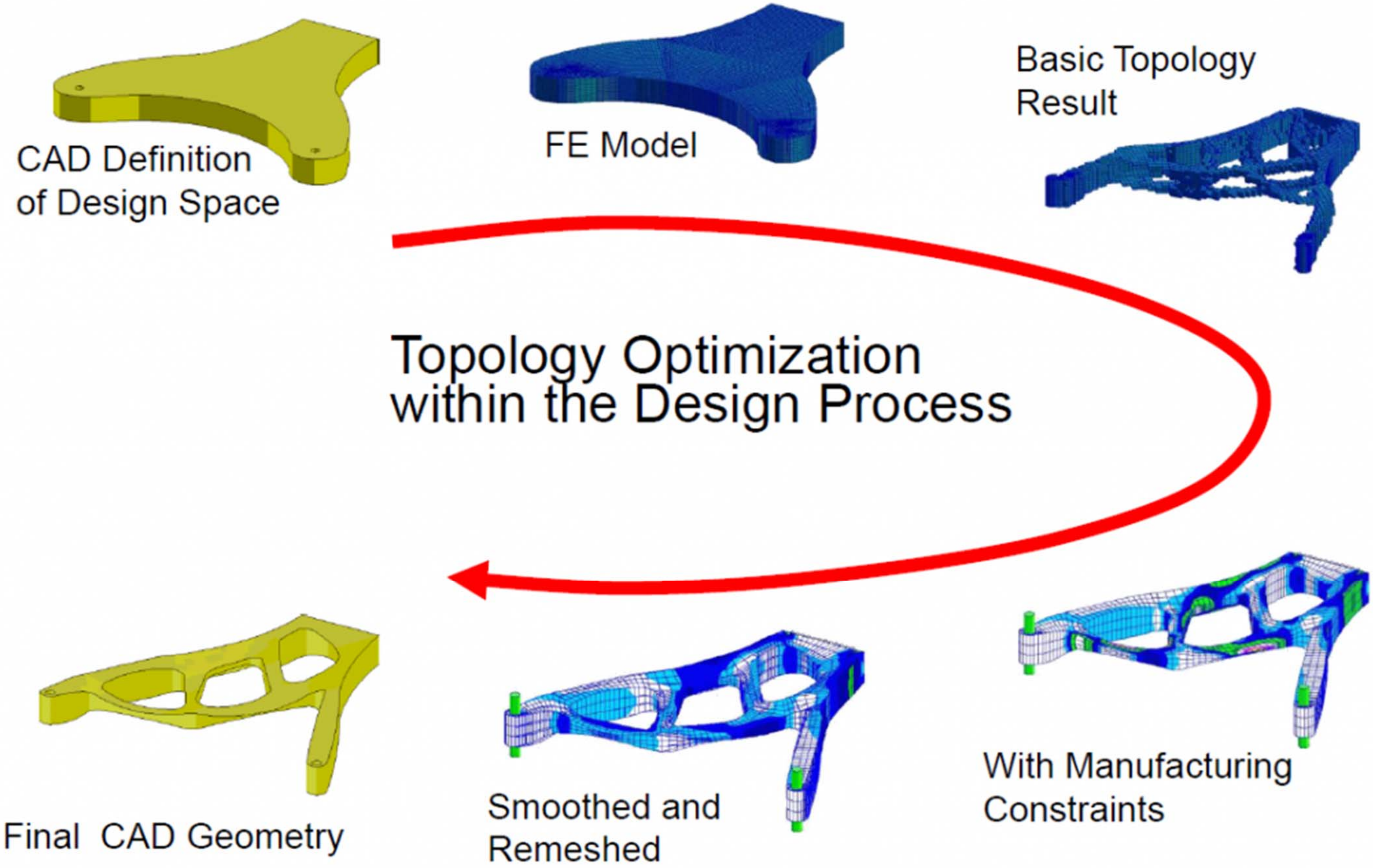
Because this value is greater than the actual tangential load of 962 lb, the design should be satisfactory, provided that the conditions defined for the application of Equation (10–36) are met.

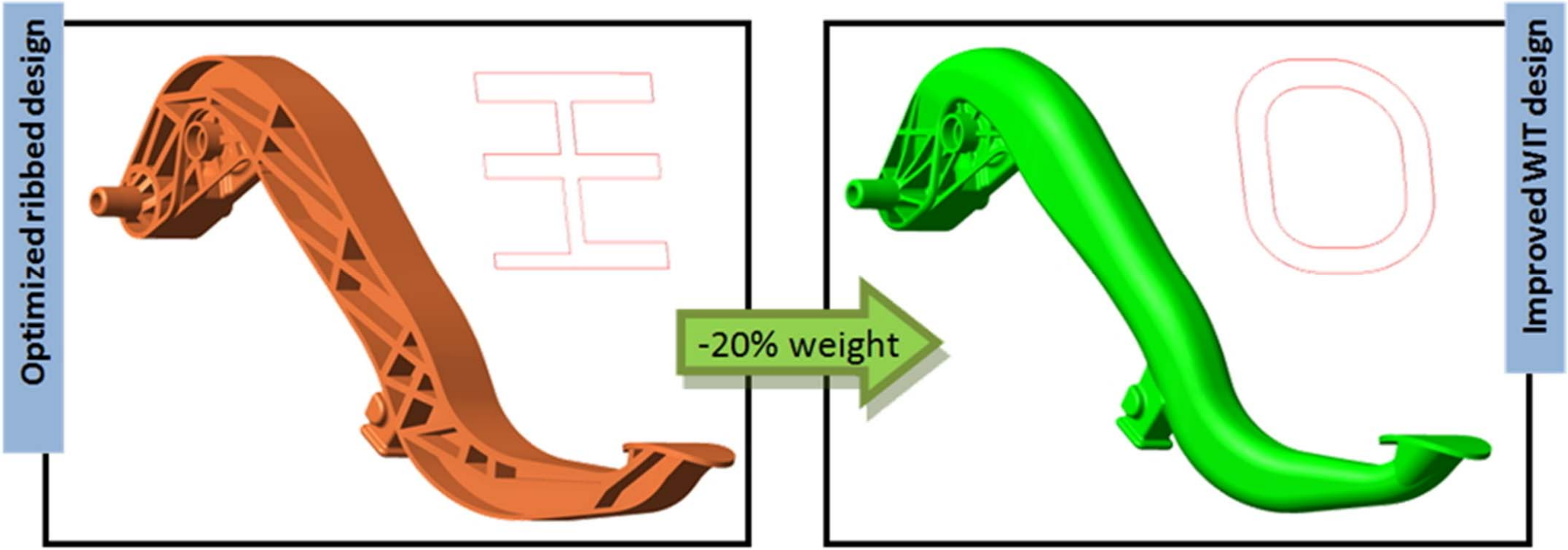
Mechanical Engineering Design II

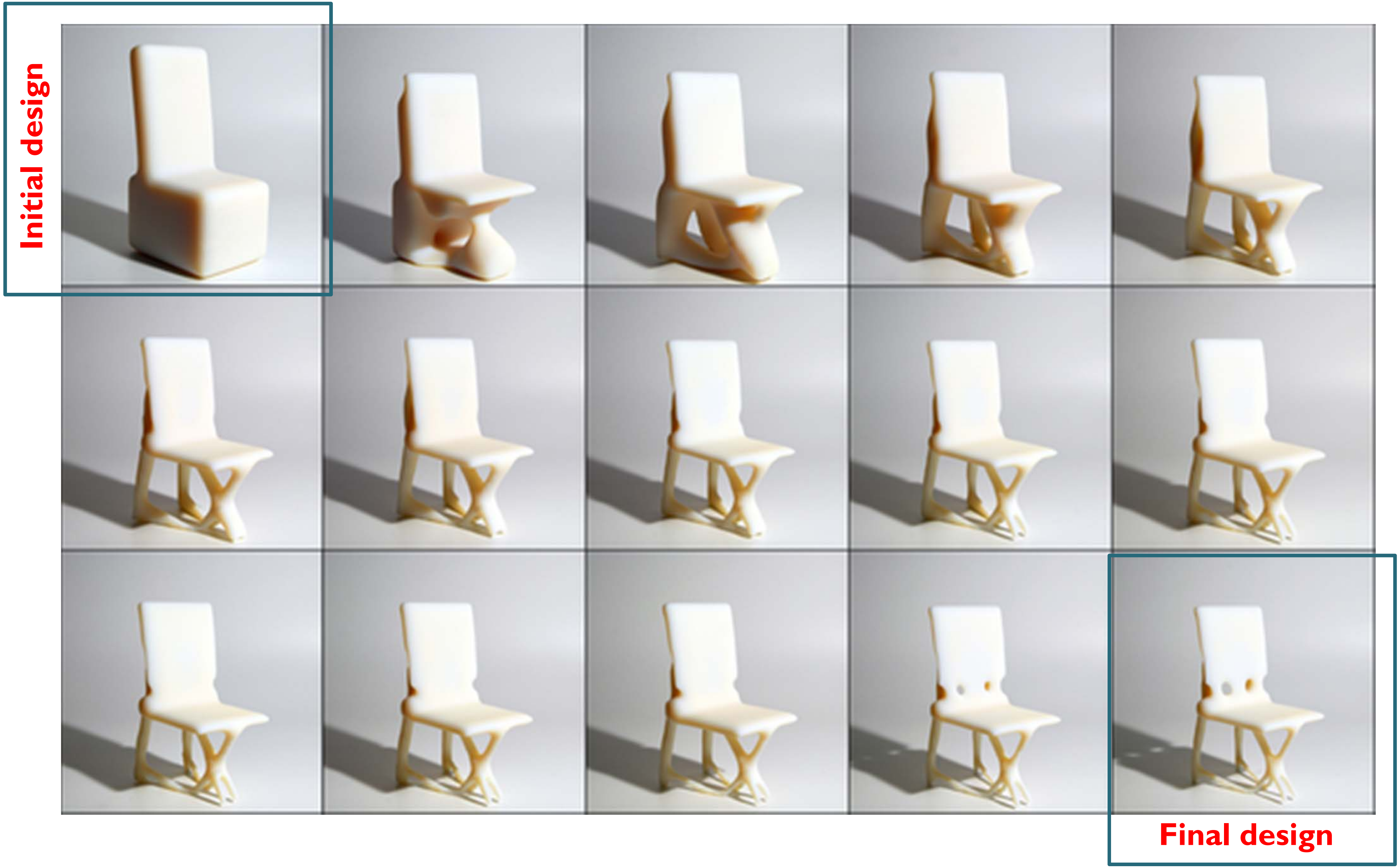
Twenty-three Lecture

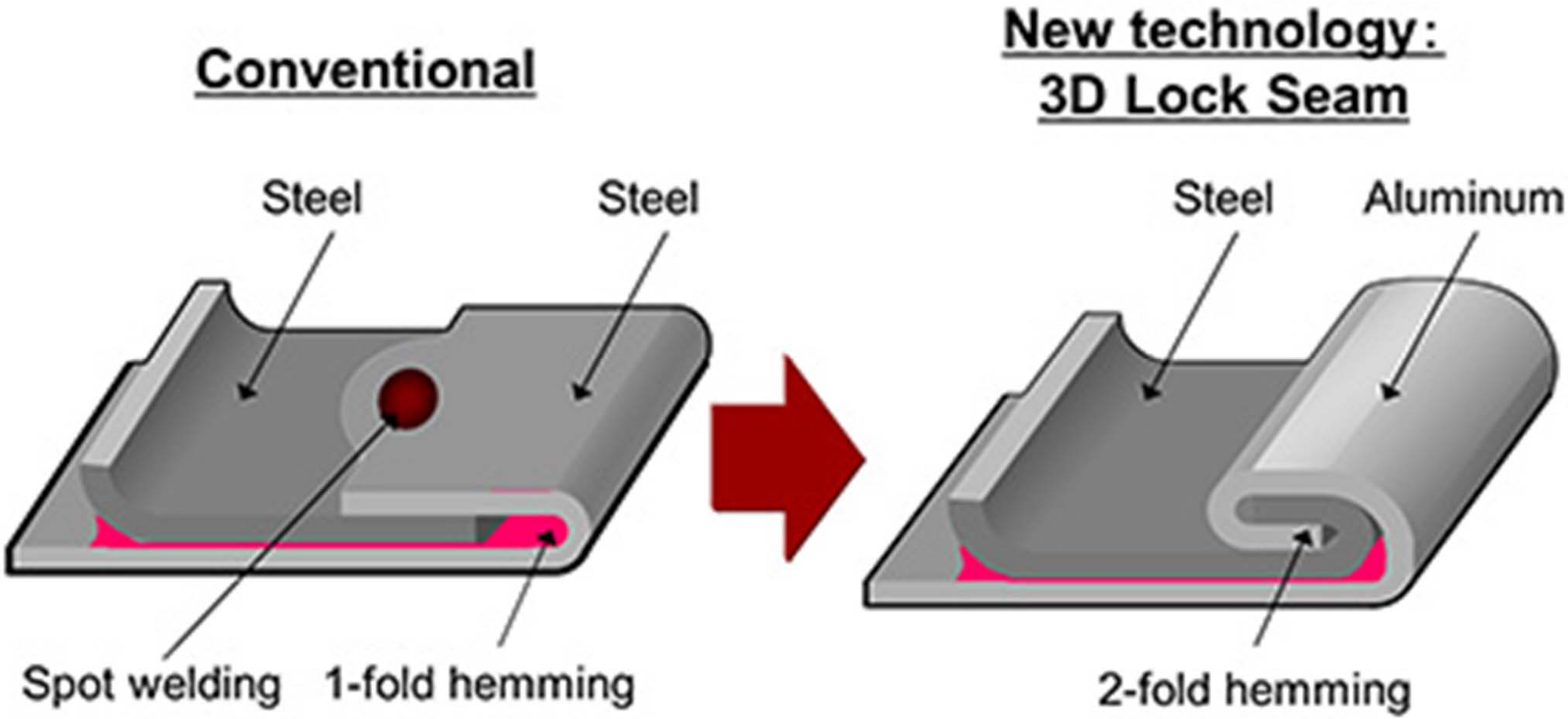
Introduction to Optimum Design


Design Optimization
CAD Definition
of Design Space
minimization
Undesirable effects
stresses
noise
vibration
deflection
weight
cost
time
Desirable effects
power transmitted
energy absorbed
overload capacity
factor of safety
maximization
Final CAD Geometry


CAD Definition
of Design Space
FE Model
Basic Topology
Result
Topology Optimization
within the Design Process
Final CAD Geometry
Smoothed and
Remeshed
With Manufacturing
Constraints


Optimized ribbed design
-20% weight
Improved WIT design

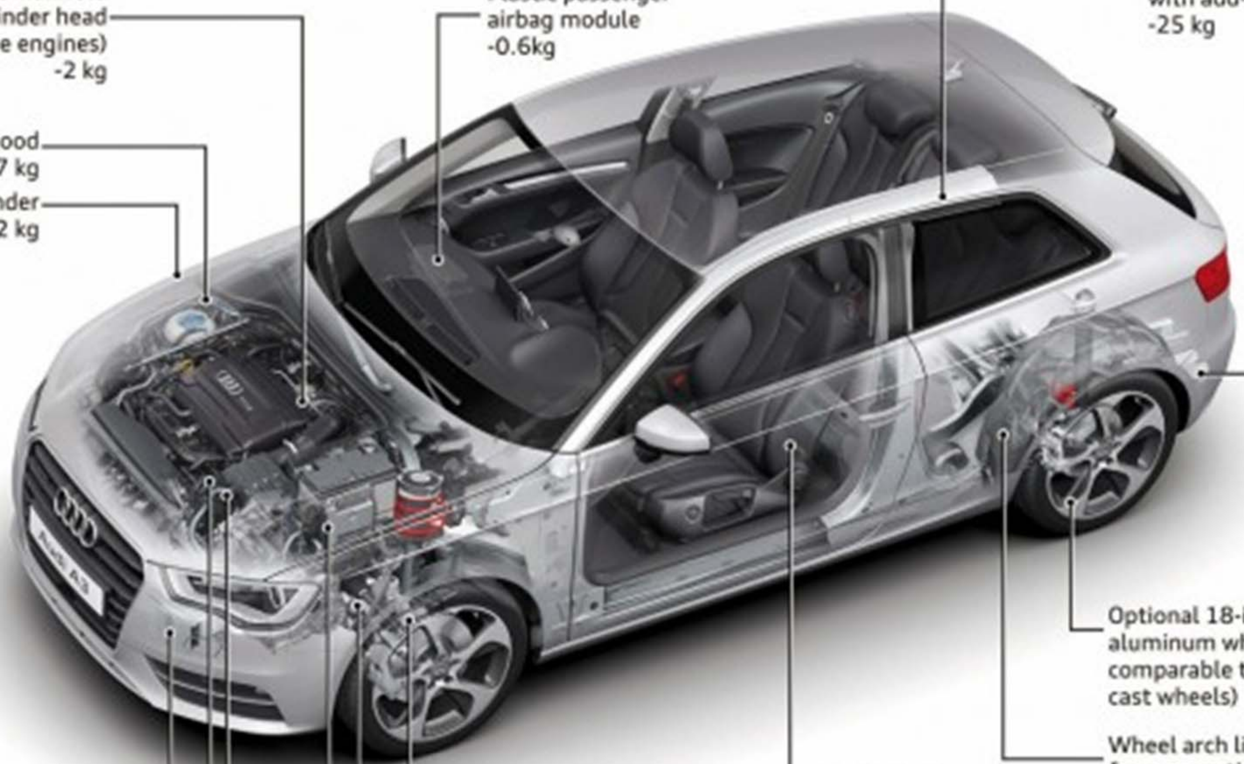

Initial design
Final design


Conventional
New technology:
3D Lock Seam
Steel
Steel
Steel
Aluminum
Spot welding
1-fold hemming
2-fold hemming

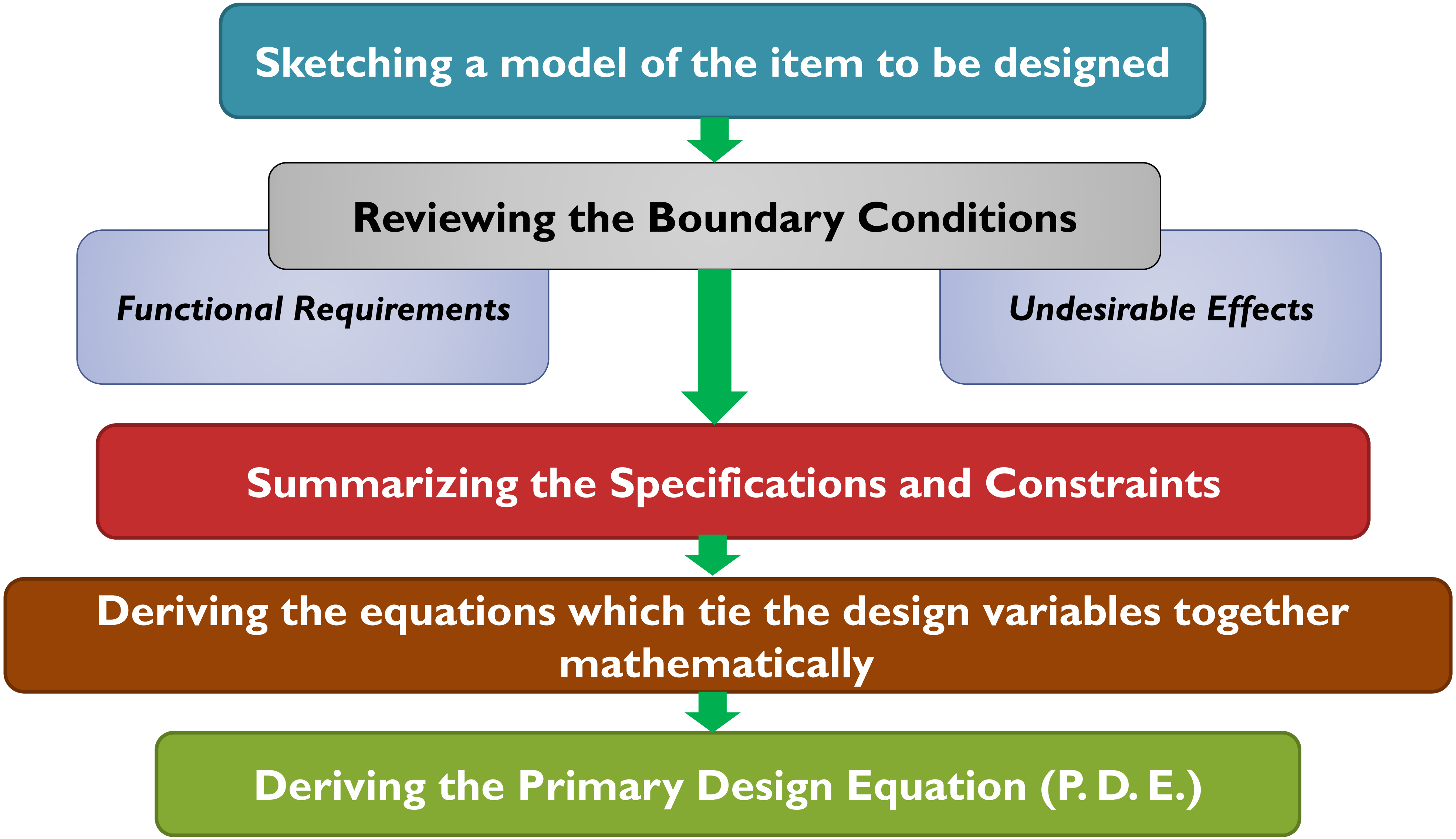
Weight reduction in the Audi A3

Main details

04/12

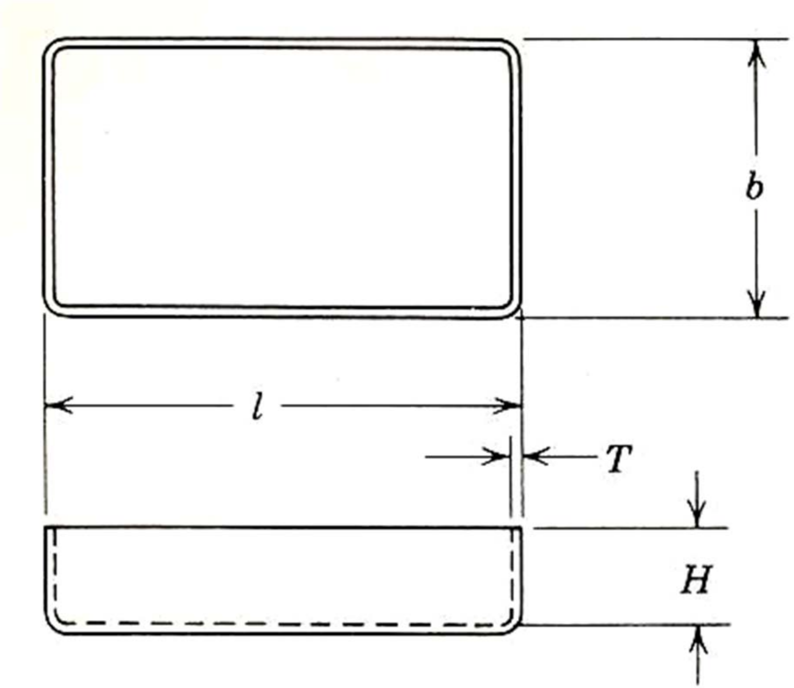


Method of Optimum Design, MOD



The Basic Design Problem:

Design a *plastic tray capable of holding a specified volume of liquid, (V), such that the liquid has a specified depth of (H), and the wall thickness of the tray is to be a specified thickness, (T). The tray is to be in large quantities.*



Adequate Design Solution:

$$V = b * l * H \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

1. **It is possible to know the value of (*l*) by choosing a value for (*b*).**
2. **It is possible to choose a certain type of material for the tray.**
3. **It is possible to choose an appropriate manufacturing method.**

Optimum Design Solution:

Since the tray is manufactured in large quantity, then:

1. **The most significant undesirable effect for this problem is *COST*.**
2. **The objective of this design is to minimize *COST*.**

The design should be the best one with respect to the following:

- **Geometry**
- **Material**
- **Manufacturing method**

The cost (C) of a tray may be written as:

$$C = C_o + C_t + C_l + C_m \text{.................................}(2)$$

, which is called the *Primary Design Equation* (*P. D. E.*), where:

C = total cost
C_o = overhead cost
C_t = tooling cost
C_l = labour cost
C_m = plastic material cost
Assume Vacuum-forming techniques will be our available manufacturing method. Hence, (*Co, Ct,* and *Cl*) are independent of reasonable geometrical shapes and feasible plastic materials.

$$C_m = c.\left((b.l) + (2.b.H) + (2.l.H)\right).T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

where: c = a unit volume of tray material, (ID / m3).

From equation (1) and equation (3a):

$$C_m = c.\left(\left(\frac{V}{H}\right) + (2.b.\mathrm{H}) + \left(\frac{2.V}{b}\right)\right).\mathrm{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(3b)}$$

$$\frac{\partial C_m}{\partial b} = c.T.\left((2.H) - \left(2.\frac{V}{b^2}\right)\right) = 2.c.T.\left(H - \frac{V}{b^2}\right) = 0$$

$$\therefore H = \frac{V}{b^2}$$

$$\therefore b_{opt.} = \sqrt{\frac{V}{H}}$$

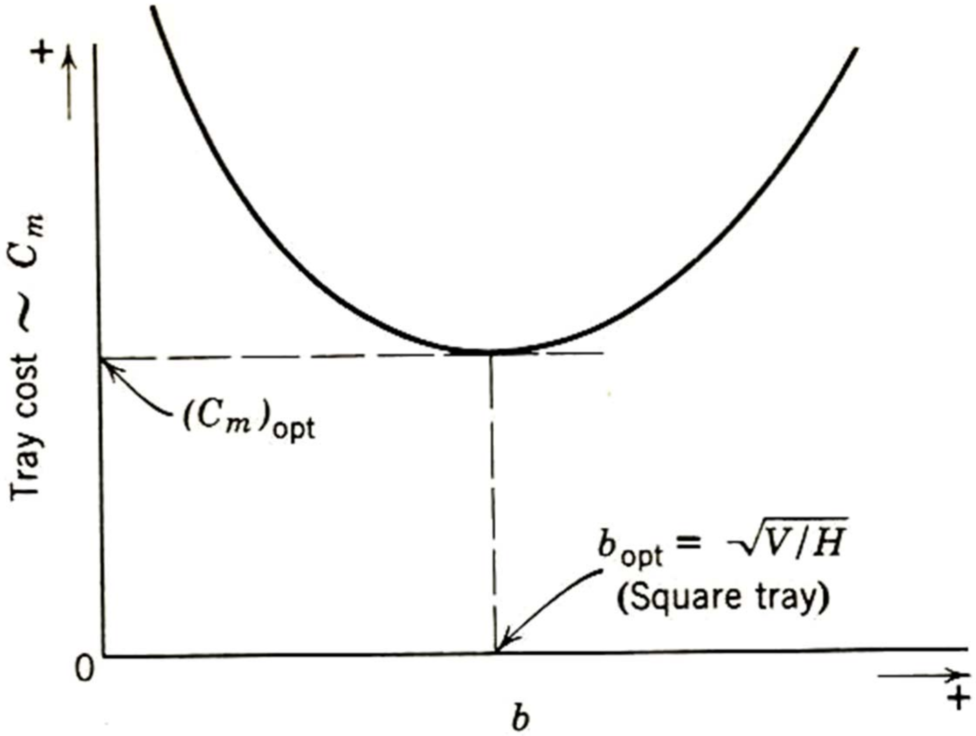
Now, $V = b * l * H$

$$\therefore l = \frac{V}{H.b} = \frac{V}{H.\sqrt{\frac{V}{H}}} = \sqrt{\frac{V}{H}}$$

$$\therefore l_{opt.} = b_{opt.} = \sqrt{\frac{V}{H}}$$

$$C_{m\,(opt)} = c.\left(\left(\frac{V}{H}\right) + \left(4.\sqrt{V.H}\right)\right).T$$

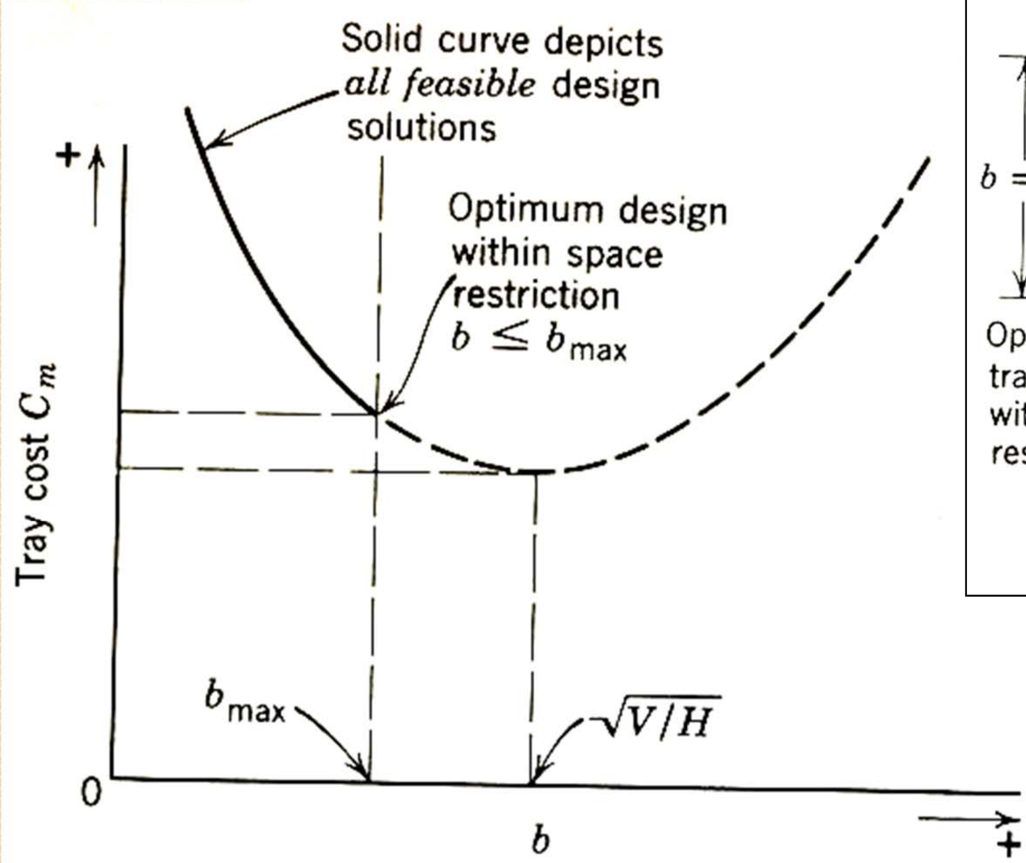
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(3c)$$

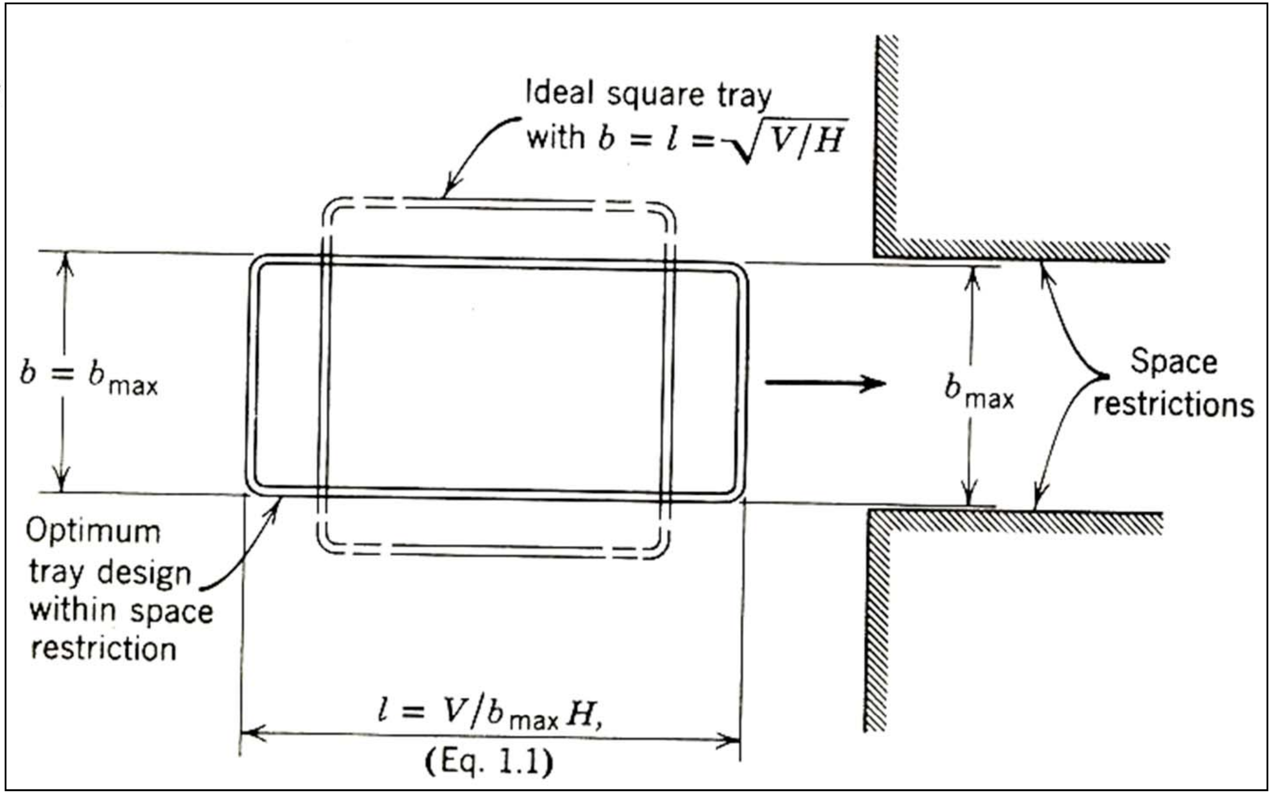


Optimum Design for Existing Restrictions:

Let:

$$b \le b_{max} \quad \text{and} \quad l \le l_{max}$$





Mechanical Engineering Design II

Twenty-four & Twenty-five Lectures

Summary of Design Equations in Optimum Design

Typical Schematic Representation of Optimum Design:

$$\overbrace{(\text{P.D.E.})}^{\substack{\text{Quantity to be}\\ \text{maximized or to}\\ \text{be minimized by}\\ \text{optimum design}}} = f\overbrace{\left[\begin{pmatrix}\text{Functional}\\ \text{requirement}\\ \text{parameters}\end{pmatrix}, \overbrace{\begin{pmatrix}\text{Material}\\ \text{parameters}\end{pmatrix}, \begin{pmatrix}\text{Geometrical}\\ \text{parameters}\end{pmatrix}}^{\substack{\text{Design parameters}\\ \text{(uniquely define element)}}}\right]}^{\text{Three } \textit{independent} \text{ groups}}$$

Individually specified or limited parameters

Individual parameters dependent on each other because of physics and chemistry of the material

Individually independent parameters

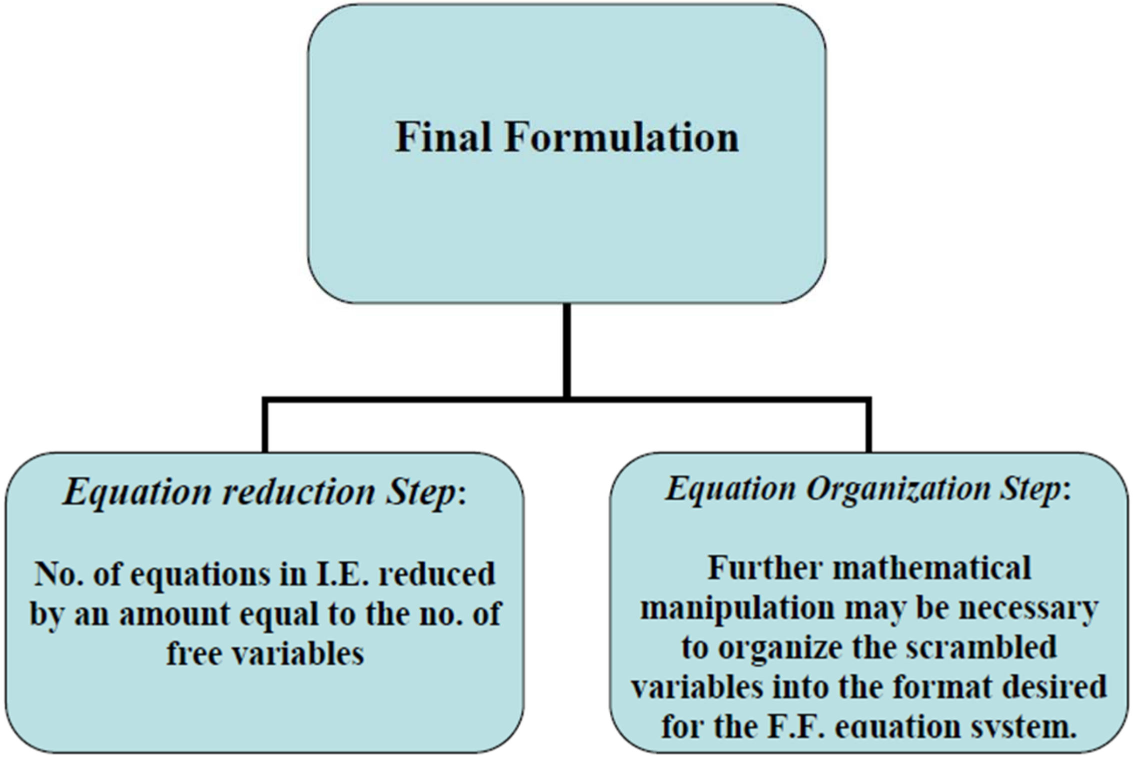
$$\underbrace{(\text{S.D.E.})}_{\substack{\text{Specified or}\\ \text{limited quantity}}} = g\left[\underbrace{\begin{pmatrix}\text{Functional}\\ \text{requirement}\\ \text{parameters}\end{pmatrix}}_{\substack{\text{Independent}\\ \text{group}}}, \underbrace{\begin{pmatrix}\text{Material}\\ \text{parameters}\end{pmatrix}, \begin{pmatrix}\text{Geometrical}\\ \text{parameters}\end{pmatrix}}_{\substack{\textit{Groups} \text{ are dependent on specified or}\\ \text{limited subsidiary design quantities } \textit{and}\\ \text{on optimum design of element}}}\right]$$

$$\underbrace{h_1}_{\substack{\text{Lower limit,}\\ \text{depending on}\\ \text{factors external}\\ \text{to element}}} \leq \underbrace{(P)}_{\substack{\text{Limited design}\\ \text{parameter or}\\ \text{limited (S.D.E.)}\\ \text{quantity}}} \leq \underbrace{h_2}_{\substack{\text{Upper limit,}\\ \text{depending on}\\ \text{factors external}\\ \text{to element}}}$$

Basic Procedural Steps for M.O.D.:

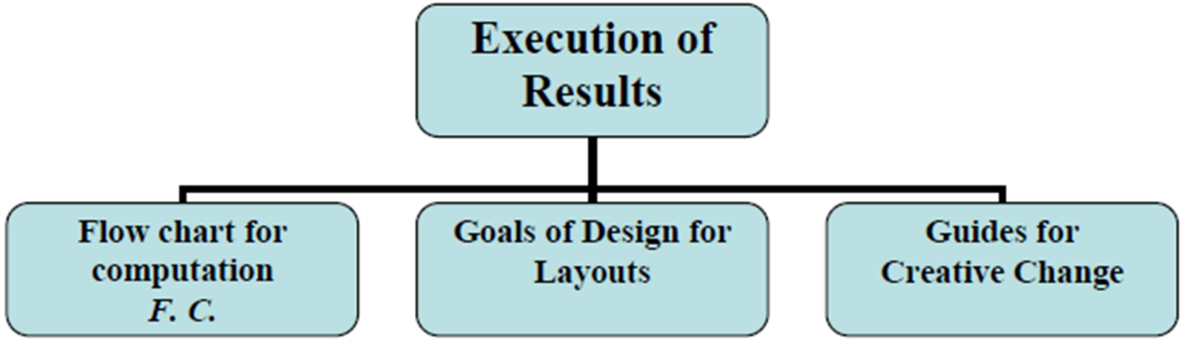
Successive steps of a systematic plan lead to the specification of the optimum design, which is summarized as follows:

1. **Initial Formulation, (*I.F.*): it is the summary of the initial system equations including (*P.D, E.*), (*S.D.E.*), and (*L.E.*)'s.**
2. **Final Formulation, (*F.F.*): it is a suitable transformation of (*I.F.*) for the use in the variation study step.**



3. Variation Study, (*V.S.*): in this step, the (*F.F.*) equation system is considered simultaneously along with the Constraints for general determination of the point's potential for Optimum Design. The sketching of Variation Diagram facilitates this step.

4. Execution of Results:



5. Evaluation of Optimum Design: here, the design is analyzed to determine what has been achieved numerically for the optimization quantity in order to confirm our acceptance of the design.

Types of Variables in I.F.:

- **Constraints parameters in number, (n_c), defined as the ones having either regional constraints or discrete value, and directly imposed by (*L.E.*)'s in the (*I.F.*).**

- **Free variables in number, (n_f), defined as, the one with no constraints directly imposed on them through (*L.E.*)'s in the (*I.F.*).**

- **Total number of the variables: $n_v = n_c + n_f$**

Types of Problems in M.O.D.:

There are basically, three classes of problems encountered in application of (*M.O.D.*). They are summarized briefly below:

1. *Case of Normal Specifications, N.S.*:

P.D.E. **is single, often an independent material selection factor, (*M.S.F.*), is recognized.**

N.S. types of problems is: [$n_f >= N_s$], where *Ns* = the number of (*S.D.E.*)'s.

2. *Case of Redundant Specifications, R.S.*:

- **Ignoring some of constraints on selected parameters that are called Eliminated Parameters.**
- **The *P.D.E.* designated as equation (I).**
- **The Eliminated Parameters must be expressed by what is called as the relating equations and designated as (II, III, IV, and so on), in (*F.F.*).**
- **The test for the (*R.S.*) type problem is [$n_f < N_s$].**

3. *Case of Incompatible Specifications*:

This is, in reality, nothing more than a special form of (*R.S.*).

There is merely no design solution that satisfy all constraints, and speed.

If boundary values changes, the domain of feasible design is opened and the optimum design can be determined by the (*M.O.D.*).

General Planning (I.F.) to (F.F.) in M.O.D.:

One of the most difficult details of execution lies in the transformation of the (*I.F.*) to (*F.F.*). Three general items are helpful in this respect. They will be outlined briefly below for the (R.S.) type of problem:

1. *Exploratory Calculations*:

$D_{vs} = n_v - N_s + 1$, where:

D_{vs} = number of dimensions required for a (*V.S.*) , where: $A_t = \frac{n_c!}{[n_e!(n_c - n_e)!]}$

A_t = number of different approaches

n_c = number of constraints variables

n_e = number of eliminated parameters

$n_e = N_s - n_f$

$n_r = n_c - n_e$, where: n_r = number of related parameters.

2. *Choosing the Approach*:

- Choice of the particular approach to take for derivation for the specific (*F.F.*), should be made after thought.
- Choose the simplest approach.

3. *General Format of (F.F.)*:

Summarizing the (F.F.) equations can be very helpful.

Example (1): Case of Normal Specifications

Simple tensile bar, (mass production manufacturing)

L = specified length
P = constant force
Optimum design required to minimizing cost

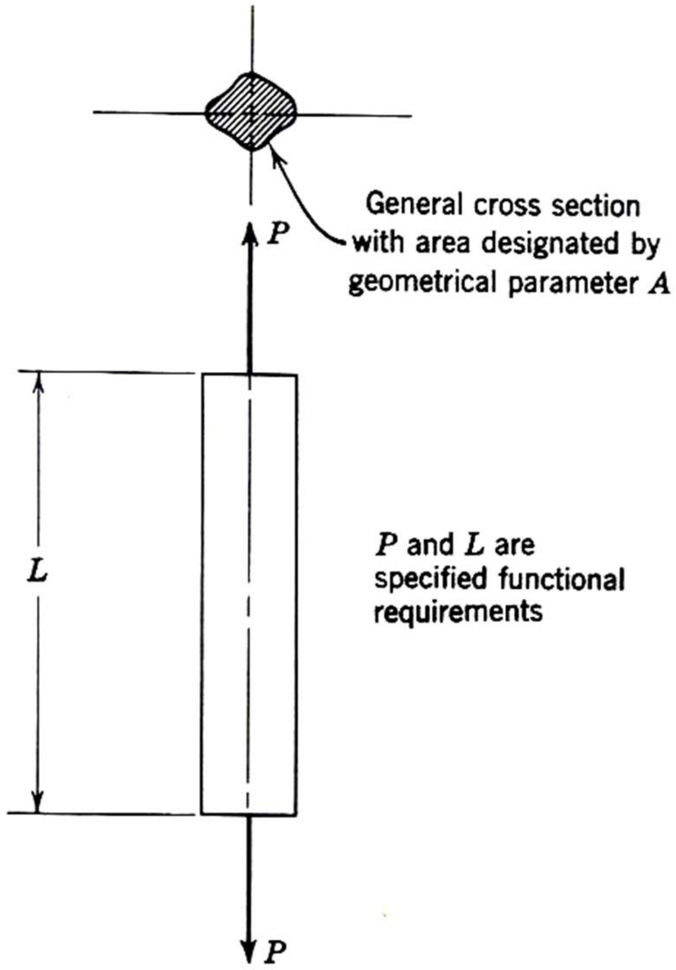
- Now, the (P.D.E.) is:

Cm = material cost =*c.V* = *c.(A.L)*, where:
Cm = cost of the bar material,
c = unit volume material cost

- The undesirable effect are the stress =σ=*P*/*A*

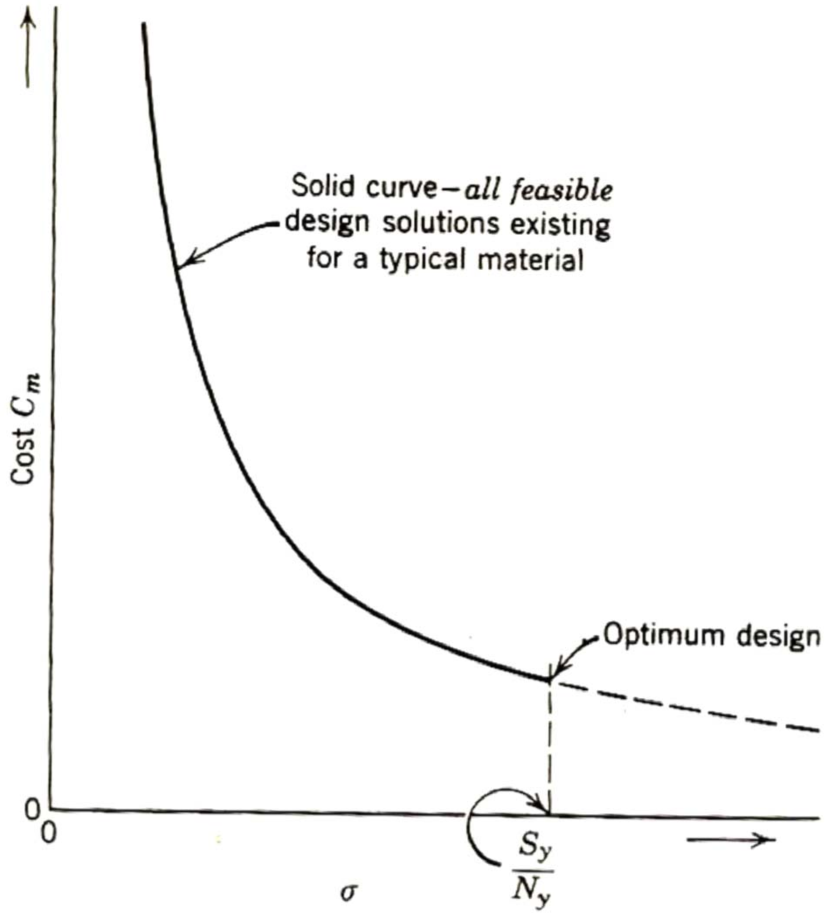
, which is the (S.D.E.).

- The limit equations are: $\sigma \leq \frac{S_y}{N_y}$



- Now, *P*, *L* and *Ny* are the Functional Requirements,
- *c* & *Sy* are the Material Parameters,
- *A* is the Geometrical Parameter,
- σ &*Cm* are the Undesirable Parameters.

- **Now, number of free variables = 1, which is (*A*), number of (S.D.E.) = 1, therefore $n_f = N_s$**



- **Now combine the (*S.D.E.*) with the (*P.D.E.*), eliminating the unlimited and unspecified geometrical parameter (*A*). thus:**

$$c_m = c \cdot L \cdot \left(\frac{P}{\sigma}\right),$$

$$c_m = c \cdot L \cdot \left(\frac{P}{S_y/N_y}\right) = (P \cdot L \cdot N_y) \cdot \left(\frac{C}{S_y}\right)$$ **which is the (*P.D.E.*).**

***P*, *L* and *Ny* are specified functional requirement, which is probably, cannot be changed.**

$\left(\frac{C}{S_y}\right)$, **is the (M.S.F.) (Material Selection Factor) which is the independent group.**

- **Finally, for determining the optimum value of (*C.S.A.*), (*A*), we would calculate very simply the optimum value of the area (*A*).**

Example (2): Case of Redundant Specifications

Simple tensile bar, (mass production manufacturing), with: $A \geq A_{min}$

Now, the (*I.F.*) is:

$$C_m = C \cdot L \cdot A \qquad \textbf{(P.D.E.)}$$

$$\sigma = \frac{P}{A} \qquad \textbf{(S.D.E.)}$$

$$\sigma \leq \frac{S_y}{N_y} \qquad \textbf{(L.E.)}$$

$$A \geq A_{\min} \qquad \textbf{(L.E.)}$$

- **In this case, it is impossible to combine (*S.D.E.*) with (*P.D.E.*). Now:**

$n_f = 0$ **and** $N_s = 1$

Therefore, the case is a Redundant Specification.

- **There are two approaches:**

1. **Ignore the (*S.D.E.*)**
2. **Ignore the (*L.E.*), on stress or in (*A*).**

***Exploratory calculations*:**

There are two approaches possible for (*F.F.*)'s.

Our (*V.S.*) will be two dimensional in character.

- **Say the simplest approach:**

  σ is the eliminated parameter

  ***A* is the related parameter.**

$$n_f = o \ \& \ N_s = 1 \ \& \ n_v = n_c + n_f = 2$$

$$n_e = N_s - n_f = 1 - 0 = 1 \ \& \ n_r = n_c - n_e = 2 - 1 = 1$$

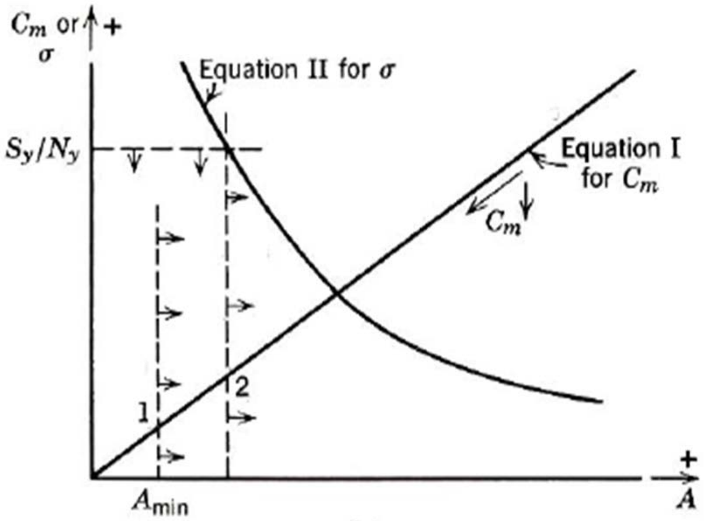
$$D_{vs} = n_v - N_s + 1 = 2 - 1 + 1 = 2$$

$$A_t = \frac{n_c\cdot!}{[n_e!\cdot(n_c - n_e)!]} = \frac{2\cdot!}{1\cdot!\cdot(2-1)\cdot!} = 2$$

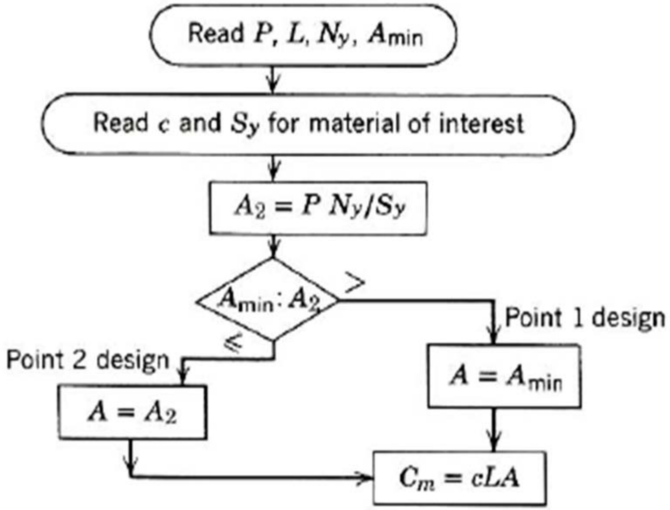
The (F.F.) is the same as the (I.F.)

$$C_m = C \cdot L \cdot A \qquad (I)$$

$$\sigma = \frac{P}{A} \qquad (II)$$



(a)



(b)

Example (3): Case of Redundant Specifications

Simple tensile bar, (mass production manufacturing), with:

$A \geq Amin$,
$\Delta \geq \Delta max$,
$Lmin \leq L \leq Lmax$

The (*I.F.*) is:

$$C_m = cLA \qquad \text{(P.D.E.)}$$

$$\sigma = \frac{P}{A} \qquad \text{(S.D.E.)}$$

$$\Delta = \frac{PL}{(AE)} \qquad \text{(S.D.E.)}$$

$$\sigma \leq \frac{S_y}{N_y} \qquad \text{(L.E.)}$$

$$A \geq A_{\min} \qquad \text{(L.E.)}$$

$$\Delta \leq \Delta_{\max} \qquad \text{(L.E.)}$$

$$L_{\min} \leq L \leq L_{\max} \qquad \text{(L.E.)}$$

The (*V.S.*) are:

$$n_f = o < N_s = 2$$

$$n_v = n_c = 4$$

$$n_e = 2 - 0 = 2$$

$$n_r = 4 - 2 = 2$$

$$D_{vs} = 4 - 2 + 1 = 3$$

$$A_t = \frac{n_c \cdot !}{[n_e ! \cdot (n_c - n_e)!]} = \frac{4 \cdot !}{2 \cdot ! \cdot (4 - 1) \cdot !} = 6$$

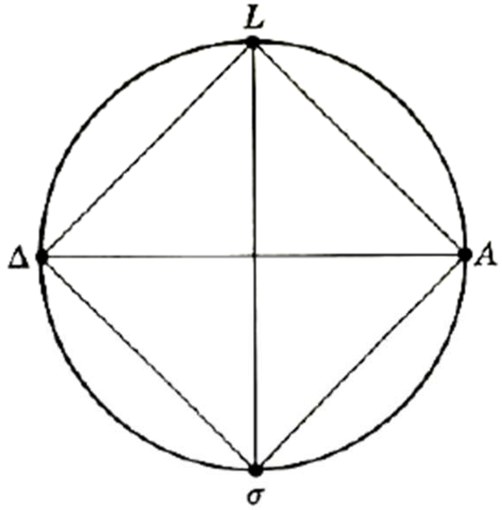


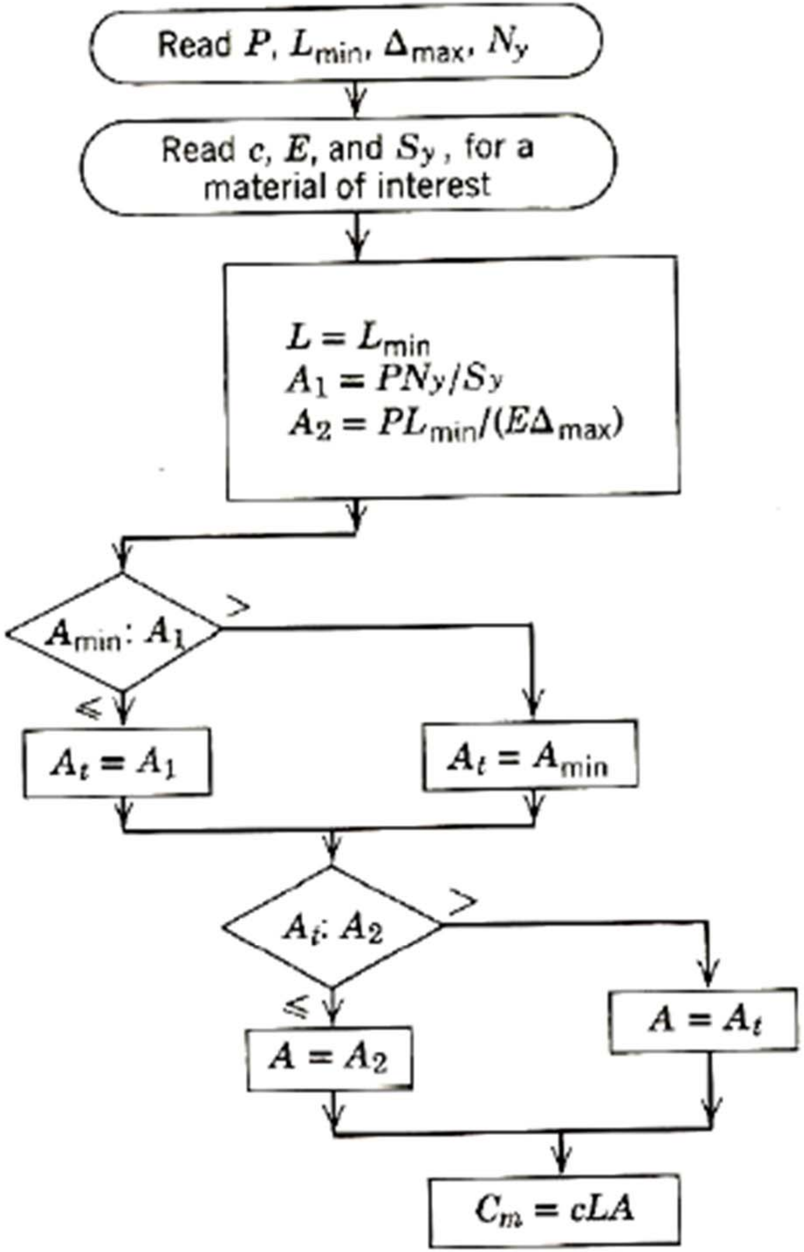
Figure 6.11 Circle diagram for example 6.4.

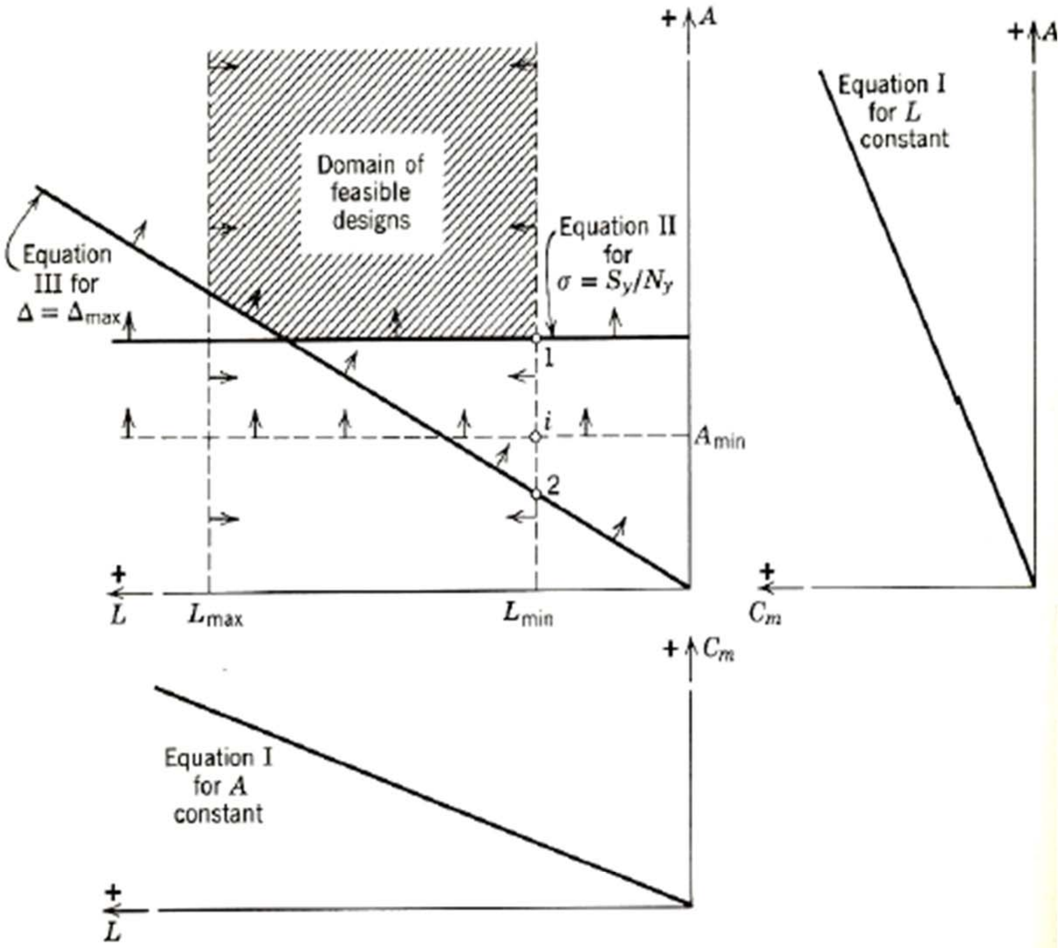
Therefore, the (*F.F.*) are:

$$C_m = C \cdot L \cdot A \qquad (I)$$

$$\sigma = \frac{P}{A} \qquad (II)$$

$$\Delta = \frac{P \cdot L}{A \cdot E} \qquad (III)$$


Read P, L_{min}, Δ_{max}, N_y
Read c, E, and S_y, for a material of interest
$L = L_{min}$
$A_1 = PN_y/S_y$
$A_2 = PL_{min}/(E\Delta_{max})$
$A_{min} : A_1$
$>$
$\leqslant$
$A_t = A_1$
$A_t = A_{min}$
$A_t : A_2$
$>$
$\leqslant$
$A = A_2$
$A = A_t$
$C_m = cLA$


Domain of feasible designs
Equation III for $\Delta = \Delta_{max}$
Equation II for $\sigma = S_y/N_y$
1
i
2
A_{min}
$+ A$
$+ L$
L_{max}
L_{min}
Equation I for L constant
$+ A$
$+ C_m$
$+ C_m$
Equation I for A constant
$+ L$