Subject : Mechanics I
Weekly Hours : Theoretical: 2 UNITS:5
Tutorial: 1
Experimental : 1

موضوع : ميكانيك 1
الساعات الأسبوعية : نظري :2 الوحدات :5
مناقشة : 1
عملي :1

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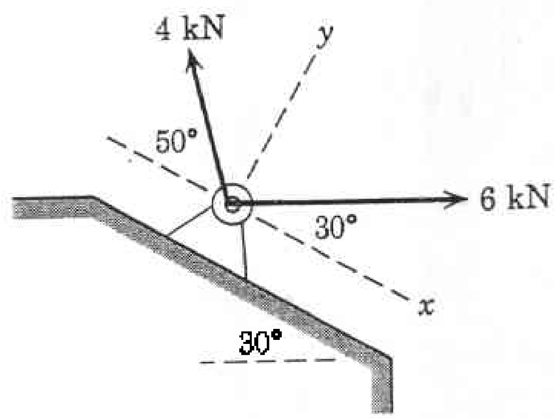
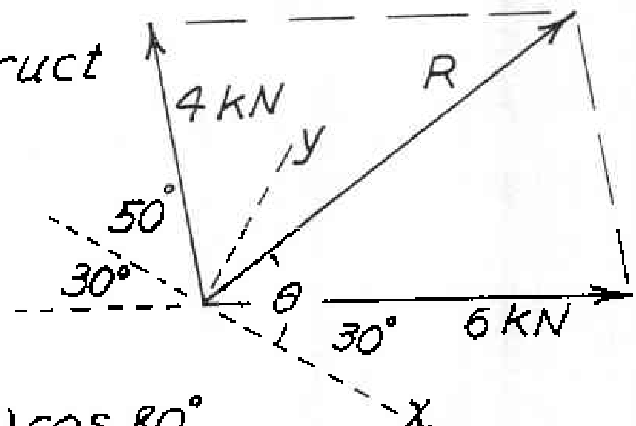
Replace the two forces by a single equivalent force $\underline{R}$ and find the angle θ between $\underline{R}$ and the x-axis. Solve both geometrically and by using unit vectors $\underline{i}$ and $\underline{j}$.



Geometric

Graphical: construct parallelogram & measure R & θ.



Trigonometric:

Law of cosines

$R^2 = 4^2 + 6^2 - 2(4)(6)\cos 80°$

$= 43.7$, $\boxed{R = 6.61 \text{ kN}}$

$4^2 = (6.61)^2 + 6^2 - 2(6.61)(6)\cos(\theta - 30°)$

$\theta - 30° = \cos^{-1} 0.8029 = 36.6°$, $\boxed{\theta = 66.6°}$

Vector algebra

$R_x = 6\cos 30° - 4\cos 50° = 2.63$ kN

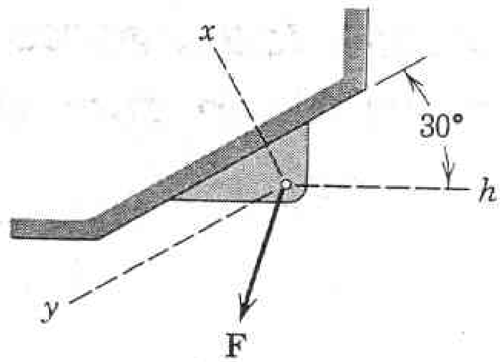
$R_y = 6\sin 30° + 4\sin 50° = 6.06$ kN

$\boxed{\underline{R} = 2.63\underline{i} + 6.06\underline{j} \text{ kN}}$, $\theta = \tan^{-1}\dfrac{6.06}{2.63} = \boxed{66.6°}$

JLM

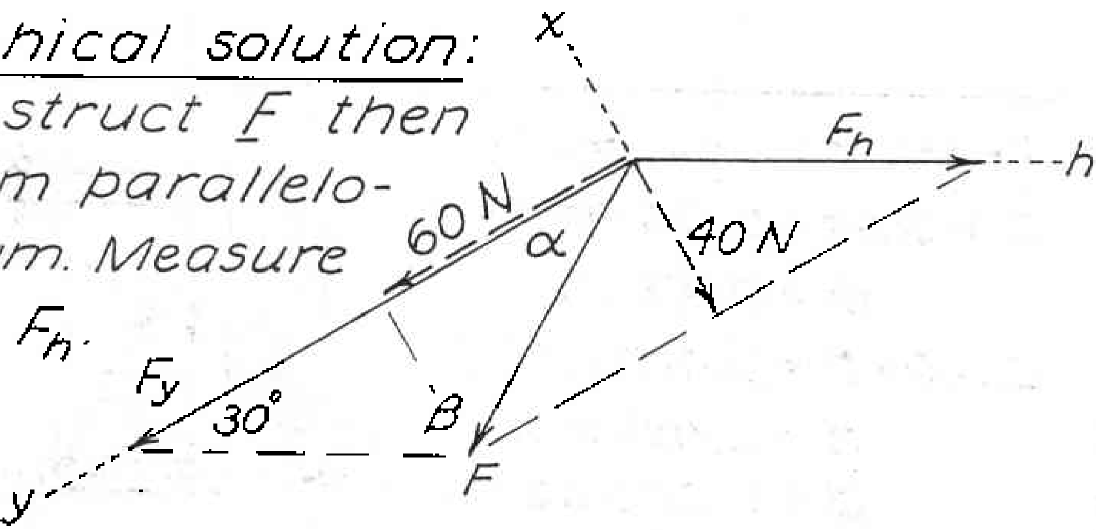
ART. 2/3 RECTANGULAR COMPONENTS (2-D)

Force $\underline{F}$ in rectangular components is given by $\underline{F} = -40\underline{i} + 60\underline{j}$ N. Determine the non-rectangular components of $\underline{F}$ in the y- and h-directions.



Graphical solution:

Construct $\underline{F}$ then form parallelogram. Measure F_y & F_h.



Trigonometric solution:

$$\alpha = \tan^{-1}\frac{40}{60} = 33.7^\circ,\quad F = \sqrt{(40)^2 + (60)^2} = 72.1 \text{ N}$$

Law of sines $\dfrac{72.1}{\sin 30^\circ} = \dfrac{F_h}{\sin 33.7^\circ}$, $\boxed{F_h = 80.0 \text{ N}}$

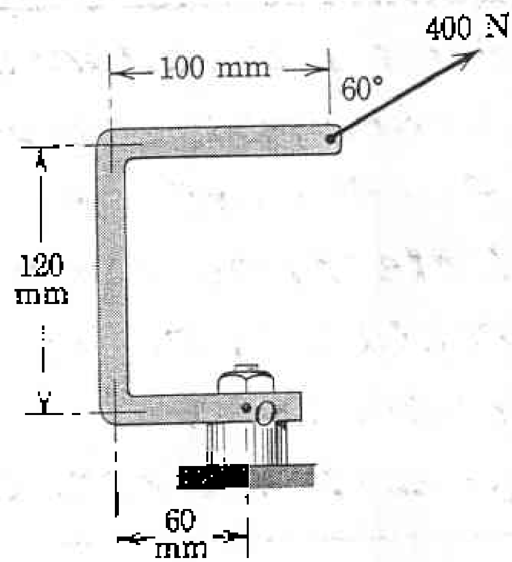
$$\beta = 180 - 30 - 33.7 = 116.3^\circ$$

$$\frac{F_y}{\sin 116.3^\circ} = \frac{72.1}{\sin 30^\circ},\quad \boxed{F_y = 129.3 \text{ N}}$$

JLM

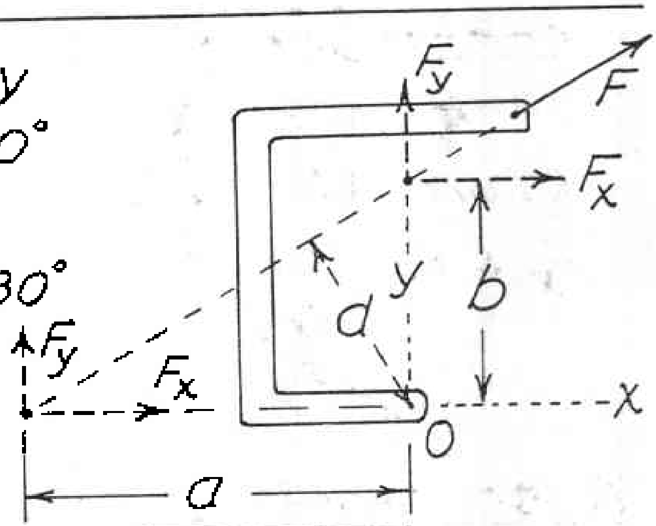
ART. 2/4 MOMENT (2-D)

Calculate the moment of the 400-N force about point O in five different ways.



From the geometry

$$a + 0.04 = 0.120 \tan 60^\circ$$
$$a = 0.168 \text{ m}$$
$$0.120 - b = 0.040 \tan 30^\circ$$
$$b = 0.0969 \text{ m}$$
$$d = b \cos 30^\circ$$
$$= 0.0839 \text{ m}$$



(I) $M_O = Fd = 400(0.0839) = \boxed{33.6 \text{ N·m}}$

(II) $M_O = 400(0.12 \sin 60^\circ - 0.040 \cos 60^\circ) = \boxed{33.6 \text{ N·m}}$

(III) $M_O = F_x b = 400 \sin 60^\circ (0.0969) = \boxed{33.6 \text{ N·m}}$

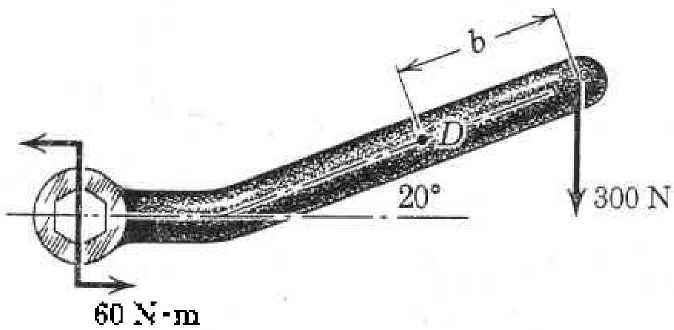
(IV) $M_O = F_y a = 400 \cos 60^\circ (0.168) = \boxed{33.6 \text{ N·m}}$

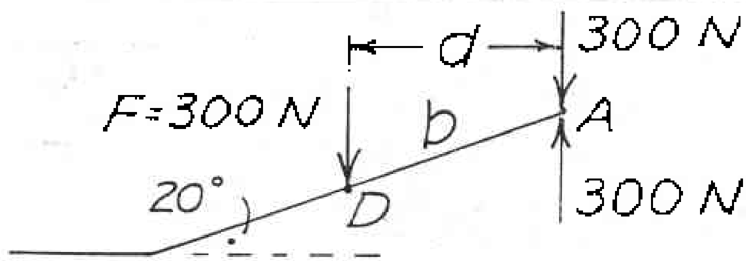
(V) $\underline{M}_O = \underline{r} \times \underline{F} = (0.04\underline{i} + 0.12\underline{j}) \times 400(\underline{i} \sin 60^\circ + \underline{j} \cos 60^\circ)$

$= 8\underline{k} - 41.6\underline{k} = \boxed{-33.6\underline{k} \text{ N·m}}$

JLM

Replace the force and couple acting on the wrench by a single equivalent force F applied at D. Determine b.





Replace 60-N·m couple by an equivalent couple consisting of two 300-N forces a distance d apart placed to cancel the given force. Thus, resultant is $\boxed{F = 300\ N}$ located at D where

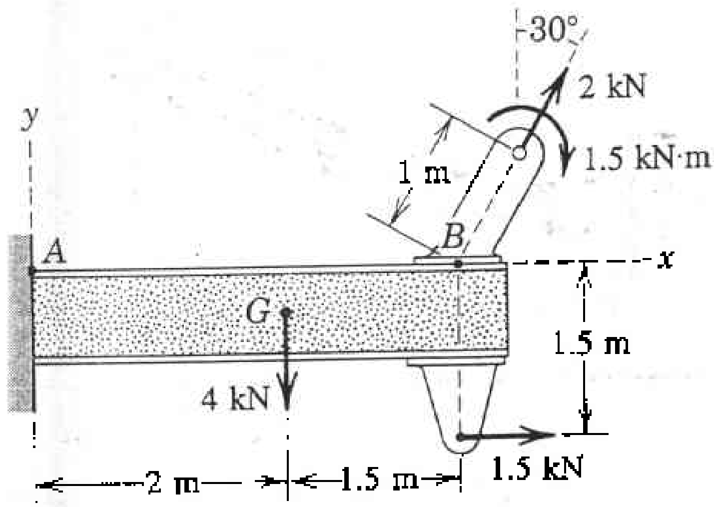
$$M_A = Fd;\quad 60 = 300d,\quad d = 0.2\ m$$

$$b = 0.2/\cos 20^\circ = 0.213\ m \text{ or } \boxed{b = 213\ mm}$$

JLM

ART. 2/6 RESULTANTS (2-D)

Represent the resultant of the three forces and one couple by an equivalent force $\underline{R}$ at A and a couple M. Find M and the magnitude and direction of $\underline{R}$.

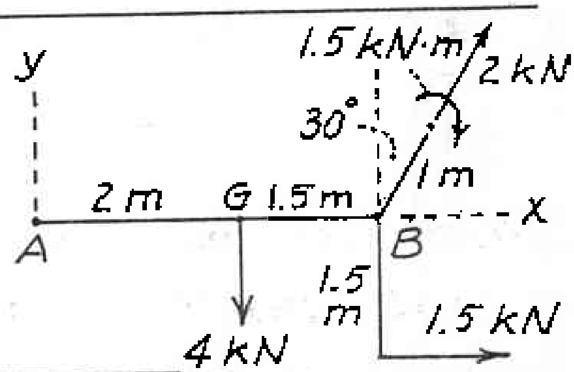


$R_x = \Sigma F_x = 2\sin 30° + 1.5$

$= 2.5 \text{ kN}$

$R_y = \Sigma F_y = 2\cos 30° - 4$

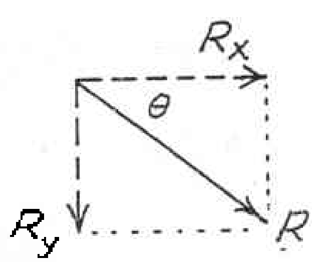
$= -2.27 \text{ kN}$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.5^2 + 2.27^2} = \boxed{3.38 \text{ kN}}$

$M = \Sigma M_A = 1.5 + 4(2) - 2\cos 30°(2+1.5) - 1.5(1.5)$

$= \boxed{1.188 \text{ kN·m CW}}$



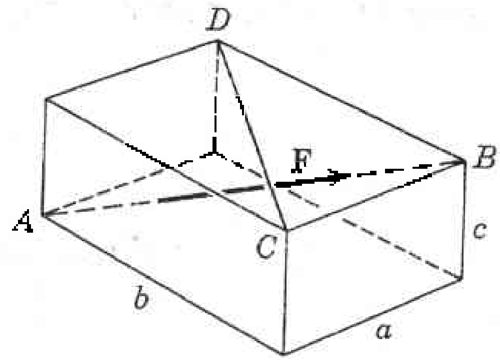
$\theta = \tan^{-1}\frac{2.27}{2.5}$

$= \boxed{42.2°}$

JLM

ART. 2/7 RECTANGULAR COMPONENTS (3-D)

For $a = 3$ m, $b = 6$ m, $c = 2$ m, $F = 10$ kN, determine the magnitudes of the components of F along AC and AD and the projection of F along DC.



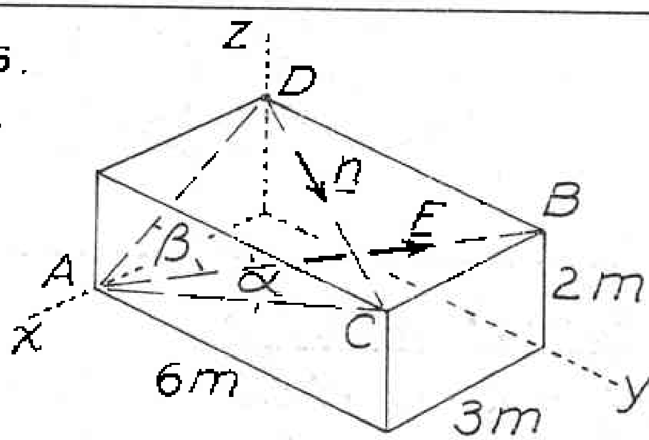
Choose x-y-z axes.

$AB = \sqrt{3^2 + 6^2 + 2^2} = 7$ m

$AC = \sqrt{2^2 + 6^2} = 2\sqrt{10}$ m

$AD = \sqrt{2^2 + 3^2} = \sqrt{13}$ m

$DC = \sqrt{3^2 + 6^2} = 3\sqrt{5}$ m



$$F_{AC} = F\cos\alpha = 10\,\frac{2\sqrt{10}}{7} = \boxed{9.04 \text{ kN}}$$

$$F_{AD} = F\cos\beta = 10\,\frac{\sqrt{13}}{7} = \boxed{5.15 \text{ kN}}$$

Let $\underline{n}$ = unit vector along DC $= \frac{3}{3\sqrt{5}}\underline{i} + \frac{6}{3\sqrt{5}}\underline{j}$

$$= \frac{1}{\sqrt{5}}(\underline{i} + 2\underline{j})$$

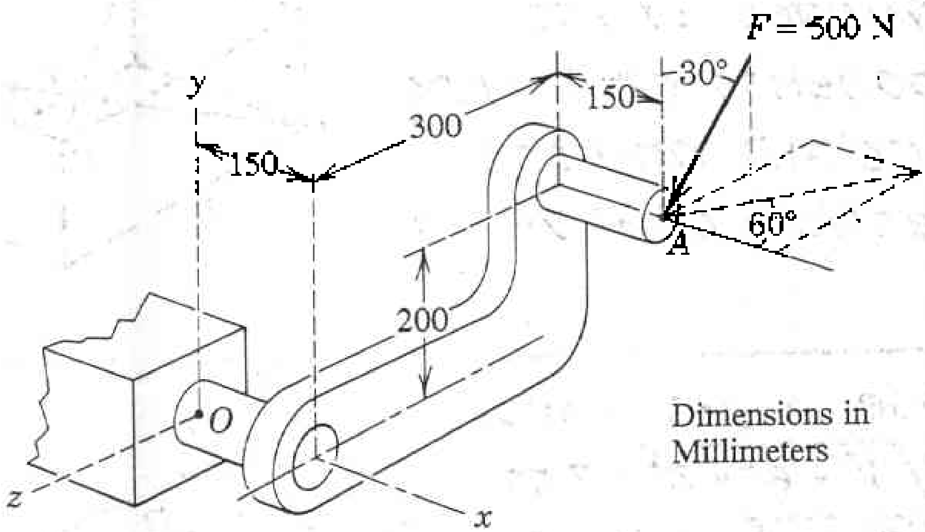
$$\underline{F} = 10\left(\frac{-3}{7}\underline{i} + \frac{6}{7}\underline{j} + \frac{2}{7}\underline{k}\right) \text{ kN}$$

$$F_{DC} = \underline{F}\cdot\underline{n} = \frac{10}{7}(-3\underline{i} + 6\underline{j} + 2\underline{k})\cdot\frac{1}{\sqrt{5}}(\underline{i} + 2\underline{j})$$

$$= \frac{10}{7\sqrt{5}}(-3 + 12) = \boxed{5.75 \text{ kN}}$$

JLM

Determine the moment of the 500-N force F about the x-axis.



Scalar solution

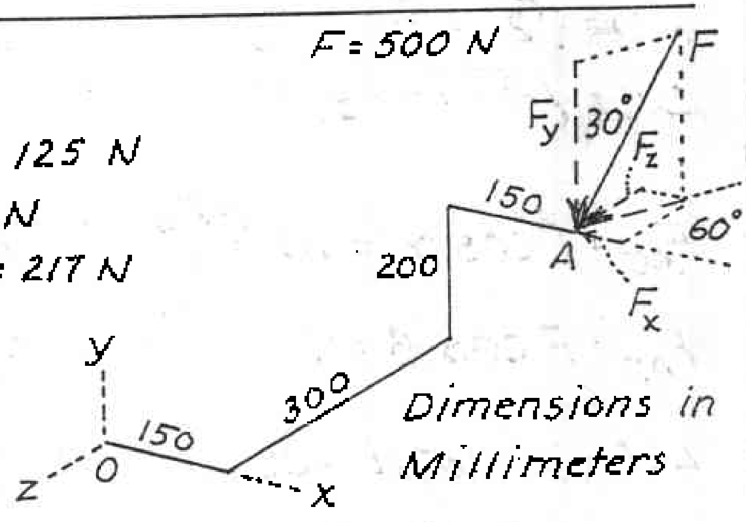
$|F_x| = 500 \sin 30° \cos 60° = 125$ N

$|F_y| = 500 \cos 30° = 433$ N

$|F_z| = 500 \sin 30° \sin 60° = 217$ N

$M_x = -433(0.3) + 217(0.2)$

$= \boxed{-86.6 \text{ N·m}}$



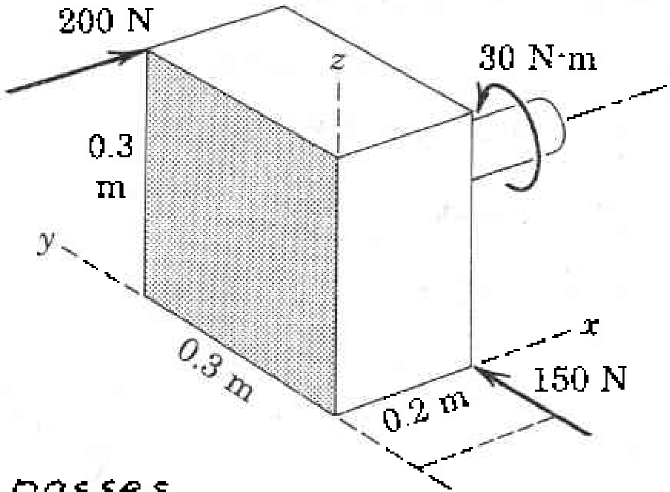
Vector solution

$\underline{M}_o = \underline{r}_A \times \underline{F}$ where $\underline{r}_A = 0.3\underline{i} + 0.2\underline{j} - 0.3\underline{k}$ m

$$M_x = \underline{M}_o \cdot \underline{i} = \underline{r}_A \times \underline{F} \cdot \underline{i} = \begin{vmatrix} 0.3 & 0.2 & -0.3 \\ -125 & -433 & 217 \\ 1 & 0 & 0 \end{vmatrix} = \boxed{-86.6 \text{ N·m}}$$

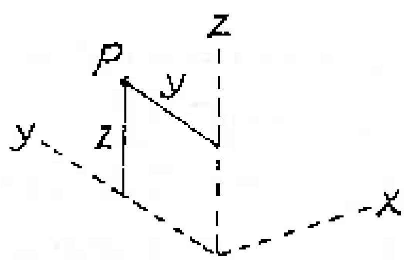
JLM

Replace the two forces and couple by a wrench. Find the moment $\underline{M}$ of the wrench and the coordinates of point P in the y-z plane through which the force of the wrench passes.



$\underline{R} = \Sigma \underline{F} = 200\underline{i} + 150\underline{j}$ N

Assume positive wrench so direction cosines of $\underline{M}$ are those of $\underline{R}$ or 0.8, 0.6, 0



$$\Sigma \underline{M}_P = 200(0.3 - z)\underline{j} - 200(0.3 - y)\underline{k} + 150z\underline{i} + 150(0.2)\underline{k} - 30\underline{i}$$

$$= (-30 + 150z)\underline{i} + (60 - 200z)\underline{j} + (-30 + 200y)\underline{k} \quad \text{N·m}$$

Equate direction cosines of $\Sigma \underline{M}_P$ & $\Sigma \underline{F}$ & get

$$\left.\begin{array}{l} (-30+150z)/M = 0.8 \\ (60-200z)/M = 0.6 \\ (-30+200y)/M = 0 \end{array}\right\}$$

where M equals the magnitude of $\Sigma \underline{M}_P$

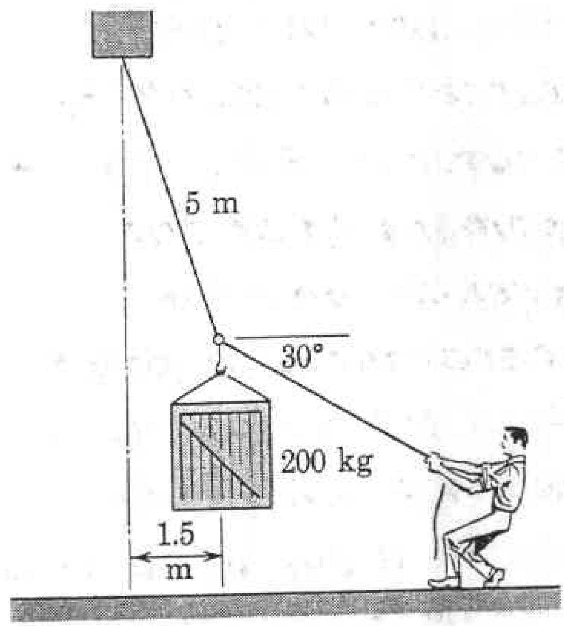
Solve & get $y = 0.15$ m or $\boxed{y = 150 \text{ mm}}$

$z = 0.264$ m or $\boxed{z = 264 \text{ mm}}$

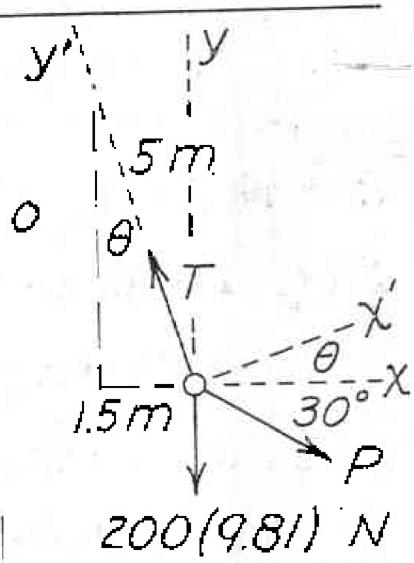
& $M = (-30 + 150[0.264])/0.8 = 12$ N·m, $\boxed{\underline{M} = 12(0.8\underline{i} + 0.6\underline{j}) \text{ N·m}}$

JLM

Determine the pull P on the rope exerted by the man to hold the crate in the position shown. Also find the tension T in the upper rope.



Solution (I) x-y axes



$\Sigma F_x = 0$: $0.866P - \frac{1.5}{5}T = 0$

$\Sigma F_y = 0$: $-0.5P - 200(9.81) + 0.954T = 0$

Solve simultaneously & get

$\boxed{P = 871\text{ N},\ \ T = 2513\text{ N}}$

Solution (II) x'-y' axes

$\Sigma F_{x'} = 0$:

$P\cos(30° + 17.45°) - 200(9.81)\frac{1.5}{5} = 0$

$\boxed{P = 871\text{ N}}$

$\theta = \sin^{-1}\frac{1.5}{5} = 17.45°$

$\cos 17.45° = 0.954$

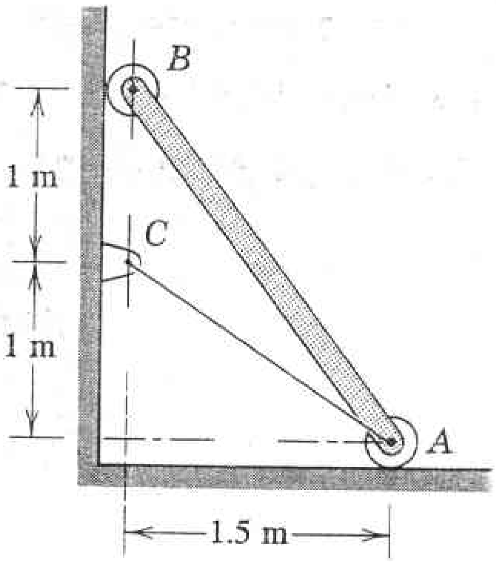
$\Sigma F_{y'} = 0$:

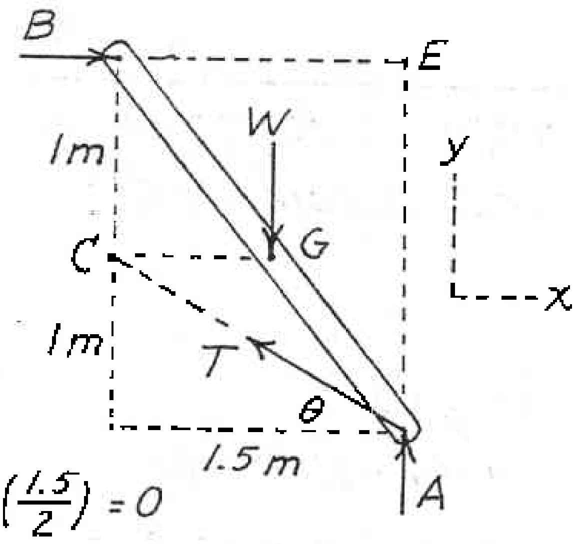
$T - 871\sin(30° + 17.45°) - 200(9.81)\cos 17.45° = 0$

$\boxed{T = 2513\text{ N}}$

JLM

The uniform 40-kg bar with small end rollers is supported by the horizontal and vertical surfaces and by wire AC. Calculate the tension T in the wire and the forces at A and B. Solve by using two moment equations and one force equation.





$W = mg = 40(9.81) = 392\ N$

$\theta = \tan^{-1}\frac{1}{1.5} = 33.7^\circ$

$\circlearrowright\!+\ \Sigma M_A = 0:\quad 2B - 392\left(\frac{1.5}{2}\right) = 0$

$\boxed{B = 147.2\ N}$

$\circlearrowright\!+\ \Sigma M_E = 0:\quad (T\cos 33.7^\circ)2 - 392\left(\frac{1.5}{2}\right) = 0$

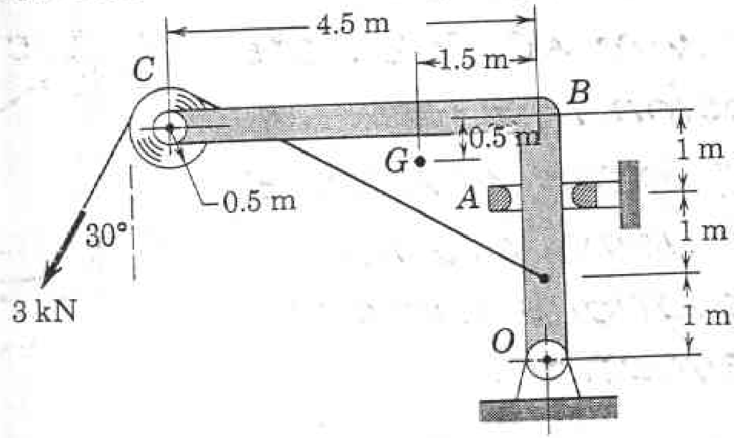
$\boxed{T = 176.9\ N}$

$\Sigma F_y = 0:\quad A + 176.9\sin 33.7^\circ - 392 = 0$

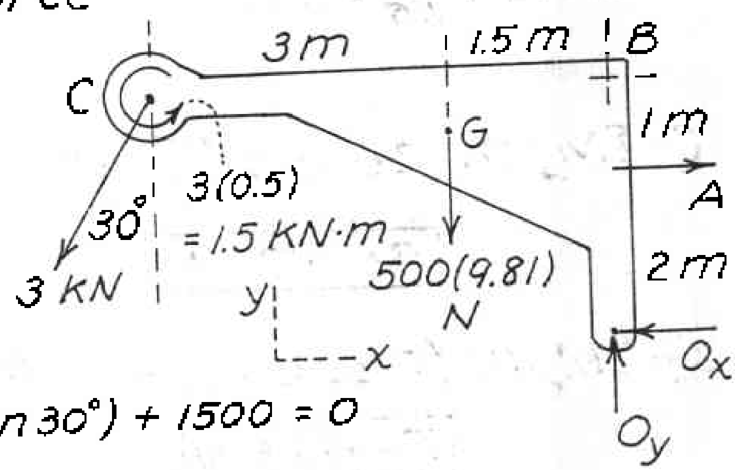
$\boxed{A = 294\ N}$

JLM

Member OBC and sheave C have a mass of 500kg with mass center at G. Calculate the magnitude of the force supported by the pin at O. Collar A provides horizontal support only.



Replace force by force and couple at C.



$\Sigma M_O = 0$:

$500(9.81)(1.5) - 2A$

$+ 3000(4.5\cos 30° + 3\sin 30°) + 1500 = 0$

$A = 12\ 524\ N$ or $A = 12.52\ kN$

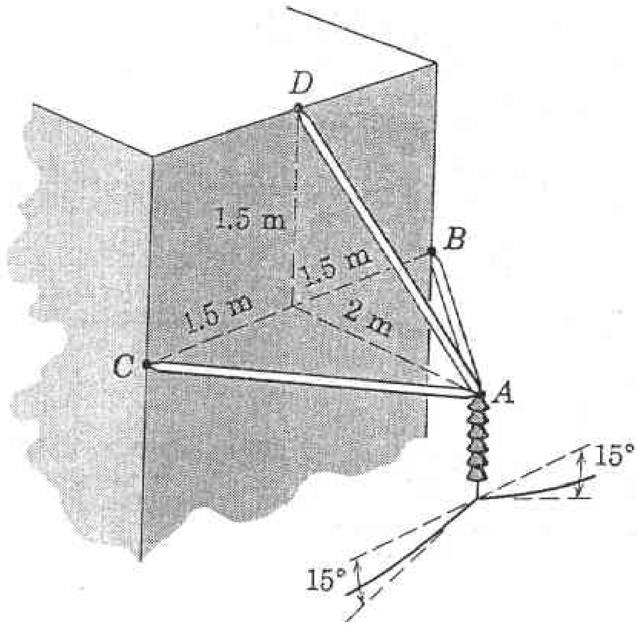
$\Sigma F_x = 0$: $12.52 - 3\sin 30° - O_x = 0$, $O_x = 11.02\ kN$

$\Sigma F_y = 0$: $O_y - 500(9.81) - 3\cos 30° = 0$, $O_y = 7.50\ kN$

$O = \sqrt{(11.02)^2 + (7.50)^2} = \boxed{13.34\ KN}$

JLM

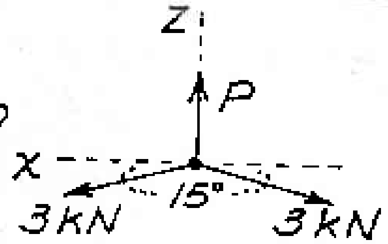
A high-voltage power line is suspended as shown. Tension in the line at the insulators is 3 kN. Calculate the tension T in link AD and the compression C in links AB and AC.

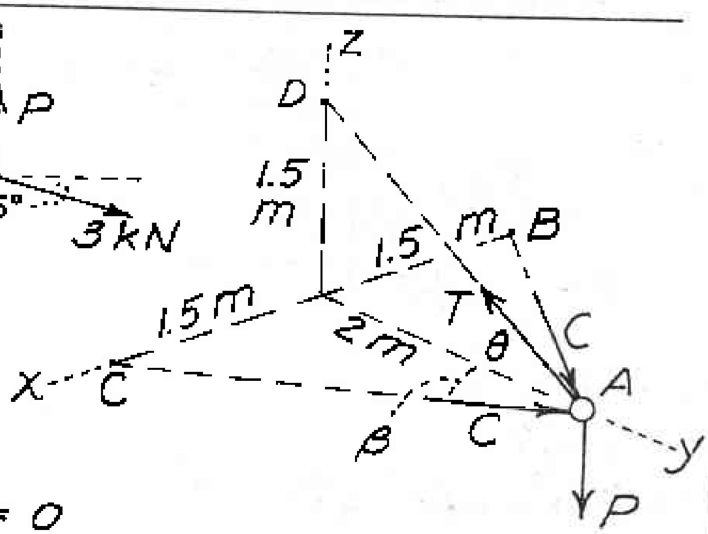


$\Sigma F_z = 0$:

$P - 2(3)\sin 15° = 0$

$P = 1.553$ kN





$\overline{AC} = \overline{AD} = \sqrt{2^2 + (1.5)^2}$

$= 2.5$ m

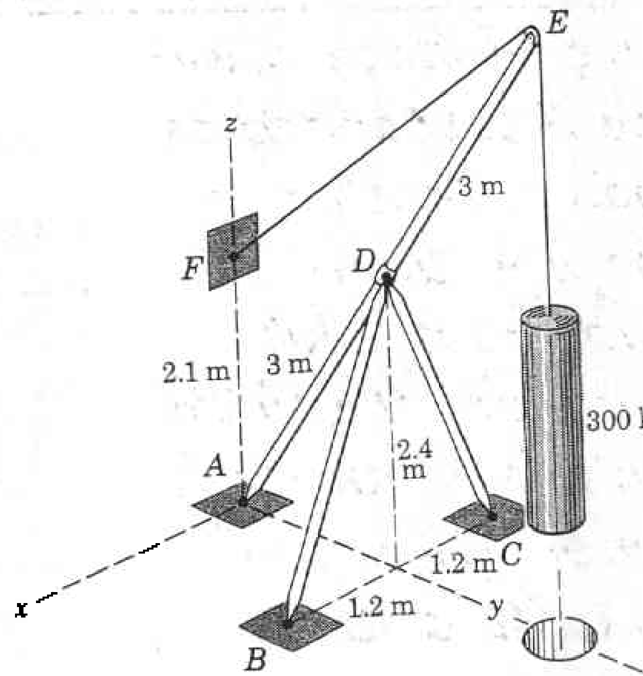
$\Sigma F_z = 0$: $T \sin\theta - 1.553 = 0$

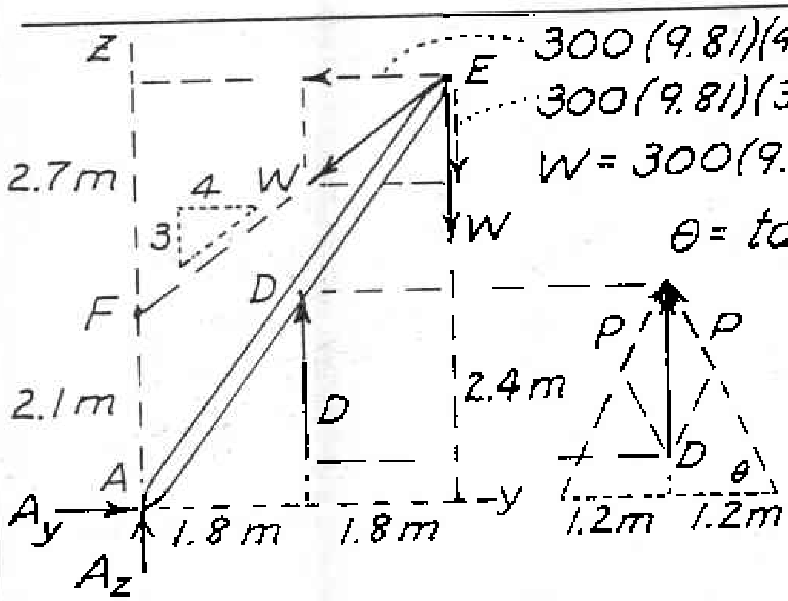
$T = \dfrac{1.553}{1.5/2.5}$, $\boxed{T = 2.59 \text{ kN}}$

$\Sigma F_y = 0$: $2C\cos\beta - 2.59\cos\theta = 0$

$C = \dfrac{2.59(2/2.5)}{2(2/2.5)}$, $\boxed{C = 1.29 \text{ kN}}$

Connections at A, B, C, D are ball & socket joints. Neglect weight of members. Find compression P in legs BD & CD and magnitude of force at A.





$300(9.81)(4/5) = 2354$ N

$300(9.81)(3/5) = 1766$ N

$W = 300(9.81) = 2943$ N

$\theta = \tan^{-1} 2.4/1.2 = 63.4°$

$2P\sin\theta = D$

$P = \dfrac{D}{2\sin\theta} = \dfrac{\sqrt{5}}{4}D$

$\Sigma F_y = 0: A_y - 2354 = 0, \quad A_y = 2354$ N

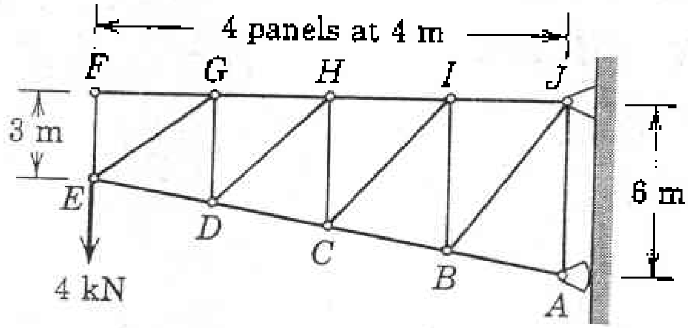
$\Sigma M_G = 0: \underbrace{1.8A_z - 2354(4.8) + 2943(1.8) + 1766(1.8) = 0,}_{A_z = 1570 \text{ N}}$

$\Sigma F_z = 0: 1570 + 4P/\sqrt{5} - 2943 - 1766 = 0, \quad \boxed{P = 1755 \text{ N}}$

$A = \sqrt{(2354)^2 + (1570)^2} = \boxed{2830 \text{ N}}$

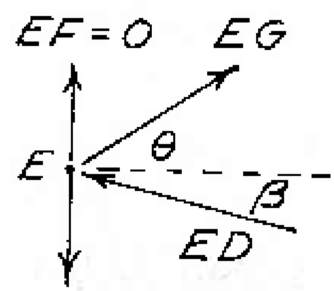
JLM

Determine the forces in members FG, EG, and GD for the simple truss.



By inspection of joint F, $FG = EF = 0$

Joint E



$\theta = \tan^{-1}\frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$\beta = \tan^{-1}\frac{3}{16} = 10.62°$

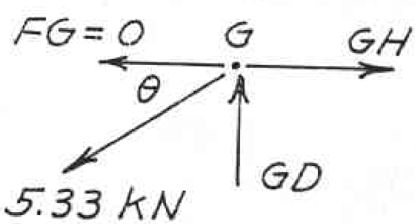
4 kN

$\Sigma F_x = 0$: $EG\left(\frac{4}{5}\right) - ED\cos 10.62° = 0$

$\Sigma F_y = 0$: $EG\left(\frac{3}{5}\right) + ED\sin 10.62° - 4 = 0$

Solve to obtain $\boxed{EG = 5.33 \text{ kN } T}$

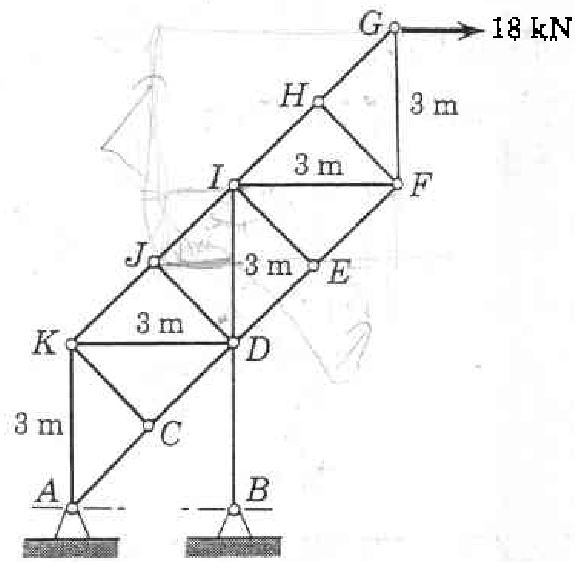
Joint G

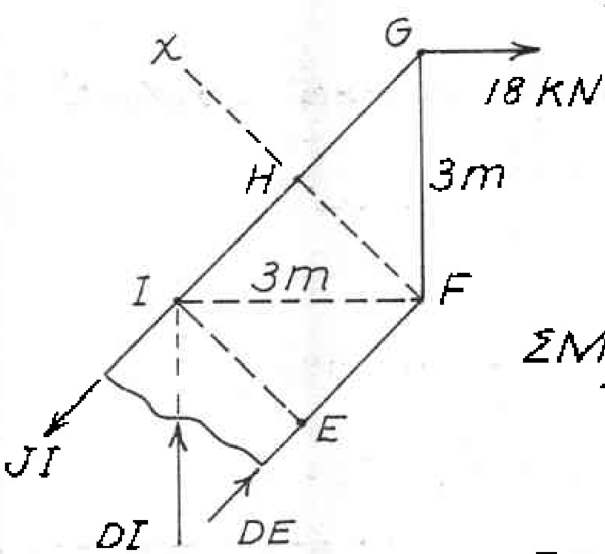


$\Sigma F_y = 0$: $GD - 5.33\left(\frac{3}{5}\right) = 0$

$\boxed{GD = 3.20 \text{ kN } C}$

Determine the forces in members DI, DE, and EI for the simple truss.





FBD of entire section

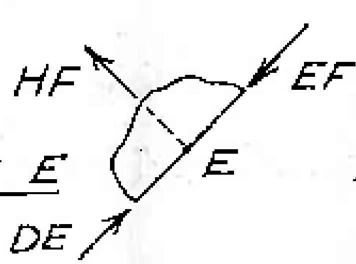
$$\Sigma M_I = 0: \quad DE\,(3\cos 45^\circ) - 18(3) = 0$$

$$\boxed{DE = 25.5 \text{ KN } C}$$

$$\Sigma F_x = 0: \quad DI \cos 45^\circ - 18\cos 45^\circ = 0$$

$$\boxed{DI = 18 \text{ KN } C}$$

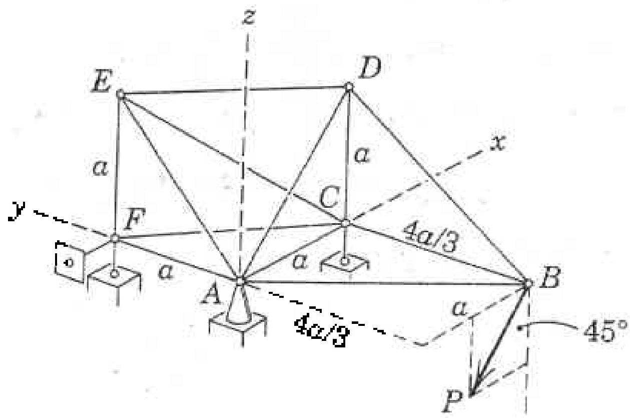
Joint E



$$\Sigma F_x = 0: \quad \boxed{EI = 0}$$

JLM

Determine the forces in members AD, BD, CD, & ED of the space truss loaded and supported as shown. Verify the adequacy of internal stability.



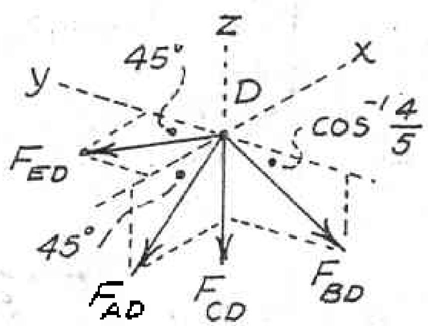
No. of members $m = 12$; No. of joints $j = 6$

$(m + 6 = 18) = (3j = 18)$ so members are adequate in number and comprise rigid tetrahedrons.

Joint B: $\Sigma F_z = 0$ gives $\frac{3}{5}F_{BD} - \frac{P}{\sqrt{2}} = 0$, $\boxed{F_{BD} = \frac{5P}{3\sqrt{2}} \quad (T)}$

All unknown forces taken (+) tension

Joint D:



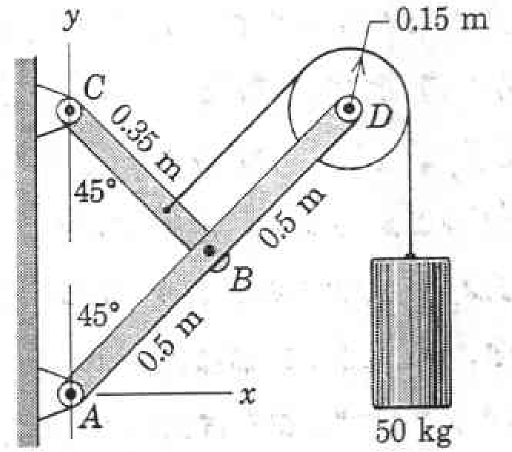
$$\begin{cases} \underline{F}_{BD} = F_{BD}\left(-\frac{4}{5}\underline{j} - \frac{3}{5}\underline{k}\right) = \frac{P}{3\sqrt{2}}(-4\underline{j} - 3\underline{k}) \\ \underline{F}_{CD} = F_{CD}(-\underline{k}) \\ \underline{F}_{AD} = F_{AD}\left(-\frac{\underline{i}}{\sqrt{2}} - \frac{\underline{k}}{\sqrt{2}}\right) = \frac{F_{AD}}{\sqrt{2}}(-\underline{i} - \underline{k}) \\ \underline{F}_{ED} = F_{ED}\left(\frac{\underline{j}}{\sqrt{2}} - \frac{\underline{i}}{\sqrt{2}}\right) = \frac{F_{ED}}{\sqrt{2}}(-\underline{i} + \underline{j}) \end{cases}$$

$\Sigma \underline{F} = \underline{0}$ yields

$$\begin{cases} \underline{i}\text{-terms: } & -F_{AD}/\sqrt{2} - F_{ED}/\sqrt{2} = 0 \\ \underline{j}\text{-terms: } & -4P/(3\sqrt{2}) + F_{ED}/\sqrt{2} = 0 \\ \underline{k}\text{-terms: } & -P/\sqrt{2} - F_{CD} - F_{AD}/\sqrt{2} = 0 \end{cases}$$

Solve & get $\boxed{F_{AD} = -\frac{4P}{3} \quad (C)}$ $\boxed{F_{CD} = \frac{P}{3\sqrt{2}} \quad (T)}$ $\boxed{F_{ED} = \frac{4P}{3} \quad (T)}$

Determine the total force (shear) supported by the pin at B for the loaded frame.



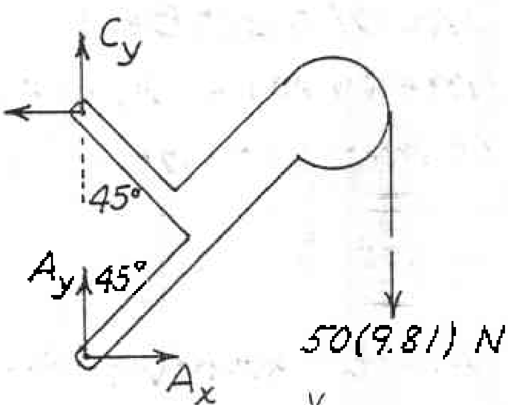
From FBD of entire frame

$\Sigma M_A = 0$:

$$C_x(0.5\sqrt{2}) - 50(9.81)\left(\frac{1}{\sqrt{2}} + 0.15\right) = 0$$

$$C_x = 595 \text{ N}$$

$\Sigma F_x = 0$: $A_x - 595 = 0$, $A_x = 595$ N

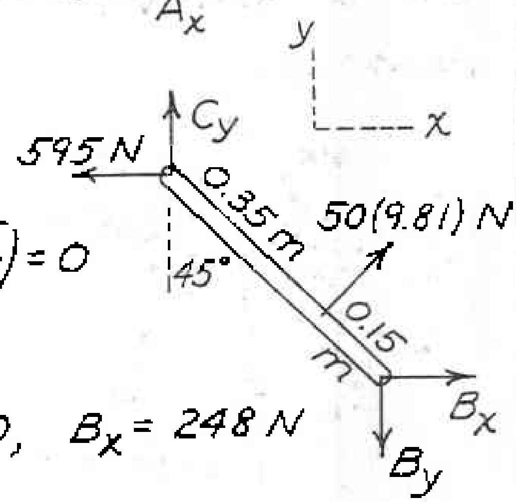


From FBD of member BC

$\Sigma M_B = 0$:

$$595\left(\frac{0.5}{\sqrt{2}}\right) - 50(9.81)(0.15) - C_y\left(\frac{0.5}{\sqrt{2}}\right) = 0$$

$$C_y = 386 \text{ N}$$



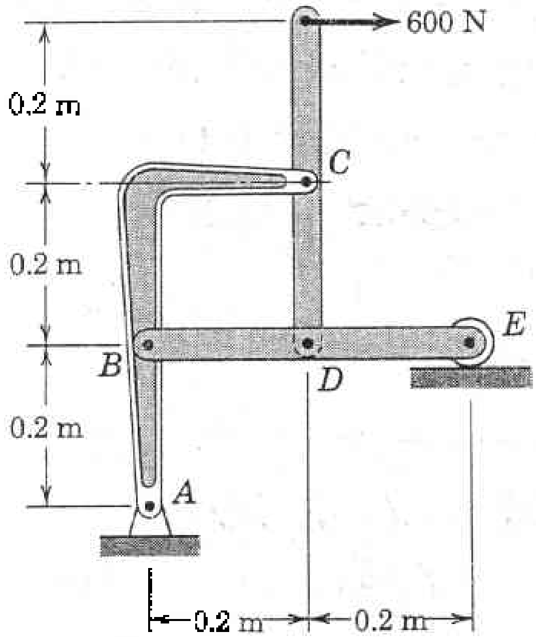
$\Sigma F_x = 0$: $B_x + 50(9.81)/\sqrt{2} - 595 = 0$, $B_x = 248$ N

$\Sigma F_y = 0$: $386 + 50(9.81)/\sqrt{2} - B_y = 0$, $B_y = 733$ N

Total force (shear) $B = \sqrt{(248)^2 + (733)^2} = \boxed{774 \text{ N}}$

JLM

Determine the magnitude of the force supported by the pin at C.



<u>Entire frame</u>

$\Sigma M_A = 0: \quad 0.4E - 0.6(600) = 0$

$E = 900\ N$

$\Sigma F_x = 0: \quad A_x = 600\ N$

$\Sigma F_y = 0: \quad A_y = 900\ N$

<u>Link CD</u> $\Sigma M_D = 0:$

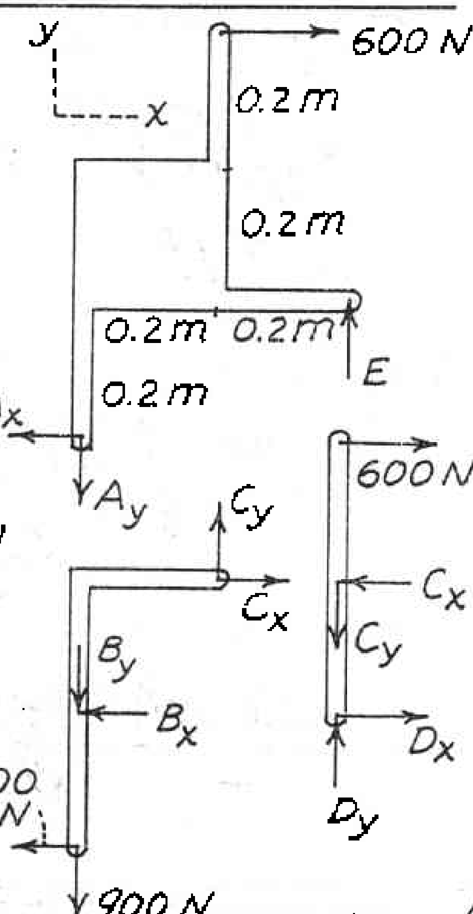
$C_x(0.2) - 600(0.4) = 0, \quad C_x = 1200\ N$

<u>Link ABC</u> $\Sigma M_B = 0:$

$C_y(0.2) - 600(0.2) - 1200(0.2) = 0$

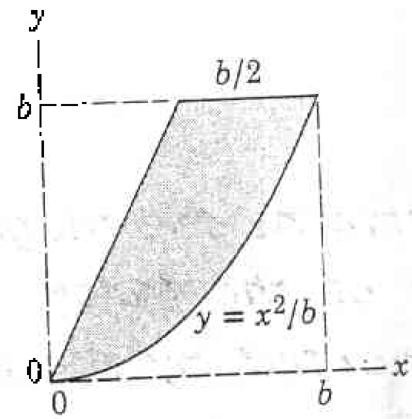
$C_y = 1800\ N$

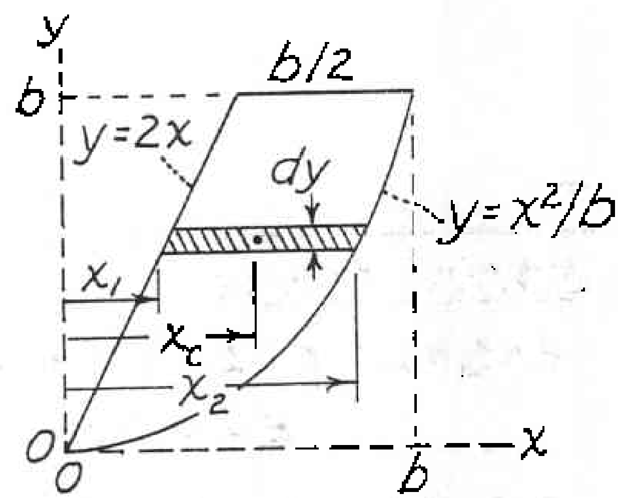
$C = \sqrt{(1200)^2 + (1800)^2} = \boxed{2160\ N}$



JLM

Determine the x-coordinate of the centroid of the shaded area.





$$dA = (x_2 - x_1)\,dy$$
$$= (\sqrt{by} - y/2)\,dy$$
$$A = \int_0^b (\sqrt{by} - y/2)\,dy$$
$$= \left[\tfrac{2}{3}\sqrt{b}\,y^{3/2} - y^2/4\right]_0^b$$
$$= \frac{5}{12}b^2$$

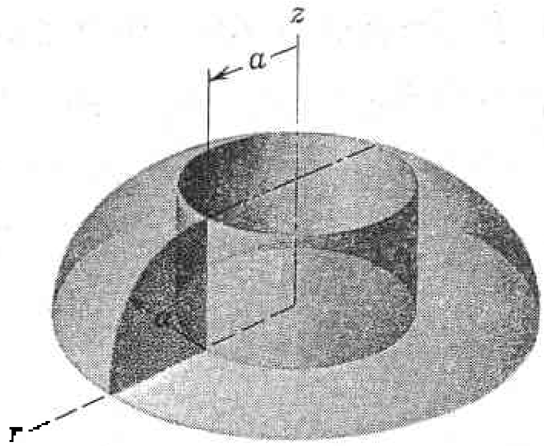
$$x_c = \tfrac{1}{2}(x_1 + x_2)$$

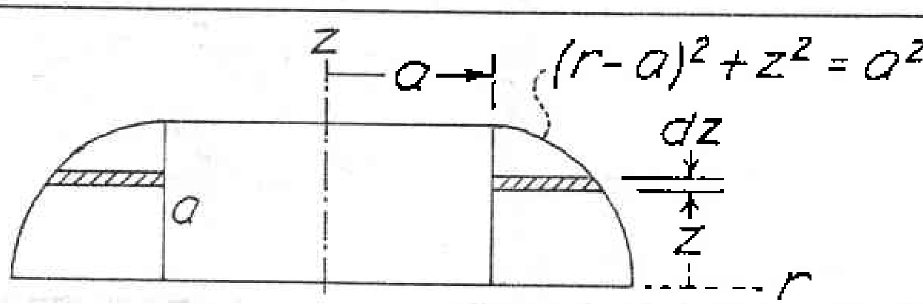
$$\int x_c\,dA = \int_0^b \tfrac{1}{2}(x_1+x_2)(x_2-x_1)\,dy = \tfrac{1}{2}\int_0^b (x_2^2 - x_1^2)\,dy$$
$$= \tfrac{1}{2}\int_0^b (by - y^2/4)\,dy = \tfrac{1}{2}\left(\frac{by^2}{2} - \frac{y^3}{12}\right)_0^b = \frac{5}{24}b^3$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{5b^3/24}{5b^2/12} = \boxed{\frac{b}{2}}$$

JLM

Determine the z-coordinate of the mass center of the solid obtained by revolving the quarter-circular area about the z-axis.





Differential element is a washer of radii r and a and thickness dz with volume

$$dV=\pi(r^2-a^2)dz=\pi(a^2-z^2+2a\sqrt{a^2-z^2})dz$$

$$\int z\,dV=\int_0^a \pi(a^2z-z^3+2az\sqrt{a^2-z^2})\,dz$$

$$=\pi\left[\frac{a^2z^2}{2}-\frac{z^4}{4}+\frac{2a}{3}\sqrt{(a^2-z^2)^3}\right]_0^a=\frac{11}{12}\pi a^4$$

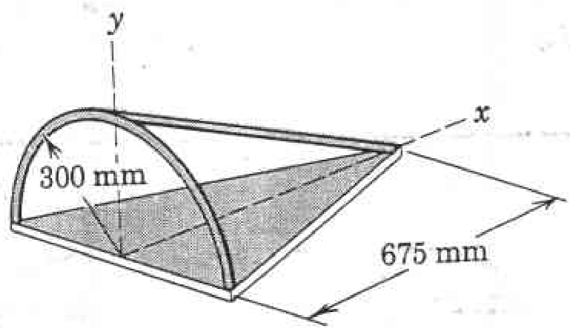
$$\int dV=\int_0^a \pi(a^2-z^2+2a\sqrt{a^2-z^2})\,dz$$

$$=\pi\left[a^2z-\frac{z^3}{3}+a(z\sqrt{a^2-z^2}+a^2\sin^{-1}\frac{z}{a})\right]_0^a=\pi a^3\left(\frac{2}{3}+\frac{\pi}{2}\right)$$

$$\bar{z}=\frac{\int z\,dV}{\int dV}=\frac{(11/12)\pi a^4}{\pi a^3(2/3+\pi/2)},\quad \boxed{\bar{z}=\frac{11a}{2(4+3\pi)}=0.410a}$$

JLM

The semicircular and straight bars are made from stock with a mass of 7.5 kg per meter of length and are welded to the triangular plate made from material with a mass of 100 kg per square meter of area. Calculate the coordinates of the mass center of the assembly.



Part	m kg	$\bar{x}$ mm	$\bar{y}$ mm	$m\bar{x}$ kg·mm	$m\bar{y}$ kg·mm
Circular bar	2.25π	0	$600/\pi$	0	1350
Brace	5.54	338	150	1870	831
Base	20.25	225	0	4556	0
sums	32.86			6426	2181

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{6426}{32.86} = \boxed{195.6 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{2181}{32.86} = \boxed{66.4 \text{ mm}}$$

JLM

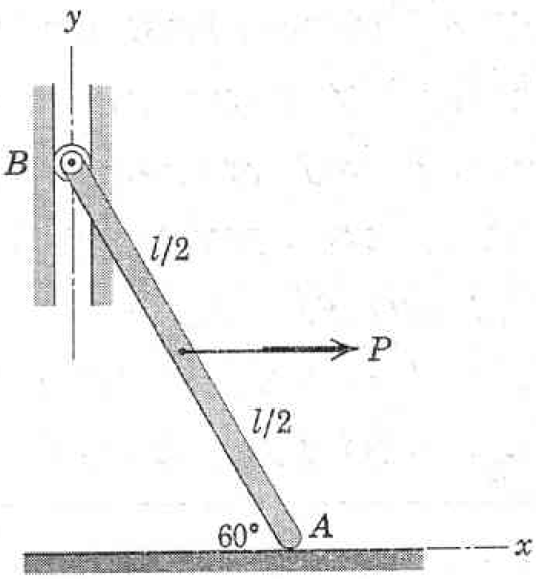
ART 6/3 DRY FRICTION

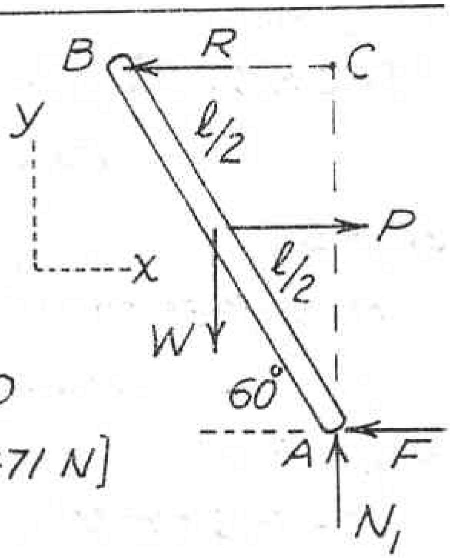
NO. 1

Uniform 60-kg bar AB is subjected to force P. Smooth guides at B. At A, $\mu_s = 0.8$.

(a) If $P = 400$ N, find friction force at A.

(b) Find P required to cause slippage at A.





$W = mg = 60(9.81) = 589$ N

(a) $P = 400$ N. Assume equil.

$\Sigma F_y = 0$: $N_1 - 589 = 0$, $N_1 = 589$ N

$\Sigma M_C = 0$: $400 \frac{\ell}{2} \sin 60°$

$+ 589 \frac{\ell}{2} \cos 60° - F\ell \sin 60° = 0$

$\boxed{F = 370 \text{ N}} < [\mu_s N_1 = 0.8(589) = 471 \text{ N}]$

so assumption is valid

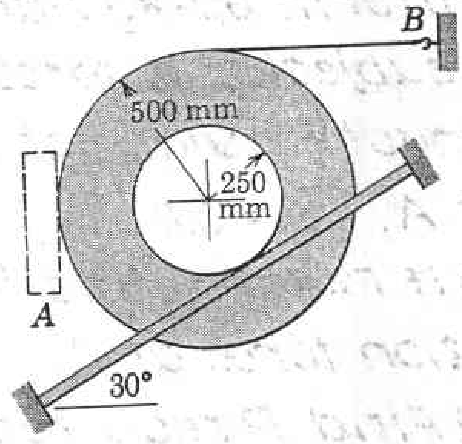
(b) $F = \mu_s N_1 = 471$ N

$\Sigma M_C = 0$; $P \frac{\ell}{2} \sin 60° + 589 \frac{\ell}{2} \cos 60° - 471(\ell \sin 60°) = 0$

$\boxed{P = 602 \text{ N}}$

JLM

The hubs of the uniform 50-kg wheel rest on inclined rails. If support at A is removed, determine the friction force acting on the wheel if $\mu_s = 0.50$, $\mu_k = 0.40$. What would happen if $\mu_s = 0.30$ & $\mu_k = 0.25$?

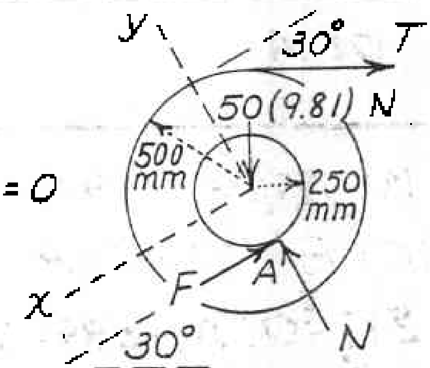


First, assume equilibrium.

$\Sigma M_A = 0$:

$T(500 + 250\cos 30°) - 50(9.81)(250\sin 30°) = 0$

$T = 85.6\ N$



$\Sigma F_y = 0$:

$N - 50(9.81)\cos 30° - 85.6\sin 30° = 0$

$N = 468\ N$

$\Sigma F_x = 0$:

$-F - 85.6\cos 30° + 50(9.81)\sin 30° = 0, \quad F = 171\ N$

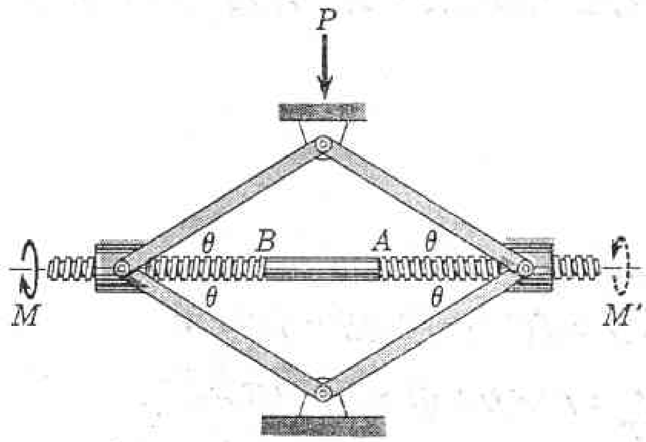
Since $(F_{needed} = 171\ N) < (F_{s\,max} = \mu_s N = 0.50(468) = 234\ N)$, equilibrium assumption is valid & $\boxed{F = 171\ N}$

If $\mu_s = 0.30$, $F_{s\,max} = 0.30(468) = 140.4\ N < 171\ N$

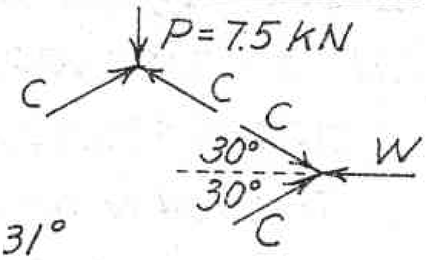
so wheel will slip. But $F \neq 0.25(468)\ N$ since $N \neq 468\ N$ under accelerating conditions.

JLM

Each screw of the jack has a mean diameter of 21 mm and a lead of 8 mm, one a right-hand and the other a left-hand thread. For $\theta = 30°$ determine (a) the torque M required to raise the load $P = 7.5$ kN and (b) the torque M' required to lower the load. The coefficient of friction is $\mu = 0.20$.



For equilibrium



$W = 2C\cos 30°,\ P = 2C\sin 30°$

so $W = P\,\text{ctn}\,30° = 7.5\sqrt{3} = 12.99$ kN

Friction angle $\phi = \tan^{-1} 0.20 = 11.31°$

Helix angle $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{8}{2\pi(21/2)} = 6.91°$

$M = 2Wr\tan(\phi + \alpha) = 2(12.99)(21/2)\tan(11.31° + 6.91°)$

(a) $M = 89.8$ kN·mm or $\boxed{M = 89.8\ \text{N·m}}$

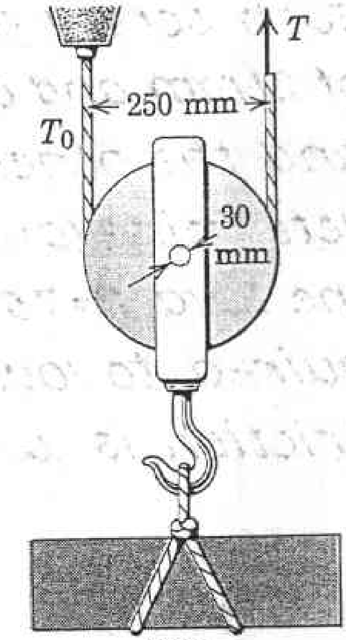
$M' = 2Wr\tan(\phi - \alpha) = 2(12.99)(21/2)\tan(11.31° - 6.91°)$

(b) $M' = 21.0$ kN·mm or $\boxed{M' = 21.0\ \text{N·m}}$

JLM

ART. 6/6 JOURNAL BEARINGS

The coefficient of kinetic friction between the 30-mm-diameter pin and the pulley is 0.25. Calculate the tension T required to (a) raise the load and (b) to lower the load at a constant speed. Neglect the mass of the pulley.



$\phi = \tan^{-1} 0.25 = 14.04°$

$r_f = r \sin\phi = 0.015 \sin 14.04° = 0.00364$ m

$L = 500(9.81) = 4905$ N

(a) To raise load: $\Sigma M_B = 0$

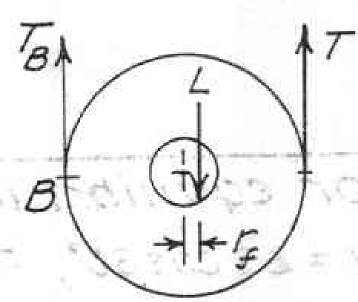
$0.25T - 4905(0.125 + 0.00364) = 0$

$T = 2524$ N or $\boxed{T = 2.52 \text{ kN}}$

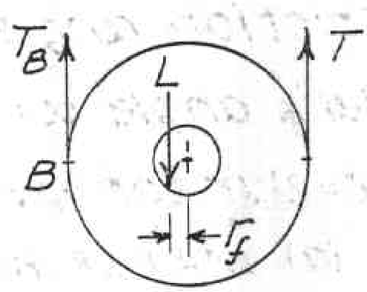
(b) To lower load: $\Sigma M_B = 0$

$0.25T - 4905(0.125 - 0.00364) = 0$

$T = 2381$ N or $\boxed{T = 2.38 \text{ kN}}$



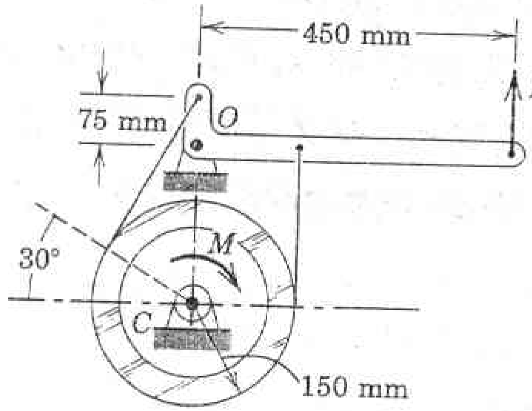
To raise

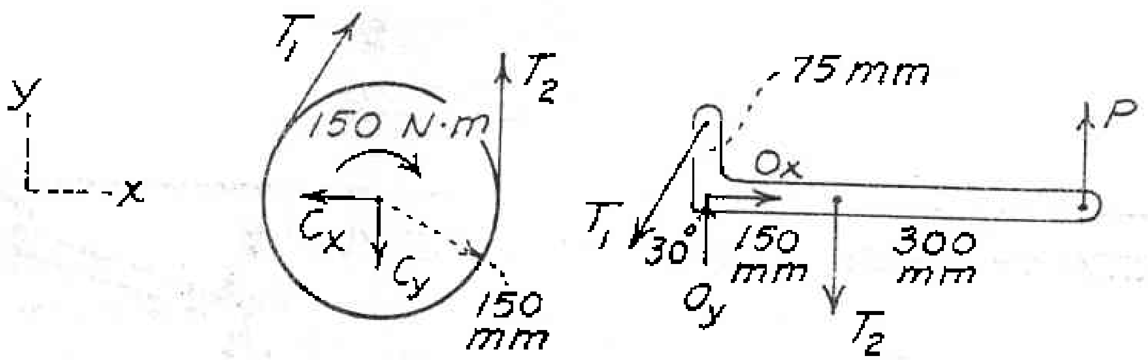


To lower

JLM

Calculate the force P on the handle of the differential band brake that will prevent the flywheel from turning on its shaft to which the torque $M = 150\ \text{N}\cdot\text{m}$ is applied. The coefficient of friction between the band and the flywheel is $\mu = 0.40$.





Band $\quad T_2 = T_1 e^{\mu\beta} \qquad T_2 = T_1 e^{0.40\,\frac{7\pi}{6}} = 4.33\,T_1 \quad \cdots (1)$

Flywheel $\quad \Sigma M_C = 0; \quad 150 + (T_1 - T_2)(0.150) = 0$

$$T_2 - T_1 = 1000\ \text{N} \quad \cdots (2)$$

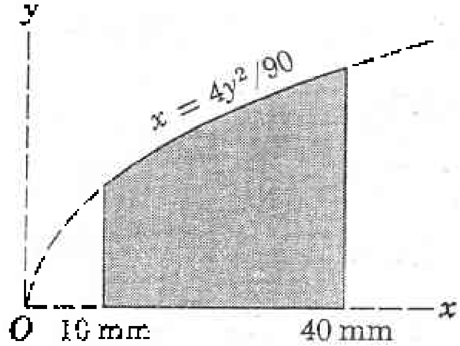
Handle $\quad \Sigma M_O = 0; \quad 0.150\,T_2 - (T_1 \sin 30°)(0.075) - 0.450\,P = 0$

Solve (1) & (2) & get $T_1 = 300\ \text{N}, \quad T_2 = 1300\ \text{N}$

Solve for P & get $\quad \boxed{P = 408\ \text{N}}$

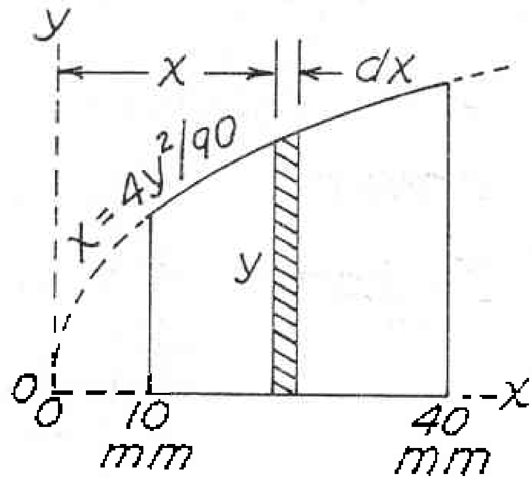
JLM

Calculate the moment of inertia of the shaded area about the x- and y-axes. Also find the radius of gyration k_x.



For rectangular area about its base $I = \frac{1}{3}bh^3$ so

$$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}(90x/4)^{3/2}dx$$
$$= \frac{9}{8}(10x)^{3/2}dx$$



$$I_x = \frac{9}{8}(10)^{3/2}\int_{10}^{40} x^{3/2}dx$$
$$= \frac{9}{8}(10)^{3/2}\left(\frac{2}{5}\right)x^{5/2}\Big]_{10}^{40} = \frac{9(31)}{20}(10)^4 = \boxed{13.95(10)^4 \text{ mm}^4}$$

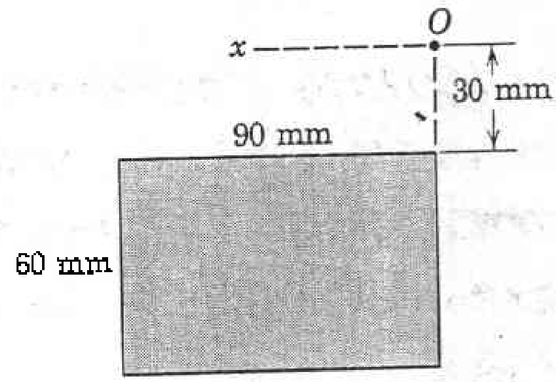
$$\text{Area } A = \int y\,dx = \frac{3}{2}\sqrt{10}\int_{10}^{40} x^{1/2}dx = \frac{3}{2}\sqrt{10}\,\frac{2}{3}x^{3/2}\Big]_{10}^{40} = 700 \text{ mm}^2$$

$$k_x = \sqrt{I_x/A} = \sqrt{13.95(10)^4/700} = \boxed{14.12 \text{ mm}}$$

$$I_y = \int x^2 dA = \int x^2 y\,dx = \frac{3}{2}\sqrt{10}\int_{10}^{40} x^{5/2}dx$$
$$= \frac{3}{2}\sqrt{10}\left(\frac{2}{7}\right)x^{7/2}\Big]_{10}^{40} = \boxed{54.43(10)^4 \text{ mm}^4}$$

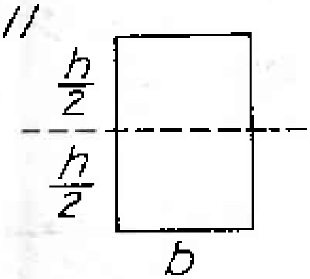
JLM

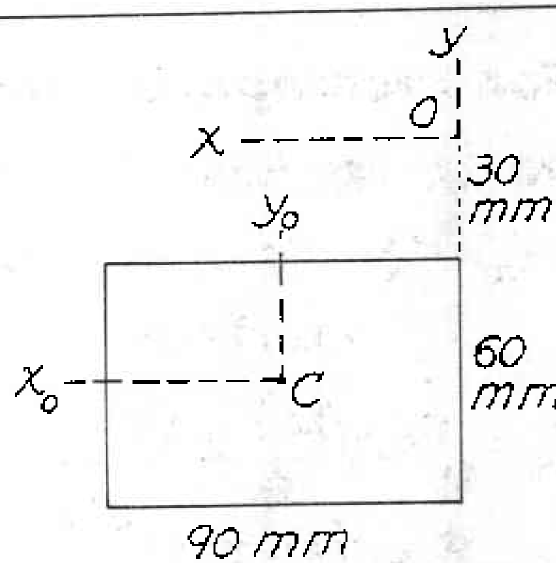
Calculate the moment of inertia of the rectangular area about the x-axis and find the polar moment of inertia about point O.



For rectangular area recall

$\bar{I} = \frac{1}{12} b h^3$





$$I_x = \bar{I}_x + A d_x^2$$

$$= \frac{1}{12}(90)(60)^3 + (90)(60)(30+30)^2 = \boxed{21.06(10)^6 \text{ mm}^4}$$

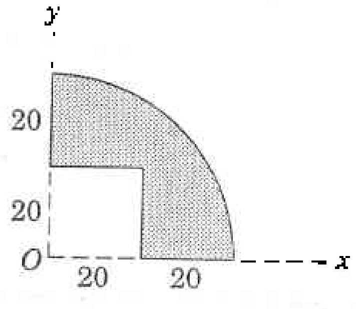
$$I_y = \bar{I}_y + A d_y^2$$

$$= \frac{1}{12}(60)(90)^3 + (90)(60)(45)^2 = 14.58(10)^6 \text{ mm}^4$$

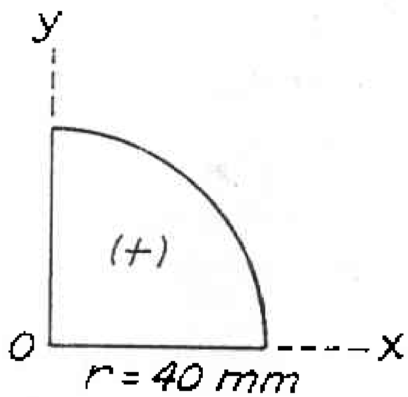
$$I_z = I_x + I_y = 21.06(10)^6 + 14.58(10)^6 = \boxed{35.64(10)^6 \text{ mm}^4}$$

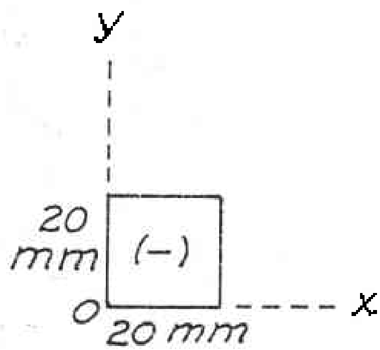
ART. A/3 COMPOSITE AREAS

Compute the moment of inertia about the x-axis and the polar radius of gyration about O for the area shown.



Dimensions in millimeters





For quarter circular area $A = \frac{\pi}{4}(40)^2 = 1257\ mm^2$

$$I_x = I_y = \frac{1}{4}\left(\frac{\pi}{4}r^4\right) = \frac{\pi}{16}(40)^4 = 503(10^3)\ mm^4$$

$$I_z = I_x + I_y = 2(503)(10^3) = 1005(10^3)\ mm^4$$

For square area $A = -20(20) = -400\ mm^2$

$$I_x = I_y = -\frac{1}{3}bh^3 = -\frac{1}{3}(20)(20)^3 = -53.3(10^3)\ mm^4$$

$$I_z = I_x + I_y = -2(53.3)(10^3) = -106.7(10^3)\ mm^4$$

For net area $I_x = (503 - 53.3)(10^3) = \boxed{449(10^3)\ mm^4}$

$$k_z = k_o = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{(1005 - 106.7)(10^3)}{(1.257 - 0.4)(10^3)}} = \boxed{32.4\ mm}$$

JLM