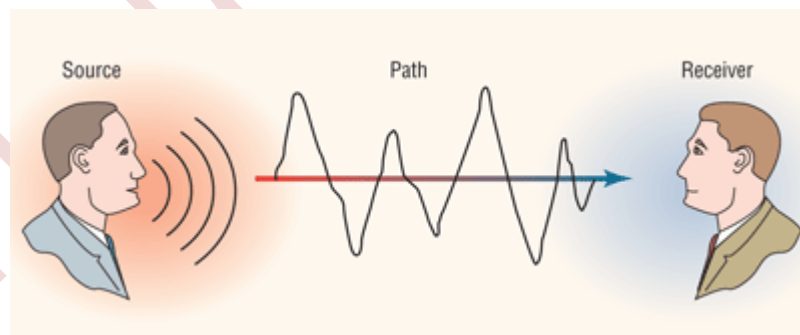


MEASUREMENT OF SOUND & NOISE



Introduction

Sound is a form of energy transmitted from a vibrating source. The vibrating matter creates small, repetitive pressure disturbances that are imparted to the air along a path and reach a receiver, the ear. Sound and vibration are created by a **source**, are transmitted along one or more **paths**, and reach a **receiver**.

Measurement unit:

dB SCALE: Acoustic parameters are expressed as logarithmic ratio of the measured value to a reference value. The Bell (B) is a unit of measurement invented by Bell Labs and named after Alexander Graham Bell.

The Bell was too large, so the deciBel (dB), equal to 0.1 is B.

Sound characteristics:

Sound is a propagating disturbance in a fluid (gas or liquid) or in a solid. Ear drums sense these small changes in the barometric pressure of the air, distinguishing sounds based on amplitude and pitch. Amplitude refers to the level of energy that reaches the ear which corresponds to how loud we perceive sound. Pitch is the relative quality or the frequency of the sound that reaches the ear, helping a person to identify the source of the sound.

In systems, the source of sound is a combination of different processes, such as turbulence from the fan(s) and mechanical sounds from the motor(s), etc. Frequency, measured in Hertz (Hz), is the number of oscillations (cycles) completed per second by a vibrating object. The sound that humans hear covers a frequency range of about 20 Hz to about 20,000 Hz. Sounds at different frequencies behave differently, causing human ears to react to them differently as well.

<20Hz	20Hz to 20000Hz	> 20000Hz
Infrasonic	Audio Range	Ultrasonic

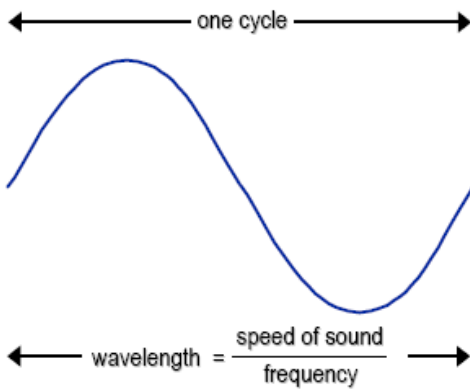
The speed of a longitudinal wave in a fluid is a function of the fluid's density and bulk modulus of elasticity. In air, at room temperature, the speed of sound is about 340 m/s; in water, about 1500 m/s. In solids, there are several different types of waves, each with a different speed.

Wavelength

The wavelength of sound in a medium is the distance between successive maxima or minima of a simple harmonic disturbance propagating in that medium at a single instant in time. Wavelength, speed, and frequency are related by:

$$\lambda = \frac{c}{f}$$

where: λ = wavelength, m, c = speed of sound, m/s, f = frequency, Hz



The speed of sound transmission depend on the physical property of the medium.

For air, the speed varies slightly with temperature change.

The speed of sound = a constant (344 m/s) in consideration of narrow temperature range in HVAC system.

Sound traveling through the air at a frequency of 200 Hz has a wavelength of 1.7 m.

$$\text{wavelength} = \frac{344 \text{ m/s}}{200 \text{ Hz}} = 1.7 \text{ m}$$

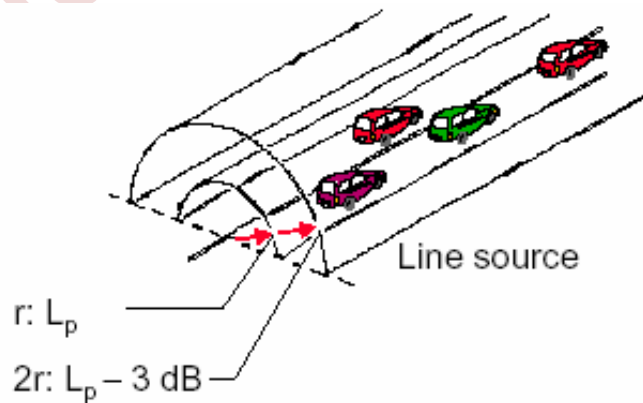
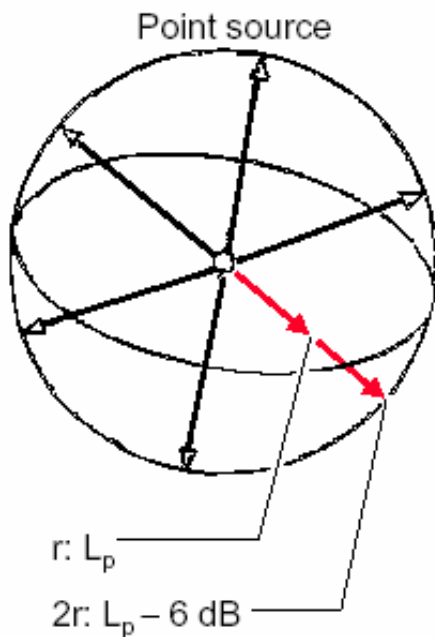
Sound Pressure: evaluation of harmfulness and annoyance of noise sources

Sound Intensity: location & rating of noise sources rate of energy flow per unit area

Sound Power: for noise rating of machines unique descriptor of noisiness of source

Types of sources sound:

- 1- Point source:
- 2- Line source:
- 3- Plane source:



SOUND BITS:

Unless there is a 3 dB difference in SPL, human beings cannot distinguish the difference in the sound, Sound is perceived as doubled in its loudness when there is 10dB difference in the SPL.

(Remember 6dB change represents doubling of sound pressure)

Ear is not equally sensitive at all frequencies:

highly sensitive at frequencies between 2kHz to 5kHz less at other freq.

This sensitivity dependence on frequency is also dependent on SPL.

levels

Magnitude of sound and vibration is almost always expressed in *levels*. As shown in the following equations, the level L is based on the common (base 10) logarithm of a ratio of the magnitude of a physical property (e.g., sound pressure) to a reference magnitude of the same type of property:

$$L = 10 \log \left(\frac{A}{A_{ref}} \right)$$

Where A is the magnitude of the physical property of interest and A_{ref} is the reference. Note that the ratio is dimensionless. In this equation, a factor of 10 is included to convert bels to decibels (dB). This basic equation describes levels of power, intensity, and energy, which are related to the square of other physical properties, such as sound pressure and vibration acceleration. Therefore, levels of magnitude of these quantities can be written as:

$$L = 10 \log \left(\frac{P^2}{P_{ref}^2} \right) = 20 \log \left(\frac{P}{P_{ref}} \right)$$

Where P is the physical quantity, such as the magnitude of acoustic pressure. Numerically, the decibel is ten times the base 10 logarithm of the ratio of two like quantities proportional to acoustical power or energy.

Sound Pressure and Sound Pressure Level

Sound waves in air are variations in pressure above and below atmospheric pressure. The human ear responds to a large range of sound pressures. Sound pressure is typically measured in Pascals (Pa), which creates a range of pressure values so wide that it is more convenient to use a logarithmic scale. Therefore, the decibel (dB) scale is preferred because it collapses a large range of pressure values to a more manageable, easier to analyze range. The sound pressure level is measured in dB above a standard reference level and given by:

$$L_p = 10 \log \left(\frac{P^2}{P_{ref}^2} \right) = 20 \log \left(\frac{P}{P_{ref}} \right)$$

Here “ P ” represents the sound pressure being measured and “ P_{ref} ” is the reference sound pressure, typically 20 μ Pa, which is generally considered the threshold of human hearing.

Sound pressure level is relatively easy to measure and thus is used by most noise codes and criteria. (The human ear and microphones are pressure-sensitive.)

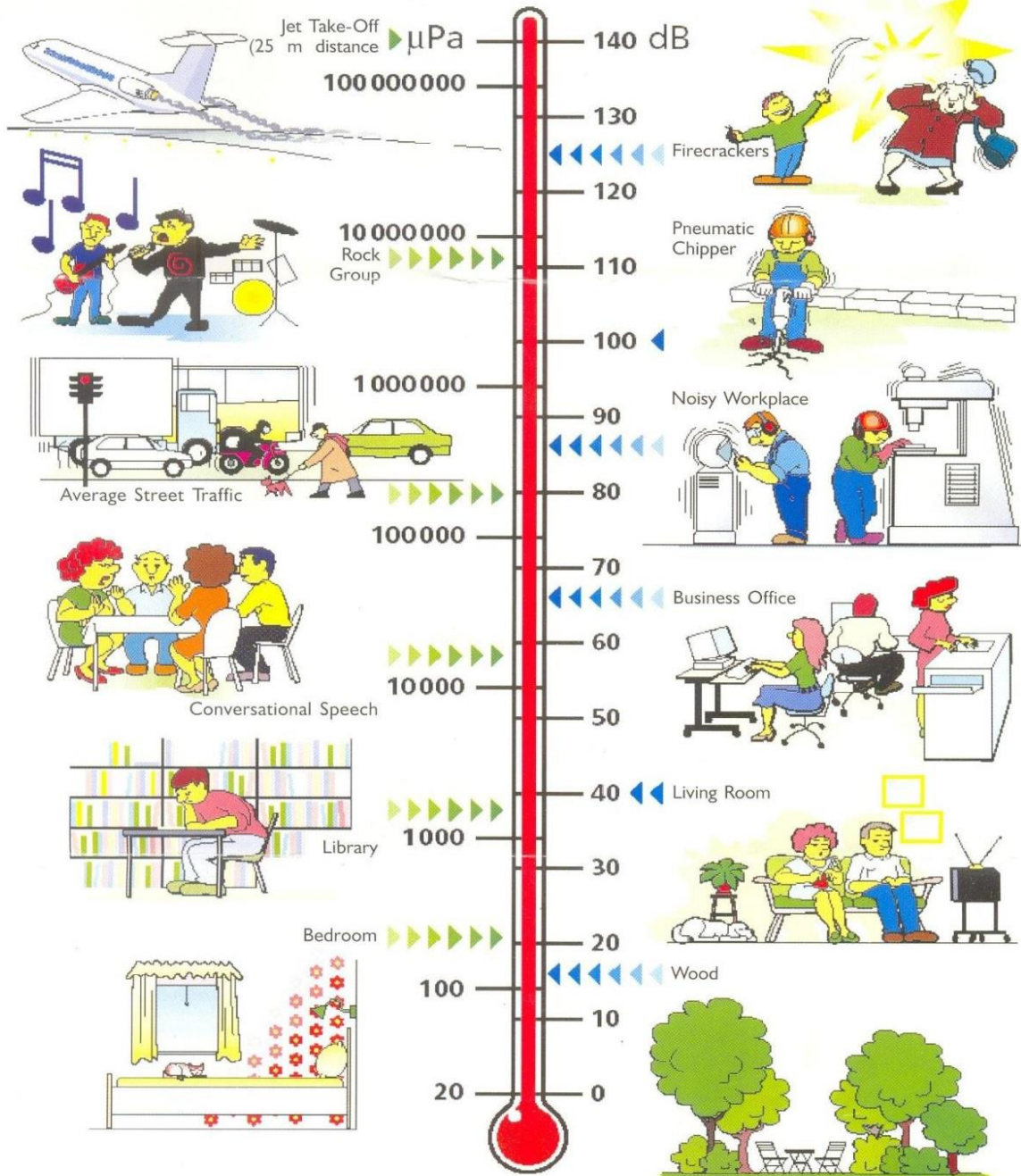
Why log ratio?

- Logarithmic scale compresses the high amplitudes and expands the low ones
- The other reason: Equal relative modifications of the strength of a physical stimulus lead to equal absolute changes in the salience of the sensory events (Weber-Fechner Law) and can be approximated by a logarithmic characteristics

source	SP Pa	SPL dB re20 μPa	Subjective reaction	source	SP Pa	SPL dB re20 μPa	Subjective reaction
Military jet takeoff at 30 m	200	140	Extreme danger	Conversational speech at 1 m	0.02	60	
Artillery fire at 3 m	63.2	130		Window air conditioner at 3 m	0.006	50	Moderate
Passenger jet takeoff at 15 m	20	120	Threshold of pain	Quiet residential area	0.002	40	Quiet
Loud rock band	6.3	110	Threshold of discomfort	Whispered conversation at 2 m	0.0006	30	
Automobile horn at 3 m	2	100		Buzzing insect at 1 m	0.0002	20	Perceptible
large diesel engine	0.6	90	Very loud	Threshold of good hearing	0.00006	10	Faint
Accelerating diesel truck at 15 m	0.2	80		Threshold of excellent youthful hearing	0.00002	0	Threshold of hearing
Freight train at 30 m	0.06	70	loud				

SOUND PRESSURE

SOUND PRESSURE LEVEL



Sound Power and Sound Power Level:

Sound ratings are typically provided in terms of the sound power of a source, which is its rate of emission of acoustical energy and is expressed in watts. Sound power does not depend on the distance of observation location from the source but it does depend on operating conditions. The sound power level, L_w , is defined by:

$$L_w = 10 \log \left(\frac{W}{10^{-12}} \right)$$

Here “W” is the sound power emitted by the source in watts and 10^{-12} is the reference power.

Mechanical equipment is rated in terms of sound power level in order to provide a common reference measurement that is independent of distance and the acoustical conditions of the environment. When attempting to measure sound power level ratings, an engineer will find that he cannot measure these ratings directly. Instead, sound power level ratings are calculated from several sound pressure measurements created by a source in a particular test environment using one of four common methods: free-field, reverberation room, progressive wave (in-duct), and sound intensity.

Typical average decibel levels (dBA) of some common sounds:

activation	SPL dB	SP W	activation	SPL dB	SP W
Threshold of hearing	0	10^{-12}	Food blender (3 feet), shouting	90	10^{-3}
Human breath	10	10^{-11}	Subway (inside)	94	
Rustling leaves	20	10^{-10}	Diesel truck (30 feet), heavy truck at high way speed	100	10^{-2}
Quiet whisper (3 feet)	30	10^{-9}	Power mower (3 feet)	107	
Quiet home	40	10^{-8}	Large HVAC fan	110	10^{-1}
Quiet street, office air diffuser	50	10^{-7}	Pneumatic riveter (3 feet)	115	
Normal conversation, electronic equipment ventilation fan	60	10^{-6}	Chainsaw (3 feet)	117	
Inside car, voice, conversation level	70	10^{-5}	Amplified Rock and Roll (6 feet), small aircraft engine	120	1
Loud singing (3 feet)	75		Jet plane (100 feet), large pipe organ	130	10
Automobile (25 feet)	80	10^{-4}	Jet aircraft at takeoff	160	10^4
Motorcycle (30 feet)	88		Space shuttle launch	200	10^8

Sound Intensity and Sound Intensity Level:

The **sound intensity** I at a point in a specified direction is the rate of flow of sound energy (i.e., power) through unit area at that point. The unit area is perpendicular to the specified direction, and the units of intensity are watts per square meter. **Sound intensity level** L_I is expressed in dB with a reference quantity of 10^{-12} W/m^2 , thus:

$$L_I = 10 \log \left(\frac{I}{10^{-12}} \right)$$

The instantaneous intensity I is the product of the pressure and velocity of air motion (e.g., particle velocity), as shown here:

$$I = pv$$

Both pressure and particle velocity is oscillating, with a magnitude and time variation. Usually, the time-averaged intensity I_{ave} (i.e., the net power flow through a surface area, often simply called “the intensity”) is of interest.

Combining Sound Levels:

To estimate the levels from multiple sources from the levels from each source, the intensities (not the levels) must be added. Thus, the levels must first be converted to find intensities, the intensities summed, and then converted to a level again. Because sound pressure levels are usually good approximations for sound intensity levels, the combination of two levels L_1 and L_2 produces a level L_{sum} given by:

$$L_{sum} = 10 \log (10^{L_1/10} + 10^{L_2/10})$$

Where $10^{L_i/10}$ is p_i^2/p_{ref}^2 .

This process may be extended to combine as many levels as needed using the following equation:

$$L_{sum} = 10 \log \left(\sum_i 10^{L_i/10} \right)$$

To remove background noise, the levels are unlogged and the square of the background sound pressure subtracted from the square of the sound pressure for the combination of the source and background noise:

$$L_p(\text{source}) = 10 \log (10^{L(\text{comb})/10} - 10^{L(\text{bgd})/10})$$

Where $L_{(bgd)}$ is the sound pressure level of the background noise, measured with the source of interest turned off. If the difference between the levels with the source on and off is greater than 10 dB, then background noise levels are low enough that the effect of background noise on the levels measured with the source on can be ignored.

Octave Bands

Human ear perception: sounds at frequencies 20 to 16,000 Hz.

HVAC system sounds 45 to 11,200 Hz (11,156 data points).

HVAC sounds frequencies → smaller ranges (octave bands).

The highest frequency in the band is two times the lowest frequency.

Center frequency = square root of the product of the lowest and highest frequencies in the band.

The frequency range (45 to 11,200 Hz) → eight octave bands with center frequencies of 63, 125, 250, 500, 1,000, 2,000, 4,000, and 8,000 Hz.

octave band	center frequency (Hz)	frequency range (Hz)
1	63	45 to 90
2	125	90 to 180
3	250	180 to 355
4	500	355 to 710
5	1,000	710 to 1,400
6	2,000	1,400 to 2,800
7	4,000	2,800 to 5,600
8	8,000	5,600 to 11,200

Analyzing Sound Ratings

The purpose of developing sound ratings is to help determine whether or not the equipment will cause a sound problem. Sound becomes noise when it is too loud, unexpected, contains unwanted tones (e.g. a whine, whistle, or hum), or is unpleasant. Sound only has to be unwanted for it to be noise, not necessarily just loud. Humans respond differently to each particular frequency of sound, making the ear more receptive to certain frequencies than others. As mentioned before, sound is a combination of frequencies. This creates a problem for measuring the impact of each sound on human hearing since each frequency will be perceived differently by the ear.

MEASURING SOUND

Instrumentation: The basic instrument for measuring sound is a sound level meter, which comprises a microphone, electronic circuitry, and a display device. The microphone converts sound pressure at a point to an electronic signal, which is then processed and the sound pressure level displayed using analog or digital circuitry. Sound level meters are usually battery-operated, light, handheld units with outputs that vary in complexity depending on cost and level of technology.

Sophisticated sound measurements and their procedures should be carried out by individuals experienced in acoustic measurements. At present, there are only a few noise standards that can be used to measure interior sound levels from mechanical equipment (e.g., ASTM Standards E1573 and E1574).

The infections on sound measurement:

1. Sound reflecting from neighboring surface.
2. Sound transitioning by base or background noise.
3. Other sound sources and Magnitude of sound sources and Directivity of source.
4. Room volume
5. Room furnishings and surface treatments
6. Distance from sound sources to point of observation.

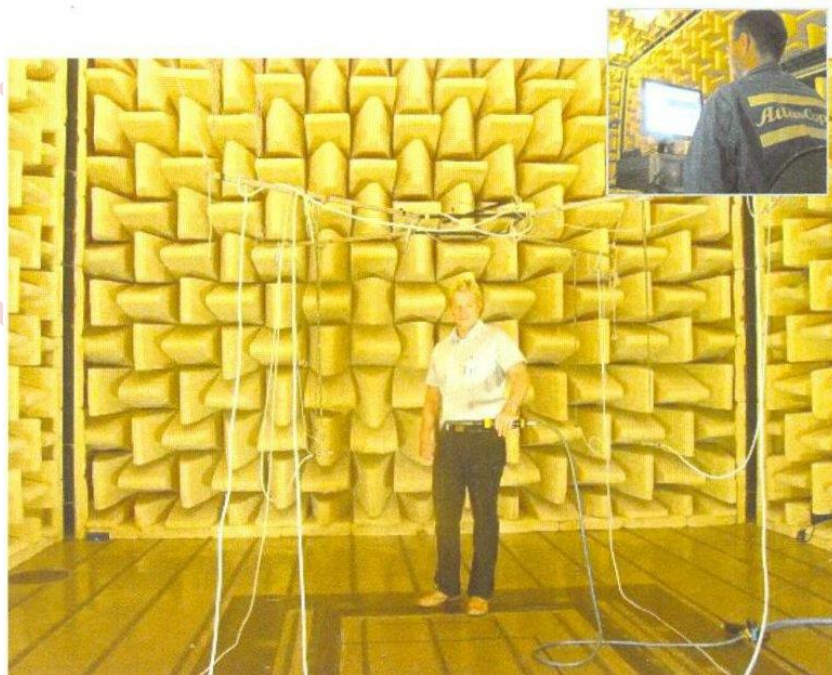
Measuring location:

Outdoor measuring: When measuring the noise, the **background noise** from other sources (occupants, wind, nearby traffic, elevators, etc.) must be determined. Sometimes the sound from a particular piece of equipment must be measured in the presence of background sound from sources that cannot be turned off, such as automobile traffic or certain office equipment. Important issues such as the effect of large, nearby sound-reflecting surfaces and weather conditions such as wind, temperature, and precipitation must be addressed.

Indoor measuring: a specified room noise criterion must demonstratively be met. Measurement procedures for obtaining the data to demonstrate compliance are often not specified which can lead to confusion when different parties make measurements using different procedures, because the results often do not agree. The problem is that most rooms exhibit significant point-to-point variation in sound pressure level.

When a noise has no audible tonal components, the differences in measured sound pressure level at several locations in a room may be as high as 3 to 5 dB. However, when audible tonal components are present, especially at low frequencies, the variations due to standing waves that occur at frequencies of resonance may exceed 10 dB. These are generally noticeable to the average listener when moving through the room.

Although commissioning procedures usually set precise limits for demonstrating compliance, the outcome can unfortunately be controversial unless the measurement procedure has been specified in detail. At the time of writing, there was no general agreement in the industry on an acoustical measurement procedure for commissioning systems.



Controlling sound:

- 1- Enclosures and barriers
- 2- Partitions
- 3- Attenuation and plenums

HUMAN RESPONSE TO SOUND

Noise may be defined as any unwanted sound. Sound becomes noise when it:

- + Is too loud—the sound is uncomfortable or makes speech difficult to understand
- + Is unexpected (e.g., the sound of breaking glass)
- + Is uncontrolled (e.g., a neighbor's lawn mower)
- + Happens at the wrong time (e.g., a door slamming in the middle of the night)
- + Contains unwanted pure tones (e.g., a whine, whistle, or hum)
- + Contains unwanted information or is distracting (e.g., an adjacent telephone conversation or undesirable music)
- + Is unpleasant (e.g., a dripping faucet)
- + Connotes unpleasant experiences (e.g., a mosquito buzz or a siren wail)
- + Is any combination of the previous examples

To be noise, sound does not have to be loud, just unwanted. In addition to being annoying, loud noise can cause hearing loss, and, depending on other factors, can affect stress level, sleep patterns, and heart rate.

To increase privacy, broadband sound may be radiated into a room by an electronic sound-masking system that has a random noise generator, amplifier, and multiple loudspeakers. Noise from such a system can mask low-level intrusive sounds from adjacent spaces. This controlled sound may be referred to as *noise*, but not in the context of unwanted sound; rather, it is a broadband, neutral sound that is frequently unobtrusive. It is difficult to design air conditioning systems to produce noise that effectively masks low level intrusive sound from adjacent spaces without also being a source of annoyance.

Types of noise:

Random noise: is an oscillation, the instantaneous magnitude of which cannot be specified for any given instant. The instantaneous magnitudes of a random noise are specified only by probability distributions, giving the fraction of the total time that the magnitude, or some sequence of magnitudes, lies within a specified range (ANSI *Standard S1.1*). There are three types of random noise: white, pink, and red.

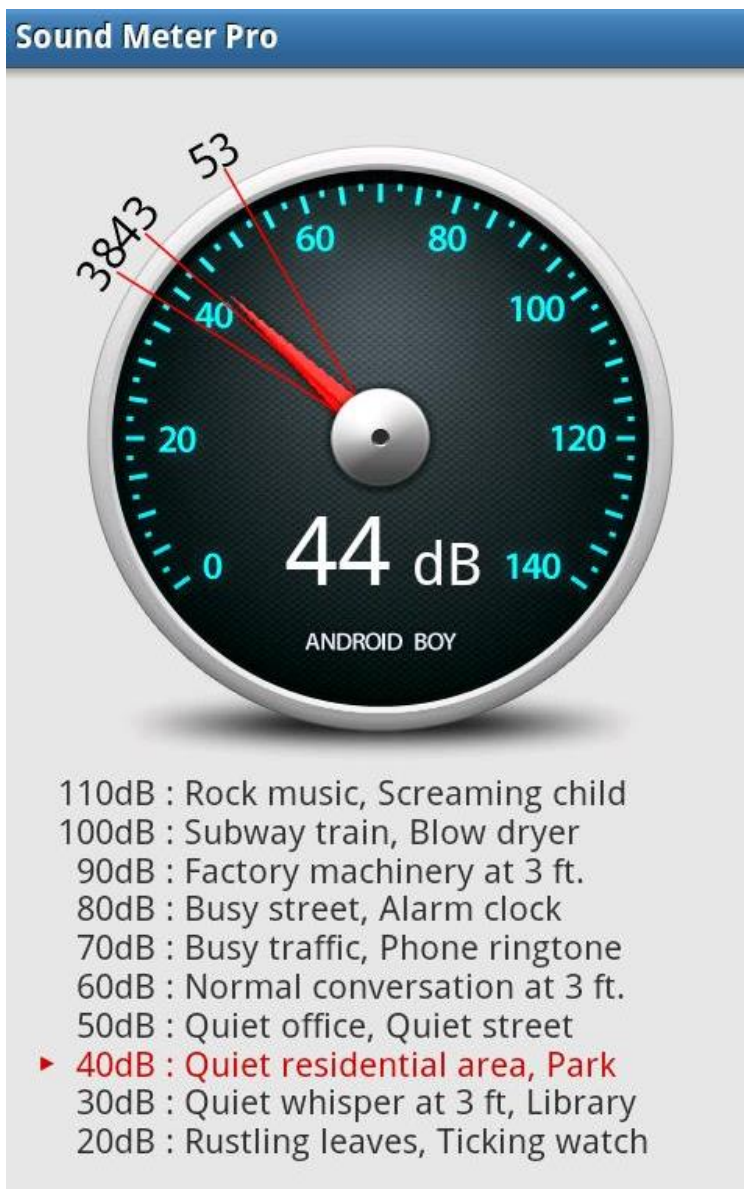
White noise: has a continuous frequency spectrum with equal energy per hertz over a specified frequency range. Because octave bands double in width for each successive band, for white noise the energy

also doubles in each successive octave band. Thus white noise displayed on a 1/3 octave or octave band chart increases in level by 3 dB per octave.

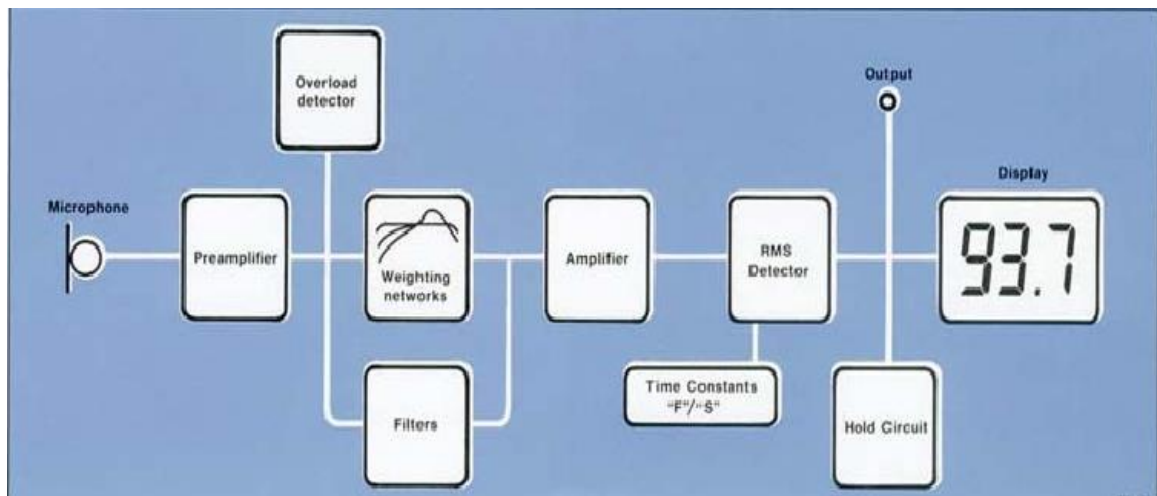
Pink noise: has a continuous frequency spectrum with equal energy per constant-percentage bandwidth, such as per octave or 1/3 octave band. Thus pink noise appears on a 1/3 octave or octave band chart as a horizontal line.

Red noise: has a continuous frequency spectrum with octave band levels that decrease at a rate of 4 to 5 dB per octave with increasing frequency. Red noise is typical of noise from well-designed HVAC systems.

Measuring devices:



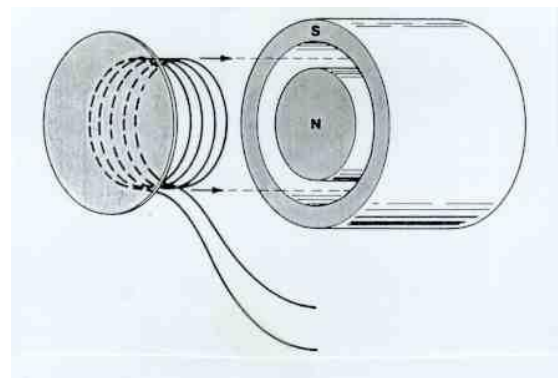
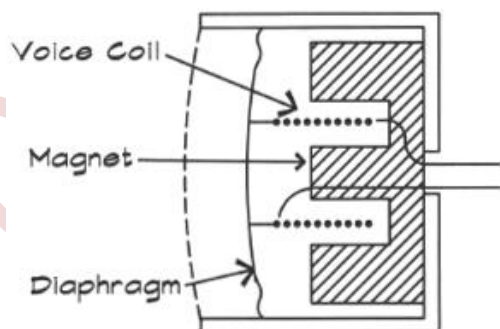
Microphone:



Basic Microphone Types:

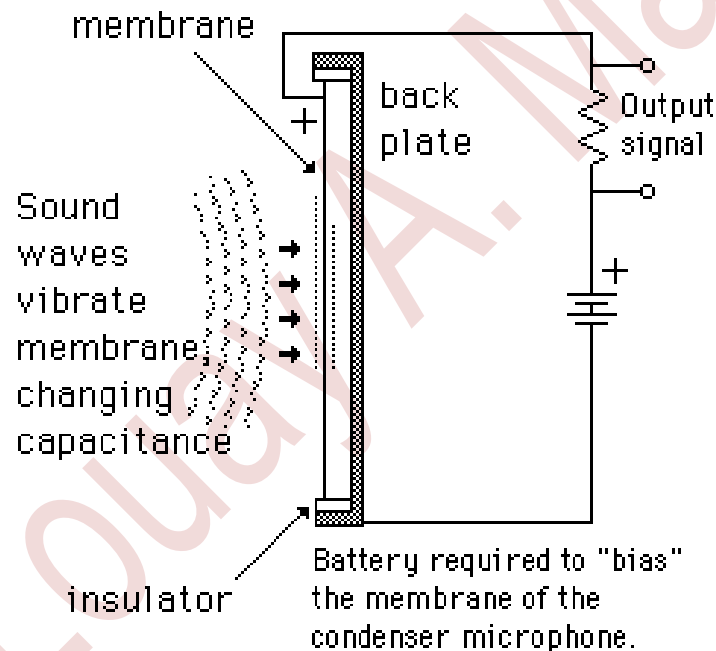
1. Dynamic (moving coil)

- ✚ Sound pressure on the diaphragm causes the voice coil to move in a magnetic field
- ✚ The induced voltage mimics the sound pressure
- ✚ Comments
 - Diaphragm and coil must be light
 - Low output impedance – good with long cables
 - Rugged



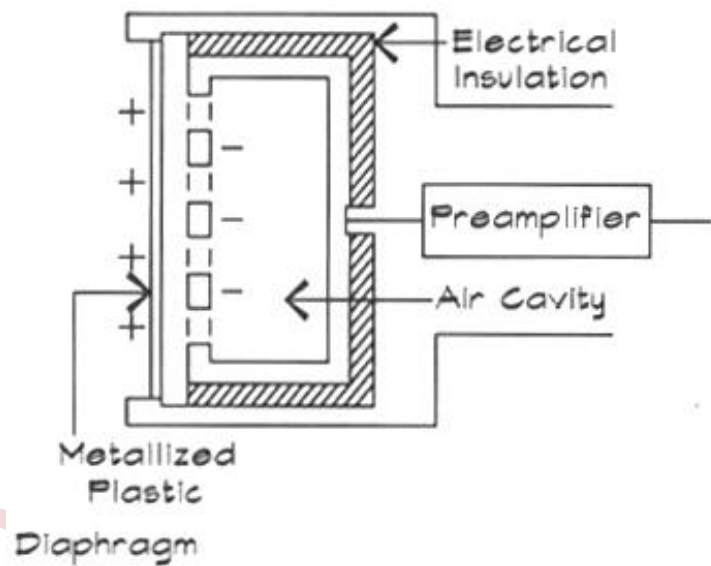
2. Condenser (capacitor)

- + Diaphragm and back plate form a capacitor
- + Incident sound waves move the diaphragm, change the separation distance, change the capacitance, create current
- + Comments
 - Requires a DC polarizing voltage
 - High sensitivity
 - Flat frequency response
 - Fragile
 - High output impedance, nearby pre-amp is necessary



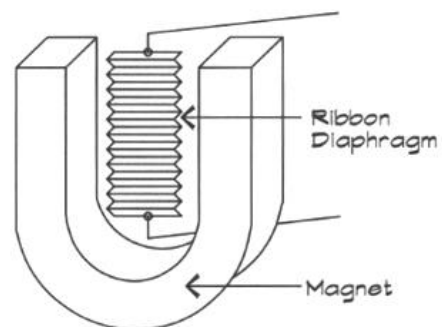
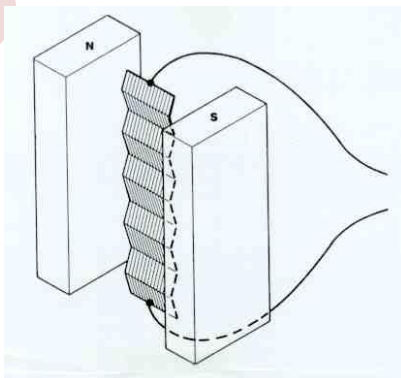
3. Electric:

- + Same basic operation principle as the condenser mic
- + Polarizing voltage is built into the diaphragm
- + Comments:
 - High sensitivity
 - Flat frequency response
 - Fragile
 - High output impedance, nearby pre-amp is necessary



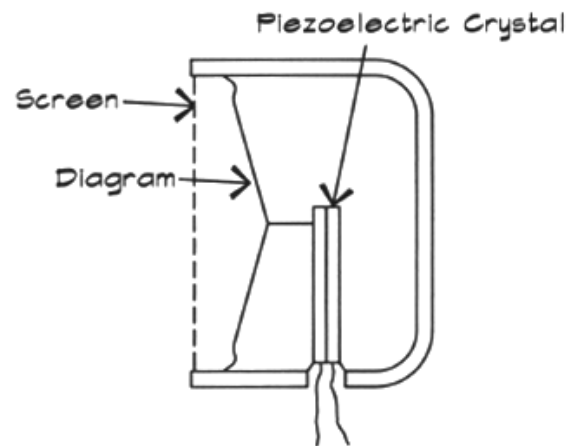
4. Ribbon

- + Conductive ribbon diaphragm moving in a magnetic field generates an electric signal
- + Comments
 - Lightweight ribbon responds to particle velocity rather than pressure
 - Both sides are exposed resulting in a bidirectional response
 - Sensitive to moving air
 - Easily damaged by high sound-pressure levels

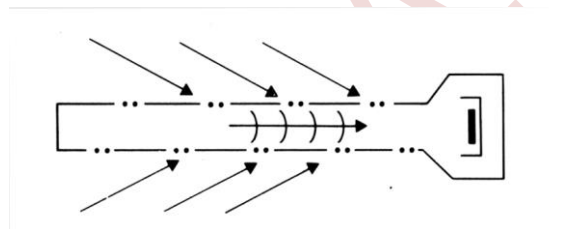


5. Piezo-electric (crystal or ceramic):

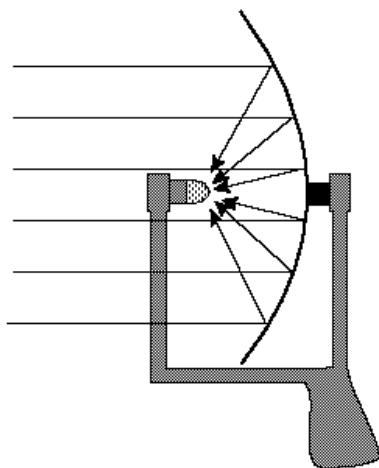
- + Diaphragm mechanically coupled to a piezoelectric material
- + Piezo (lead zirconate titanate (PZT), barium titanate, rochelle salt) generates electricity when strained
- + Comments:
 - No polarization voltage
 - Generally rugged



6. Shotgun Microphone:



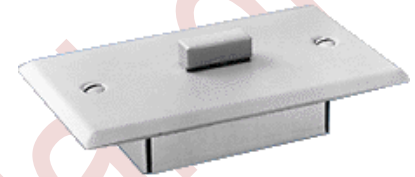
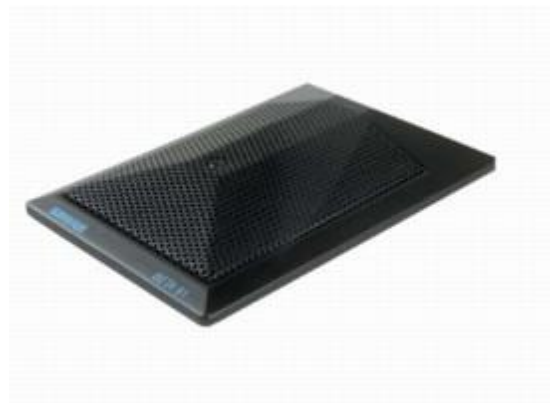
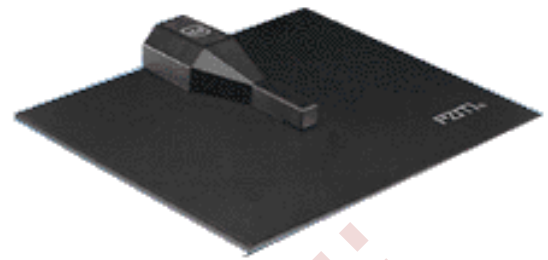
7. Parabolic Microphone



8. Contact Microphones



9. Pressure Zone Microphone (PZM)



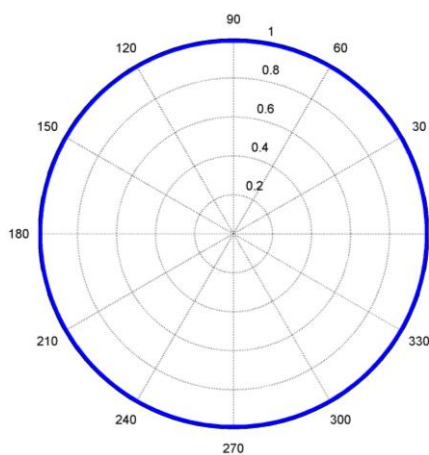
Directivity Patterns:

Single-diaphragm microphones are typically constructed to have one of a variety of directivity patterns

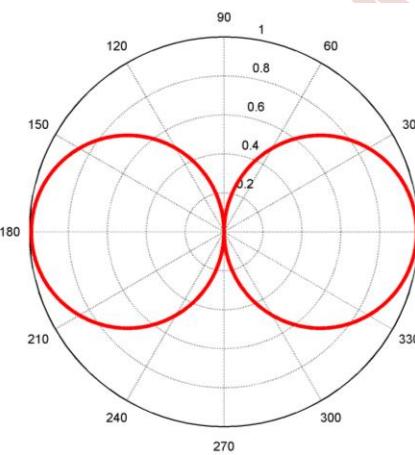
- Omni directional
- Bidirectional
- Cardioid
- Hyper cardioid
- Super cardioid
- All five

Directivity in 2 dimensions:

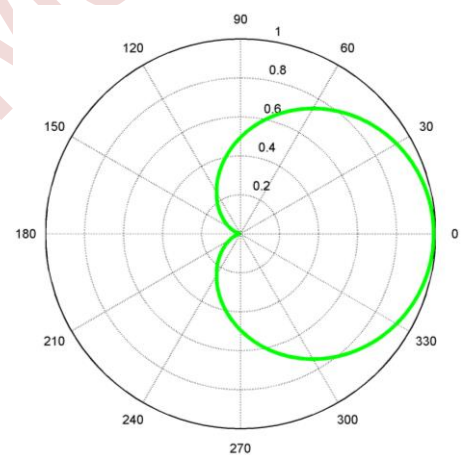
Omnidirectional



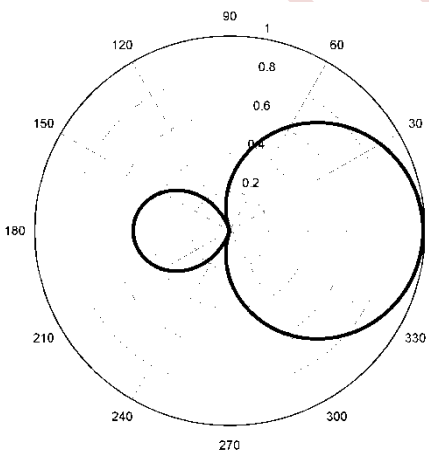
Bidirectional



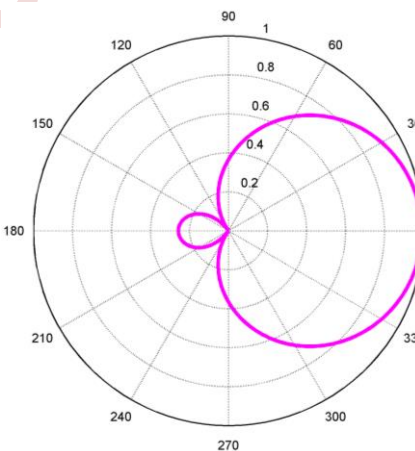
Cardioid



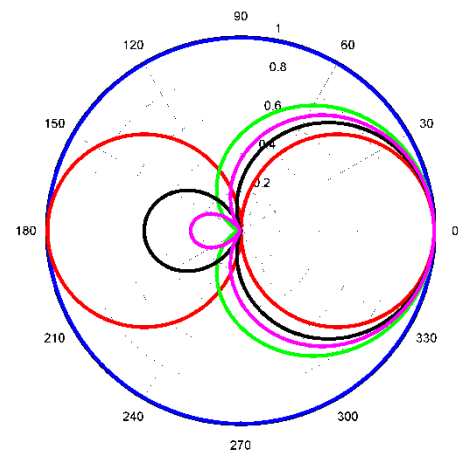
Hyper cardioid



Super cardioid

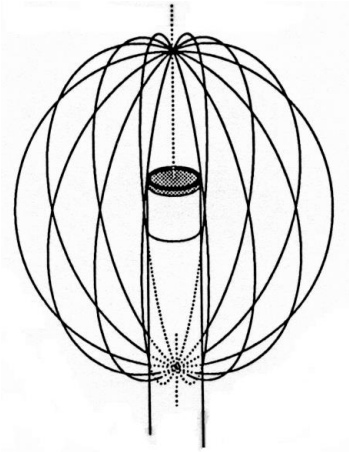


All Five

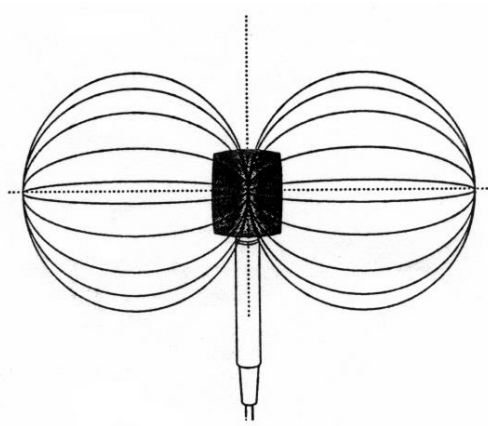


Directivity in 3 dimensions:

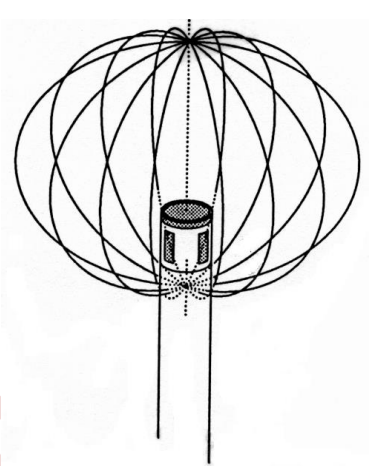
Omnidirectional



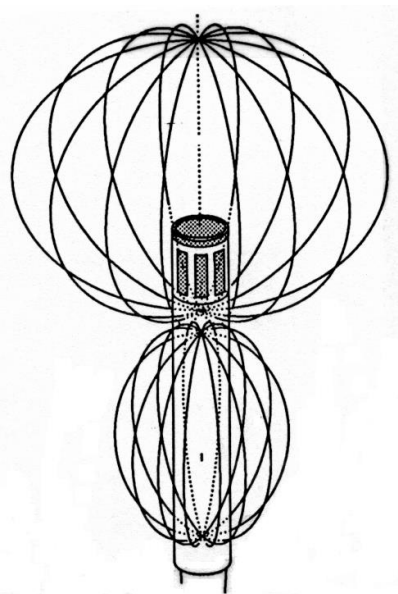
Bidirectional



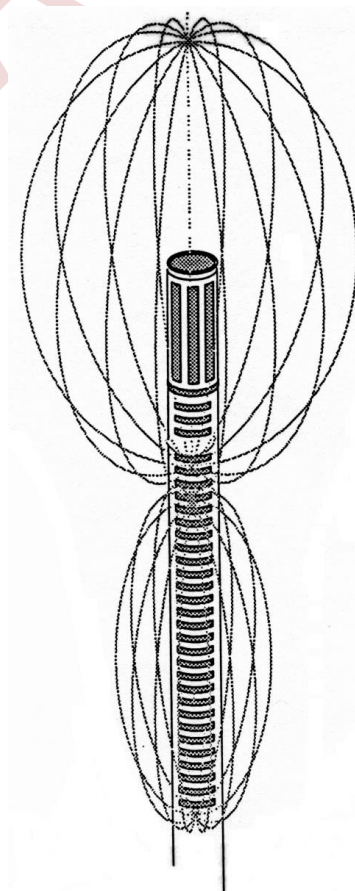
Cardioid



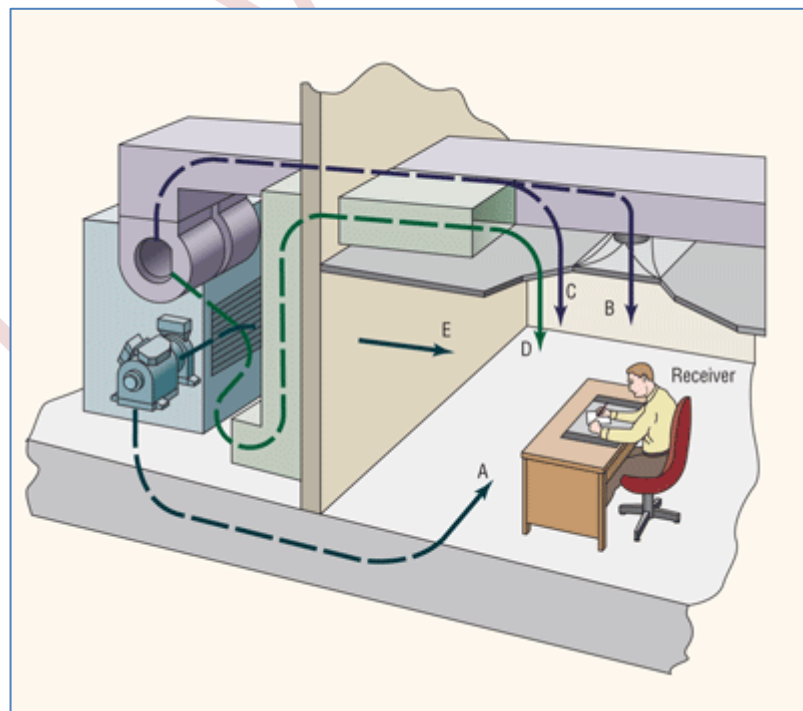
Hyper cardioid



Super cardioid



HVAC ACOUSTIC FUNDAMENTALS



The lecture is depend on McQuay HVAC Acoustic Fundamentals

Introduction:

Occupant comfort is the goal of all HVAC designers. Sound (or noise) is a key parameter in measuring comfort, in addition to temperature, humidity and Indoor Air Quality (IAQ). While acoustics consultants are usually involved in critical applications (such as performing arts centers), the task of creating a comfortable acoustic environment in most other applications falls on the HVAC engineer. This is because most background sound sources are generated by the HVAC equipment.

General

Sound is defined as a disturbance in an elastic medium that can be detected by the human ear. The medium can be gas, liquid or solid. Noise is undesirable sound or sound without value. The pressure waves (or sound) act on the inner ear, which is what we hear. The best sound is not necessarily no sound. In an open office concept, background sound offers privacy for conversations. The quality of sound is also important. Tonal sounds are usually not desirable. The following list will help better understand sound.

- The amplitude of the sound wave represents the loudness and is measured in decibels (Pascal's). The louder the sound, the larger the amplitude. The loudest atmospheric sound has zero atmospheric pressure at the low point and two times atmospheric pressure at the high point. This is 194 dB.
- The frequency of sound represents the pitch and is measured in Hertz. The higher the frequency, the higher the sound. The human hearing range is from about 16 Hz to 16,000 Hz. below 30 Hz, sound can be felt as well as heard.
- The wavelengths for sound can vary from 70 feet (21.3 m) at 16 Hz to 0.07 feet (0.02 m) at 16,000 Hz. This is important because sound absorbing materials tend to work well when their dimensions are close to the wavelength. Therefore, a 1-inch (25 mm) ceiling tile is effective at absorbing higher frequency sounds, but low frequency sounds are much more difficult to attenuate.
- The human ear can respond to very wide range of sound levels. At the low end, the ear is sensitive to sound pressure waves as little as 0.00002 Pa. At the high end, the human ear can hear about 20 Pa without pain, which is 1,000,000 times louder. This is a key reason why Decibel logarithmic scales are used.
- The speed of sound is dependent on the density of the medium it is travelling through. The lower the density, the slower the sound wave. At standard atmospheric conditions, the speed of sound (Mach 1) is 764 miles per hour (1120 feet per second, 341 m/s).

- Sound waves do not actually pass through walls or other solid objects. Instead, they impinge on the exterior surface of the wall or object, causing it to vibrate. This, in turn, causes the air molecules in the space to vibrate. What is actually happening is that the sound wave is making the wall or object move!

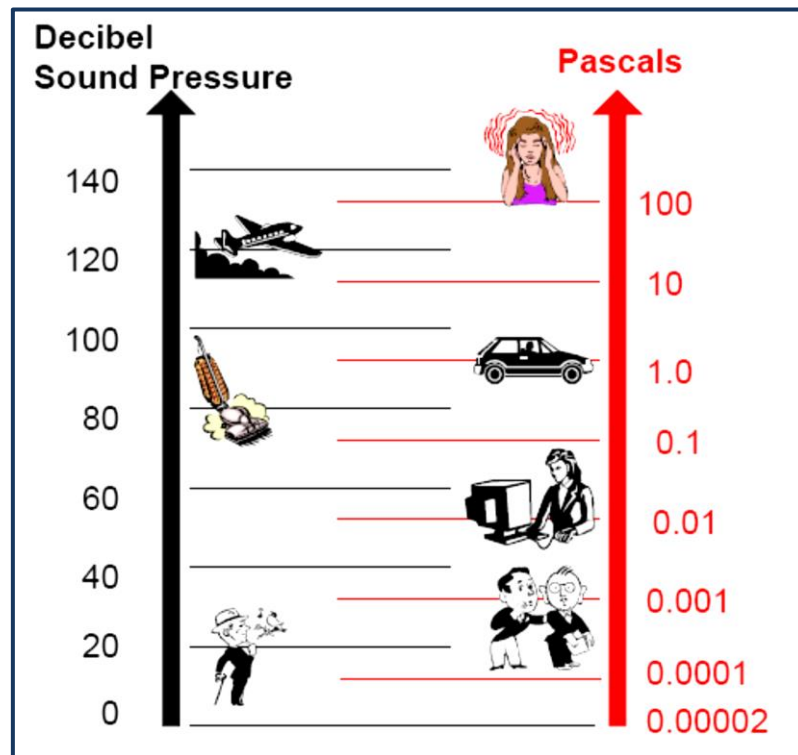


Figure show the typical sound pressure levels

Wavelength and Frequency

The wavelength of sound in air is given by;

$$\lambda = c_0 / f$$

Where

λ is the wavelength in feet (m).

c_0 is the speed of sound, which is 1120 feet per second (341 m/s) at sea level.

f is the frequency in Hz.

Decibels

The very large range in sound pressure makes a logarithmic scale more convenient. Decibels (dB) are always referenced to base signal. Knowing the base reference is critical because the term “decibels” in acoustics is used for sound pressure and sound power. In the case of sound pressure, the reference is 0.00002 Pascals, which is the threshold of hearing.

$$L_p = 20\log(P/0.00002)$$

Where

L_p is the sound pressure in Decibels (dB)

P is the sound pressure in Pascals.

For sound power the reference is 10^{-12} Watts.

$$L_w = 10\log(W/10^{-12})$$

Where

L_w is the sound power in Decibels (dB)

W is the sound power in Watts

Decibel Example

Calculate the loudest possible sound at standard atmospheric pressure (101.3 kPa)?

$$L_p = 20\log(101,300/0.00002)$$

$$= 194 \text{ dB RE } 20\mu \text{ Pa}$$

Note: $20\mu \text{ Pa}$ is another way to write 0.00002 Pa .

Decibel Addition and Subtraction

Since Decibels are logarithmic, they cannot simply be added. For instance, $40 \text{ dB} + 40 \text{ dB}$ is not 80 dB , it is 43 dB . Decibels can be added as follows:

$$L_s = 10\log(10^{L_1/10} + 10^{L_2/10} + 10^{L_3/10} + \dots)$$

Decibels can be quickly added together with an accuracy of about 1 dB by using the relationship shown in

Table 1-Decibel Addition Chart.

When Two Decibel Values Differ By	Add The Following Number To The Higher Number
0 or 1 dB	3 dB
2 or 3 dB	2 dB
4 to 9 dB	1 dB
10 dB or more	0 dB

Decibel subtraction is accomplished as follows:

$$L_s = 10\log(10^{L_1/10} - 10^{L_2/10} - 10^{L_3/10} - \dots)$$

Decibel Addition Example

Add the follow values together;

L1 = 80 dB

L2 = 82 dB

L3 = 84 dB

L4 = 93 dB

L5 = 72 dB

$L_{Total} = 10 \log(10^{80/10} + 10^{82/10} + 10^{84/10} + 10^{93/10} + 10^{72/10}) = 94 \text{ dB}$

Using **Table 1- Decibel**

Addition Chart

Between L1 and L2, there is a difference of 2 dB so add 2 dB to L2 for a total of 84 dB.

Between 84 dB and L3, there is a difference of 0 dB so add 3 dB to 84 dB for a total of 87 dB.

Between 87 dB and L4, there is a difference of 6 dB so add 1 dB to L4 for a total of 94 dB.

Between 94 dB and L5, there is a difference of 22 dB so add 0 dB to 94 dB for a total of 94 dB.

Sound Pressure vs. Sound Power

What you hear is sound pressure. It is the fluctuation in the atmospheric pressure that acts on your eardrum. However, sound pressure is dependent on the surroundings, making it a difficult means to measure the sound level of equipment. Sound power is the sound energy released by a sound source. It cannot be “heard”, but it can be used to estimate the sound pressure levels if the space conditions are known.

Sound pressure and sound power are best explained with an example. Consider a 5 kW electric baseboard heater. The 5 kW rating is a clear, definable measure that can be used to compare one heater against another. This is the equivalent of Sound Power. However, it not possible to know whether a 5 kW heater is sufficient to keep the occupant warm and comfortable unless the temperature of the space is known. The temperature of the space is the equivalent of sound pressure. If the 5 kW heater is used in a small, single room addition to a house, it will probably provide a comfortable temperature. If the heater is used in the Toronto Sky dome, it is unlikely to offer any comfort to occupants. In each application the same size heater (or sound power level) provides very different thermal comfort results (or sound pressure level).

Knowing the Sound Power levels for a piece of equipment (e.g. a fan coil) will allow a fair and direct comparison of two models. It will not, however, indicate whether the sound level will be acceptable until the space is defined. Knowing the sound pressure of a piece of equipment (e.g. a cooling tower) will allow two models to be compared (assuming the same conditions were used for testing both units). However, unless the actual space where the product is used has the same properties as the test conditions, the sound pressure level provided will not be what the occupant experiences.

In this manual, **Sound Pressure** will be indicated by L_p RE 20 μ Pa and **Sound Power** will be indicated by L_w RE 10⁻¹² W.

Octave Bands

Acoustic analysis is performed over a wide frequency range. It is broken down into 10 Octave bands labeled by the center frequency. The Bandwidth covers all sound from 0.707 of the center frequency to 1.414 times the center frequency. **Table 2 - Octave Band Properties**, shows the 10 octave bands and their properties. In many cases, the 16 and 31 Hz bands are not used because sound data is not available. Several publications reference the octave bands by the numbers 1 through 8, starting with the 63 Hz band. This can lead to misunderstanding and is not recommended. Always refer to an octave band by its center frequency (e.g. 63 Hz).

Table 2 - Octave Band Properties

Center Frequency	16	31.5	63	125	250	500	1000	2000	4000	8000
Max Freq. (Hz)	23	44	88	88	175	350	700	1400	2800	5600
Min. Freq. (Hz)	11	22	44	175	350	700	1400	2800	5600	11200
Wavelength (ft)	70	36	18	8.98	4.48	2.24	1.12	0.56	0.28	0.14

Acoustic analysis and evaluations are based on the sum of sound pressure or sound power measured over the all of the frequencies of the octave band. In some cases, a finer analysis is required and it is common to hear reference to 1/3 octave band analysis. With this approach the bands are broken down into thirds so there are 30 values to consider in the analysis.

Tonal Sounds

A pure tone is the sound pressure or power associated with a single frequency. For example, there is often a 60 Hz sound component when based on 60 Hz electric frequency. Identifying a pure tone can help in acoustic analysis. For example, if there was a 60 Hz pure tone and only octave band analysis is considered, then the 60 Hz pure tone energy would be included in the 63 Hz octave band. It would not be evident that the sound energy in this band was due the electric frequency. Equipment such as fans, compressors and pumps can produce tonal sounds.

Caution should be taken when evaluating tonal products because they are not modeled well by octave band analysis. For instance, a tonal product such as a double helix screw compressor cannot be evaluated using the processes described in this manual or by the Acoustic Analyzer.

Human Response to Sound

The goal of acoustic analysis and design is to provide a satisfactory environment for occupants. It is necessary to consider occupant response to sound and noise as part of the analysis in the same way as occupant response to temperature and humidity are considered.

The following lists key issues about human response to sound that must be considered.

- Humans are more sensitive to sound around the 1000 Hz octave band. These are the frequencies of voice communication. For example, a 60 dB sound pressure in the 1000 Hz band will *seem* louder to an occupant than a 60 dB sound pressure in the 63 Hz band.
- It takes a very large change in sound levels to be perceived as a change to humans. Reducing the sound pressure by 50% does not *seem* 50% quieter. **Table 3 – Perceived Sound Level Changes** shows the perceived change relative to the actual change in sound levels.
- Humans acclimate to constant sounds and they are sensitive to changes in sound levels. For example, constantly operating the fan on a fan coil will be less of an issue than cycling the fan on and off.
- Pure tones can stand out and be uncomfortable for occupants. A broad spectrum, bland sound is just as important as the overall sound level.
- Occupants can become fixated on a sound level once it is considered irritable. This may require an even greater improvement in sound level to satisfy the occupants than if the sound had been attenuated in the first place.

Sound change level (dB)	Acoustic energy loss	Relative loudness
0	0	Reference
-3	50%	Perceptible change
-6	75%	Noticeable change
-10	90%	Half as loud
-20	99%	1/4 as loud
-30	99.9%	1/8 as loud
-40	99.99%	1/16 as loud

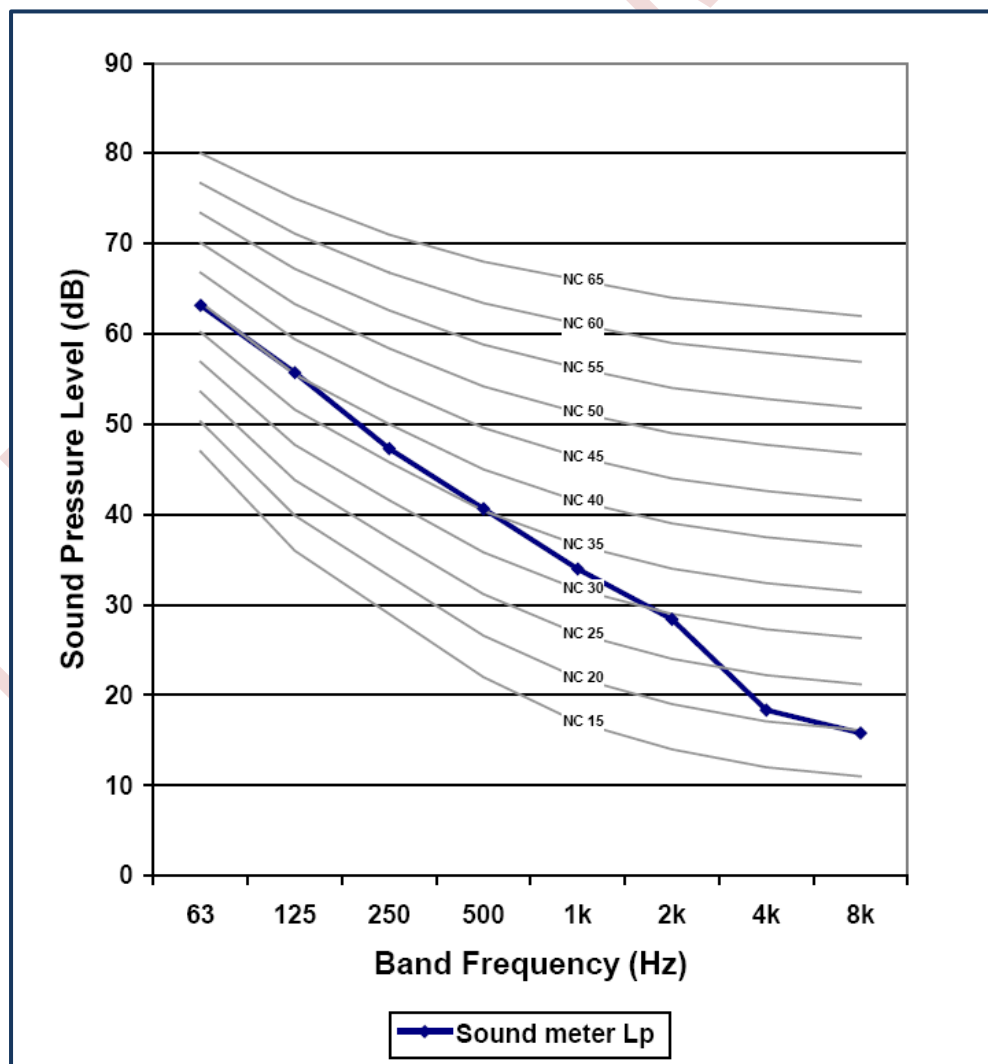
Sound Pressure Evaluation Criteria

There several recognized sound pressure criteria that are used to evaluate sound pressure levels and occupant comfort. The criteria attempt to take into account the human response to frequency, overall spectrum and level. Several of the most common in the HVAC industry are discussed here. Other criteria include Balanced Noise Criteria Method (NCB), RC mark II, Composite Noise Rating (CNR), etc. For more information on these criteria, refer to the ASHRAE Applications Handbook.

Noise Criteria (NC)

Noise Criteria or NC curves are the most common standard for indoor spaces. They were developed to take into account human response to sound pressure levels in different octave bands. Since humans are less sensitive to sound pressure in the lower bands, a higher sound pressure level was deemed acceptable. The shape of the curves can be seen in **Figure 2 - NC Curves**.

Figure 2 - NC Curves



NC curves are based on the 63 through 8000 Hz octave band values. When the octave band values of the space sound pressure are known, they are plotted on the NC curve. **Table 4 - Sound Pressure Levels For Each NC Level** shows the sound pressure levels for each NC level in tabular form.

Table 4 - Sound Pressure Levels For Each NC Level

NC level	Band							
	63	125	250	500	1000	2000	4000	8000
15	47	36	29	22	17	14	12	11
20	51	40	33	26	22	19	17	16
25	54	44	37	31	27	24	22	21
30	57	48	41	35	31	29	28	27
35	60	52	45	40	36	34	33	32
40	64	56	50	45	41	39	38	37
45	67	60	54	49	46	44	43	42
50	71	64	58	54	51	49	48	47
55	74	67	62	58	56	54	53	52
60	77	71	67	63	61	59	58	57
65	80	75	71	68	66	64	63	62

The NC value is not an “average” that is based on a curve drawn through the data points. Instead, it is the highest NC value in any one octave band. This is called the tangent method. An issue with the tangent method is that it is possible to have three distinct sound spectrums where the highest single NC level in any band could be the same value for all three spectrums. In this case, all three would have the same NC value, but they would sound quite different.

Another issue with the NC method is that it does not evaluate low frequency sounds below 63 Hz, which can be the most troublesome and the most difficult to attenuate.

NC Calculation Example

Using the rules described above and the following sound pressure levels, calculate the NC value?

The sound pressure levels in dB RE 20μPa:

Band	63	125	250	500	1000	2000	4000	8000
Lp	63	56	47	41	34	28	18	16

Using the Tangent Rule, the highest NC level in any one band occurs in the 125 Hz band.

The NC level in this band is NC 40.

Note: This curve is plotted in **Figure 2 – NC Curves**.

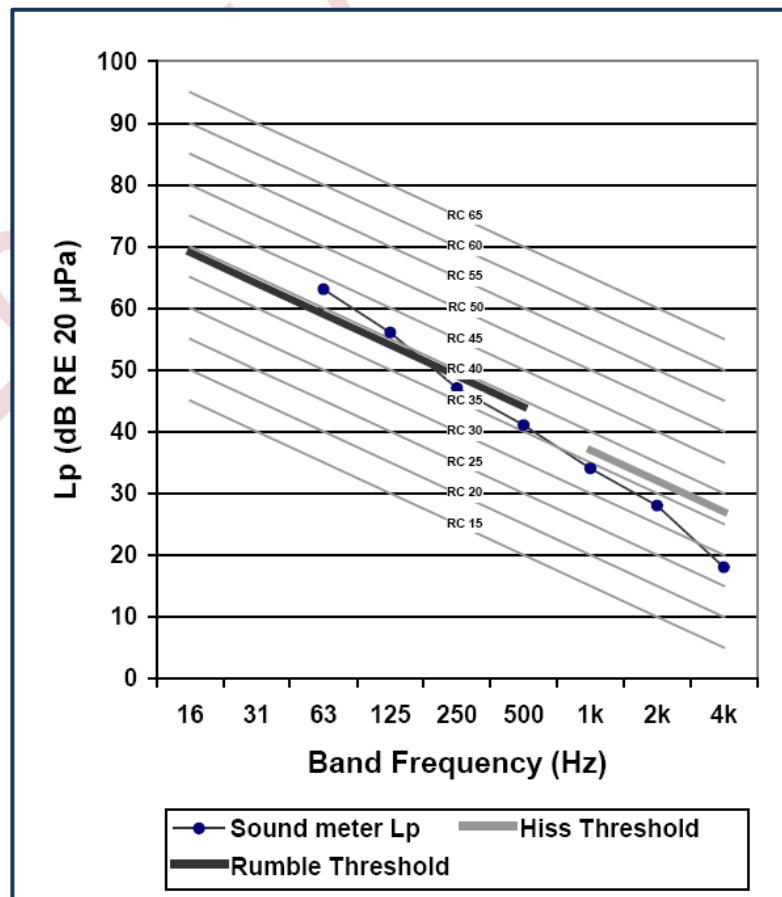
Room Criterion (RC)

Room criterion or RC curves are currently the most favored method for determining sound levels by ASHRAE. The RC system was developed to overcome the shortcomings of the NC system. They account for spectrum shape as well as sound level. The RC system also uses the 16.5 and 31 Hz bands, which allows the criteria to account for acoustically produced vibration in light building construction. However, the RC calculations should only be performed when valid data exists. If the 16.5 and 31 Hz data are not available, it should not be extrapolated from available values for RC calculations.

RC curves are based on 16 Hz through 4000 Hz octave bands. The RC value is the average of the 500, 1000 and 2000 Hz bands. The slope of each line is -5 dB per octave band as shown in **Figure 3- RC Curve**.

In addition to the numerical value, an RC criterion also includes one or more letters to denote specifics about the spectral shape of sound. The letter R denotes Rumble. This occurs whenever any specific RC value is more than 5 dB greater than the standard curve values in the 500 Hz and below octave bands. The letter H indicates Hiss. This occurs whenever any specific RC value is more than 3 dB greater than the standard curve values above the 500 Hz octave band. If the specific dB levels are between the dead bands, then the letter N is used to denote neutral. This sound will not have any identity with frequency and will sound bland.

Figure 3 - RC Curve



RC Calculation Example

Using the rules described above and the sound power data from the NC example, calculate the RC value?

The RC value is the average of the 500, 1000 and 2000 Hz values.

$$RC = (41+34+28)/3 = 34$$

The slope of each RC line is -5 dB per octave band.

Refer to the curve plotted above in **Figure 3 – RC Curve**. The sound meter Lp line is plotted on the RC chart and shows the sound measurement points.

The rumble threshold line is plotted on the chart from 500 Hz and down, 5 dB above RC 34. If any point at or below 500 Hz exceeds this line, then an R for rumble is added. In this case there is rumble.

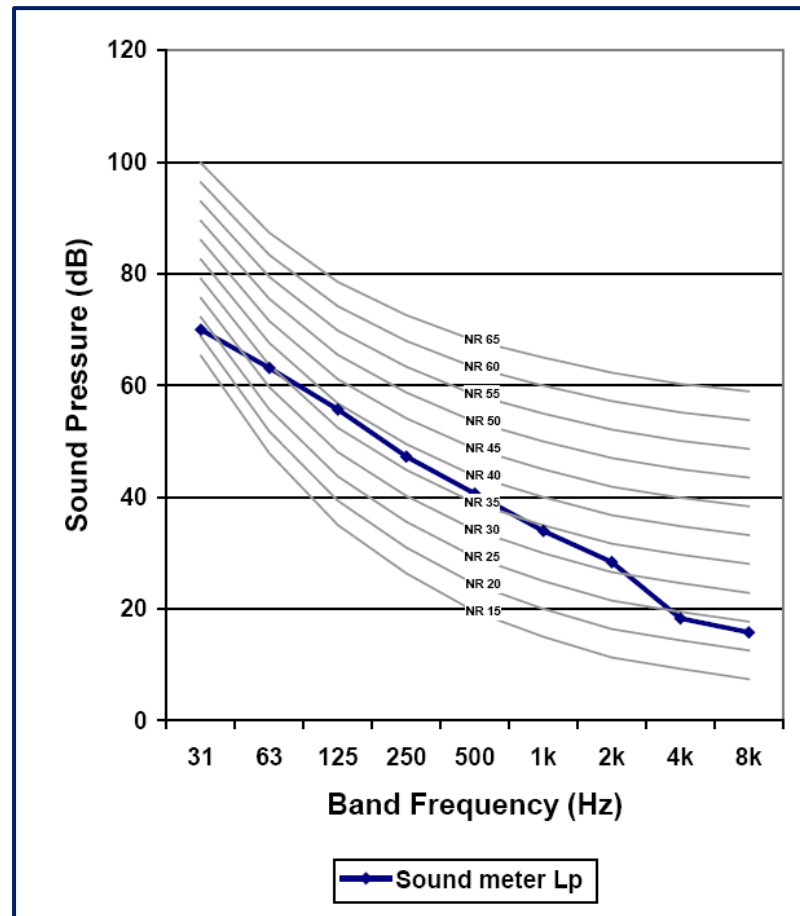
The hiss threshold line is plotted on the chart above 500 Hz, 3 dB above RC 34. If any point above 500 Hz is above this line, then an H for hiss is added. In this case there is no hiss.

Noise Rating (NR) Curves

Table 5 - Sound Pressure Level For Each NR Level

NR level	Octave Band								
	31	63	125	250	500	1000	2000	4000	8000
NR 0	55	36	22	12	5	0	- 4	- 6	- 8
NR 10	62	43	31	21	15	10	7	4	2
NR 20	69	51	39	31	24	20	17	14	13
NR 30	76	59	48	40	34	30	27	25	23
NR 40	83	67	57	49	44	40	37	35	33
NR 50	89	75	66	59	54	50	47	45	44
NR 60	96	83	74	68	63	60	57	55	54
NR 70	103	91	83	77	73	70	68	66	64
NR 80	110	99	92	86	83	80	78	76	74
NR 90	117	107	100	96	93	90	88	86	85
NR 100	124	115	109	105	102	100	98	96	95
NR 110	130	122	118	114	112	110	108	107	105
NR 120	137	130	126	124	122	120	118	117	116
NR 130	144	138	135	133	131	130	128	127	126

Figure 4 - NR Curves



Noise Rating curves were developed by International Organization for Standardization (ISO) and are similar to NC curves. They are based on sound pressure and the different curves represent different rooms and uses. NR levels are based on the tangent method so the highest NR value in any octave band is the NR level. NR curves are based on 31 through 8000 Hz octave bands.

Table 6 - Comparison of Sound Rating Methods

Method	Overview	Considers Speech Interference	Evaluates Sound Quality	Components Currently Rated by Method
NC	Can rate components No quality assessment Does not evaluate low frequency rumble	Yes	No	Air Terminals Diffusers
RC	Used to evaluate systems Should not be used to evaluate components Can be used to evaluate sound quality Provides some diagnostic capability	Yes	Yes	
NR	Can rate components No quality assessment Does not evaluate low frequency rumble	Yes	No	
dBA	Can be determined using sound level meter No quality assessment Frequently used for outdoor noise ordinances	Yes	No	Cooling Towers Water Chillers Condensing Units

Acceptable Sound Levels:

Table 7 - Design Guidelines for Sound Levels in Unoccupied Spaces

Space		RC(N)	NC
Private residences, apartments, condominiums		25-35	25-35
Hotels/Motels			
	Individual rooms or suites	25-35	25-35
	Meeting/banquet rooms	25-35	25-35
	Halls, corridors, lobbies	35-45	35-45
	Service, support areas	35-45	35-45
Office Buildings			
	Executive and private offices	25-35	25-35
	Conference rooms	25-35	25-35
	Teleconference rooms	25max	25max
	Open plan offices	30-40	30-40
	Circulation and public lobbies	40-45	40-45
Hospitals and Clinics			
	Private rooms	25-35	25-35
	Wards	30-40	30-40
	Operating Rooms	25-35	25-35
	Corridors	30-40	30-40
	Public areas	30-40	30-40
Performing Arts			
	Drama theaters	25 max	25 max
	Concert and recital halls	A	A
	Music teaching studios	25 max	25 max
	Music practice rooms	25 max	25 max
Laboratories			
	Testing/Research, minimal speech communication	45-55	45-55
	Research, extensive phone use, speech communication	40-50	40-50
	Group teaching	35-45	35-45
Churches, Mosques, Synagogues		25-35	25-35
	With critical music programs	A	A
Schools			
	Classrooms up to 750 ft ²	40max	40max
	Classrooms over 750 ft ²	35max	35max
	Lecture rooms for than 50	35max	35max
	Libraries	30-40	30-40
Courtrooms			
	Unamplified speech	25-35	25-35
	Amplified speech	30-40	30-40
Indoor Stadiums and gymnasiums			
	School and College gymnasiums and natatoriums	40-50	40-50
	Large seating capacity spaces	45-55	45-55

Acceptable sound levels depend on the use of the space. For example, the level in an office environment is established based on speech requirements. ASHRAE recommends that important space sound levels be specified by RC(N), while less stringent spaces can use NC levels. **Table 7 - Design Guidelines For Sound** Lists recommended RC(N) and NC levels for various spaces.

HVAC Equipment Acoustics

HVAC systems and equipment generate sound at all frequencies and power levels. **Figure 7 - Frequency Ranges** shows the frequency ranges where various types of HVAC equipment may create noise issues.

Knowing the frequency range that is not acceptable can help identify the source.

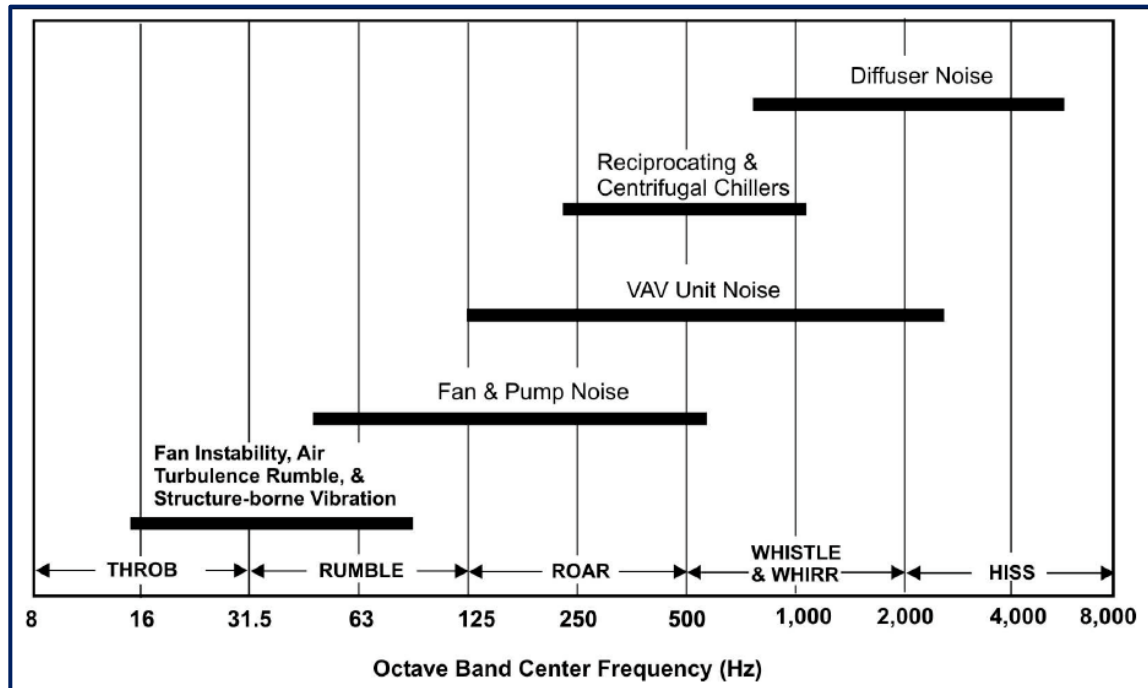


Figure 7 - Frequency Ranges Where Various HVAC Equipment Affect Sound Levels

Calculating Sound Pressure from Sound Power

Sound from a Point Source:

As soon as sound energy is released from a source, it interacts with the environment and creates sound pressure in a *source – path – receiver* arrangement. The most fundamental example is a point sound source emanating energy uniformly in all directions. Sound waves will travel uniformly in a spherical manner from a point sound source. The sound pressure level will decrease as a function of distance as follows;

$$L_p = L_w + 10\log(Q/4\pi d^2) + k$$

Where

d is the distance in feet (m) from the source to the measurement point

Q is the directivity factor, which for spherical radiation is 1

K is a constant whose value is 10.5 for I-P and 0.5 for SI

Ideally, this works out to a 6 dB drop in sound levels for every doubling of distance.

Directivity:

When a sound source is near a reflecting surface, the sound energy is concentrated and the sound waves are focused in a particular direction. This changes the *directivity* of the sound waves as shown in **Figure 8 - Sound Directivity Q**, directivity is referred to as Q and can vary from 1 to 8. An example of directivity is a point sound source at ground level in an open field. Here the ground is a reflecting surface, which concentrates the sound energy in a hemispherical pattern. The value of Q in this case is 2. When the sound source is near a reflecting surface such as the ground, the sound equation then becomes;

$$L_p = L_w + 10\log(1/2\pi d^2) + k$$

Where

d is the distance in feet (m) from the source to the measurement point

k is a constant whose value is 10.5 for I-P and 0.5 for SI

These equations are used extensively for outdoor sound analysis.


	<p>Diversity factor $Q=1$</p>
	<p>Diversity factor $Q=2$ Energy density doubled compared with 1 i.e directivity=3 dB</p>
	<p>Diversity factor $Q=4$ Energy density doubled compared with 2 i.e directivity=6 dB</p>
	<p>Diversity factor $Q=8$ Energy density doubled compared with 3 i.e directivity=9 dB</p>

Figure 8 - Sound Directivity Q

Figure 9, Graph of Sound Pressure vs. Distance in a Free Field, page 18, is a graphical means to convert a sound power source into a sound pressure level at a prescribed distance. This is only applicable in an open field. The diagonal lines represent sound power levels. Using a sound power level of 100 dB at a distance of 100 ft., the sound pressure level is about 63 dB. You can also start with a sound pressure level and estimate the sound level at a different distance. For example, start with a sound pressure level of 75 dB at 10 ft. Follow a constant sound power line (90 dB) until you reach 40 feet then read horizontally back to see the new sound pressure (60 dB) at the greater distance.

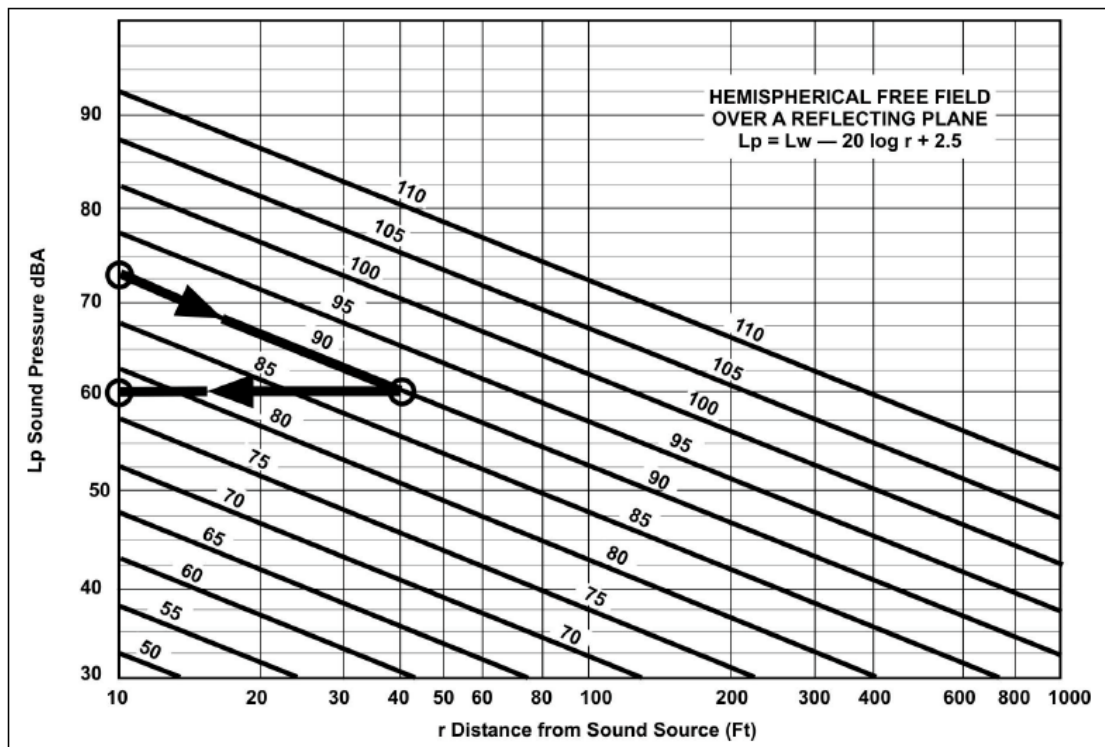


Figure 9, Graph of Sound Pressure vs. Distance in a Free Field

Sound from a Line Source

A line source is a collection of point sources that radiate sound in a cylindrical pattern. The equation that relates line sound power to sound pressure is;

$$L_p = L_w + 10\log(Q/\pi dL) + k$$

Where

d is the distance from the source to the measurement point

L is the length of the sound source in feet (m)

K is a constant whose value is 10.5 for I-P and 0.5 for SI

Line sources in HVAC systems include duct breakouts, where the breakout sound energy radiates away from the ducting and potentially into an occupied space. Traffic noise from a highway can also be considered a line source.

Sound from a Plane Source

A plane source is a surface that radiates sound into a space. In close proximity to the wall, the sound level does not change, making plane sound sources an issue to attenuate. The equations that relate sound power from a plane source to sound pressure are;

$$\text{When } d < b/\pi \quad L_p = L_w + 10\log(\pi/(4bc)) + k$$

$$\text{When } b/\pi < d < c/\pi \quad L_p = L_w - 10\log(d) - 10\log(4c) + k$$

$$\text{When } d > c/\pi \quad L_p = L_w - 20\log(d) - 11 + k$$

Where

d is the distance from the source to the measurement point

c is the larger dimension of the wall in feet (m)

b is the shorter dimension of the wall in feet (m)

k is a constant whose value is 10.5 for I-P and 0.5 for SI

An example is sound transmission through a wall separating a mechanical room from occupied space.

Usually the height of a room is less than either the length or the width. When this occurs and either the ceiling or the floor is the sound source, then the sound level will be uniform through the space and it will be a challenge to attenuate.

Sound Pressure in a Confined Space

When a point sound source emanates sound waves in a confined space, part of the sound energy reflects off the surfaces and back into the space. This contains the sound energy and makes the sound analysis more complex. Close to the sound source, the receiver is in the *near* sound field where no sound waves have been reflected. Further away, the receiver is in the *reverberant* sound field where there is a combination of direct and reflected sound waves. The basic formula that calculates sound pressure in a confined space is;

$$L_p = L_w + 10\log(Q/(4\pi d^2) + 4/R) + k$$

Where

d is the distance in feet (m) from the source to the measurement point

R is the Room Constant in Ft^2 (m^2)

Q is the Directivity factor

k is a constant whose value is 10.5 for I-P and 0.5 for SI

Room constant is a factor that measures a room's ability to absorb sound. For example, if R equaled infinity, then the walls, floor, ceiling, etc. would absorb all sound and the sound energy would behave as if it were in a free field. Indoor Sound Analysis provides a more detailed description of Room constant. When $Q/(4\pi d^2)$ is dominant, then the receiver is in the near field as most of the sound energy is coming directly from the source. When $4/R$ is dominant, the receiver is in the reverberant field.

Sound Level vs. Distance in a Confined Space:

is based on a 30 ft by 30 ft by 9 ft (10 m by 10 m by 3 m) room with a single 100 dB sound power source against one wall. The classic equation shows the drop off in sound level as you move out of the direct field and into the reverberant field (around 10 ft (3 m)). Beyond 10 ft (3 m), there is little drop in sound level. The Thompson and Schultz equations are considered more accurate in real world applications and show the sound level continuing to drop off as you move away from the source. The diffuse equation is included as a reference and a show how sound drops off in a free field. In this case, the sound level drops 6 dB for every doubling of distance.

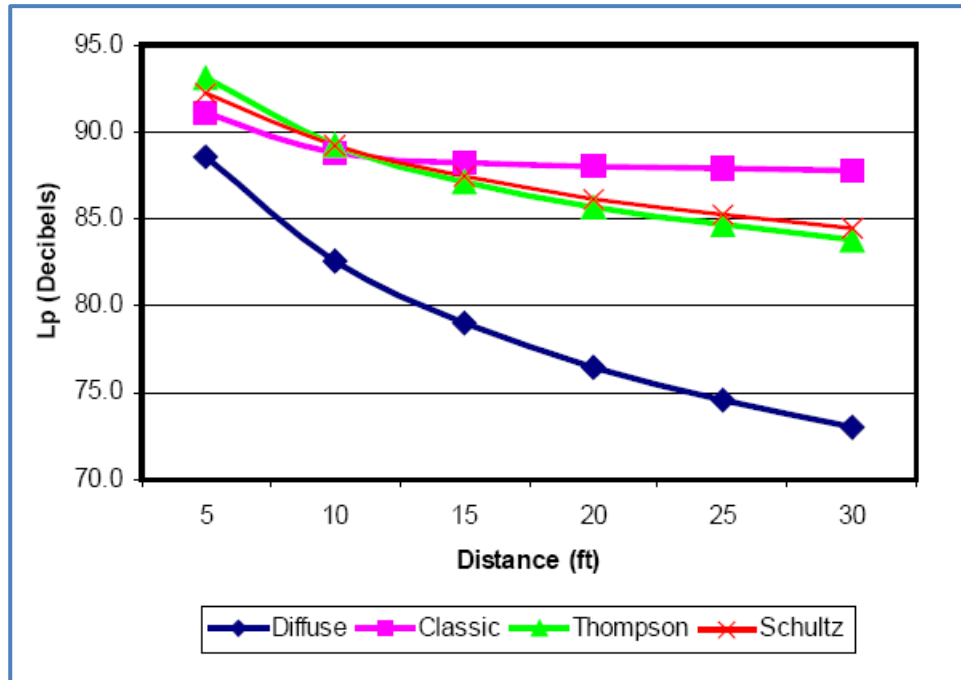


Figure 11 - Sound Level vs. Distance in a Confined Space

Sound Transmission through a Wall

In the previous section it was shown that the sound waves will reflect off the walls, floor, etc. and back into the space. However, not all the sound energy is reflected. Some of the sound energy is absorbed and some is transmitted through the wall and into the next space. Any energy that is either absorbed or transmitted will lower the sound level in the space that has the sound source. However, the adjacent space will now have an additional sound source (the common wall) which will radiate sound energy and add to the sound level. The basic equation that considers sound transmission is;

$$NR = TL - 10\log (S_w/R)$$

Where

- NR is the noise reduction in dB
- TL is the Transmission Loss of the wall in dB
- S_w is the common wall area in ft^2 (m^2)
- R is the Room Constant of the receiving room in Ft^2 (m^2)

This relationship is used to consider areas such as mechanical room, which can transmit sound into occupied spaces. This will be covered in more detail in future sections.



OUTDOOR SOUND ANALYSIS

BASICS

Directivity

Outdoor sound analysis is closest to a free field analysis where the directivity factor is 2. A point sound source (e.g. an air-cooled chiller) releases sound energy in a hemispherical pattern. The sound pressure level will depend on the distance from the source. In actuality, many sound sources have some level of directivity associated with them. For instance, an air-cooled chiller sends a significant amount of sound vertically from the condenser fans.

In critical applications, it is necessary to account for directivity. Figure 12 – Example of Sound Directivity shows the sound profile emanating from an air cooled chiller on a roof with one reflecting wall. In this case, the sound energy has greater concentration from the top fan deck. Accounting for this level of directivity is not easily done since the calculations become more involved and the sound profiles are not readily available. One method is to have a five point profile of the sound source. This profile indicates increases and decreases in the sound power level from the overall sound power levels for the four sides and vertical directions. With this information, it is possible to account for the orientation of the sound source. For example, a chiller with a control panel at one end may have a lower sound power level in that direction because the panel acts like a barrier. Orienting the unit so that the panel faces the property line may help reduce the sound pressure level at the property line.



Figure 12 - Example of Sound Directivity

Atmospheric and Anomalous Affects

Air absorbs sound energy, particularly in the higher frequencies. The amount of absorption is dependent on temperature and humidity. Acoustic Analyzer uses a molecular absorption coefficient (α_m) in dB per 1000 ft. (305 m) based on ambient temperature and humidity. A standard day is generally considered 59°F (15°C) and 70% relative humidity.

There are also small scale anomalous effects of refraction, sound interference, etc. that lowers the sound level – particularly at large distances (over 500 feet (150 m)). The Acoustic Analyzer accounts for the anomalous effects using the molecular absorption coefficient (α_a) in dB per 1000 feet (305 m). When these two factors are added to Eq. ;

$$L_p = L_w + 10\log(1/2\pi d^2) - d(\alpha_m + \alpha_a)/1000 + k$$

Where

- d is the distance from the source to the measurement point in feet (m)
- α_m is the molecular absorption coefficient
- α_a is the anomalous affects coefficient
- k is a constant whose value is 10.5 for I-P and 0.5 for SI

Trees and Shrubs

Trees and shrubs tend to break up and scatter sound waves. This can have both a positive and a negative effect. On the positive side, they can act as a barrier and reduce sound levels on the opposite side of the sound source. However, trees and shrubs can also disperse sound waves into *shadow* areas. For example, consider a sound barrier on a highway that runs through a residential neighborhood. Trees extending above the barrier can disperse sound down behind the barrier and toward the residences.

Wind, Temperature and Precipitation

Wind, temperature and precipitation can all bend sound waves and influence sound levels at large distances. Their effects are short term and generally not included in acoustic evaluations. However, they can explain differences in field testing.

Open Field Analysis

Open field or standard analysis is the most basic evaluation. An example would be rooftop equipment with no barriers or reflecting walls.

Calculating the Path Length

The first requirement is to calculate the distance from the source to the receiver. Referring to **Figure 13 - Standard Open Field Analysis**, the distance (d) from the source to the receiver is;

$$d = (D^2 + (EQH - MH)^2)^{\frac{1}{2}}$$

Where

d is the distance from the source to the receiver in feet (m)

D is the horizontal distance from the source to the receiver in feet (m)

EQH is the height to the source in feet (m)

MH is the height to the measurement point in feet (m)

Often D is the distance to the closest property line. It could also be the distance to an adjacent building on the property. The two heights are usually measured from the base of the building.

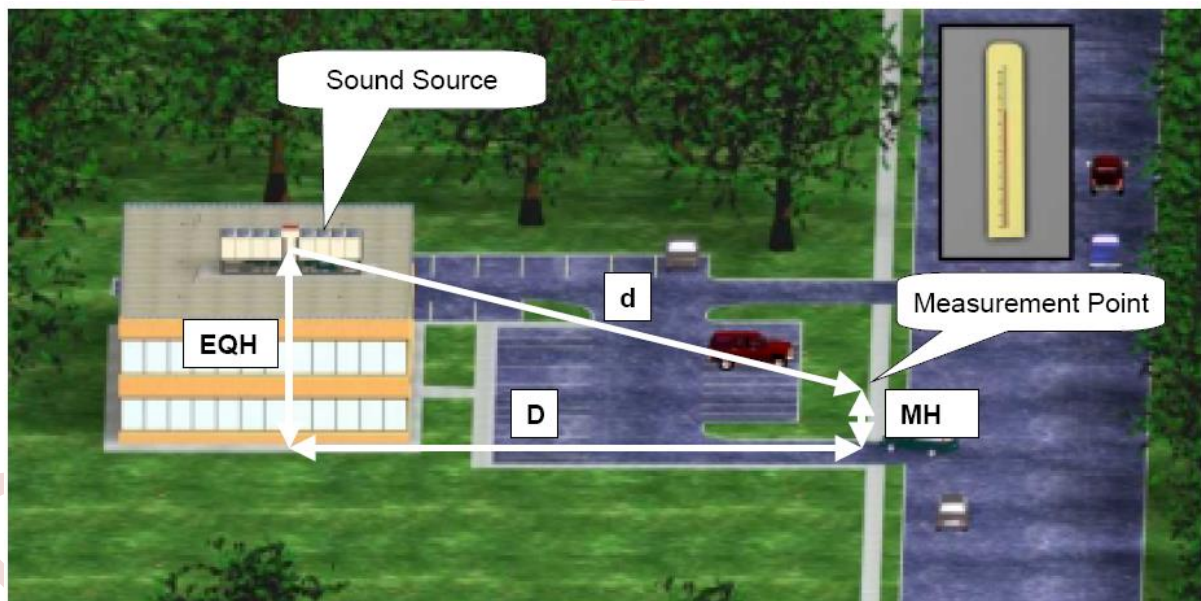


Figure 13 - Standard Open Field Analysis

Sound Barriers

Sound barriers can be installed to reduce the sound levels and hide equipment from view. The barrier creates an “acoustic shadow” that reduces sound levels on the opposite side of the source. The reduction in sound level, or *Insertion Loss*, is based on the path length difference. The path length difference equals the path around the barrier minus direct path from the source to the receiver in feet (or meters). Refer to Figure 14 - Sound Barrier.

Ideal Barrier : shows the insertion loss for an ideal barrier. An ideal barrier has a transmission loss at least 10 dB in all frequencies greater than the insertion loss expected of it. Note the best possible insertion loss is about 24 dB. This is due to scattering and refraction of sound into the shadow area. The best sound barriers surround the equipment on all four sides. Where the barrier is open, it should extend horizontally beyond the ends of the equipment to at least three times the path length difference over the top of the barrier. Reflecting walls can reduce the effectiveness of a barrier by reflecting sound into the shadow area.

Note: An ideal barrier has a transmission loss at least 10 dB in all frequencies greater than the insertion loss expected of it. Note the best possible insertion loss is about 24 dB.

Table 10 - Insertion Loss for Ideal Barrier

Path length Difference	Insertion Loss Per Octave Band (Hz)								
	31	63	125	250	500	1000	2000	4000	8000
ft									
0	0	0	0	0	0	0	0	0	0
0.01	5	5	5	5	5	6	7	8	9
0.02	5	5	5	5	5	6	8	9	10
0.05	5	5	5	5	6	7	9	10	12
0.1	5	5	5	6	7	9	11	13	16
0.2	5	5	6	8	9	11	13	16	19
0.5	6	7	9	10	12	15	18	20	22
1	7	8	10	12	14	17	20	22	23
2	8	10	12	14	17	20	22	23	24
5	10	12	14	17	20	22	23	24	24
10	12	15	17	20	22	23	24	24	24
20	15	18	20	22	23	24	24	24	24
50	18	20	23	24	24	24	24	24	24

Calculating the Path-Length Difference

the path-length difference δ can be calculated as follows;

$$d = (D^2 + (EQH - MH)^2)^{1/2}$$

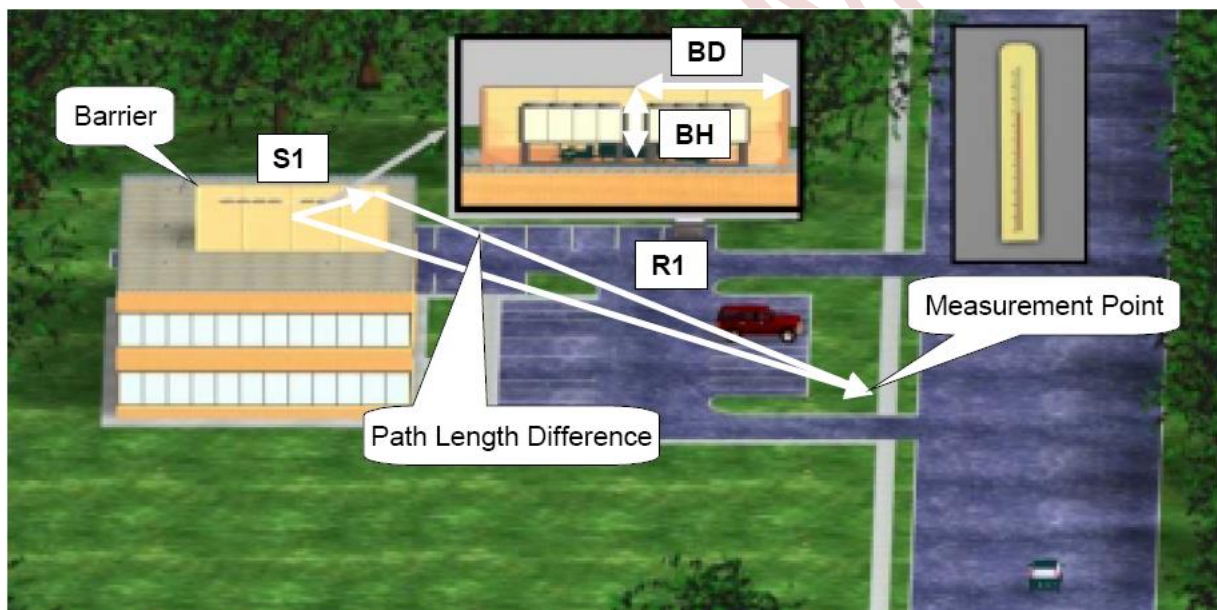
$$S1 = (BD^2 + BH^2)^{1/2}$$

$$R1 = ((D-BD)^2 + (EQH+BH)^2)^{1/2}$$

$$\delta = S1 + R1 - d$$

Where

- d is the distance from the source to the receiver in feet (m)
- D is the horizontal distance from the source to the receiver in feet (m)
- EQH is the height to the source in feet (m)
- MH is the height to the measurement point in feet (m)
- BD is the horizontal distance from the source to the barrier in feet (m)
- BH is the vertical distance from the source to the top of the barrier in feet (m)
- S1 is the distance from the source to the top of the barrier in feet (m)
- R1 is the distance from the top of the barrier to the receiver in feet (m)
- δ is the path length difference (m)



Reflecting Walls

When a large vertical surface is located near a sound source, it can reflect and concentrate sound energy into the open field portion. In addition, reflecting walls can diminish the effect of barriers by reflecting sound into the shadow zone of a barrier.

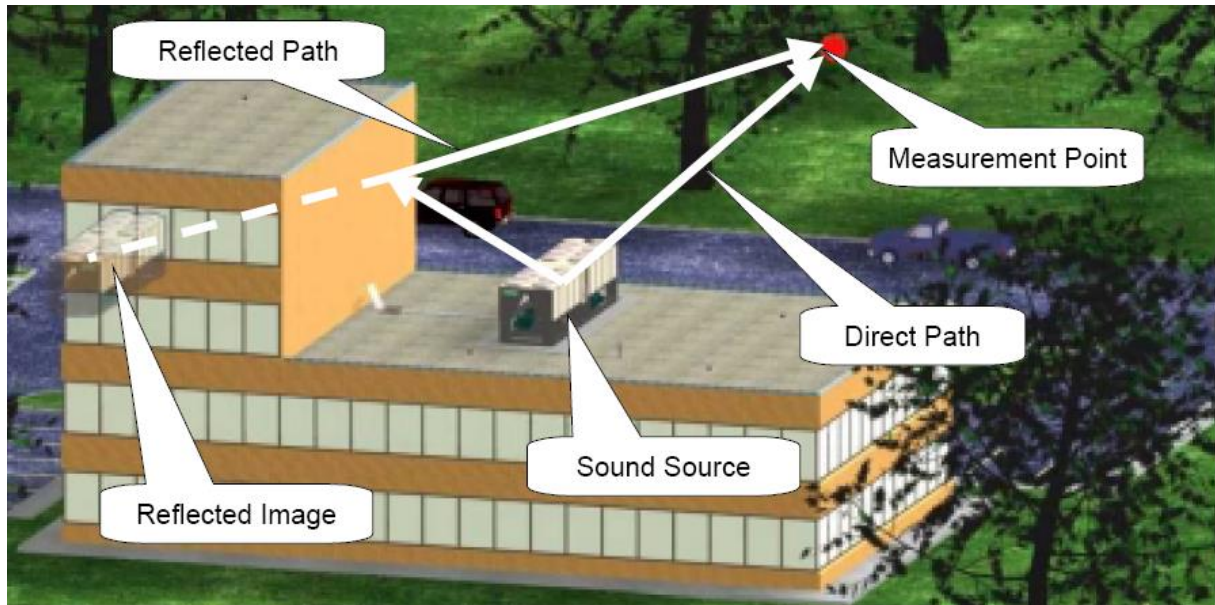


Figure 15 - Reflected Sound Waves

Sound waves reflect off walls in a similar manner as light waves reflecting in a mirror (refer to **Figure 15 - Reflected Sound Waves**). The reflected sound waves must travel from the sound source to the wall, where they are reflected before they travel to the receiver. The longer path is used to calculate the attenuation.

The additional sound level due to reflected sound is calculated as follows;

$$\Delta L_p = 3.00 - 9.29 \log(SR1/d) + 10.13(\log(SR1/d))^2 - 3.84(\log(SR1/d))^3$$

Where

d is the distance from the source to the receiver in feet

SR1 is the reflected distance in feet.

Additional reflecting walls can be included by calculating a ΔL_p for each wall and adding the results to the sound pressure levels at the receiver.



INDOOR SOUND ANALYSIS

ZONED COMFORT SYSTEMS

Sound in a Room

As discussed earlier, sound in a room will reflect off the walls, creating a near field where sound is dominated by direct sound from the source and a reverberant field where sound is dominated by the reflected sound energy. The Classic equation looks like;

$$L_p = L_w + 10 \log(Q / (4\pi d^2) + 4/R) + k$$

Where

- d is the distance from the source to the measurement point
- R is the Room Constant in ft^2 (m^2)
- Q is the Directivity factor
- k is a constant whose value is 10.5 for I-P and 0.5 for SI

Room Constant and Sound Absorption Coefficient

Eq. above introduces the Room Constant measured in ft^2 (m^2). It is a factor that describes the room's ability to absorb sound and is defined as;

$$R = S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_3 \dots + A_1 + A_2 + \dots + 4mV$$

Where

- S_1, S_2, S_3 etc. are the areas of surfaces 1, 2, 3 in ft^2 (m^2)
- $\alpha_1, \alpha_2, \alpha_3$ etc. are the *sound absorption coefficients* of surfaces 1, 2, 3 etc.
- $+ A_1 + A_2 \dots$ etc. are lumped groups of known absorbers
- m is the air absorption coefficient
- V is the volume of the room in ft^3 (m^3)

Sound Absorption Coefficient

The sound absorption coefficient is a property of the surface material. A surface with absorption coefficient of 1.0 would absorb all sound energy that it contacts. A coefficient of 0 would reflect all sound incident that it contacts. Sound Absorption coefficients vary with different bands, so the calculations must be performed for each band being considered. *Table 11 - Typical Sound Absorption Coefficients*, lists several common materials and their typical sound absorption coefficients. The term *4mV* represents the amount of sound energy the air in the space absorbs. It is most important in the 4000 and 8000 Hz bands. In some literature, the term *Sabin* is used to describe a room constant of 1 square foot (A metric Sabin is based on 1 m^2) with a sound absorption coefficient of 1.0.

Number of Occupants

Occupants in a space can have a large effect on the sound level. For example, nearly 75% of a concert hall's total absorption comes from the audience. Furniture also improves sound absorption by diffracting sound waves or absorbing them. Often, the goal of the designer is to obtain an acceptable background sound level in the space, which would mean there were no occupants.

However, it is important that the designer knows that occupants improve sound absorption. In critical applications such as performing arts centers, accounting for the audience is required.

Table 11 - Typical Sound Absorption Coefficients

Material	Octave Bands						
	63	125	250	500	1000	2000	4000
Brick, unglazed	0.02	0.03	0.03	0.03	0.04	0.05	0.07
Brick, unglazed, painted	0	0.01	0.01	0.02	0.02	0.02	0.03
Carpet on concrete	0.01	0.02	0.06	0.14	0.37	0.6	0.65
Carpet on foam rubber	0.06	0.08	0.27	0.39	0.34	0.48	0.65
Concrete Block, light, porous	0.25	0.36	0.44	0.31	0.29	0.39	0.25
Concrete Block, dense, painted	0.07	0.1	0.05	0.06	0.07	0.09	0.08
8" Acoustic Block	0.47	0.67	0.64	0.51	0.75	0.77	0.69
12" Acoustic Block	0.67	0.95	0.89	0.55	0.74	0.81	0.72
Concrete or Terrazzo Floor	0	0.01	0.01	0.015	0.02	0.02	0.02
Linoleum, asphalt, rubber or cork tile on concrete	0.01	0.02	0.03	0.03	0.03	0.03	0.02
Wood Floor	0.1	0.15	0.11	0.1	0.07	0.06	0.07
Glass	0.25	0.35	0.25	0.18	0.12	0.07	0.04
Curtain Wall	0.126	0.18	0.06	0.04	0.03	0.02	0.02
Closed Curtains	0.05	0.07	0.31	0.49	0.75	0.7	0.6
Drywall on Stud Wall	0.2	0.29	0.1	0.05	0.04	0.07	0.09
Marble or Glazed Tile	0	0.01	0.01	0.01	0.01	0.02	0.02
Drywall on Brick wall	0.01	0.013	0.015	0.02	0.03	0.04	0.05
Wood Paneling	0.2	0.28	0.22	0.17	0.09	0.1	0.11
2" 3 pcf Fiberglass Insulation	0.15	0.22	0.82	1	1	1	1
3" 3 pcf Fiberglass Insulation	0.48	0.53	1	1	1	1	1
4" 3 pcf Fiberglass Insulation	0.76	0.84	1	1	1	1	0.97
Suspended Ceiling, 3/4 to 1" Acoustic Tile	0.4	0.58	0.59	0.69	0.86	0.84	0.75
Suspended Ceiling, 3/4 to 1" Acoustic Tile	0.49	0.73	0.71	0.76	0.89	0.75	0.58

Calculating Room Constants

One method to calculate the Room Constant is to use equation 17 and add all of the surface areas and their sound absorption coefficients. This level of detail is most easily done with software like the McQuay Acoustic Analyzer™. A simplified method is to use the average sound absorption coefficients in **Table 12 - Average Sound Absorption Coefficients for Typical Receiving Rooms**, in the following equation;

$$R = S \cdot \alpha_T / (1 - \alpha_T)$$

And

$$\alpha_T = \alpha + 4mV/S$$

Where

S is the total surface area of the room in ft² (m²)

α is the average room absorption coefficient from Table 12

Table 12 - Average Sound Absorption Coefficients for Typical Receiving Rooms

Type of room	Octave Bands						
	63	125	250	500	1000	2000	4000
Dead	0.26	0.30	0.35	0.40	0.43	0.46	0.52
Medium Dead	0.24	0.22	0.18	0.25	0.30	0.36	0.42
Average	0.25	0.23	0.17	0.20	0.24	0.29	0.34
Medium Live	0.25	0.23	0.15	0.15	0.17	0.20	0.23
Live	0.26	0.24	0.12	0.10	0.09	0.11	0.13
Air absorption coefficient							
m(1/ft)	0	0	0	0	0	0.0009	0.0029

Note: Most rooms where an HVAC designer would be interested in (office space, classrooms, etc.) would be rated as medium dead.

Major Factors in Room Constants:

Figure 16 - Typical Sound Absorption Coefficients shows absorption coefficients of typical wall materials. From this figure, it is clear that suspended ceiling tiles play an important role. Curtains and carpeting can also help absorb sound. In high-rise residential applications with poured concrete ceilings, the carpets and curtains are typically the best absorbers in the space. Hard surfaces such as glass and concrete can reflect sound back into the space, making it very “live”. An example is a gymnasium or natatorium.

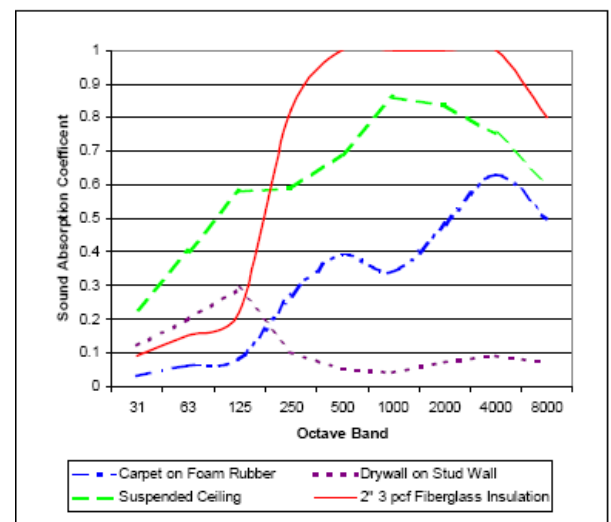


Figure 16 - Typical Sound Absorption Coefficients

Thompson and Schultz Equations

Equation 12 shows the fundamental relationship between sound power and sound pressure in an enclosed space. Based on empirical data, two equations are commonly used to provide more accurate results. These are the Thompson equation and the Schultz equation. Here is the Thompson equation;

$$L_p = L_w + 10 \log(Q \cdot e^{-md} / (4\pi d^2) + (MFP/d) \cdot (4/R)) + 10 \log(N) + k$$

Where

- Q is the directivity factor, which is usually 2.
- m is the air absorption coefficient
- d is the distance from the source to the receiver in feet (m)
- MFP is the mean free path in feet (m)
- R is the Room constant in ft² (m²)
- N is the number of point sound sources.

k is a constant whose value is 10.5 for I-P and 0.5 for SI

The mean free path is defined as;

$$MFP = 4 \cdot V / S$$

Where

- V is the room volume in ft³ (m³)
- S is the total surface area of the room in ft² (m²)

Here is the Schultz equation;

$$L_p = L_w - 10 \log(d) - 5 \log(V) - 3 \log(f) + 10 \log(N) + k$$

Where

- d is the distance from the source to the receiver in feet (m)
- V is the room volume in ft³ (m³)
- f is the center Band frequency in Hz
- N is the number of point sound sources
- K is a constant whose value is 25 for I-P and 12 for SI

The Schultz equation can be used for a single point sound source such as a diffuser or return air opening. The equation will work for about three point sources (N=3). For an array (four or more) of distributed ceiling diffusers, the multiple ceiling array equation can be used for a receiver point 5 feet above the floor;

$$L_p = L_{ws} - 27.6 \log(h) - 5 \log(X) - 3 \log(f) + 1.3 \log(N) + k$$

Where

- L_{ws} is the sound power level associated with a single diffuser in dB RE 10-12 watts
- H is the ceiling height in feet (m)
- X is the ratio of the floor area served by each diffuser divided by square of the ceiling height
- K is a constant whose value is 30 for I-P and 15.8 for SI

When to Use Each Equation

So far, this manual has listed three point source equations to estimate the sound pressure level in a confined space (Classic, Thompson and Schultz). While the classic equation is good for showing the relationship between near field and reverberant field, it is not used for actual sound calculations. Both the Thompson and Schultz equations produce acceptable results when properly applied. The Thompson equation is based on the Classic equation with modifications based on empirical data. It also requires a Room Constant, which means the space must be understood. The Thompson equation works well with live spaces such as gymnasiums and churches.

The Schultz equation is completely empirical. It is referenced in the *ASHRAE Applications Handbook* and in *ARI Standard 885, Procedure for Estimating Occupied Space Sound Levels in the Application of Air Terminals and Air Outlets*. The Schultz equation is easier to use and works well in many typical applications (medium dead spaces).

In addition to point source equations, there are also line source equations that are used for duct breakout calculations. There are also plane source equations, which are used for wall transmission calculations. When to apply each equation is dependent on the room type (live vs. Dead) and the type of sound source (breakout duct vs. Point source). Some equations are easier to use manually.

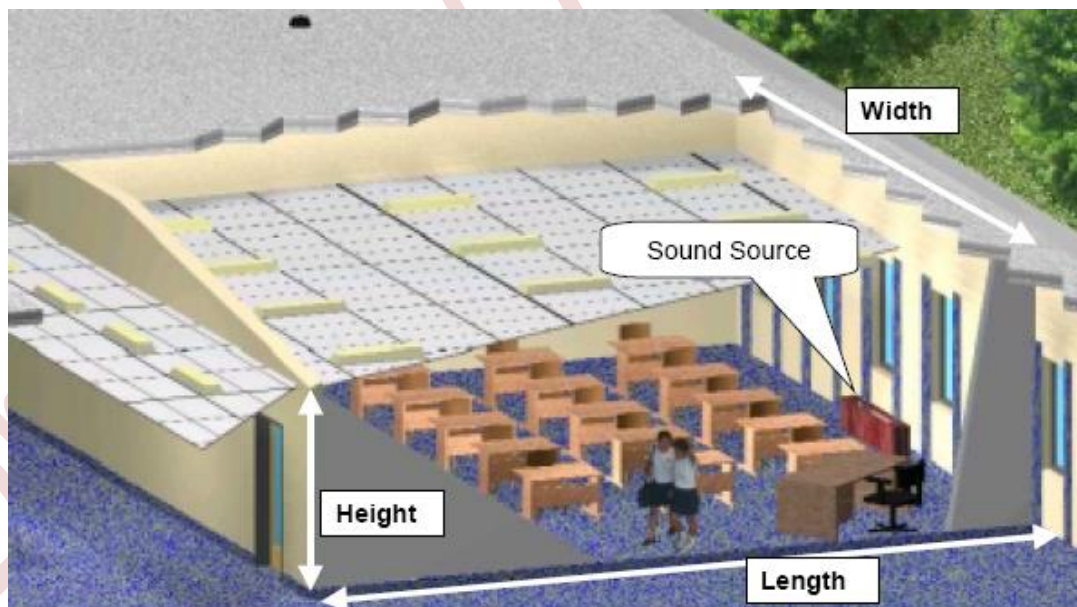


Figure 17 - Decentralized Unit in the Space

INDOOR SOUND ANALYSIS

Ducted Zoned comfort systems

Ducted zoned comfort systems are common in HVAC design. Ceiling WSHPs, fan coils and unit ventilators are good examples. Acoustically evaluating a ducted system introduces the concept of multiple pathways. The ducting is both an attenuator and a sound source. The sound energy from the equipment is divided into discharge and radiated sound. Ceiling plenums are commonly used with these systems, and they offer attenuation that must also be considered.

Radiated and Discharge Sound Power

Most ducted HVAC product sound data will include both radiated and discharge sound power levels. The radiated sound data is used to evaluate the sound energy that emanates from the product as if it was a single point sound source. If the unit is not above the space (a classroom HVAC unit in the corridor, for example) then radiated sound may not be important. When the unit is above the occupied space, then radiated sound will have to be considered. This is covered in **Radiated Sound Path**.

Discharge sound power data is the sound energy that is focused into the ductwork. This sound energy will be attenuated by the ductwork with the remaining sound energy being dispersed into the occupied space at the diffusers. Discharge sound power can also break out of the ductwork, creating a path to the occupied space.

Return Air vs. Supply Air Sound Power

When considering the discharge sound energy, the designer will want to know the sound energy focused in the discharge air side (supply) and the return air side (return). Discharge and return sound power levels may be available from the manufacturer. When only the total discharge sound power levels are known, conventional wisdom is to divide the sound power in half (lower the values by 3 decibels) and assume half the energy goes one way and half the other. But this may not be true.

Multiple Path Concepts:

Figure 18 – Multiple Path Concept shows the many sound paths that can exist with a ducted system. To estimate the sound level in the space, each of these paths should be checked. The sound levels from all the paths are logarithmically added together to obtain the final space sound pressure level.



Figure 18 - Multiple Path Concept

Ceiling Plenums

When a ceiling plenum is used, it can have a greater effect on the sound levels in the space. First, when calculating the Room Constant, acoustic tile is typically the most important surface in absorbing sound energy, particularly in the higher frequency bands. The second advantage of a ceiling plenum is its ability to “trap” sound released in the ceiling plenum. Several sound paths require the sound energy to pass through the ceiling plenum such as return air, breakout and radiated sound paths. ASHRAE has done a considerable amount of testing on ceiling plenums and has developed the following process for estimating the plenum attenuation.

Table 13 - Ceiling Plenum Attenuation with T bar Suspension

Description	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
No Suspended Ceiling	0	0	0	0	0	0	0	0
Mineral Fiber 1 lb. Density	3	6	8	10	16	21	36	21
Mineral Fiber 0.5 lb. Density	3	5	7	9	15	20	23	18
Glass Fiber 0.1 lb. Density, 5/8" Thick	3	5	6	7	7	8	9	7
Glass Fiber 0.6 lb. Density, 2" Thick	4	7	8	11	15	19	25	20
Glass Fiber with TL Backing 0.6 lb. Density, 2" Thick	4	7	8	12	27	22	29	23
Drywall Ceiling	8	11	15	15	17	17	18	14
Double Drywall Ceiling	14	17	21	21	23	23	24	19

The following qualifications have been made when using these values:

- The plenum is at least 3 ft (1 m) deep.
- The plenum space is either wide (over 30 ft (9 m)) or lined with insulation.
- The ceiling has no significant penetration directly under the unit.

For conditions other than these, the sound attenuation may be less. For instance, tests have shown that a 2-ft (0.6 m) deep plenum will be 5 to 7 dB louder below 500 Hz.

Duct Sound Path:

The discharge and often the return air sound energy must pass through a duct system prior to being released into the space. The duct can attenuate the sound, but it can also generate sound energy.

Consider a splitter-damper that causes turbulence and makes noise as the air flows around it. This is referred to as **regenerated** noise.

Figure 19 – Duct Path Attenuation, a typical supply duct arrangement for a terminal unit (such as a WSHP). In this case, there is also a return air elbow. As the sound energy leaves the HVAC unit, each section of duct will generally attenuate or lower the sound level. The sound energy level at the duct opening close to the unit will be higher than the duct opening further from the unit because there are fewer attenuating components.

The process of estimating the attenuation requires considering each component, estimating their sound attenuation and **arithmetically** subtracting the attenuation from the sound power. If a component causes regenerated noise, then the noise source is **logarithmically** added to the sound energy level at that point in the duct. Therefore, it is important to calculate attenuation and regenerated noise in the correct order from supply to discharge.



Figure 19 - Duct Path Attenuation

This room will be used for several of the upcoming examples. It is 40 ft by 50 ft by 8 ft (12 by 15 by 2.4 m). The supply air volume is 2000 cfm distributed through 6 lay in type diffusers.

Duct Component Attenuation:

ASHRAE and other sources have developed tables and equations to estimate the attenuation of duct fittings. These can be found in the ASHRAE Applications Handbook. The McQuay Acoustic Analyzer uses algorithms based on this data. The following is an overview of several attenuating components found in ducted systems. Additional components are also described in **Central System Duct Sound Path**.

Table 14 – Typical Duct Attenuation Table

Description	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
6 x 6	0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1
12 x 12	0.35	0.2	0.1	0.06	0.06	0.06	0.06	0.06
12 x 24	0.04	0.2	0.1	0.05	0.05	0.05	0.05	0.05
24 x 24	0.25	0.2	0.1	0.03	0.03	0.03	0.03	0.03
48 x 48	0.15	0.1	0.07	0.2	0.2	0.2	0.2	0.2
72 x 72	0.1	0.1	0.05	0.2	0.2	0.2	0.2	0.2

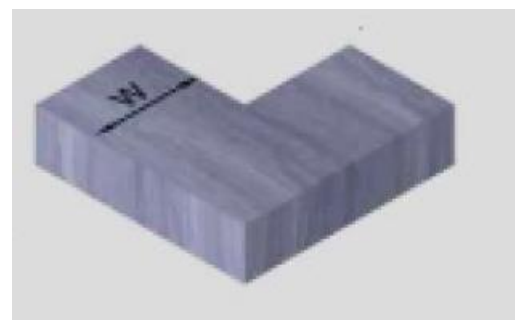
Straight Ducting

Table 14, shows a typical table for a rectangular, unlined duct. The attenuation is based on length of duct in feet (meters). Round, oval and rectangular ducts behave quite differently, so it is important to use the correct data. Round duct has a much lower attenuation than a rectangular duct of the same size. However, round duct has much lower breakout. Duct lining on the interior surface also significantly improves duct performance, particularly in the higher frequencies. It is important to understand how much ducting is lined. Additional straight duct insertion loss tables are listed in **Appendix 3 – Various Acoustic Properties of Materials**.

Elbows:

Elbows can be square (mitered) or round with a round or rectangular cross-sectional profile. They can be insulated or have turning vanes. From an acoustical perspective, a square elbow with no turning vanes is best choice if the regenerated noise is not too high. This elbow can reflect sound back up the duct. There are tables for elbows that are usually based on the elbow width (see **Figure 20**). Additional elbow insertion loss tables are in **Appendix 3 – Various Acoustic Properties of Materials**.

Figure 20 – Typical Elbow Dimension



Duct Branches

Figure 21 – Duct Branch

When sound energy reaches a branch in the duct, the sound energy is split. If the duct velocities all remain constant, then the ratio of the sound energy split is the same as the ratio of the duct areas. If the duct areas on the discharge side have a different cross-sectional area than the supply duct, then the change in cross-sectional areas will cause sound energy to be reflected back up the duct. This will occur for frequencies with plane waves.

Plane waves occur at frequencies below;

$$f_{co} = Co / 2a$$

Where

f_{co} is maximum frequency (Hz) that the reflection will occur

Co is the speed of sound in air (1120 fps [341 m/s])

a is the larger cross sectional dimension (ft) (m) of a rectangular duct

The attenuation for a duct branch is given as follows;

$$\Delta L_{Bi} = 10 \log * \left[\frac{\sum \frac{S_{Bi}}{S_M - 1}}{\sum \frac{S_{Bi}}{S_M - 1}} \right]^2 + 10 \log * \left[\frac{S_{Bi}}{\sum S_{Bi}} \right]$$

Where

ΔL_{Bi} is the branch attenuation in dB

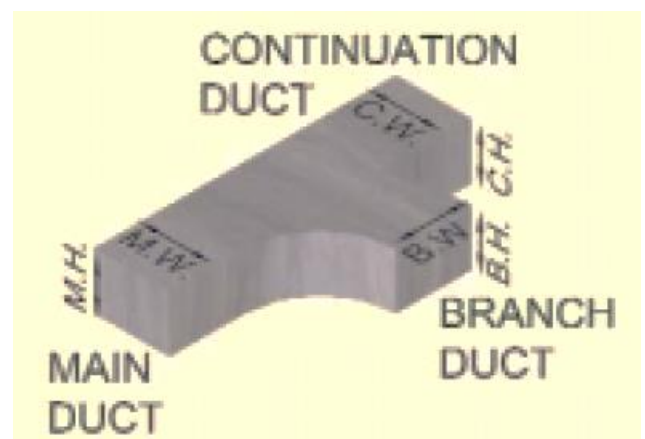
S_{Bi} is the cross sectional area (ft² [m²]) of branch i.

$\sum S_{Bi}$ is the sum of the cross sectional areas (ft² [m²]) of the branches continuing on from the main duct.

S_M is the cross sectional area (ft² [m²]) of the main feeder duct

The first term in the equation is related to the plane wave reflection. The second term is related to the division of the sound power based on the ratio of air flows.

Figure 21 – Duct Branch



Flex Duct

Flex duct is evaluated for duct attenuation in the same way as regular duct. However, its properties are different enough to require special tables and equations. Flex duct is a very good sound attenuator, but it also allows a significant amount of duct breakout (see **Duct Breakout Sound Path**). For this reason, it is a good idea to limit flex duct to no more than 5 ft. (1.5 m).

Table 15 - Flex Duct Attenuation

Diameter by length	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
4" by 3'	2	3	3	8	11	11	7	5
5" by 3'	2	3	4	8	10	10	7	5
6" by 3'	2	3	4	8	10	10	7	5
7" by 3'	2	3	5	8	9	10	6	5
8" by 3'	2	3	5	8	9	9	6	5
9" by 3'	2	3	6	8	9	9	6	5
10" by 3'	2	3	6	8	9	9	5	4
12" by 3'	2	2	5	8	9	8	5	4
14" by 3'	1	2	4	7	8	7	4	3
16" by 3'	1	1	2	6	7	6	2	2

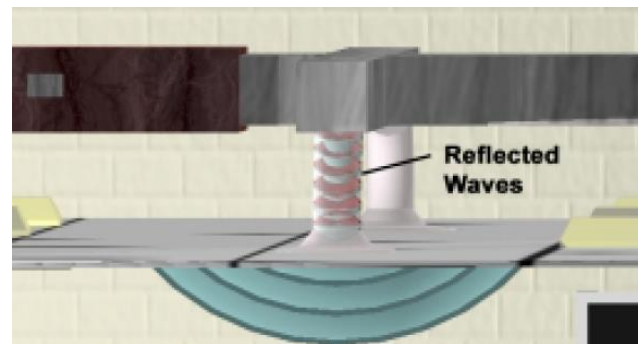
End Reflection:

When low frequency sound waves travelling in confined spaces (ductwork) suddenly undergo a large change in cross sectional area, some of the waves are reflected back up the ductwork. This creates a significant amount of low frequency attenuation referred to as **End Reflection**. The attenuation is greater when the duct terminates in free space versus ending flush with a wall since a ceiling tile array is basically transparent to low frequency sound, a duct terminating in the ceiling grid is considered to be terminating in free space. For example, a supply duct ending at a diffuser mounted in the ceiling grid is considered to end in free space.

Table 16 and **Table 17** show the end reflection for an equivalent size round duct. The Acoustic Analyzer uses an algorithm to estimate similar attenuation.



Figure 22 a) End reflection in a wall



b) End reflection in a free space

Table 17 - End Reflection Duct Terminated in Wall

Size diameter	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
6	18	13	8	4	1	0	0	0
8	16	11	6	2	1	0	0	0
10	14	9	5	2	1	0	0	0
12	13	8	4	1	0	0	0	0
16	10	6	2	1	0	0	0	0
20	9	5	2	1	0	0	0	0
24	8	4	1	0	0	0	0	0
28	7	3	1	0	0	0	0	0
32	6	2	1	0	0	0	0	0
36	5	2	1	0	0	0	0	0
48	4	1	0	0	0	0	0	0
72	2	1	0	0	0	0	0	0

Table 16 - End Reflection Duct Terminated in Free Space

Size diameter	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
6	20	14	9	5	2	1	0	0
8	18	12	7	3	1	0	0	0
10	16	11	6	2	1	0	0	0
12	14	9	5	2	1	0	0	0
16	12	7	3	1	0	0	0	0
20	10	6	2	1	0	0	0	0
24	9	5	2	1	0	0	0	0
28	8	4	1	0	0	0	0	0
32	8	3	1	0	0	0	0	0
36	6	3	1	0	0	0	0	0
48	5	2	1	0	0	0	0	0
72	3	1	0	0	0	0	0	0

Converting Ducted Sound Power to Sound Pressure:

The processes described above will allow the designer to estimate the sound power at each duct opening in the space. This technique is used for both supply and return duct paths. Once the sound power has been estimated, the designer will have to convert the sound power to sound pressure by taking into account the room effect. For up to three point sound sources such as a supply diffuser or a return air opening, the Schultz equation can be used. For multiple diffusers (more than four) the multiple diffuser array equation can be used. Also, the Thompson equation can be used for either a single or a multiple point array.

Duct Breakout Sound Path:

Estimating the Breakout Sound Power As sound energy travels down a duct, some of the energy escapes through the duct wall. We refer to this as breakout. Breakout can be a significant sound source when evaluating sound levels in a room, and it can result from a duct passing near an occupied space that does not serve that area. This is particularly true near mechanical rooms where main supply and return ducts pass over occupied spaces as they enter the mechanical room. **Figure 23 - Duct Breakout**, shows sound emanating from the duct surface. Breakout is often the source of low frequency rumble in the space.

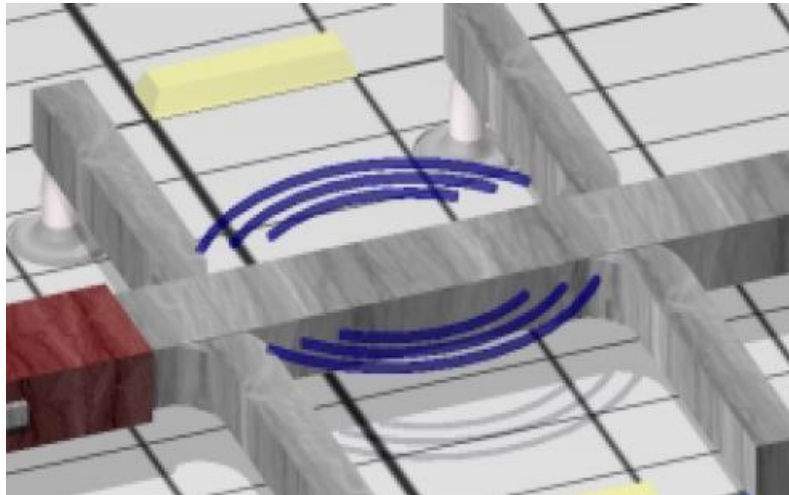


Figure 23 - Duct Breakout

The sound energy that breaks out from the duct is defined as;

$$L_{w_r} = L_{w_i} + 10 \log[S/A] - TL_{out}$$

Where

L_{w_r} is the sound power radiated from the duct in decibels

L_{w_i} is the sound power in the duct in decibels

S is the outer surface area of the duct in inches²

A is the cross-sectional area of the duct in inches²

TL_{out} is the breakout transmission loss in dB

Breakout in Rectangular Ducts

For rectangular ducts;

$$S = 24 \cdot L \cdot (w+h)$$

$$A = w \cdot h$$

Where

L is the duct length in feet (m)

Breakout in Circular Ducts

For circular ducts;

$$S = 12 \cdot L \pi d$$

$$A = \pi d^2 / 4$$

Where

d is the diameter in inches

L is the duct length in feet (m)

Table 18 - Breakout TL_{out} for Rectangular Duct

Duct Size	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
12x12	21	24	27	30	33	36	41	45
12x24	19	22	25	28	31	35	41	45
12x48	19	22	25	28	31	37	43	45
24x24	20	23	26	29	32	37	43	45
24x48	20	23	26	29	31	39	45	45
48x48	21	24	27	30	35	41	45	45
48x96	19	22	25	29	35	41	45	45

Table 18, is typical TL_{out} values for rectangular, long seam and spiral ducts that can be used to estimate breakout. The Tables do not account for sound energy attenuation along the duct, so they should only be used for ducts that are 20 to 30 feet (7 to 10 m) in length. Acoustic Analyzer uses algorithms to calculate breakout TL_{out} . Notice that the TL_{out} for round ducts are significantly higher than for rectangular ducts. Using round ducts can reduce breakout, but rectangular ducts are better at attenuating sound. Using rectangular duct can reduce discharge sound levels.

Insulation, such as acoustic lining for discharge sound attenuation, has little effect on breakout. To reduce breakout sound levels, the duct mass can be increased with a heavier gauge metal, or by lagging material to the outside of the duct.

Estimating the Breakout Sound Pressure in a Space

Once the breakout sound power level has been estimated, the sound pressure level in the space can be calculated. The equations for estimating breakout sound pressure are based on the basic equation for a line source. A line source equation is used instead of a point source equation (e.g. Schultz or Thompson) because duct breakout is essentially a long line across the top of the room.

$$L_p = L_w - 10 \log (\pi d L) + k$$

Where

d is the distance from the source to the measurement point

L is the length of the sound source in feet (m)

K is a constant whose value is 10.5 for I-P and 0.5 for SI

Return Air Sound Path

Estimating Return Air Path Sound Power

The return sound path depends on several factors. For most HVAC equipment, the return sound energy comes from a portion of the fan sound energy. This information is not always made available, so it is up to the designer to identify the sound energy levels (Refer to **Return Air vs. Supply Air Sound Power**).

Return Air Sound Power Attenuation

Return air systems may or may not include ducting. In some cases ducting is required to bring air to the unit. This is common in classroom applications where the HVAC unit is in the corridor and the corridor wall is fire rated. In an open plenum approach, ducting is not required but often considered as a sound attenuating solution. This can be a very good idea because the supply air sound energy and the return air sound energy are about equal and there is not a lot of attenuation on the return air side.

If the ducting ends in the plenum space or there is no ducting, then the plenum can offer attenuation as described in **Ceiling Plenums**. Return air systems that are ducted directly to the space will channel the sound energy to the space and not take advantage of the plenum. Ducted return air systems enjoy the same attenuation benefits from ducting as supply air systems. The ducting elbows and other fittings, end reflection, etc., all come into play.

Estimating Return Air Sound Pressure in a Space

The first step in estimating return air sound pressure in a space is to identify if there is ducting and/or other sound attenuating features. If this is the case, then identify whether the ducting system path leads directly to the occupied space, or if the ducting system ends in the ceiling plenum.

If the return air path is simply the back of the HVAC unit, then the *source - path - receiver* concept becomes HVAC return air sound power levels – ceiling plenum attenuation (if any) – room effect calculation such as Schultz for a single point or Thompson.

If the HVAC unit has some ducting, such as a return air elbow, then the process is HVAC return air sound power levels – duct fitting attenuation – ceiling plenum attenuation (if any) – Room effect calculation such as Schultz for a single point or Thompson.

If the HVAC unit is ducted directly to the occupied space, then the process is HVAC return air sound power levels – duct fitting attenuation – Room effect calculation such as Schultz for a single point or Thompson. If there is a return air grille, then a new sound source may be created. This can be evaluated in a similar manner as shown in **Diffuser Sound Path later**.

Diffuser Sound Path

As air passes through diffusers it generates sound (noise), so diffusers become a sound source. The sound level is proportional to the air discharge velocity, which also affects the diffuser throw. This sound energy tends to occur in the higher frequencies.

Diffuser Catalog Data

Diffuser catalogs generally rate products for air volume in cfm (m^3/h), throw in fpm (m/s), pressure drop in inches or (Pa) and NC level.

To provide sound performance such as an NC value requires defining the space so the diffuser sound power data can be converted from sound power to sound pressure and plotted on a NC chart. Also, consider that the cataloged tables are only for one diffuser. If there is more than one diffuser, then the sound power levels (and the NC level) will increase.

Estimating Diffuser Sound Pressure in a Space

Each diffuser will be a sound power source. Where possible, it is best to use the sound power data as provided in the figure above. The sound energy is then converted into sound pressure in the same manner as the sound power for ducted discharge. The Thompson or Schultz equation can be used. If there are four or more diffusers, then the multiple ceiling array equation can be used.

The distance used from the diffuser to the measurement point should be carefully considered. Two locations should be checked – directly under a diffuser and the center of the room.

Radiated Sound Path

When a terminal unit or decentralized HVAC unit is placed near the occupied space (such as in the ceiling plenum) radiated sound should be considered. Most equipment manufacturers can provide radiated sound power levels for their equipment (fan coils, VAV boxes, etc.). For most products it is appropriate to include the environmental correction factor.

Estimating Radiated Sound Pressure in a Space

The radiated sound path is similar to the return air sound path. The ceiling plenum (if any) can provide considerable attenuation. Converting from sound power to sound pressure can be accomplished with either the Schultz or Thompson equation.

Evaluating All the Sound Paths

The final step in evaluating sound pressure levels in a room is to evaluate how all of the various sound paths act together to create the sound level within the space. The various sound paths must be added together logarithmically. The total L_p is the logarithmic sum of all the sound paths. The total L_p can then be used with the sound criteria of choice (such as NC or RC levels).

Sound Path Evaluation Example

Consider the previous sound path examples and evaluate whether the space can meet NC 35.

List the results of the five sound paths and total the sound pressure in the space using decibel addition.

Using the 125 Hz band as an example:

$$\text{Total } L_p = 10 \log (1034/10 + 1040/10 + 1025/10 + 1047/10 + 1055/10) \\ = 55 \text{ dB}$$

	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
Discharge	49	71	59	53	41	27	26	23
Radiated	49	71	59	53	41	27	35	23
Supply Fan L_p	8	34	23	10	0	0	0	0
Return Duct L_p	17	40	22	6	0	0	0	0
Diffuser L_p	40	25	28	27	27	11	0	6
Breakout L_p	28	47	30	16	0	0	0	0
Radiated L_p	35	55	41	33	14	0	0	0
Total L_p	41	55	41	34	27	11	3	6
Requested NC level	60	52	45	40	36	34	33	32
Required Attenuation	0	3	0	0	0	0	0	0

The NC level is 38 and is set by the level in the 125 Hz band. This demonstrates the common advice in HVAC acoustics that if you can attenuate the 125 Hz band, then the system should be okay. This can be helpful if the calculations are being done by hand.

If the 125 Hz sound pressure level was lowered 3 dB, then the requested NC 35 criteria would be met. The radiated sound level is setting the 125 Hz sound pressure level. The designer can now consider equipment options (perhaps two smaller units) or relocating the unit so the sound level is reduced.

INDOOR SOUND ANALYSIS

Central systems

General

Central systems such as air handling units serving multiple spaces are similar to the decentralized systems. Many of the sound calculation techniques used in decentralized systems can be used with central systems.

The larger scale of equipment generally means larger sound power sources, but the equipment is usually more removed from the occupants. The large fan sound energy involved in central systems introduces the concept of silencers which are added to the HVAC system to reduce sound energy. Often the designer is attempting to evaluate the sound energy transmitted through ducting to the space to estimate the insertion loss required of the silencer to meet the required sound criteria. Many silencer manufacturers develop software that is focused on evaluating ducting systems for sizing silencers.

The high level of sound energy in a mechanical room creates a need to evaluate sound transmission through the wall into occupied spaces. This topic will be discussed in detail.

Regenerated noise is caused when airflow creates a sound source in the ducting system. Regenerated noise becomes an issue with larger systems with higher air flows and air velocities.

Multiple Paths

Central System Multiple Paths shows a central system in a mechanical room serving several spaces. The mechanical room impacts the building with sound energy from the fan systems that is distributed throughout the building by the ductwork, and by the transmitted sound energy from the equipment (e.g. chiller).

The space being considered is evaluated in a similar manner to a decentralized system serving a room.

For instance a central VAV system will have the following sound paths:

Ducted sound energy path (Path 1)

Return air sound path (Path 3)

Diffuser sound path (Path 4)

Radiated sound path with a VAV box being the radiated sound source (Path 5)

Duct breakout path (Path 2)

In addition, a central system can introduce duct breakout from the main supply and return ducts (Path 6 and 7) because air is being distributed throughout the building and may pass over the space being considered. In addition, transmitted sound from the mechanical room (Path 8) may need to be considered.

Central systems can also serve large spaces such as an open plan office environment. These will be discussed in the following sections.

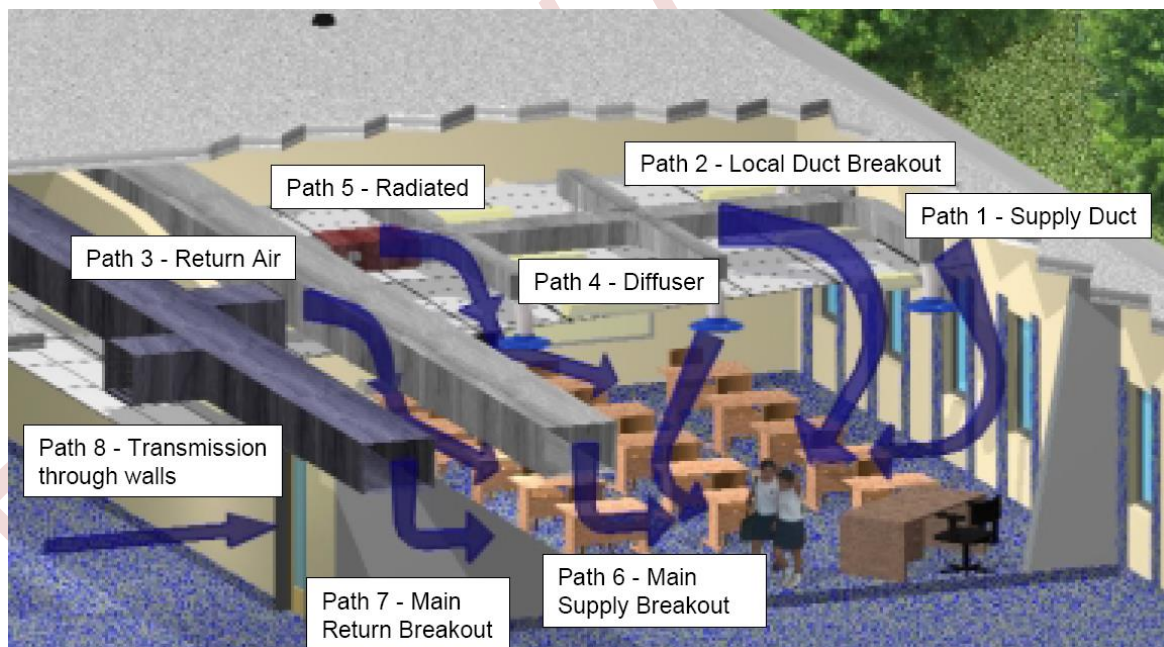


Figure 25 - Central System Multiple Paths

Central System Duct Sound Path

The ducted sound path for central systems is evaluated in the same manner as for decentralized systems (Refer to **Duct Sound Path**). The main differences are that several spaces are served by a single central system and the source sound energy levels are usually much larger. Having several occupied spaces introduces the issue of which spaces need to be evaluated. Acoustic treatment of ducted systems will affect the sound levels of all spaces downstream of the treatment.

Usually the treatment (such as a duct silencer) is installed at or near the beginning of the duct system so that all spaces will have reduced sound levels. In this case, it becomes necessary to choose the “worst case” space and evaluate it.

It is not always obvious which space(s) need to be evaluated and experience is usually the best teacher. There are two key parameters to watch for when choosing. The first is the distance through the duct work from the sound source. The ductwork will naturally attenuate, so spaces further away from the source are generally better. Conversely, spaces close the source should be reviewed.

The second key issue is the percentage of supply air delivered to the space. Sound energy is almost proportional to the percentage of supply air. This means that a space close to the mechanical room that only receives a small percentage of the supply air may not be the worst case. Conversely, a large space far from the mechanical room may need to be reviewed.

It is also important to balance the supply duct sound path with other sound paths when evaluating which spaces need to be reviewed. Consider the return air paths and main duct breakouts when choosing spaces.

Air Handling Units and Return Fans

Central systems usually have larger fans, which are the main sound power source in the ducted system. Fans are, for the most part, the sound energy source for both supply and return duct systems.

Fans are used when air movement is required. This can include indoor and outdoor air handling units, vertical self-contained units, applied or packaged rooftop air conditioning units. Return fans can be either inline, base mounted fans directly connected to the ductwork or cabinet fans where the fan is in a cabinet.



Indoor A.H.U



outdoor A.H.U

ASHRAE developed a methodology to estimate sound power levels from fans based on the physical and performance parameters of the fan. This is no longer supported by ASHRAE (since 1995), but it is still widely used.

Most air moving equipment today is tested for sound power levels and the data is readily available. It is recommended that this data be used when evaluating air movement systems. Individual fans are usually tested to *AMCA Standard 300-85*. This provides the total fan power of just the fan in eight octave bands from 63 to 8000 Hz. A common practice is to divide the sound power in half (subtract 3 dB from the cataloged sound power) with one half going to the supply and the other half going the return connection of the fan. This is not, by any means, necessarily true. The conservative answer is to use the total sound power for each direction.

Individual fan sound power is used for inline return air fans and fans used in custom air handling units. The air handling unit itself can provide a significant amount of sound attenuation. With custom air handling units, it is possible to either test the final unit to account for the unit attenuation, or to estimate the sound attenuation. What the designer should be looking for is the sound power levels at all duct connections to the air handling unit, and the radiated sound power through the casing and into the mechanical room/outdoor air area (Refer to **AHU Sound Power Data**).

UNIT SOUND	63	125	250	500	1000	2000	4000	8000
Radiated	73	72	67	65	57	45	38	30
Unit discharge	91	93	91	92	86	80	76	68
Unit return	88	90	85	85	74	64	53	44

Commercial or applied products are often tested to *ARI Standard 260, Sound Rating of Ducted Air Moving and Conditioning Equipment*. This standard has the advantage of testing the fan in the actual application (e.g. the fan is tested as installed in an air handling unit) as opposed to the near ideal conditions provided by the AMCA standard. If a unit fan is tightly fitted into the cabinet, it is possible for the fan to be 5 to 10 dB louder than indicated by the AMCA test standard below 250 Hz.

Duct Silencers

Duct silencers (sometimes referred to as sound attenuators) are devices that are designed to absorb or cancel sound energy in ductwork. Silencers can be static devices designed to absorb sound or dynamic devices designed to cancel out sound waves. Static silencers are either dissipative or reactive. Dissipative silencers have sound absorbing material, such as fiberglass, that is usually encased in perforated liners. Reactive silencers do not have sound absorbing material. Instead, they attenuate sound using the Helmholtz resonator principle. Dynamic or active silencers electronically generate a sound wave that is equal in amplitude, but opposite in phase to cancel out the sound source.

Most silencers used in typical HVAC applications are the dissipative type. They are rated in terms of the insertion loss they provide and their air pressure drop (refer to **Figure 33 - Typical Silencer Selection Showing Generated Noise**). Generally, the larger the air pressure drop, the more regenerated noise they create.

Silencers are usually installed near the sound sources, such as the air handling unit or return fan. In some cases the manufacturer can integrate the silencer into the air handling or rooftop unit and provide a complete package.

Where mechanical room break-in may be a concern (e.g. sound energy entering the duct system and using it as a conduit to travel throughout the building), the silencer should be installed in the ductwork in the mechanical room as close the wall as possible. This will allow the silencer to attenuate any break-in sound from the mechanical room.

Straddling the wall is a very good option, but often the mechanical room wall is fire rated and requires a fire damper. Placing silencers outside the mechanical room wall is also possible, but the silencers are a sound source and the radiated sound may enter an occupied space. HVAC designers are often focused on silencer selection as a key part of their HVAC design.

Controlling supply air sound in the occupied spaces is often a key aspect to controlling the overall sound level in the space. Many silencers manufacturers provide software modeling tools to estimate the required insertion loss for silencer selection. Attenuating the

supply sound source does not always provide acceptable sound levels in the space. There are many other sound paths to the space that must be considered.



Duct Plenums

Plenums are common in central system designs. For example, a rooftop unit may have a return air plenum just below the roof deck to draw air from the ceiling space. Also, the outdoor inlet to an indoor air handling unit is often a plenum. Plenums can provide significant sound attenuation because the abrupt changes in cross sectional area cause end reflection, and there can be a lot of surface area for sound absorbing material.

The transmission loss associated with a plenum can be described as follows:

$$TL = -10 \log[S_{out} [(Q \cos \theta) / (4\pi R^2) + (1 - \alpha_{avg}) / (S \alpha_{avg})]]$$

Where

TL is the transmission loss in dB

S_{out} is the area of the outgoing duct in ft^2 (m^2)

Q is the directivity which equals 2 if the inlet is near the center of the side it is located on. Q equals 4 if the inlet is in a corner of the side it is located on.

R is the distance between the centers of the inlet and outlet in ft (m).

S is the total inside surface area of the plenum minus the inlet and outlet areas in ft^2 (m^2)

θ is the angle of the vector r and the horizontal plan and can be written as

$$r = (rh^2 + rv^2)^{1/2}$$

$$\cos \theta = rh/r$$

α_{avg} is the average absorption coefficient of the plenum lining as given by $\alpha_{avg} = (S_1 \alpha_1 + S_2 \alpha_2) / S$

S_1 and α_1 are the surface area and coefficient of any bare or unlined inside surfaces.

S_2 and α_2 are the surface area and coefficient of any bare or lined inside surfaces.

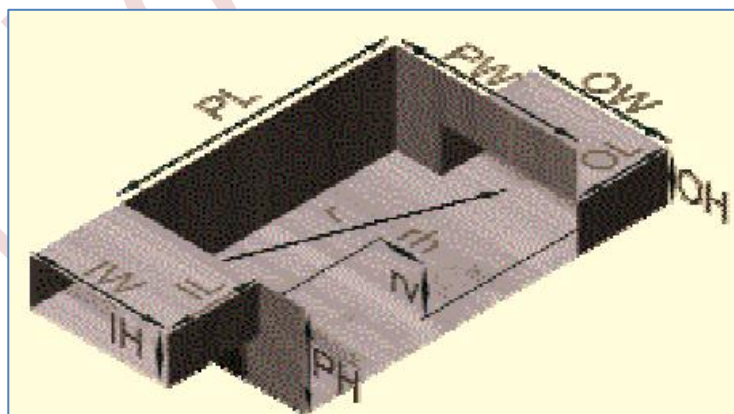


Figure 29 - Typical Plenum with Required Dimensions

Table 21 - Absorption Coefficients For Plenum Materials:

Material	Octave Bands							
	63	125	250	500	1000	2000	4000	8000
Concrete	0.01	0.01	0.01	0.02	0.02	0.02	0.03	
Bara sheet metal	0.04	0.04	0.04	0.05	0.05	0.05	0.07	
3 lb. density Fiberglass insulation	0.02	0.03	0.22	0.69	0.69	0.96	0.99	
4 lb. density Fiberglass insulation	0.18	0.22	0.82	1.00	1.00	1.00	1.00	
5 lb. density Fiberglass insulation	0.48	0.53	1.00	1.00	1.00	1.00	1.00	
6 lb. density Fiberglass insulation	0.76	0.84	1.00	1.00	1.00	1.00	0.97	

Table 21 shows typical absorption coefficients for common plenum materials. The eq. in page 65 assumes the plenum is a large enclosure and will only work if the wavelength of sound is small as compared to the characteristic dimensions of the plenum. At frequencies where plane waves exist, the equation is not valid. Plane wave propagation occurs at frequencies below the cutoff frequency (f_{co}):

$$f_{co} = c/2a$$

Or

$$f_{co} = 0.586 c/d$$

Where

- f_{co} is the cutoff frequency in Hz
- c is the speed of sound in air (1125 fps)
- a is the larger cross sectional dimension of a rectangular duct in ft (m)
- d is the diameter of a round duct in ft (m).

Where plane waves do occur, the plenum can be treated as an acoustically lined expansion chamber. These calculations would require computer. *An algorithm for HVAC Acoustics* contains the necessary equations.

valuating Supply Duct Paths

Figure 30 - Central System Duct Plan, shows a typical central duct system for an office building that will be used to illustrate several types of acoustic analysis. The following two examples show the supply duct calculations required to estimate the required Insertion Loss for the silencer. Several silencer manufacturers provide software specifically for this type of analysis.

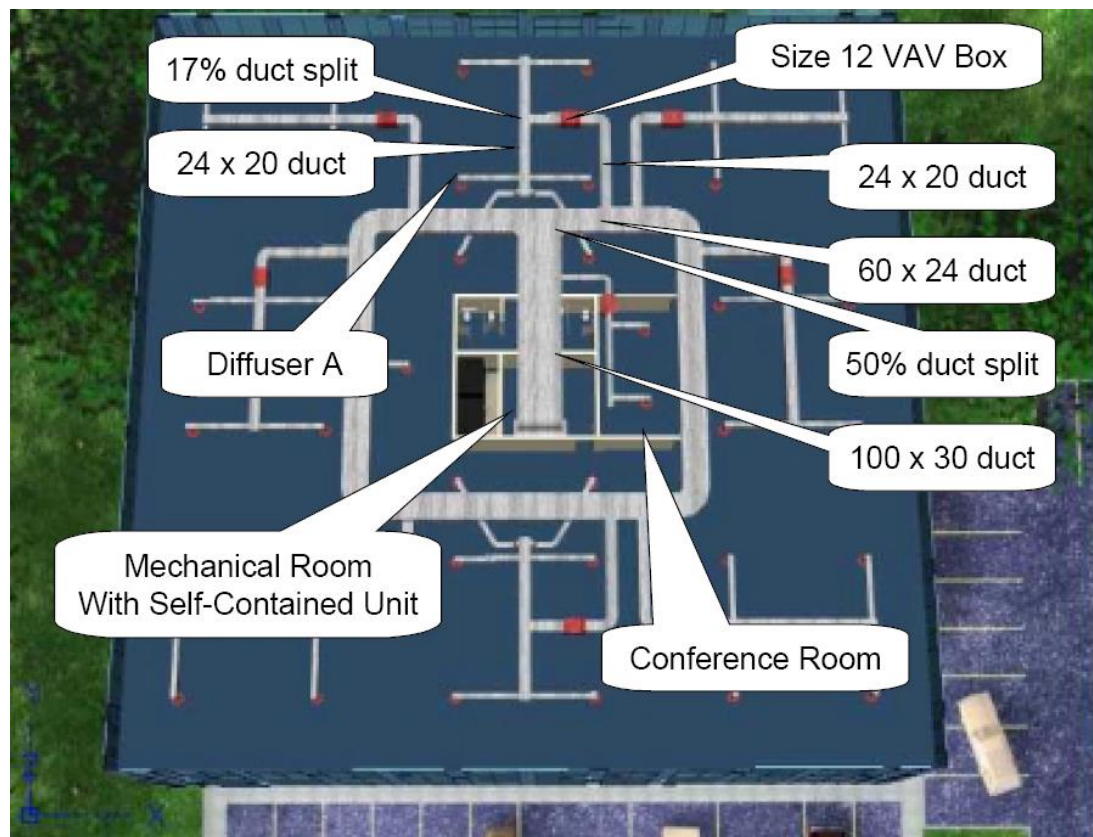


Figure 30 - Central System Duct Plan

Reviewing these examples shows the following:

- While the open office area is large and has many sound sources, it does not require a silencer. This is the result of good duct design and an effective discharge plenum. The conference room also does not require a silencer. Even though the conference room branch duct appears to be closely connected to the main supply air duct, the small amount of air being drawn by it (3%) introduces only a small amount of sound power to the room. Further improvement can be made by connecting the conference room branch duct to the ring supply duct instead of the main supply trunk.
- While good equipment and ducting design may resolve ducted sound transmission, it does not mean the space is acceptable. None of the other sound paths have been considered (return air, duct breakout, diffuser sound, etc.). Only sizing silencers is not enough to evaluate whether the space will be acceptable.

- Either the large ceiling array equation or the Schultz equation provided 10 dB sound attenuation in all octave bands for the space. But many silencer sizing programs default to 10 dB in each band. In addition, the NC levels listed for diffusers are usually based on 10 dB per band.
- Even if the supply fan were perfectly attenuated by a silencer, the sound level from ducted sound in the open office area would be NC 27. This sound power is coming from the VAV box, which is downstream of the silencers. Again, there is usually more than one source for sound energy.

Sound energy in a space can pass through the walls, floor and ceiling and affect other spaces. An example is a mechanical room next to an occupied space. The equipment in the mechanical room can be heard in the occupied space and may be unacceptable. Figure 31 - Transmitted Sound Considering larger central systems introduces the concept of transmitted sound. Often the impact of mechanical rooms needs to be evaluated because the sound transmitted from them is another sound path that is logarithmically added to the other sound paths to estimate the sound level in the occupied space.

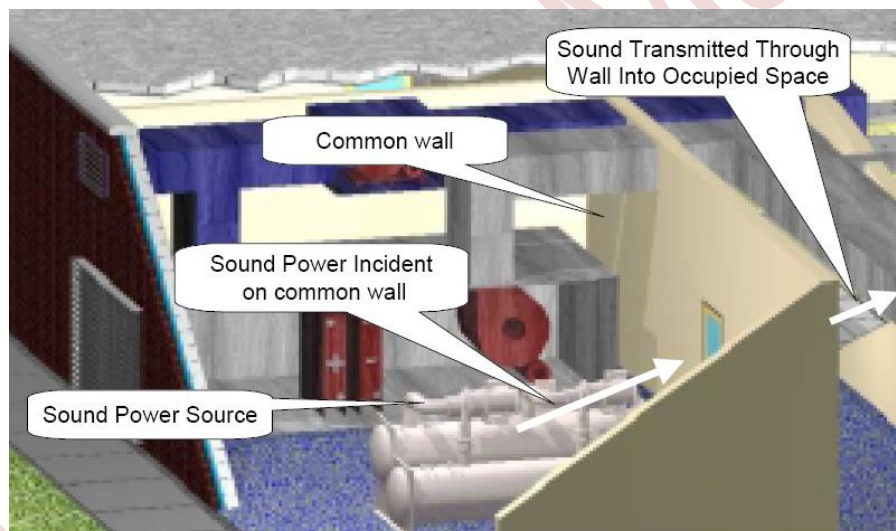


Figure 31 - Transmitted Sound

Estimating Transmitted Sound

A sound energy source such as a chiller, pump or air handling unit emits sound energy into the source room. If this space is a mechanical room, the surfaces are usually very hard and the room is acoustically live. A certain portion of the emitted sound energy acts on the common wall between the source room and the receiver room. How much energy acts on the common wall will depend on the source location with respect to the wall and the source room properties.

The common wall will reflect some sound energy, absorb some sound energy and transmit some sound energy. The transmitted sound energy will pass through the wall and act on the receiver room. From the receiver room's perspective, the common wall is a sound source. Surface sound sources create a very evenly distributed sound level in the adjacent space. As an occupant backs away from the common wall, the sound

level will not change until the distance from the wall is almost equal to the larger dimension of the common wall. Beyond this point, the sound level will start to decrease at a rate that is dependent on the source room properties.

The following equations¹⁴ can be used to estimate sound levels due to transmission. The process breaks down into the following steps:

- i. Estimate the sound power incident on the common wall from the sound source.
- ii. Estimate the sound power transmitted through the wall
- iii. Estimate the effect of the sound power on the receiving room

Eq. below can be used to estimate the sound power incident on the common wall.

$$LW_{\text{wall}} = LW_{\text{source}} + 10 \log [S_W [(1 - \alpha_{\text{avg}}) / (S_M \alpha_{\text{avg}}) + 1 / (4S_W + 4\pi l^2)]]$$

Where

- LW_{wall} is the sound power incident on the common wall in dB.
 LW_{source} is the sound power emitted by the source in dB.
 S_W is the area of the common wall in ft^2 (m^2)
 α_{avg} is the average absorption coefficient of the surfaces of the source room
 S_M is the surface area of the source room in ft^2 (m^2)
 l is the distance from the source to the wall.

The average absorption coefficient can be calculated using **below Eq.** and the values listed in **Table 11 -**

Typical Sound Absorption Coefficients.

$$\alpha_{\text{avg}} = (S_1 \alpha_1 + S_2 \alpha_2) / S_M$$

Where

- S_1 is the floor and ceiling area
 α_1 is the absorption coefficient of floor and ceiling
 S_2 is the total wall area
 α_2 is the absorption coefficient of the wall

Mechanical rooms are often treated with sound absorbing material to reduce the sound energy. This can be accounted for by using **Eq.**

$$\alpha_a = [PC \alpha_3 + (100 - PC) \alpha_{\text{avg}}] / 100$$

Where

- PC is the percent of total room surface area covered by sound absorbing material
 α_3 is the absorption coefficient of the sound absorbing material

The next phase is to estimate the sound energy that actually transmits through the wall. **Table 22 - Transmission Loss Values** shows transmission loss values that were obtained under ideal conditions.

Eq. below shows how to estimate the transmitted sound energy.

$$LW_{\text{room}} = LW_{\text{wall}} - TL$$

Where

LW_{wall} is the sound power incident on the common wall in dB.

LW_{room} is the sound power that passes through the common wall in to the room.

TL is the transmission loss value for the specific wall type.

The quality of wall construction is a major factor in how much sound energy is allowed to pass through the wall. Because the transmission loss values are taken under ideal conditions, it is appropriate to take into account the wall construction as shown in **following Eq.**

$$TL = -10 \log [(1 - \tau) \cdot 10^{-TL/10} + \tau]$$

Where

τ is correction coefficient as listed in **Table 23 - Correction Coefficients For Wall Construction.**

Table 22 - Transmission Loss Values

Description	Insertion Loss Per Octave Band (Hz)								
	31	63	125	250	500	1000	2000	4000	8000
4" poured concrete	29	35	36	36	41	45	50	54	58
8" poured concrete	34	36	37	41	45	49	53	57	61
12" poured concrete	36	36	38	44	48	51	55	59	63
8" Hollow Core Concrete Block	29	35	36	36	41	45	49	53	57
12" Hollow Core Concrete Block	32	36	36	37	43	47	51	55	59
4" Metal Stud Wall with 5/8" Drywall	8	11	20	30	37	47	40	44	35
4" Metal Stud Wall, 5/8" Drywall and 2" 3 lb/ft ³ fiberglass insulation	9	14	23	40	45	53	47	48	38
4" Metal Stud Wall with 2 layers 5/8" Drywall	11	19	27	40	46	52	48	48	38
4" Metal Stud Wall, 2 layers 5/8" Drywall and 2" 3 lb/ft ³ fiberglass insulation	13	24	32	43	50	52	49	50	40
1/8 in Single Pane Glass	2	8	13	19	23	27	27	27	31
1/4 in Single Pane Glass	7	14	20	25	27	28	28	31	34
1/4 in Double Pane Glass, 1/2 in air gap	13	18	23	26	29	34	31	34	38
Wood Hollow core Door	0	2	7	12	17	18	19	22	30
Wood Solid Core Door	12	17	18	19	22	30	35	39	43
2 in Acoustic Metal Door	23	25	31	34	37	39	43	47	45
4 in Acoustic Metal Door	27	29	34	36	40	45	49	51	40
Two 4 in Acoustic Doors, 32 in Separation	42	48	54	60	67	75	84	90	95

Table 23 - Correction Coefficients For Wall Construction

Quality of Construction		τ
Excellent	No acoustic leaks or penetrations	0.00001
Good	Very few acoustic leaks and penetrations	0.0001
Average	Many acoustic leaks and penetrations	0.001
poor	Many acoustic leaks and visible holes	0.01

The last step is to estimate the sound pressure in the space, given the sound power that passes through the wall. In this case, the sound source is a plane (the wall). Near the wall there will be no directionality. The

sound level will remain relatively constant as the distance changes. At greater distances the sound will start to decrease. This usually occurs when the area of a hemisphere with a radius equal to the distance from the wall becomes greater than the area of the common wall. Beyond this point, the sound source behaves more like a point source than a plane source (see **Comparing Point, Line and Plane Sources**).

For distances where $2\pi d^2 < S_w$, **following Eq.** can be used.

$$L_{p_{\text{room}}} = L_{w_{\text{room}}} + 10 \log [1 / S_w + 4(1 + \alpha_{\text{avg}}) / S_R \alpha_{\text{avg}}] + k$$

Where

S_R is the total surface area of the receiving room in ft^2 (m)

S_w is the surface area of the common wall in ft^2 (m)

α_{avg} is the average absorption coefficient for the receiving room which can be found in **Table 12 – Average Sound Absorption Coefficients for Typical Receiving Rooms**.

k is a constant whose value is 10.5 for I-P and 0.5 for SI

For distances where $2\pi d^2 > S_w$, following **Eq.** should be used.

$$L_{p_{\text{room}}} = L_{w_{\text{room}}} + 10 \log (1 / (2\pi d^2) + (MFP/d) * (4/R)) + k$$

Where

D is the distance from the common wall to the receiver in feet (m)

MFP is the mean free path in feet calculated before.

R is the Room constant in ft^2 (m^2)

K is a constant whose value is 10.5 for I-P and 0.5 for SI

Main Duct Breakout

As a duct passes through an occupied space, sound energy can escape and affect the occupants. The process to account for this is covered in detail in **Duct Breakout Sound Path**. Central systems should consider both the local ductwork that serves the space and the main supply and return ducts. The main trunks handle large air volumes and often have a significant amount of sound energy within them. If this sound energy breaks out, it can add to the background sound level in the space. This can be very problematic with low frequency fan noise.

The following example shows the calculation process using the same example as the supply duct calculations. In this case, the duct breakout from the main duct is actually louder than the sound power delivered to the diffusers. If the silencer sizing is based only on the sound level due to the ductwork, this breakout would be missed. In this case, the silencer should be sized to meet the needs of duct breakout, which is more than enough to meet the requirements of ducted discharge sound levels.

Return Air Path

Return air paths in central systems are often underestimated. A rooftop unit may only have an opening or short elbow to the ceiling plenum, which is a very short sound path. Air handling units used in gymnasiums often have no return fan and draw return air directly through a louver. While the supply air path has plenty of ductwork to help attenuate sound, the return path is very short. To make matters more difficult, a gymnasium is a very hard space.

Spaces where the return air is ducted directly from the return air grille cannot take advantage of the attenuation offered by a plenum. Low static pressure on return fans often makes them very inefficient, so they generate a fair amount of noise relative to the static pressure and motor size.

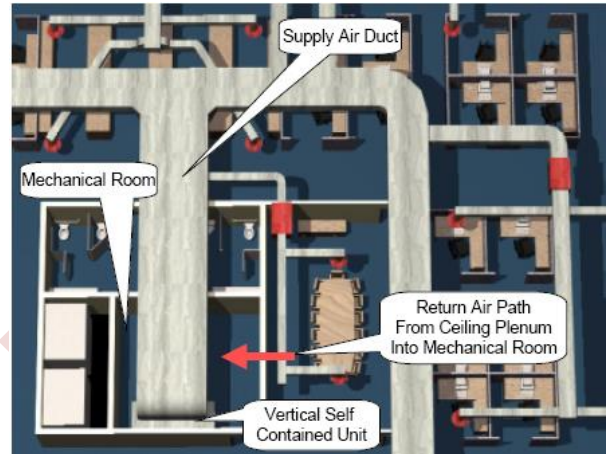
Return Air Sound Path provides details on how to calculate return air sound paths. For the most part, they are similar to supply duct sound paths. Duct breakout from return ducts is also possible and should be considered.

Return Air Openings

The following example considers a special application of a vertical self-contained unit. The common application for a self-contained unit is to use the mechanical room as the return air plenum. Outdoor air is ducted directly into the mechanical room to be picked up by the supply fan and distributed throughout the occupied space. Air returns to the mechanical room from the ceiling plenum via openings above the ceiling tile. This provides a direct path from the HVAC unit to the occupied space.

Figure 32- Return Air Path For Self-Contained Unit

Evaluating this style of mechanical room is similar to evaluating transmitted sound. A sound source (the HVAC unit) radiates sound energy into the mechanical room. The room absorbs some of the sound. The sound energy incident on the return air opening will pass through and into the ceiling plenum. The sound energy that is incident on the return air opening can be estimated as follows:



$$LW_{\text{opening}} = LW_{\text{source}} + 10 \log \left[S_W \left[(1 - \alpha_{\text{avg}}) / (S_M * \alpha_{\text{avg}}) + 1 / (4S_W + 4\pi l^2) \right] \right]$$

Where

LW_{opening}	is the sound power incident on the opening dB.
LW_{source}	is the sound power emitted by the source in dB.
S_W	is the area of the common wall in ft ² (m ²)
α_{avg}	is the average absorption coefficient of the surfaces of the source room
S_M	is the surface area of the source room in ft ² (m ²)
l	is the distance from the source to the opening.

The average absorption coefficient can be calculated using **below Eq.** and the values listed in **Table 11 -**

Typical Sound Absorption Coefficients.

$$\alpha_{\text{avg}} = (S_1 \alpha_1 + S_2 \alpha_2) / S_M$$

Where

S_1	is the floor and ceiling area
α_1	is the absorption coefficient of floor and ceiling
S_2	is the total wall area
α_2	is the absorption coefficient of the wall

Often a mechanical room treated with sound absorbing material to reduce the sound energy. This can be accounted for by using **the following Eq.**

$$\alpha_a = [PC * \alpha_3 + (100 - PC) * \alpha_{\text{avg}}] / 100$$

Where

PC is the percent of total room surface area covered by sound absorbing material

α_3 is the absorption coefficient of the sound absorbing material

Once the sound power is known in the ceiling plenum, then we can account for the attenuation for the ceiling plenum. There is minimal end reflection affect in this application.

Evaluating Multiple Sound Paths and Locations

To estimate the space sound level requires logarithmically adding all of the various sound paths. Central systems generally require that several locations be considered. In the example used through this section of the Manual, both the conference room and the open office area near the main trunk were considered. The conference room is a confined space, has transmitted sound via a common wall with the mechanical room and the return air opening is above. The open area has a large amount of supply air (duct sound power) and breakout from the main supply duct.

Sound Path Evaluation Example

Consider the previous sound path examples and evaluate whether the space can meet NC 35, both in the conference room and the open office area. List the results the sound paths for the open office area and total the sound pressure in the space using decibel addition.

Using the 125 Hz band as an example:

$$\text{Total } L_p = 10\log(10^{40/10} + 10^{50/10} + 10^{47/10} + 10^{50/10}) = 54 \text{ dB}$$

Octave Band	63	125	250	500	1000	2000	4000	8000
Supply Duct Disch.Lp	26	40	39	35	34	31	31	33
Radiated Lp	34	50	47	43	43	45	52	29
Return Duct Lp	nil	nil	nil	nil	nil	nil	nil	nil
Diffuser Lp	52	47	49	48	49	44	28	24
Main Duct Breakout Lp	57	50	44	38	25	4	0	0
Total Lp	58	54	52	50	50	48	52	35
Requested NC level	60	52	45	40	36	34	33	32
Required Attenuation	0	2	7	10	14	14	19	3

List the results the sound paths for the conference room and total the sound pressure in the space using decibel addition.

Octave Band	63	125	250	500	1000	2000	4000	8000
Supply Duct Disch.Lp	24	38	42	38	35	31	27	30
Radiated Lp	38	53	48	42	44	46	54	34
Return Duct Lp	78	68	57	67	59	58	37	46
Diffuser Lp	47	40	40	37	36	30	12	7
Transmitted Lp	50	44	37	48	46	50	45	40
Main Duct Breakout Lp	nil	nil	nil	nil	nil	nil	nil	nil
Total Lp	78	68	58	67	59	59	55	48
Requested NC level	60	52	45	40	36	34	33	32
Required Attenuation	18	16	13	27	23	25	22	16

Reviewing the two spaces in the above example shows each space has its own challenges. The open office area has diffuser sound issues that silencers will not help. The diffusers need to be reelected to achieve the desired sound level.

The conference room requires attention to the return air path. A silencer or a length of return air duct is required to reduce the return air sound level. Relocating the return air path over the bathrooms is also advisable. Adding sound absorbing material to the mechanical room would help reduce both the transmitted and return air sound levels. The conference room also requires a small main supply duct silencer or duct lining to lower the supply duct sound levels.

Regenerated Noise

General

Regenerated noise is a secondary sound source usually caused by air flowing through a device. An example is noise generated by a splitter-damper. Another common source of regenerated noise is silencers.

Evaluating Regenerated Noise

There are equations and methods available to estimate regenerated noise in branches, elbows, splitters, etc. However, they are very application sensitive. The 1999 ASHRAE Applications Handbook suggests that duct related regenerated noise can be avoided by;

- Sizing ductwork or duct configurations so that air velocity is low (Refer to **Table 24** and **Table 25**).
- Avoiding abrupt changes in duct cross-sectional area.
- Providing smooth transitions at duct branches, takeoffs and bends.

Table 24 - Max Recommend Duct Velocities to Achieve Specified Acoustic Design Criteria

Main Duct Location	Max. Velocity (fpm)		
	Design RC	Rectangular Duct	Round Duct
	45	3500	5000
In shaft or above drywall ceiling	35	2500	3500
	25	1700	2500
	45	2500	4500
Above Suspended Acoustic Ceiling	35	1750	3000
	25	1200	2000
	45	2000	3900
Duct Located Within Occupied Space	35	1450	2600
	25	950	1700

Table 25 - Maximum Recommended "Free" Supply Outlet and Return Air Opening Velocities to Achieve Specified Acoustic Design Criteria

Type of Opening	Design RC	"Free" Opening Air Velocity (fpm)
Supply Air Outlet	45	625
	40	560
	35	500
	30	425
	25	350
Return Air Opening	45	750
	40	675
	35	600
	30	500
	25	425

Various Acoustic Properties of Materials

General

The following are additional tables of acoustic properties of various HVAC components. For more information refer to the ASHRAE Applications Handbook.

Table 27 - Insertion Loss For Rectangular Unlined Duct

Duct Size Band	63	125	250	500	1000	2000	4000	8000
6 x 6	0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1
12 x 12	0.35	0.2	0.1	0.06	0.06	0.06	0.06	0.06
12 x 24	0.04	0.2	0.1	0.05	0.05	0.05	0.05	0.05
24 x 24	0.25	0.2	0.1	0.03	0.03	0.03	0.03	0.03
48 x 48	0.15	0.1	0.07	0.2	0.2	0.2	0.2	0.2
72 x 72	0.1	0.1	0.05	0.2	0.2	0.2	0.2	0.2

Table 28 - Insertion Loss For Rectangular Unlined Duct With 1 Inch Lining

Duct Size Band	63	125	250	500	1000	2000	4000	8000
6 x 6	0.49	0.6	1.5	2.7	5.8	7.4	4.3	3.4
12 x 12	0.28	0.4	0.8	1.9	4	4.1	2.8	2.2
12 x 24	0.21	0.3	0.6	1.7	3.5	3.2	2.3	1.8
24 x 24	0.14	0.2	0.5	1.4	2.8	2.2	1.8	1.4
48 x 48	0.07	0.1	0.3	1	2	1.2	1.2	0.72
72 x 72	0.07	0.1	0.2	0.8	1.7	1	1	0.8

Table 29 - Insertion Loss For Rectangular Unlined Duct With 2 Inch Lining

Duct Size Band	63	125	250	500	1000	2000	4000	8000
6 x 6	0.56	0.8	2.9	4.9	7.2	7.4	4.3	3.4
12 x 12	0.35	0.5	1.6	3.5	5	4.1	2.8	2.2
12 x 24	0.28	0.4	1.3	3	4.3	3.2	2.3	1.8
24 x 24	0.21	0.3	0.9	2.5	3.5	2.2	1.8	1.4
48 x 48	1.4	0.2	0.5	1.8	2.5	1.2	1.2	1
72 x 72	0.07	0.1	0.4	1.5	2.1	1	1	0.8

Table 30 - Insertion Loss For Round Unlined Duct

Duct Size Band	63	125	250	500	1000	2000	4000	8000
<7	0.03	0.03	0.05	0.05	0.1	0.1	0.1	0.08
7D<15	0.03	0.03	0.03	0.05	0.07	0.07	0.07	0.05
15<D<30	0.02	0.02	0.02	0.03	0.05	0.05	0.05	0.04
30<D<60	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.016

Table 31 - Insertion Loss For Round 1 Inch lined Duct

Duct Size Band	63	125	250	500	1000	2000	4000	8000
<7	0.38	0.59	0.93	1.53	2.17	2.31	2.04	1.26
7D<15	0.19	0.42	0.77	1.43	2.14	1.79	1.34	1
15<D<30	0.03	0.19	0.49	1.2	1.46	1.04	0.74	0.74
30<D<60	0	0	0.08	0.06	0.1	0.14	0.09	0.07

Table 32 - Insertion Loss For Round 2 Inch lined Duct

Duct Size Band	63	125	250	500	1000	2000	4000	8000
<7	0.56	0.8	1.37	2.25	2.17	2.31	2.04	1.26
7D<15	0.38	0.63	1.21	2.15	2.14	1.79	1.34	1
15<D<30	0.22	0.4	0.93	1.93	1.46	1.04	0.74	0.74
30<D<60	0	0	0.53	0.79	0.1	0.14	0.09	0.07

Figure 33 – Typical Elbow Fitting

The insertion loss for elbows is found by multiplying the center band frequency in kHz with the width of the elbow in inches as shown **Figure 33 – Typical Elbow Fitting**. Using 125 Hz and a round unlined 24-inch elbow as an example:

$$IL = (24 \times 125 \text{ Hz})/1000 = 3.0$$

The insertion loss is 1 dB in the 125 Hz Band.



Table 33 - Insertion Loss For Square Elbows Without Turning Vanes

	Insertion Loss	
	Unlined Elbows	Lined Elbows
fw < 1.9	0	0
1.9 < fw < 3.8	1	1
3.8 < fw < 7.5	5	6
7.5 < fw < 15	8	11
15 < fw < 30	4	10
fw > 30	3	10

Table 34 - Insertion Loss For Square Elbows With Turning Vanes

	Insertion Loss	
	Unlined Elbows	Lined Elbows
fw < 1.9	-	0
1.9 < fw < 3.8	1	1
3.8 < fw < 7.5	4	4
7.5 < fw < 15	6	7
fw > 15	4	7

Table 35 - Insertion Loss For Round Elbows Without Turning Vanes

	Insertion Loss
	Unlined Elbows
fw < 1.9	0
1.9 < fw < 3.8	1
3.8 < fw < 7.5	2
fw > 7.5	3



MEASUREMENT OF VIBRATION

Introduction:

What Is Vibration?

Vibration is the physical movement or oscillation of a mechanical part about a reference position.

Vibration is:

- ✚ Wasted energy
- ✚ A major cause of premature component failure
- ✚ Cause of aircraft noise which contributes to crew and passenger discomfort

Terminology:

Prior to any discussion of vibration, it is important to first understand the common terms used for vibration analysis and their applications.

1. **Amplitude:** Amplitude is an indicator of the severity of a vibration. Amplitude can be expressed as one of the following engineering units:

- i. **Displacement:**

- Displacement is a measure of the actual distance an object is moving from a reference point.

- ii. **Velocity:** Velocity is the rate of change in position.

- Typical velocity units are: IPS (Inches Per Second), m/sec (meters per second)
- Velocity is the most accurate measure of vibration because it is not frequency related.

- iii. **Acceleration:**

- Acceleration is the rate of change of velocity and is the measurement of the force being produced.
- Acceleration is expressed in gravitational forces or "G's", ($1G = 32.17 \text{ ft/sec/sec}$ or 9.817 m/s^2)
- Acceleration is frequency related.

2. **Frequencies:** The rate of mechanical oscillation in a period of time. Frequency can be expressed in one of the following units:

- RPM - Revolutions per Minute
- CPM - Cycles per Minute
- CPS - Cycles per Second
- Hz - Hertz, $1 \text{ Hz} = 1 \text{ Cycle per Second}$

Types of Vibration:

Vibration can be classified into one or more of the following categories:

1. Periodic:

- Repeats itself once every time period
- Result of a mass imbalance in a component or disc.
- As the component rotates, it produces a “bump” every rotation which is referred to as the once-per-revolution or “1P” vibration.
- This vibration is usually correctable by balancing.

2. Random:

- Do not repeat themselves
- Not related to a fundamental frequency.
- An example - the shock that is felt as a result of driving down the road and hitting a pothole.

3. Resonant:

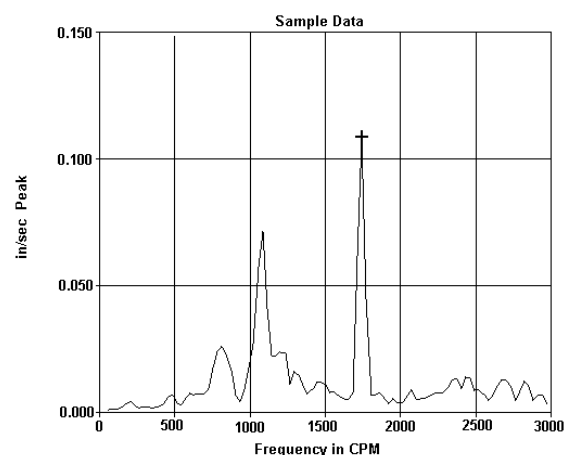
- The natural frequency at which an airframe or mechanical system is inclined to vibrate. All things have one or more resonant frequencies.
- Resonant vibrations are the result of a response in a mechanical system to a periodic driving force.

4. Harmonic:

- Exact multiples of a fundamental frequency
- Classified in terms as 1st, 2nd, 3rd.....

Bandwidth:

- Upper and lower frequency limits of the survey being acquired - either hardware set (with the use of an external band pass filter) or software controlled by the analyzer.
- Setting the frequency bandwidth is a way of eliminating vibration data or noise that is of no interest for your particular application.
- In the survey above, the frequency bandwidth is 0 CPM to 3000 CPM



Resolution:

- The resolution of a spectrum is the number of lines or points used to plot the spectrum.
- The higher the number of lines, the more data acquired.

Equipment:

Sensor: A transducer that converts mechanical motion into electronic signals.

Sensor Selection: The first consideration is manufacturer's recommendations. If none exist, then:

- Frequency Range
- Environmental conditions

Sensor Installation: Varies depending upon the application. Most manufactures provide the specific location for mounting and this should be strictly adhered to. If these recommendations are not followed, the resulting measurements may be invalid. Generally, mount in a location that provides the closest proximity to the component of interest.

How Vibration Is Analyzed:

- Time Domain - Vibration vs. Time. A vibration signal is presented as a sin wave form with all frequencies and amplitudes combining to give one overall signal.
- Frequency Domain: By applying the FFT (Fast Fourier Transform) algorithm to a Time Domain signal, it is converted to the Frequency Domain. In the Frequency Domain, each individual amplitude and frequency point is displayed.

Types of Vibration Surveys:

1. **Overall Vibration:** Outputs the sum of all vibration measured within a specified frequency range.

Used as an initial "alarm" type survey, whereby if the overall indication is above a specified value, a more detailed survey is performed to identify the possible cause.

Model 2020 ProBalancer Overall Vibration		
	Chan A	Chan B
Current:	0.29	
Maximum:	0.69	
Samples:	970	
Units:	IPS Peak	
Reset		

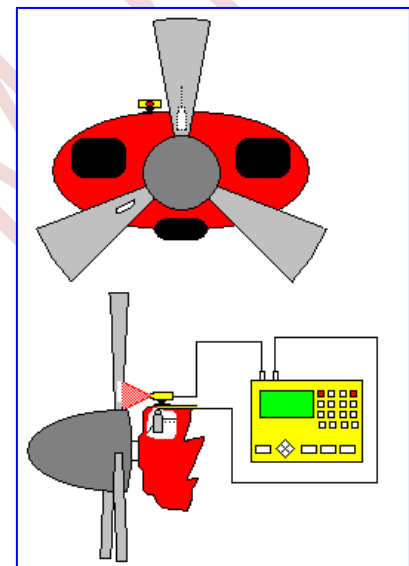
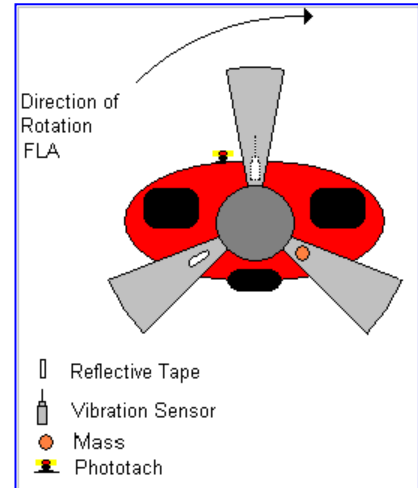
Basic Balancing:

- ✚ Mass Imbalance
- ✚ Aerodynamic Imbalance

Fundamentals of Balancing

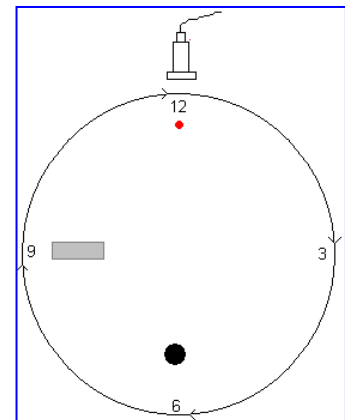
Data Collection and Processing:

- ✚ The vibration sensor is installed on the engine as near the front bearing as possible.
- ✚ The Photo tach is mounted on the cowling, behind the propeller.
- ✚ The reflective tape is applied to the back side of the target propeller blade in line with the Photo tach beam.
- ✚ The mass is located by the relative occurrence of tach trigger and mass passage at the radial sensor location.
- ✚ As the heavy spot on the propeller passes the location of the vibration sensor, the sensor generates and sends an electrical pulse to the analyzer.
- ✚ The Reflective tape triggers a response as it passes the Photo tach, which then sends an electrical signal to the analyzer.



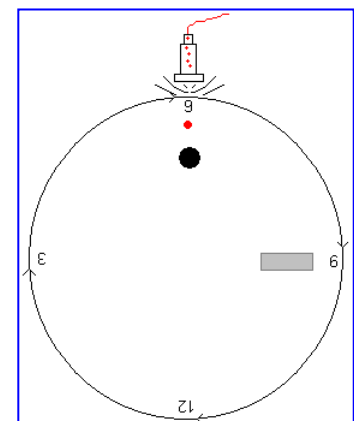
- ✚ In this illustration, the vibration sensor and Photo tach beam are co-located at the 12:00 or 0 degree position. Rotation is clock-wise from the viewer's position.

This is our starting point, elapsed time = 0

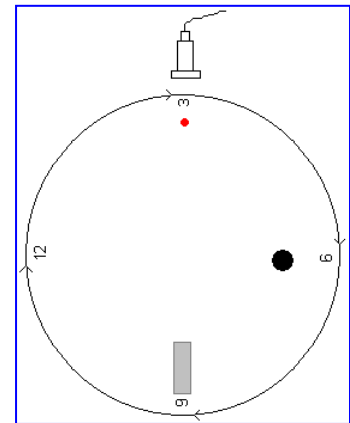


- ✚ The speed is 1 RPM. Fifteen seconds (90 degrees) of travel has occurred. In this sequence, the reflective tape has just entered the Photo tach beam to trigger the tach event.

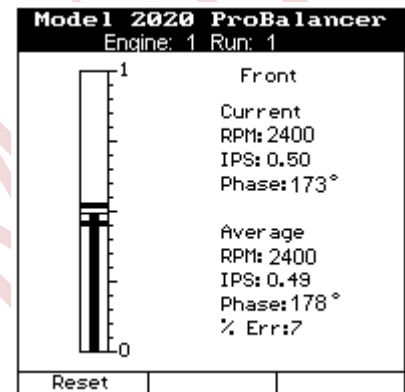
Elapsed time = 15 seconds.



- The tape and mass have both passed the 0 degree location.
The unit now waits for the exact sequence to repeat for averaging.
- Solution would be to add weight at 270 degrees.



- The process is repeated while the analyzer averages out errors caused by momentary vibration events outside the running average.



Vibration Measurement

It is apparent that the intensity of vibration can be measured in terms of displacement, velocity, or acceleration.

Acceleration is clearly the best parameter to measure at high frequencies. However, because displacements are large at low frequencies it would seem that measuring either displacement or velocity would be best at low frequencies. The amplitude of vibrations can be measured by various forms of displacement transducers. Fiber-optic-based devices are particularly attractive and can give measurement resolution as high as 1 mm. Unfortunately, there are considerable practical difficulties in mounting and calibrating displacement and velocity transducers and therefore they are rarely used. Because of this, vibration is usually measured by accelerometers at all frequencies. The most common type of transducer used is the **piezoaccelerometer**, which has typical inaccuracy levels of $\pm 2\%$. The frequency response of accelerometers is particularly important in vibration measurement in view of the inherently high-frequency characteristics of the measurement situation. The bandwidth of both potentiometer-based accelerometers:

- 1- accelerometers using variable inductance-type displacement transducers only goes up to 25 Hz.
- 2- Accelerometers that include either the LVDT or strain gauges can measure frequencies up to 150 Hz.
- 3- the latest instruments using piezoresistive strain gauges have bandwidths up to 2 kHz.
- 4- Finally, inclusion of piezoelectric crystal displacement transducers yields an instrument with a bandwidth that can be as high as 7 kHz.

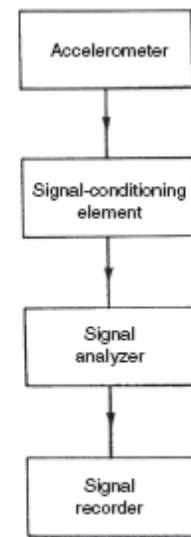
When measuring vibration, consideration must be given to the fact that attaching an accelerometer to the vibrating body will significantly affect the vibration characteristics if the body has a small mass. As well as an accelerometer, a vibration measurement system requires other elements to translate the accelerometer output into a recorded signal. The three other necessary elements are :

- i. a signal conditioning element
- ii. a signal analyzer
- iii. a signal recorder.

The signal-conditioning element amplifies the relatively weak output signal from the accelerometer and also transforms the high output impedance of the accelerometer to a lower impedance value. *The signal analyzer* then converts the signal into the form required for output. The output parameter may be displacement, velocity, or acceleration, and this may be expressed as peak value, r.m.s. value, or average absolute value. *The final element of the measurement system is the signal recorder.* All

elements of the measurement system, especially the signal recorder, must be chosen very carefully to avoid distortion of the vibration waveform.

Vibration measuring system



Calibration of Vibration Sensors

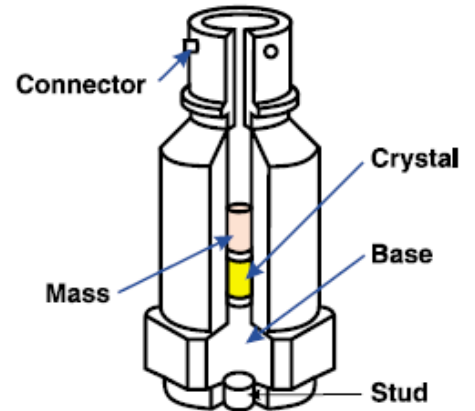
Calibration of the accelerometer used within a vibration measurement system is normally carried out by mounting the accelerometer in a back-to-back configuration with a reference calibrated accelerometer on an electromechanically excited vibrating table.

Basic Vibration Sensors:

1. Accelerometer

Types of Accelerometers:

- ✚ Piezoelectric
 - Charge mode
 - Internally amplified
- ✚ Strain Gauge
- ✚ Piezoresistive
- ✚ Variable Capacitance



Advantages

- Very wide frequency
- Wide amplitude range
- Broad temperature range
- Velocity or displacement output available
- Rugged, industrial design

Disadvantages

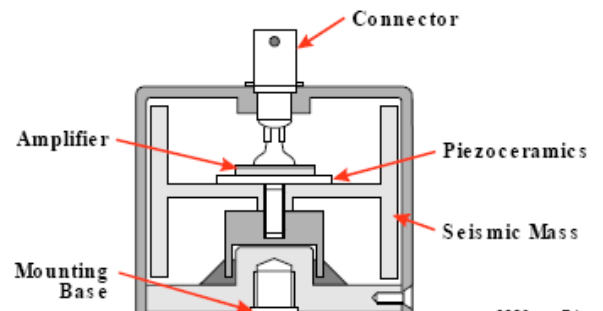
- Not responsive to 0 Hz
- Internal Amplifier limits temperature



Low Frequency Accelerometer Considerations:

High Sensitivity

- ✚ ➤ Low Noise
- ✚ ➤ Low Pass Filter
- ✚ ➤ Environmental Protection
 - Overload Protected
 - Resists Thermal transients
 - Low Strain Sensitivity
- ✚ ➤ Limited Amplitude Range



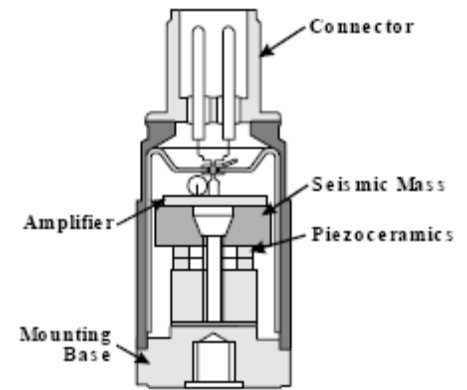
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Piezoelectric Accelerometers:

- ✚ Measures Acceleration
- ✚ Velocity or Displacement Output Available
- ✚ Very Sensitive
- ✚ Contacting
- ✚ Measures Absolute Casing Motion
- ✚ Measures Very Low Frequency
- ✚ Measures Very High Frequency

Piezoelectric Accelerometer - How It Works:

- Piezoelectric material (sensing element) is placed under load using a mass
- As 'stack' vibrates, crystal is squeezed or released
- Charge output is proportional to the force (and acceleration)
- Electronics convert charge output into voltage output



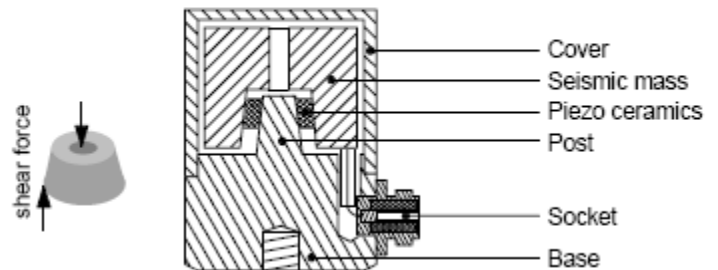
Metra offers accelerometers with three mechanical construction designs:

The reason for using different piezoelectric systems is their individual suitability for various measurement tasks and their varying sensitivity to environmental influences. The following table shows advantages and drawbacks of the three designs:

Shear:

- Low temperature transient sensitivity
- Low base strain sensitivity
- Lower sensitivity-to-mass ratio

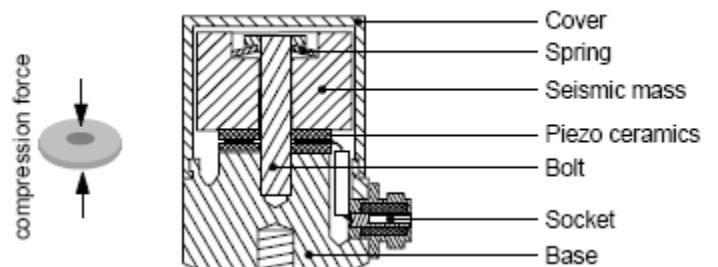
Shear Design:



Compression:

- High sensitivity-to-mass ratio
- Robustness
- Technological advantages
- High temperature transient sensitivity
- High base strain sensitivity

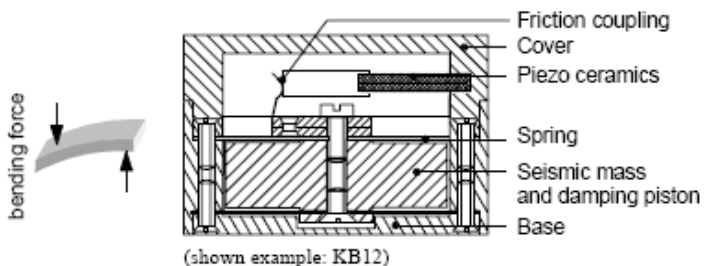
Compression Design:



Bending:

- Best sensitivity-to-mass ratio
- Fragile
- Relatively high temperature transient sensitivity

Bending Design:



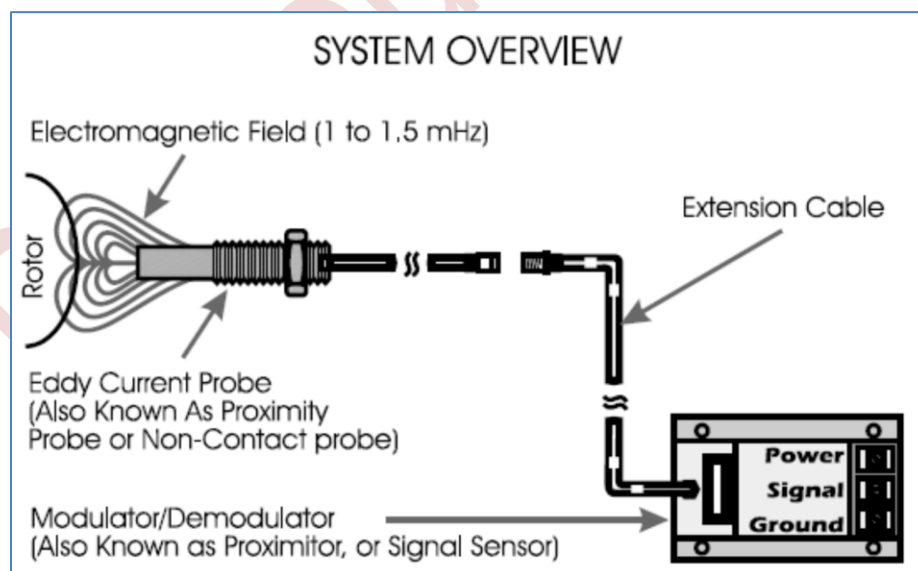
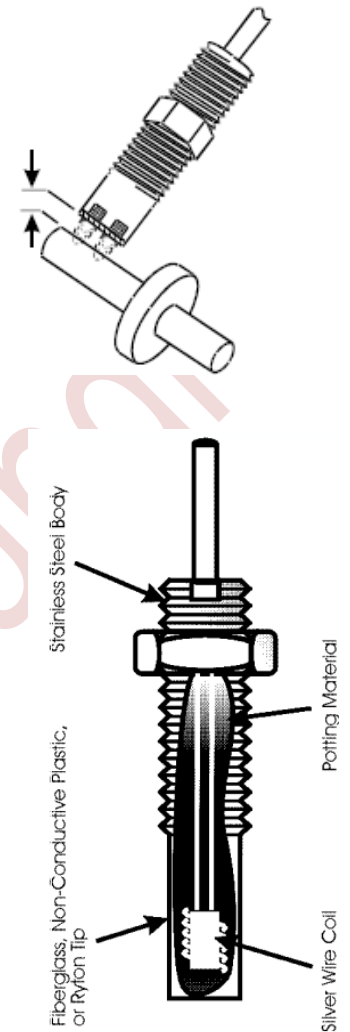
The shear design is applied in the majority of modern accelerometers due to its better performance. Compression and bending type sensors are still used in many applications.

2. Non contacting Displacement Transducer

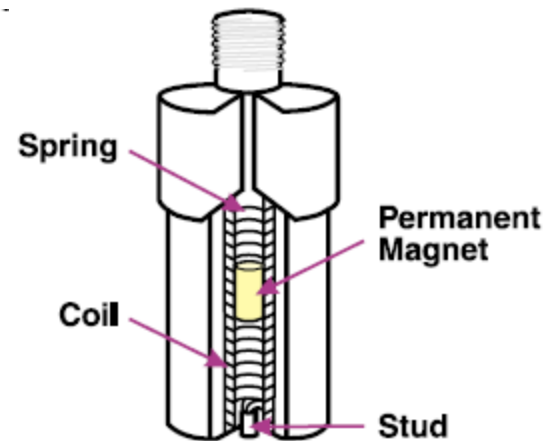
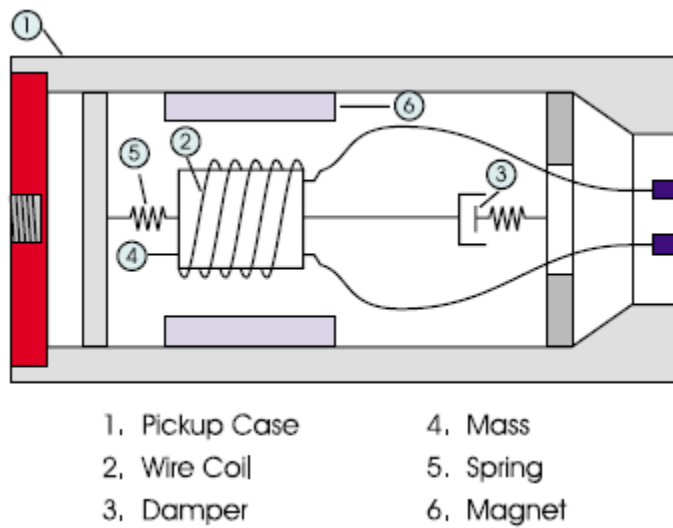
Eddy Current Probe Fabrication:

How It Works

- Three matched components - Driver, probe and extension cable
- Voltage applied to the Driver causes an RF signal to be generated
- Signal is transmitted to the probe by the extension cable
- Coil inside probe tip serves as an antenna and radiates high frequency energy into free space
- Any conductive material within the field absorbs energy and causes output of probe to decrease proportional to gap distance



3. Electrodynamics Velocity Transducer



Housing vibrates while the spring-suspended coil remains stationary
Amplitude of the output voltage is proportional to the velocity of the vibration

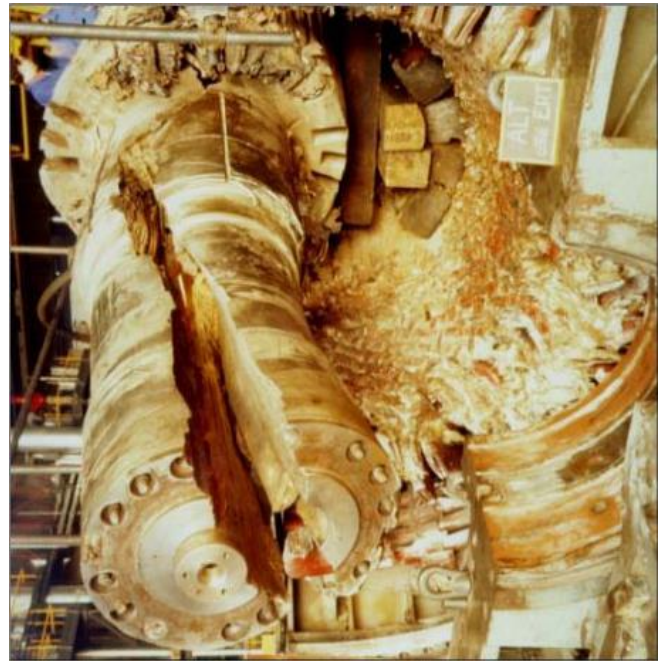
Advantages

- No external powering
- Powerful signal Output
- Easy to use (not as sensitive to mounting problems as alternative)
- Ability to operate at elevated temperatures

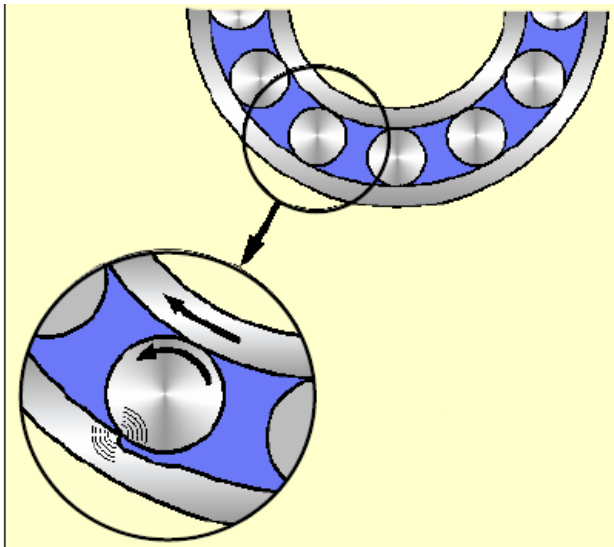
Disadvantages

- Not useful for very low frequency
- Not useful for very high frequency
- Moving parts wear
- Mounting orientation may be important
- Size
- Accuracy (resolution / noise as compared to alternatives)

Machine damage in a power station



Rolling-element bearing damage



Vibration Measurement in the past (& still today):



Modern machine diagnosis

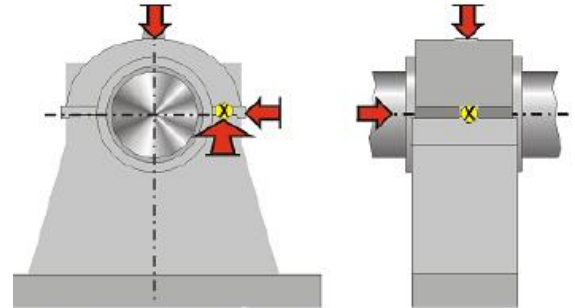
Measuring machine condition with a modern
measuring Instrument



Measuring Absolute Bearing Vibration:

General rules:

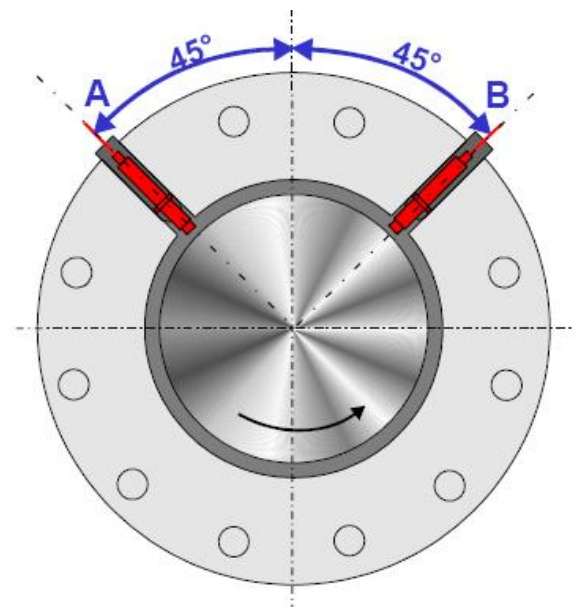
- Measurement points should be exactly defined and clearly marked
- Measuring points should be flat, clean and free of grease
- Loose paint and rusted surfaces should be cleaned or avoided
- Sensor must sit securely and not wobble
- Sensor and cable should not move during measurement



Vibration velocity sensors



Measuring Relative Shaft Vibration:



Eddy-current sensors:

Discrete type:

- Sensor with integral cable
- Calibrated extension cable
- Separate converter (oscillator)



Eddy-current sensors:



Acceleration sensors:



ELECTRICAL MEASUREMENT AND TEST INSTRUMENTS



Note: The lecture dependent on unit 35, Electrical measuring and Test Instruments, **Fundamentals of HVACR, Second Edition**

INTRODUCTION

The terms *test instrument* and *test meter* are used interchangeably in the HVACR field to describe devices that are used to detect some aspect of an electrical system's operation. Usually they are used to determine how a system is operating or why it is not operating. This information can then be used for servicing and troubleshooting.

The types and availability of test meters have increased to the point that it can be extremely difficult to determine which meter is best for a particular application (Figure 12-1). The flood of inexpensive test meters into the market has also provided what may appear to be a cost-effective way to outfit your toolkit. However, it is important to note that many of these inexpensive test meters will not be appropriate for HVACR use. Test meters should be selected based upon the standards required for their application. Never select a meter that is not rated for the application where it will be used.

This unit provides information on some of the general standards and ratings for test meters, but this information is not intended to replace any operating instructions provided by the meter's manufacturer. Always read and follow all manufacturers' operating and safety instructions; failing to do so can damage the equipment and may result in your being injured.



Figure 12-1 Assortment of analog and digital electrical test meters.

CATEGORY RATINGS

Properly designed test meters should not cause a shock, fire, arcing, or explosion. They are also designed to reduce the potential damage that can result from operator error if the meter is connected improperly. Even so, this protection does have its limits! The wrong meter used for the wrong application is extremely dangerous! That is why there are four categories designated for low-voltage test equipment. Be aware that the term *low voltage* is deceiving, because for test meters this is considered to be 1,000 volts or less. Many HVACR technicians would consider any circuit of 480 volts or above to be high voltage.

There are a number of considerations taken into account for meter ratings. These include the safe distance to the power source the technician is working and transient voltage spikes. Motors, capacitors, and power-conversion equipment such as variable speed drives can be prime generators of spikes. Transients can occur on low-voltage power circuits and can reach peak values in the many thousands of volts.

Any time you are working with electricity, there is a possibility of an overvoltage situation occurring. We use surge protectors on our computers to protect them from overvoltage. A momentary surge or spike of voltage can be several times the normal circuit voltage. These surges, called transient voltages, can be caused by electrical storms, even miles away, or they can be caused when a major load is added or dropped from the system. Examples of changes in the load could be an auto accident where a utility pole was damaged, or a large piece of equipment starting or stopping.

Category numbers range from CAT I to CAT IV. A higher CAT number is used for higher voltages and energy transients. A test meter designed to a CAT III standard can resist much higher energy transients than one designed to CAT II standards (Figure 12-2). There are also voltage ratings within categories, such as a CAT III 1,000 V test meter has superior protection when compared to a CAT III 600 V test meter.



Figure 12-2 Category ratings of two voltage levels: CAT II 1000 V and CAT III 600 V.

- **CAT I** is required for use on low-energy equipment such as protected electronic circuits and on high voltage/low-energy sources from a high-winding resistance transformer. A typical example would be a photocopier machine.
- **CAT II** is required for single-phase receptacle connected loads such as appliances, portable tools, and other household-type loads. The outlet should be located at least 30 feet from a CAT III source.
- **CAT III** is required for distribution circuits such as three-phase bus and feeder circuits, load centers, and distribution panels. These test meters are also used for permanently installed loads such as three-phase motors, large commercial lighting systems, and heavy appliance outlets with short connections to the service entrance.
- **CAT IV** is required for outside and service entrance use from the pole to the meter and the meter to the service panel. These test meters are also used for outside overhead and underground cable runs because they may be affected by lightning strikes.

Meter Selection and Test Leads

Always choose a test meter rated for the highest category (CAT) you could be working in. A higher-category meter can be used for any lower-category applications. The voltage rating for the meter must also be considered. As an example, a CAT III 600 V meter has superior transient protection compared to a CAT II 1,000 V meter even though the voltage ratings may seem to indicate otherwise.

The test leads for the meter are just as important as the meter ratings. They should be certified to a category and voltage the same or higher than the meter (Figure 12-3). Alligator clips are handy in freeing up your hands from holding the test leads, which allows you more freedom to set the meter for the proper reading (Figure 12-4).



Figure 12-3 (a) Meter test leads; (b) CAT rating listed on the test lead.

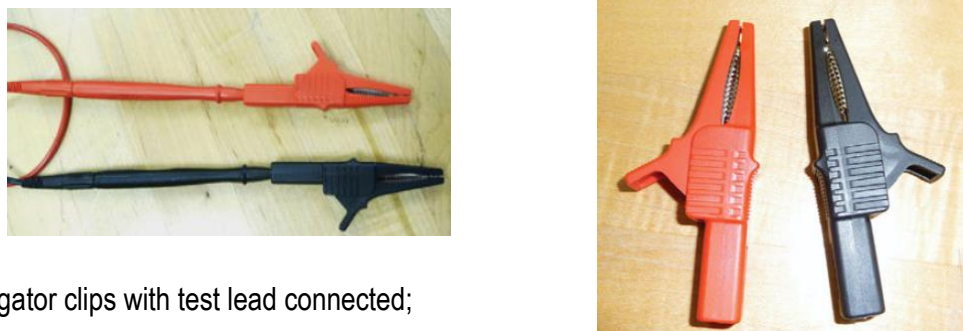


Figure 12-4 (a) Alligator clips with test lead connected;
(b) Alligator clips disconnected from test leads.

METER TYPES: ANALOG AND DIGITAL

Digital meters are commonly used in the field today and have all but replaced analog-style meters. The following comparisons between them will highlight the advantages and disadvantages of both types.

Analog Meters

Analog meters are seldom used today because digital meters are superior in almost every way. Analog meters have a mechanical needle that swings across the dial to a point over a scale (Figure 12-5). The meter reading is taken by looking directly at the face of the meter dial and comparing the needle location to the appropriate scale. Some technicians prefer this type of display and consider it superior to a digital meter (Figure 12-6).

Digital meters display discrete number values, while analog meters provide a relative reading. An example is a digital watch compared to a watch with minute and second hands. Some people prefer the moving hands. Most cars use a speedometer needle for displaying relative speed rather than a digital number. The type of display then just becomes a matter of preference for the user. Some digital meters offer both types of display.

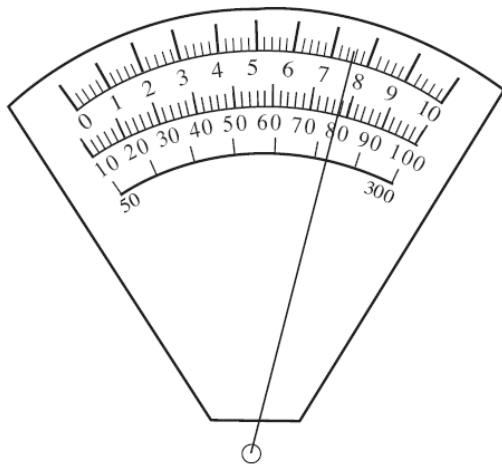


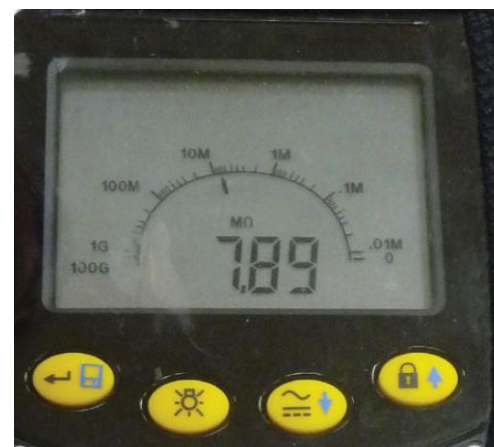
Figure 12-5 An analog millimeter has several scales, so you must know which range you are measuring and read the appropriate scale.



Figure 12-6 Analog electrical test meter.

They incorporate an analog sliding scale that appears across the digital meter face so that it is possible for the technician to have both the digital number and an analog reference (Figure 12-7).

Figure 12-7 A digital meter displaying 7.89 megohms both as a numeric value and on a sliding scale.



Other than the needle display, analog meters offer no other true advantages. They are almost never auto-ranging. If too high a voltage is being tested for the scale setting, then the meter can be damaged. If you prefer to use an analog meter, there are three important characteristics that need to be considered:

- The most accurate reading is at the midpoint of the scale. So whenever the operator has a choice of scales, the one selected should place the pointer in the most favorable (central) position.
- Analog meters periodically need to recalibrate. Most ohmmeters include some type of adjustment and instructions for calibration.
- The small coil of wire that forms part of the meter movement is sensitive to excessive current. The meter may be made completely inoperable if subjected to excessive current. In using a meter with multiple scales to choose from, always use the higher scale first and move down to the scale required.

Digital Meters

Digital meters offer a number of advantages.

- They are direct reading, so there is no need to interpret the scale.
- Digital meters can be obtained that will give accurate readings to three decimal places.
- They have no moving parts and are less likely to fail or get out of calibration than analog meters, and they are more rugged.
- They often have automatic scaling features.
- An ohmmeter to measure electrical resistance
- A megohm meter to measure very high electrical resistance (a megohm is the same as a mega ohm)
- A capacitor checker to measure capacitance, measured in microfarads (mfd)
- A wattmeter to measure electrical power

Today it is no longer necessary to carry all of these single-use meters. Multi-meters will measure voltage, current, and resistance all with one instrument. Multi-meters designed specifically for HVACR applications often provide additional measurements such as readings for capacitance, temperature, frequency, and power.

Some have a diode function to determine the forward or reverse bias of a diode or if the diode is open or shorted. Many also provide features such as the ability to hold the value on the display screen (display hold) or record min/max averages.

METER RANGE, RESOLUTION, AND ACCURACY

Range, resolution, and accuracy are three terms used to differentiate all meters. Understanding test instrument terminology will help you make the best selection of the instrument for your specific job requirements.

Range The range tells you how low and how high a reading the meter will provide for each function the meter can perform. Many meters today have an auto-ranging function (Figure 12-8). This means the meter sets the range to the best resolution for the input signal. Normally there is also a manual range mode that allows you to set the range yourself.

- Voltmeters should be able to read voltages from a fraction of a volt up to 600 V AC and DC.
- Ohmmeters should be able to read ohms from a fraction of an ohm up to 20,000 (20K) ohms.
- Mega ohm (megohm) meters should be able to read up to 40,000,000 (40M) ohms.
- Amp meters (ammeters) should be able to read AC amperage from 1 A up to 200 A, and DC amps from 200 mA to 4,000 mA, and from 0 to 20 μ A (micro amps).
- Capacitance meters should be able to check from 1 μ F (microfarad) up to 200 μ F.

Resolution The resolution tells you what units of measure are used for each function. On a digital meter specification, it may be expressed as the number of digits or decimal points. This can range from three to four digits and one to three decimal places.

Accuracy The accuracy is how close to the actual value the meter is going to read. These values are usually given as a percentage \pm (plus or minus) of the reading. Meters may vary from ± 0.5 percent up to ± 3 percent, depending on the function and scale.

Figure 12-8 Auto-ranging multi-meter.



TESTING A DE-ENERGIZED CIRCUIT

If you are testing to verify that there is no voltage present in a de-energized circuit before beginning work, there are three different test instruments that may be used. These are a noncontact proximity tester, an electrical tester, and a multi-meter.

Testing electrical circuits is an important skill that each technician needs to develop. Although practice and experience are significant, a high degree of success is obtainable by following a proven procedure, such as the following:

1. Know the unit electrically. This means understanding the proper function of each control and the sequence of the control operation. It is as important to know what a meter does not measure as well as what it does.
2. Be able to read schematic wiring diagrams and have them available.
3. Be able to use the proper electrical test instruments. Know the instrument. Read instructions carefully before using.

Testing Procedures

Always try to work on de-energized circuits whenever possible, and follow the proper lockout/tagout procedures. Wear protective gear and stand on an insulated mat. Test the circuit in the following manner.

1. Make sure the circuit is off and properly locked out and tagged out.
2. Test the meter on a known voltage source.
3. Test the circuit that should be de-energized (off).
4. Test the meter again on a known source to ensure that you didn't damage your meter or blow a fuse and get a "0" volt reading accidentally in step 3.

Try to rest or hang the meter whenever possible rather than holding it in your hands. Alligator-clip test leads also allow you to keep your hands free by connecting one at a time and leaving them in place. Connect the ground test lead first and then the hot tests lead last. Reverse this procedure to do the opposite for removal. Remove the hot test lead first and the ground test lead last. When connecting test leads, try to keep one hand in your pocket to lessen the chance of a closed circuit across your chest and through your heart.

Low-Voltage Proximity or Noncontact Voltage Tester

This type of tester does not have test leads. The tester does not touch the conductor but is placed close (within proximity) to it. The tester will light up if voltage is present (Figure 12-9). These are good for a first test but should always be followed up with a direct-contact meter. This is because proximity tester readings can be inaccurate for a variety of reasons, such as when used inside a metal enclosure. Also, proximity readings will vary by distance and the strength of the voltage field and are therefore subject to some unreliability.

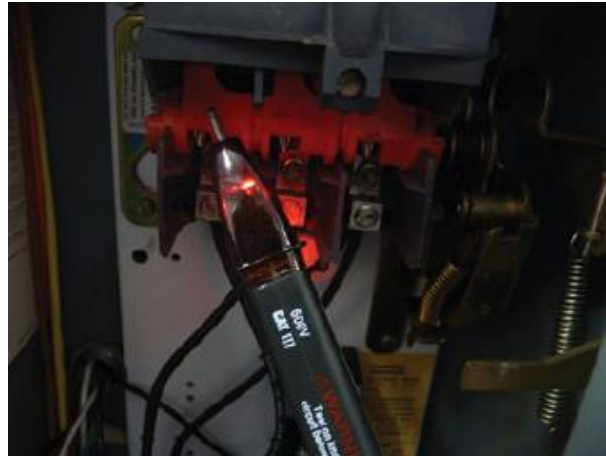


Figure 12-9 Noncontact voltage detector lights up when voltage is present.

Electrical Testers

There are two general types of voltage testers: solenoid based and electronic. Both these types have test leads that must come in direct contact with the conductors. Solenoid based testers that vibrate and light up are seldom used anymore (Figure 12-10). This is because there have been a number of safety issues involved with these, including that most are not fused. Another drawback is the sensitivity response of these testers. The circuit measured must have enough energy to move a spring-loaded slug. Therefore, low-voltage control circuits will not normally register on the indicator.

A better and more popular alternative is the electronic tester. These are fused for transient protection and are much more accurate. They have much higher input impedance so there is less input current required and the instrument remains cooler. Since they work with lower voltages, you are able to use the electronic test meter on a wider range of equipment. They are even designed to vibrate and light up to mimic the solenoid-based type.

Figure 12-10 Solenoid-based voltage testers such as the one shown are seldom used today due to safety concerns and accuracy at low voltages.



Digital Multi-meter

Multi-meters can be considered the best instrument to test for voltage, but there are a few drawbacks as compared to the more simple voltage testers described previously. One drawback is operator error in turning the multi-meter dial to the wrong function, such as amps instead of volts. This cannot happen on a proximity or electronic voltage tester.

Another drawback for an older-type meter that is not auto ranging is putting the meter into a range that is too high, which makes the voltage appear smaller than it really is. Some multi-meters are now built with a voltage testing function that allows you to start with a noncontact proximity test and then move to a contact test, with the same instrument (Figure 12-11).

Figure 12-11 Multi-meter with built-in noncontact voltage detector.



MEASURING VOLTAGE ON A LIVE CIRCUIT

It is not always possible to test only a completely DE energized circuit. Troubleshooting methods may often require that the circuit tested be “live.” Testing for proper supply voltage is usually the first thing measured when troubleshooting a circuit. There are a number of safety considerations for testing a live circuit, such as location and setup.

Location and Setup

It is never safe to work on high-energy circuits without someone else present. Make sure the test meter is rated high enough for the reading you will be taking. Assess the location before you begin, and make sure the lighting is adequate. Find a location where you will place your meter so that you do not have to hold it. Magnetic hangers allow you to position the meter so that it can be clearly read without needing to be held.

Determine if the leads will reach into the electrical cabinet without a need for extenders. Probe extenders keep your hands farther away from the conductors.

Test probes with a minimum amount of exposed metal at the probe tips, such as .12 in (4 mm) metal-tip probes, will reduce the risk of accidentally shorting between two phases (Figure 12-12).

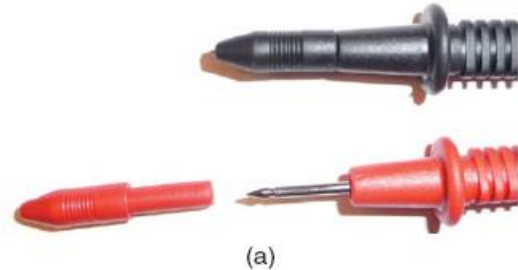
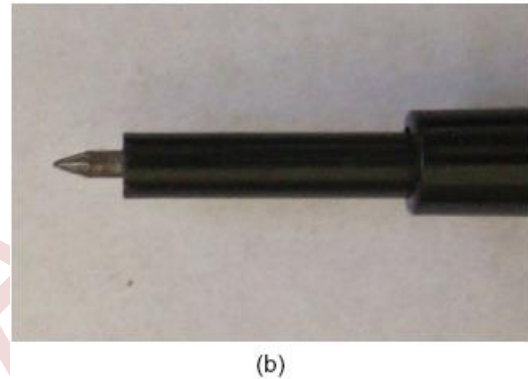


Figure 12-12 (a) Long test lead tip with cover;
(b) short test lead tip.



Attaching the Test Leads

Always use safe testing procedures as outlined in section 35.7. Whenever possible, attach the test leads to the conductors with the circuit de-energized. This can be done with alligator-clip-type test leads. Always connect the neutral test lead first and then the hot test lead last. The circuit may then be energized for the reading to be taken. Once the reading has been obtained, the circuit may once again be de-energized and the test leads removed by disconnecting the hot lead first and the grounded lead last.

If you need to probe the circuit, attach the black test lead with an alligator clip and use the red test lead to move from one conductor to the next. Keep one hand in your pocket and out of the panel so as not to offer a closed circuit path for the flow of electricity.

How to Measure Voltage

Voltmeters are connected in parallel with the load to read the voltage drop or potential difference. In testing a circuit for proper voltage, one lead goes on each side of the load, as shown in Figure 12-13 . Many test meters today are autoranging, which means a specific range of voltage does not need to be selected. However, when using a test meter that is not auto-ranging, always start to measure voltage using the highest range on the meter—for example, 600 V. When the approximate voltage is read, the meter range can then be reduced to the proper range for greater reading accuracy—for example, start at 600 V then move down to 200 V. Also, using the meter to test a higher voltage than the range of the meter could cause burnout or otherwise damage the meter. Read the instruction manual for the meter you are using. Some meters have function buttons in addition to the selector dial (Figure 12-13). On a DC circuit, the polarity of the leads must be observed. Therefore, in DC circuits, verify the correct polarity of the probes that are used before connecting the meter to the circuit.

An example of a use for a voltmeter is to determine if a hidden switch is open or closed. This is very helpful in troubleshooting. If there is power in the circuit, and the leads of the voltmeter are placed on each side of the switch, a voltage reading indicates the switch is open (Figure 11-14), and a zero reading indicates the switch is closed (Figure 12-15) (if no other open switches are in the circuit).



Figure 12-13

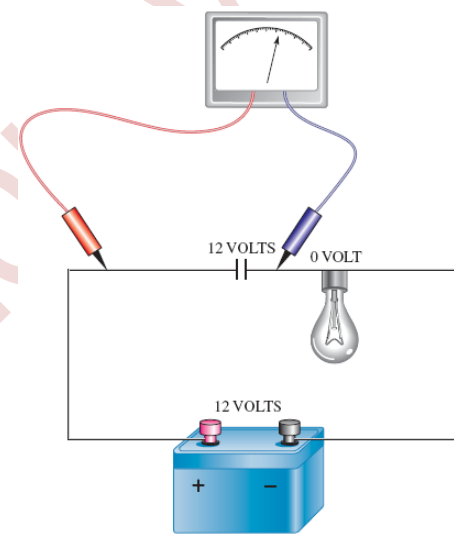


Figure 12-14

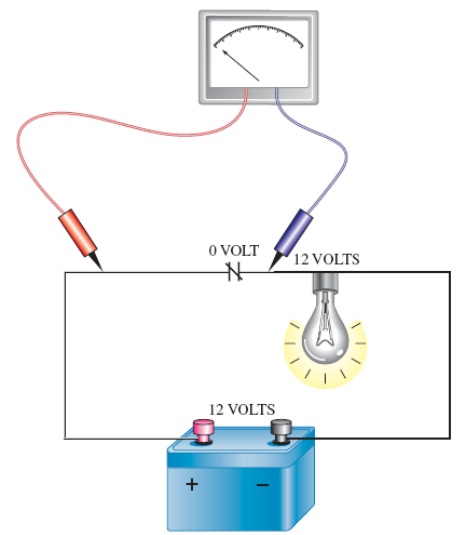


Figure 12-15

AMMETERS

Amp meters (ammeters) are used to measure current flow through a circuit. There are two types of ammeters: clamp on and inline. An advantage of the clamp-on meter is that it is not necessary to disconnect or make contact with any wires to obtain a reading; this is very convenient. The advantage of an inline meter is that it can provide very accurate current reading because all of the system current flows through the meter. Some of today's clamp-on ammeters are dual use and can serve both as inline (multi-meters) and clamp-ons (Figure 12-16). Even so, there are often times when simultaneous readings of both voltage and amperage are required, and most common meters will not be able to read both at the same time. So for troubleshooting purposes, two meters work best, and many technicians will use both a multi-meter and a clamp-on ammeter.

Figure 12-16 Clamp-on ammeters that are also a multi-meter.



Clamp-On Ammeters

The clamp-on ammeter is one of the most useful of the electric meters for HVACR technicians. It is used to measure current flow through a single wire by enclosing the wire within the jaws of the instrument. Some clamp meters are capable of measuring both AC and DC current. This instrument functions like a transformer. The primary coil is the test wire encircled by the jaws of the instrument. The secondary is a coil of wire within the instrument that is connected to the current-indicating mechanism. The current in the primary wire induces a flow of current in the secondary winding, measuring the current flow. The greater the current flow through the test wire, the greater the induced current and the greater the deflection of the needle reading on the scale.

Inline Ammeter

Inline instruments that only measure amperage are not commonly used. Instead, multi-meters are the most common electrical measuring instrument for HVACR work because they are able to do so much more. Some multi-meters have a removable clamp-on available as an accessory that may be used with the meter (Figure 12-17). When using a multi-meter as an inline meter, the proper location for its connections in a circuit is

shown in Figure 12-18. Note that it is connected in series with the circuit being tested. In DC circuits, verify the correct polarity of the ammeter used before energizing the circuit. Never connect an ammeter across a load. It will be destroyed by line current, as there is no load to limit it!



Figure 12-17

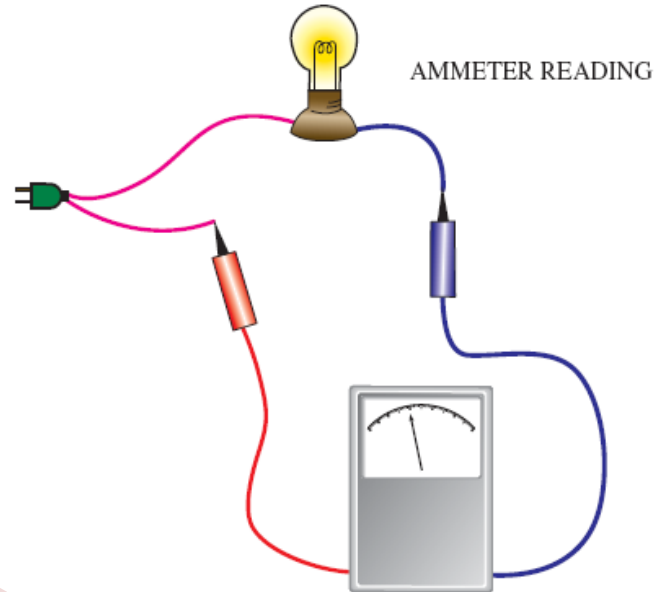


Figure 12-18

TEST METER FUSES

A somewhat common mistake for someone inexperienced in operating a test meter is to take a current measurement in line with the load and then next try to take a voltage measurement with the test leads remaining in that same position rather than placing them in parallel with the load. This effectively places a short across the voltage source. This could potentially destroy the meter. To prevent this, a fuse is located in series with the meter's test leads.

Not all fuses are the same. Even fuses of the same amperage and voltage ratings can be different from each other. Specially designed "high-energy" fuses have different circuit-interrupt characteristics. Some are filled with sand so at high temperatures the energy will melt the sand, turning it to glass to coat the element for increased protection against a meter explosion and meltdown. Using the wrong fuse or trying to jumper the fuse could result in an accident where a meter explosion could lead to serious burns to your arms, face, and clothing. Always refer to the test meter's manual, or check with the test meter manufacturer to ensure you have the correct fuse.

OHMMETERS

Ohmmeters measure the electrical resistance and can be used to check a circuit for continuity. Millimeters will have this function and so will some clamp-on ammeters. *Continuity* means there is a complete electrical path. When the test leads are touching each other there is a complete electrical path (Figure 12-19). The meter is measuring the resistance of the wires of the test leads as 0.18 ohms. When the test leads are separated there is infinite resistance as indicated by the meter reading OL (Figure 12-20).

Not every circuit that tests OK for continuity is actually good. For example, a circuit that is grounded (shorted) to the unit case would show continuity to the case. All ohmmeters can check for continuity, but some have a special position that sounds a beep when there is continuity. The ohmmeter is different from an ammeter or a voltmeter in that it uses a battery as a power supply. The battery furnishes the current needed for resistance measurements.

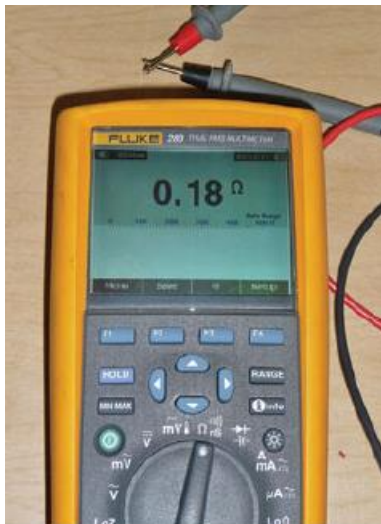


Figure 12-19

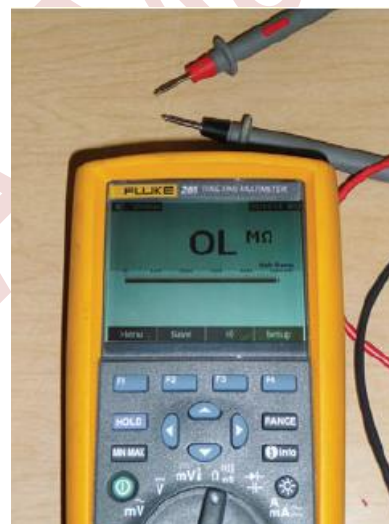


Figure 12-20

Using the ohmmeter to check for open circuits is called continuity testing. Figure 12-21 shows three diagrams representing the three possible responses that the meter can give.

- In Figure 12-21a, the meter is measuring the resistance through a pair of closed contacts in a circuit, and it registers zero. This indicates maximum current flow, or 0 Ω resistances. This is the measurement of a short circuit or a good set of closed contacts.
- In Figure 12-21b, the meter is measuring the resistance of a coil, which has a measurable resistance that is read on the meter.
- In Figure 12-21c, the meter is measuring the resistance of an open circuit, which is read on the meter as infinity. Infinity means that the resistance is so large that it cannot be measured. It means that at this point there is a lack of continuity or no current could flow.

Care must be taken to prevent errors in reading resistance when two or more circuits are connected in parallel, as shown in Figure 12-22 . The meter in the illustration is actually reading the combined resistance of two parallel resistances, HTR1 and Blower. To read only one resistance, one side of the component being tested is disconnected, as shown in Figure 12-23.

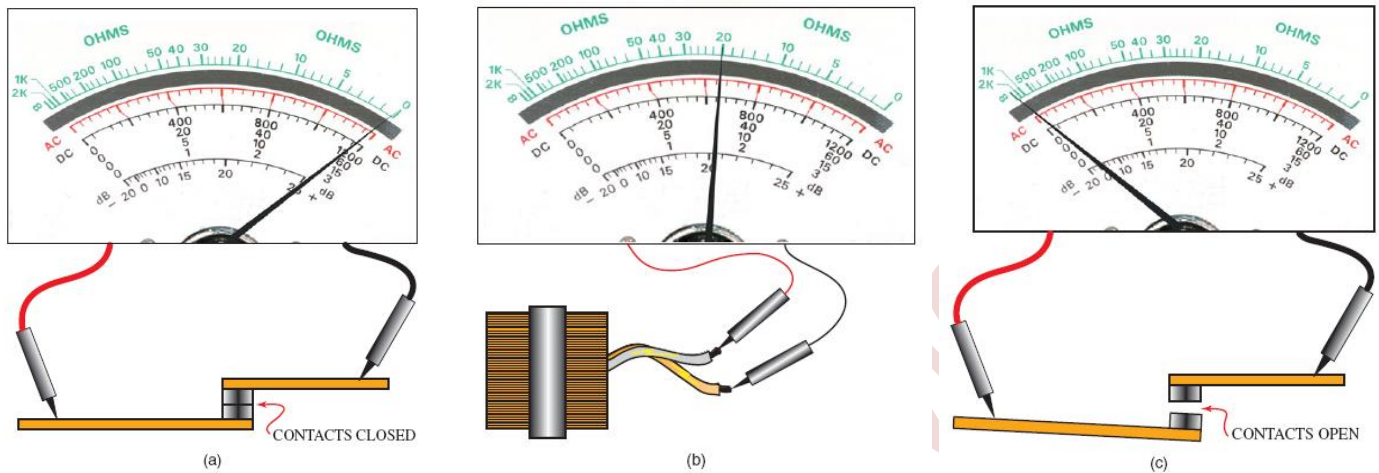


Figure 12-21 (a) Closed contacts or a short will read as 0 Ω ; (b) a coil resistor will have a resistance reading between 0 and infinity; (c) an open contact or broken circuit will have an infinite (∞) resistance.

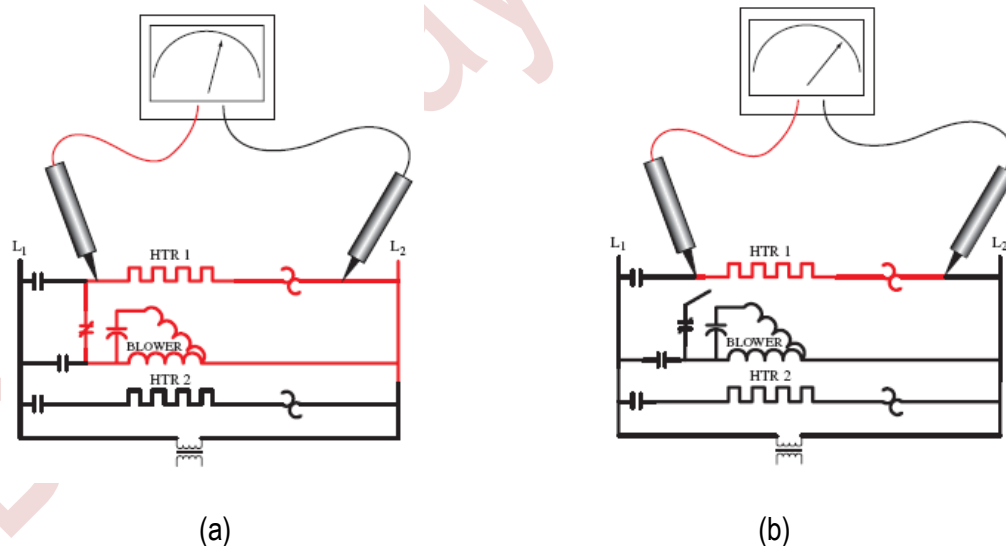


Figure 12-22:a) The ohm reading for Heater 1 would not be accurate because there are two paths, as shown in red. b) The ohm reading would be accurate because the second path is now broken at the normally closed contacts.

One caution needs to be followed: do not use an ohmmeter to test a solid state circuit unless the manufacturer specifically allows it. The internal battery voltage of the ohmmeter can damage an integrated circuit chip.

Megohmmeters (Meggers)

Some ohmmeters, often referred to as meggers (megohmmeters), must be able to read resistances of tens of millions of ohms (megohms) (Figure 12-23). Meggers are designed to generate a high voltage across their test leads and therefore require more power as compared to regular ohmmeters.

Meggers are used to test the resistance of insulation, particularly in electric motors. For testing an electric motor, the power supply must first be disconnected and the motor isolated from the circuit. Then one test lead is attached to one phase of the motor while the other test lead is attached to ground, usually with an alligator clip (Figure 12-24). The ground connection is often the motor housing. When attaching the lead to ground, make sure to scrape any paint until bare metal is showing to ensure a good connection. A high megger reading would indicate that the motor winding and insulation are good (Figure 12-25). A low megger reading would indicate a shorted motor winding (Figure 12-26).



Figure 12-23



(a)



(b)

Figure 12-24



Figure 12-25



Figure 12-26

Never touch the test leads when the megger is being used, because the megger generates a high voltage across them.

CAPACITANCE METERS

Capacitance meters measure in microfarads (Figure 12-27&28). Most capacitors are within the range for Nano farad to microfarad. One microfarad is equal to one millionth of a farad. Before testing a high-voltage capacitor, always begin by properly discharging the capacitor first. The meter measures capacitance by charging the capacitor with a known current and measuring the resulting time of charging period. From this, the capacitance is then calculated by the meter. The meter measures the circuit at 4.198 microfarads due to the capacitance of the meter leads as shown in Figure 11-29. Most meters will allow for the subtraction of the residual capacitance of the meter and leads. The meter must be connected for the proper polarity of the capacitor.



Figure 12-27

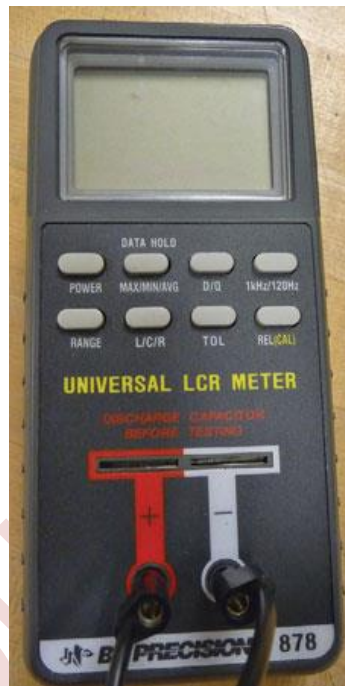


Figure 12-28



Figure 12-29

WATTMETER

Power that is measured by simply multiplying the current by the voltage is called apparent power. However, this is not a true indication of power. To mathematically calculate power in watts using the measured volts and amps, it is necessary to know the power factor. Watt meters incorporate the power factor to read true power (Figure 12-30).

The meter makes the necessary calculations for power, in accordance with the power formula:

$$\text{Watts} = \text{volts} * \text{amps} * \text{power factor}$$

Most watt meters zero themselves when not in use and automatically select the proper range when used. They can be used to measure watts on single, split-phase, and three phase power sources. Some millimeters and clamp-on ammeters have a power measurement function.



Figure 12-30

SPECIAL-PURPOSE METERS

Power meter analyzer:

There are many special-purpose meters designed for specific functions. Many digital meters have the capability to freeze and hold the reading or provide for a peak hold, a max hold, or a min/max hold. With some of these meters you can see the starting current of a motor during start-up exactly as the circuit protector sees it (Figure 12-31). As the motor starts, the meter begins recording a large number of samples in as little as 100 milliseconds time and processes the samples to display the actual starting current. This type of meter can also be used to resolve intermittent breaker trips, helping to identify whether a nuisance trip is due to a faulty breaker.



Figure 12-31

Data Logging

Data-logging instruments can be set up to record any number of data points from a few seconds to a month or more. These instruments are extremely valuable when dealing with customers' complaints like, "My unit runs all the time," "It's always hot in here," and so on. A data logger can prove or disprove these too common and often frustrating complaints. It can also help to solve troublesome intermittent problems that never seem to happen while you are working on the system. Some data loggers display the logged data directly on the equipment display panel, while others can download the data to a computer for a comprehensive graphic analysis (Figure 12-32).

The newest line of data-logging instruments prompts the technician for information. They can connect the proper test device to the data logger, where it can collect the information directly from the device. With all of the data collected and the system equipment model number entered, the data logger will provide a comprehensive evaluation of the system's operation and its current level of efficiency.

These data are automatically compared to the manufacturer's data, so the report can include specific recommendations if the system is not within the manufacturer's operating specifications.

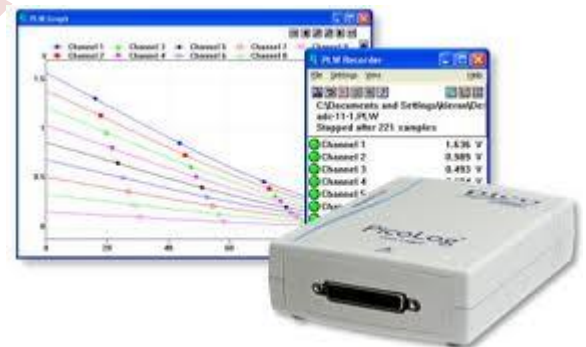


Figure 12-32



MEASUREMENT OF THERMAL CONDUCTIVITY, VISCOSITY, & HUMIDITY

Thermal conductivity:

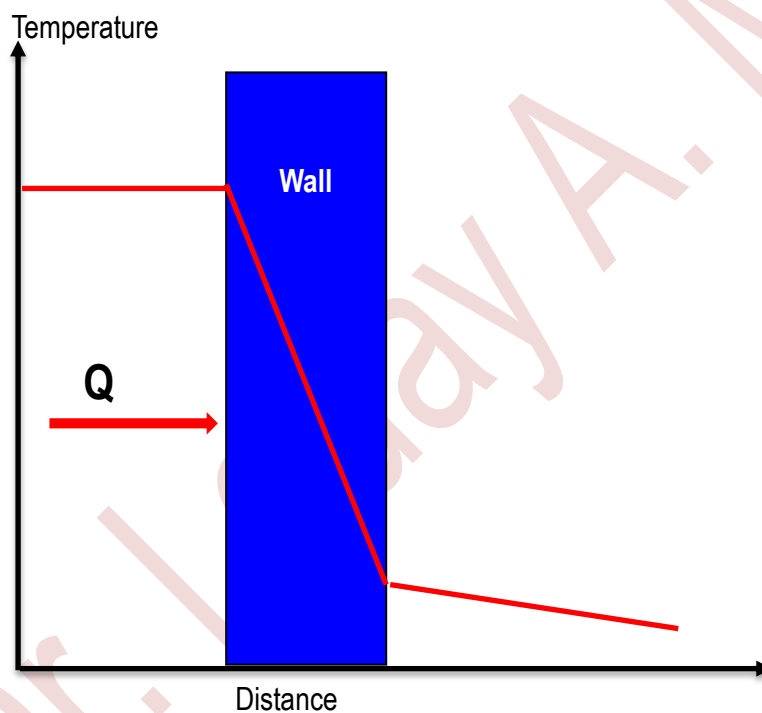
It is thermal property; the thermal conductivity is the rate of heat transfer through a material in steady state. It is not easily measured, especially for materials with low conductivity but reliable data is readily available for most common materials.

The Thermal Conductivity (k) is the measure of the ability of a material to transmit heat by conduction.

k is defined from the equation:

$$Q = -k \cdot A_c \cdot \frac{dT}{dx}$$

Where dT are the temperatures either side of an element of material with thickness dx and cross sectional area A_c . k has the units of Watts/(m °C).



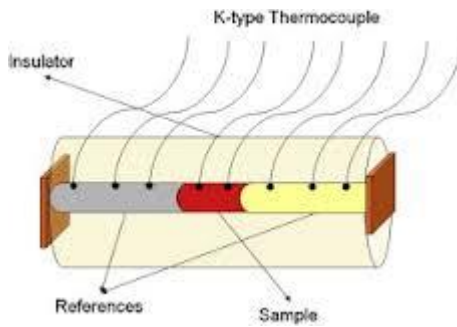
Thermal Conductivity: Comparison:



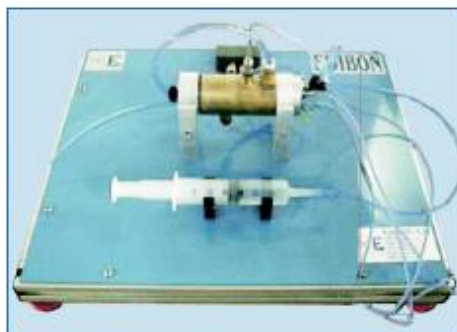
Material		k (W/m-K)	Energy Transfer Mechanism
Metals	copper	385	atomic vibrations and motion of free electrons
	Aluminum	220	
	Tungsten	178	
	Steel	52	
Ceramics	Alumina (Al_2O_3)	39	atomic vibrations
	Magnesia (MgO)	38	
	Soda-lime glass	1.7	
	Silica (cryst. SiO_2)	1.4	
Polymers	Polyethylene	0.46-0.50	vibration/rotation of chain molecules
	Teflon	0.25	
	Polystyrene	0.13	
	Polypropylene	0.12	

Measure the thermal conductivity:

Solid material:



Liquid material:



① Unit: TCLGC. Thermal Conductivity of Liquids and Gases Unit

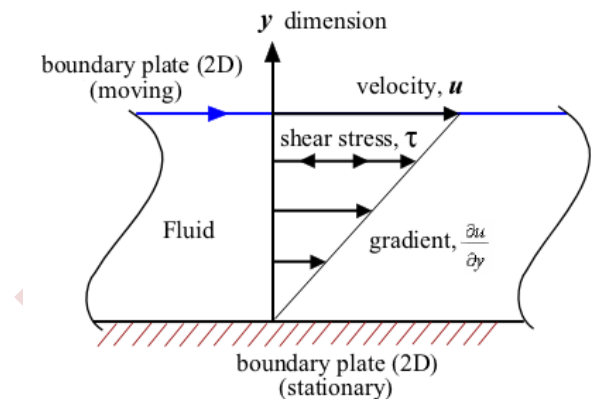


Viscosity:

Viscosity is a measure of resistance to flow or Measure of the resistance to deformation of a *fluid* under shear stress. there are two type of viscosity; dynamic viscosity($N.s/m^2$) and kinematic viscosity(m^2/s).

- Viscosity is not linked to density, but is a result of intermolecular bonding and solute concentration.
- Larger molecules like oils tend to have higher viscosities.
- Animals and plants use high- and low-viscosity liquids for important life functions.

Internal friction between layers of flow



Newtonian and Non-Newtonian Fluids:

Newtonian Fluids:

- Water
- Oil
- Gasoline
- Alcohol
- Kerosene
- Benzene

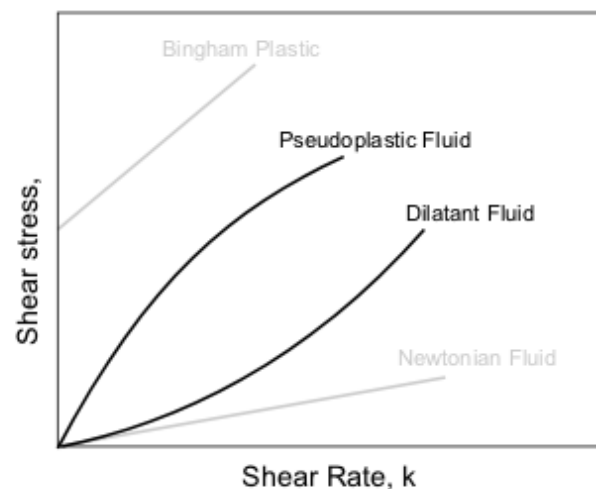
Non-Newtonian Fluids:

- Time-independent Fluids
 - Pseudo plastic (Blood Plasma, syrups, inks)
 - Dilatant (Starch in water)
 - Bingham (catsup, mustard, toothpaste)

- Time-dependent Fluids

Electro rheological (behavior changes due to electric field, particles are present)

Magneto rheological (iron powders in fluid)



Approximate Viscosities of Common Materials (At Room Temperature: 70°F)

Material	Viscosity in Centipoise(cps)
Water	1
Milk	3
SAE 10 Motor Oil	85-140
SAE 20 Motor Oil	140-420
SAE 30 Motor Oil	420-650
SAE 40 Motor Oil	650-900
Castrol Oil	1,000
Karo Syrup	5,000
Honey	10,000
Chocolate	25,000
Ketchup	50,000
Mustard	70,000
Sour Cream	100,000
Peanut Butter	250,000

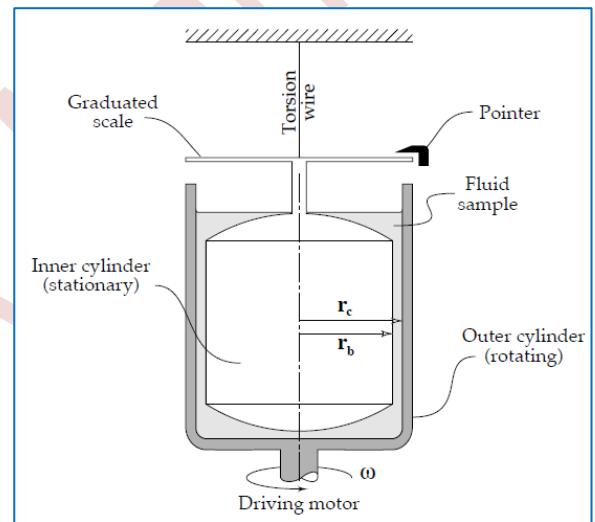
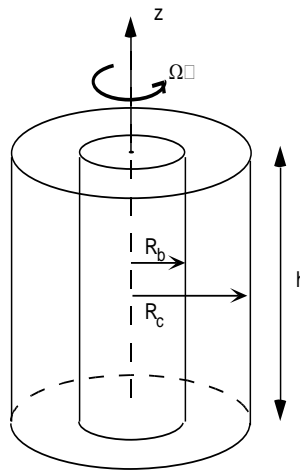
Type	Dynamic viscosity	Kinematic viscosity
SI unit	N.s/m ²	m ² /s
	cP(centipoise) P(poise) 1 P=100 g/m.s	cSt(centistokes) St(stokes)
PI unit	Lb _m /ft.h	ft ² /s

Viscosity Measurements

Rotational Viscometers

- The viscometer gives the value of the 'dynamic viscosity'. It is based on the principle that the fluid whose viscosity is being measured is sheared between two surfaces. In these viscometers one of the surfaces is stationary and the other is rotated by an external drive and the fluid fills the space in between. The measurements are conducted by applying either a constant torque and measuring the changes in the speed of rotation or applying a constant speed and measuring the changes in the torque. There are two main types of these viscometers: rotating cylinder and cone-on-plate viscometers.

Rotating cylinder viscometer:



$$\eta = M(1/r_b^2 - 1/r_c^2) / 4\pi d\omega = kM / \omega$$

where:

- η is the dynamic viscosity [Pas];
- r_b, r_c are the radii of the inner and outer cylinders respectively [m];
- M is the shear torque on the inner cylinder [Nm];
- ω is the angular velocity [rad/s];
- d is the immersion depth of the inner cylinder [m];
- k is the viscometer constant, supplied usually by the manufacturer for each pair of cylinders [m³].

Capillary viscometer:

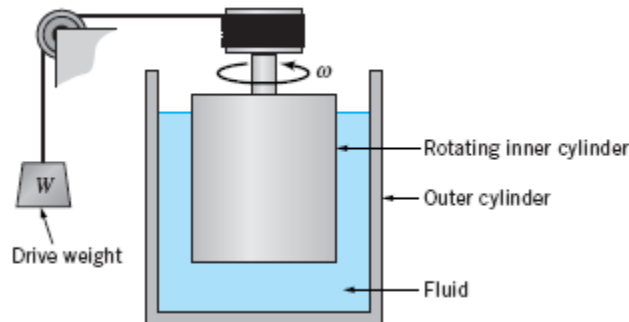
- used to measuring kinematic viscosity of the fluid based on Poiseuille's law
- or by using pressure drop to determine viscosity

$$\mu = \eta = \frac{(P_1 - P_2) \cdot D^2}{32 \cdot v \cdot L}$$



Fluid Characterization by Use of a Stormer Viscometer some fluids can be classified as Newtonian fluids; others are non-Newtonian. The purpose of this experiment is to determine the shearing stress versus rate of strain characteristics of various liquids and, thus, to classify them as Newtonian or non-Newtonian fluids.

Equipment: Stormer viscometer containing a stationary outer cylinder and a rotating, concentric inner cylinder ; stop watch; drive weights for the viscometer; three different liquids (silicone oil, Latex paint, and corn syrup).



Experimental Procedure: Fill the gap between the inner and outer cylinders with one of the three fluids to be tested. Select an appropriate drive weight (of mass m) and attach it to the end of the cord that wraps around the drum to which the inner cylinder is fastened. Release the brake mechanism to allow the inner cylinder to start to rotate. (The outer cylinder remains stationary.) After the cylinder has reached its steady-state angular velocity, measure the amount of time, t , that it takes the inner cylinder to rotate N revolutions. Repeat the measurements using various drive weights. Repeat the entire procedure for the other fluids to be tested.

Calculations: For each of the three fluids tested, convert the mass, m , of the drive weight to its weight, $W=mg$ where g is the acceleration of gravity. Also determine the angular velocity of the inner cylinder, $\omega=N/t$

Graph: For each fluid tested, plot the drive weight, W , as ordinates and angular velocity, ω as abscissas. Draw a best fit curve through the data.

Results: Note that for the flow geometry of this experiment, the weight, W , is proportional to the shearing stress, on the inner cylinder. This is true because with constant angular velocity, the torque produced by the viscous shear stress τ on the cylinder is equal to the torque produced by the weight (weight times the appropriate moment arm). Also, the angular velocity, ω , is proportional to the rate of strain, du/dy . This is true because the velocity gradient in the fluid is proportional to the inner cylinder surface speed (which is proportional to its angular velocity) divided by the width of the gap between the cylinders. Based on your graphs, classify each of the three fluids as to whether they are Newtonian, shear thickening, or shear thinning.

Capillary Tube Viscometer:

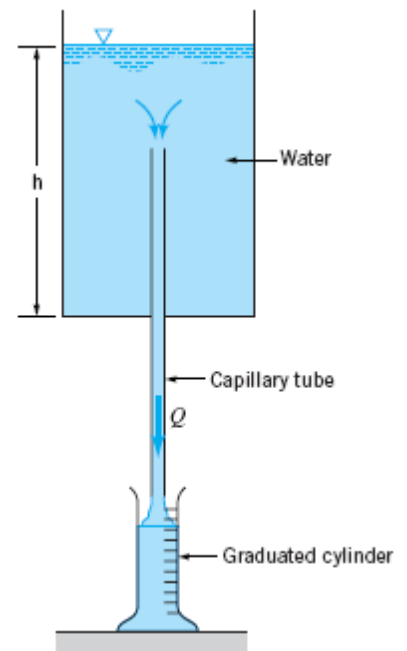
Objective: The flow rate of a viscous fluid through a small diameter (capillary) tube is a function of the viscosity of the fluid. For the flow geometry shown in Fig. the kinematic viscosity, ν is inversely proportional to the flow rate, Q . That is, $\nu = K/Q$, where K is the calibration constant for the particular device. The purpose of this experiment is to determine the value of K and to use it to determine the kinematic viscosity of water as a function of temperature.

Equipment: Constant temperature water tank, capillary tube, thermometer, stop watch, graduated cylinder.

Experimental Procedure: Adjust the water temperature to and determine the flow rate through the capillary tube by measuring the time, t , it takes to collect a volume, V , of water in a small graduated cylinder. Repeat the measurements for various water temperatures, T . Be sure that the water depth, h , in the tank is the same for each trial. Since the flow rate is a function of the depth (as well as viscosity), the value of K obtained will be valid for only that value of h .

Calculations: For each temperature tested, determine the flow rate; Use the data for the water to determine the calibration constant, K , for this device. That is, where the kinematic viscosity for water is given from Table and Q is the measured flow rate at this temperature. Use this value of K and your other data to determine the viscosity of water as a function of temperature.

Graph: Plot the experimentally determined kinematic viscosity, as ordinates and temperature, T , as abscissas.



Humidity:

The presence of water vapor in gas. The mention to humidity is done by relative humidity, absolute humidity, dew point, mixing ratio.

Relative humidity: Ratio that compares the amount of water vapor in the air with the amount of water vapor that would be present in the air as saturation.

Absolute humidity: The ratio of the mass of water vapor to the volume occupied by a mixture of water vapor and dry air g/m^3 .

Dew point: unique temperature to which air (or any gas) must be cooled in order that it shall be saturated with respect to water.

Mixing ratio: the mass of water vapor for a given mass of dry air g/kg .

Frost point: temperature that moisture just begins to condense on a surface.

Why do we need to measure humidity?

Too humid air: rotting, corrosion, bacteria.

too dry air: static electricity, skin, eyes and membrane dryness.

Humidity Instruments:

Electrical Hygrometers

- Hygroscopic films detect atmospheric moisture
- Dew point impedance from partial pressure of water vapor

Chilled mirror

- Optically sense presence of dew or frost on mirror
- Measure temperature of mirror for dew point or frost point
- Recent optical sensing improvements

Manual hygrometers

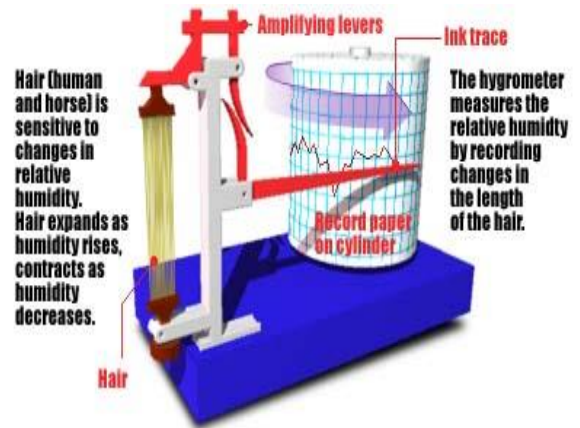
- Psychrometer
- Paired dry-bulb and wet-bulb thermometers
- Manual and semi-automated

Hygrographs

- Hygroscopic material sensor (e.g. hair)
- Mechanical link to pen

- **hair hygrometer**

- effect: detection of change of length of a human (or horse) hair in response to relative humidity changes
- hair length changes as in keratin hydrogen bonds are broken in the presence of water vapour
- slow response

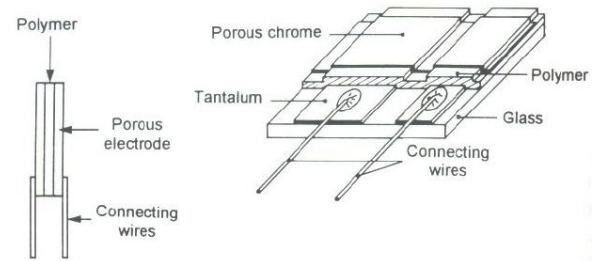


- **capacity hygrometer**

- effect: hygroscopic polymer is placed between two electrodes. In the presence of water vapour, the volume of the polymer increases, decreasing the capacity of the device
- are easily contaminated

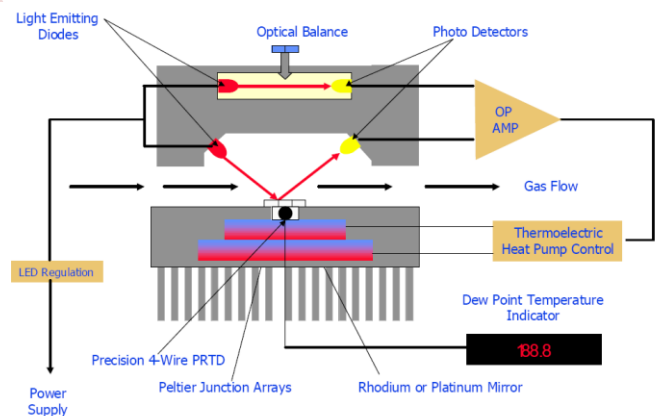
- **absorption hygrometer**

- absorption spectroscopy on H_2O can also be used to measure water vapour concentration



- **dew point hygrometer**

- effect: detection of dew on temperature controlled mirror by observation of change in reflectance
- very accurate



- psychrometer**

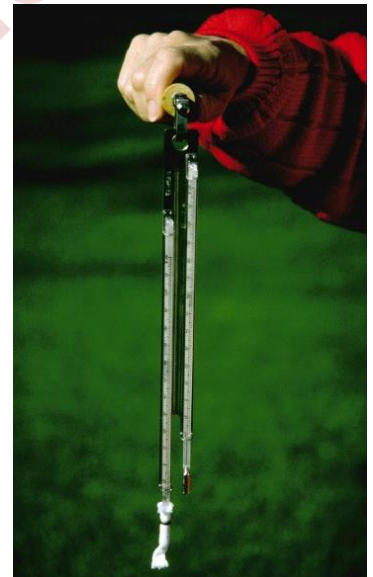
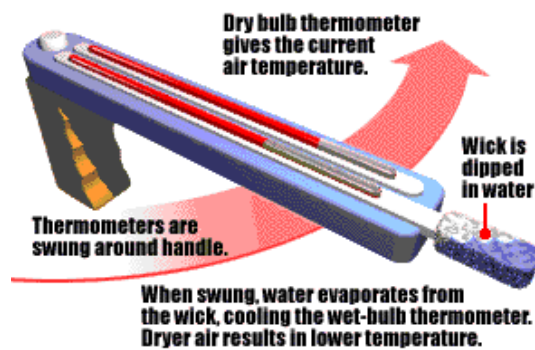
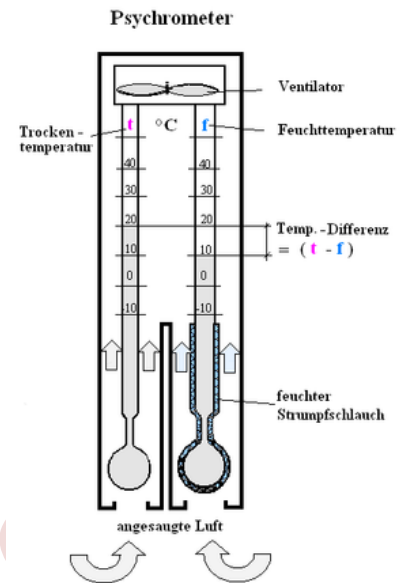
- effect: T-difference between two ventilated thermometers, one of which is covered by a wet wick (wet bulb temperature). T-difference is proportional to relative humidity

- use:

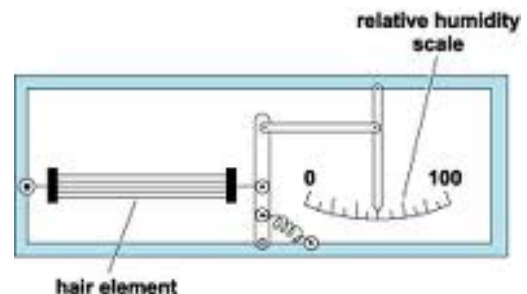
$$e = e_{\text{sat wet}} - c (T_{\text{dry}} - T_{\text{wet}})$$

water vapour partial pressure

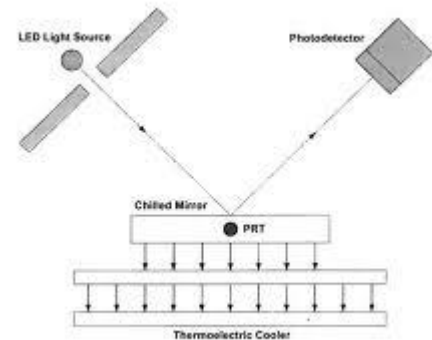
water vapour saturation pressure at T_{wet}



- Resistive hygrometer:**



- optical chilled mirror hygrometer:



- Precision-hair-hygrometer:





MEASUREMENT OF STRAIN

Strain is the amount of deformation of a body due to an applied force. More specifically, strain (ϵ) is defined as the fractional change in length, as shown in Figure 1 below.

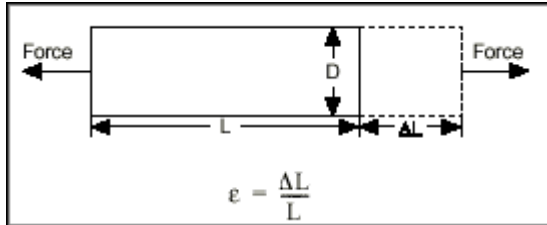


Figure 1. Definition of Strain

Strain can be positive (tensile) or negative (compressive). Although dimensionless, strain is sometimes expressed in units such as in./in. or mm/mm. In practice, the magnitude of measured strain is very small. Therefore, strain is often expressed as micro strain, which is $\epsilon \times 10^{-6}$.

When a bar is strained with a uniaxial force, as in Figure 1, a phenomenon known as Poisson Strain causes the girth of the bar, D , to contract in the transverse, or perpendicular, direction. The magnitude of this transverse contraction is a material property indicated by its Poisson's Ratio. The Poisson's Ratio ν of a material is defined as the negative ratio of the strain in the transverse direction (perpendicular to the force) to the strain in the axial direction (parallel to the force), or $\nu = \epsilon_T / \epsilon$. Poisson's Ratio for steel, for example, ranges from 0.25 to 0.3.

What is a strain gauge?

While there are several methods of measuring strain, the most common is with a strain gauge, a device whose electrical resistance varies in proportion to the amount of strain in the device. The most widely used gauge is the bonded metallic strain gauge.

The metallic strain gauge consists of a very fine wire or, more commonly, metallic foil arranged in a grid pattern. The grid pattern maximizes the amount of metallic wire or foil subject to strain in the parallel direction (Figure 2). The cross-sectional area of the grid is minimized to reduce the effect of shear strain and Poisson Strain. The grid is bonded to a thin backing, called the carrier, which is attached directly to the test specimen. Therefore, the strain experienced by the test specimen is transferred directly to the strain gauge, which responds with a linear change in electrical resistance. Strain gauges are available commercially with nominal resistance values from 30 to 3000 Ω , with 120, 350, and 1000 Ω being the most common values.

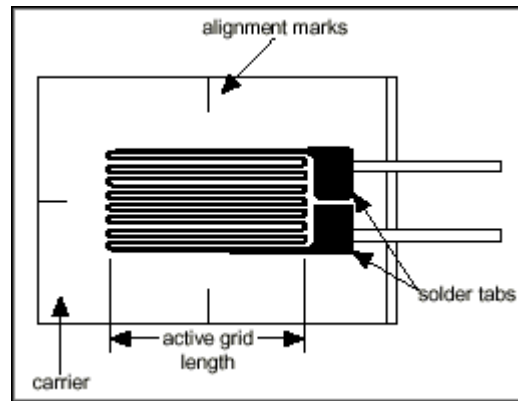


Figure 2. Bonded Metallic Strain Gauge

It is very important that the strain gauge be properly mounted onto the test specimen so that the strain is accurately transferred from the test specimen, through the adhesive and strain gauge backing, to the foil itself.

A fundamental parameter of the strain gauge is its sensitivity to strain, expressed quantitatively as the gauge factor (GF). Gauge factor is defined as the ratio of fractional change in electrical resistance to the fractional change in length (strain):

$$GF = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$$

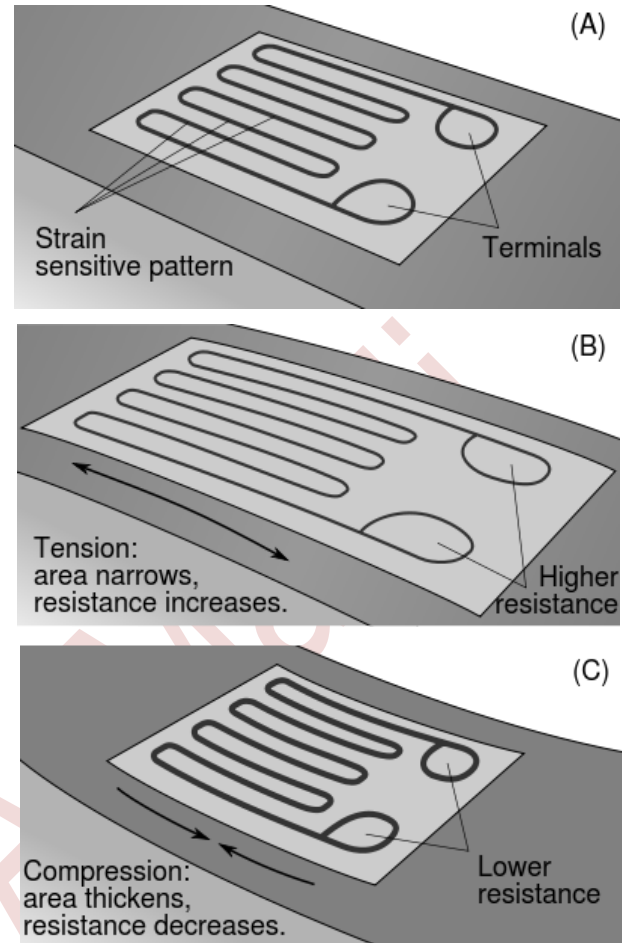
The Gauge Factor for metallic strain gauges is typically around 2.

A **strain gauge** is a device used to measure the strain of an object.

Physical operation:

A strain gauge takes advantage of the physical property of electrical conductance and its dependence on the conductor's geometry. When an electrical conductor is stretched within the limits of its elasticity such that it does not break or permanently deform, it will become narrower and longer, changes that increase its electrical resistance end-to-end. Conversely, when a conductor is compressed such that it does not buckle, it will broaden and shorten changes that decrease its electrical resistance end-to-end. From the measured electrical resistance of the strain gauge, the amount of applied stress may be inferred. A typical strain gauge arranges a long, thin conductive strip in a zig-zag pattern of parallel lines such that a small amount of stress in the direction of the orientation of the parallel lines results in a multiplicatively larger strain over the effective length of the conductor and hence a multiplicatively larger change in resistance—than would be observed with a single straight-line conductive wire. Strain gauges measure only local deformations and can be manufactured small enough to allow a "finite element" like analysis of the stresses to which the specimen is subject. This can be positively used in fatigue studies of materials.

Gauges in practice:



Strain gauge measurement:

In practice, the strain measurements rarely involve quantities larger than a few mille-strain ($\epsilon \times 10^{-3}$). Therefore, to measure the strain requires accurate measurement of very small changes in resistance. For example, suppose a test specimen undergoes a strain of $500 \mu\epsilon$. A strain gauge with a gauge factor of 2 will exhibit a change in electrical resistance of only $2 * (500 \times 10^{-6}) = 0.1\%$. For a 120 W gauge, this is a change of only 0.12 W.

To measure such small changes in resistance, strain gauges are almost always used in a bridge configuration with a voltage excitation source. The general Wheatstone bridge, illustrated below, consists of four resistive arms with an excitation voltage, V_{EX} , that is applied across the bridge.

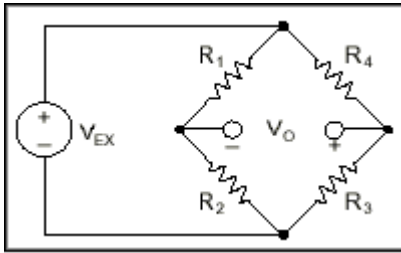


Figure 3. Wheatstone bridge

The output voltage of the bridge, V_O , will be equal to:

$$V_O = \left[\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right] \cdot V_{EX}$$

From this equation, it is apparent that when $R_1/R_2 = R_4/R_3$, the voltage output V_O will be zero. Under these conditions, the bridge is said to be balanced. Any change in resistance in any arm of the bridge will result in a nonzero output voltage.

Therefore, if we replace R_4 in Figure 3 with an active strain gauge, any changes in the strain gauge resistance will unbalance the bridge and produce a nonzero output voltage. If the nominal resistance of the strain gauge is designated as R_G , then the strain-induced change in resistance, ΔR , can be expressed as $\Delta R = R_G \cdot GF \cdot \epsilon$. Assuming that $R_1 = R_2$ and $R_3 = R_G$, the bridge equation above can be rewritten to express V_O/V_{EX} as a function of strain (see Figure 4). Note the presence of the $1/(1+GF \cdot \epsilon/2)$ term that indicates the nonlinearity of the quarter-bridge output with respect to strain.

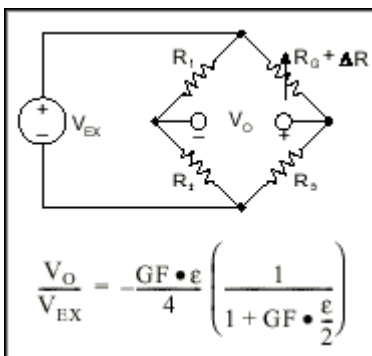


Figure 4. Quarter-Bridge Circuit

Ideally, we would like the resistance of the strain gauge to change only in response to applied strain. However, strain gauge material, as well as the specimen material to which the gauge is applied, will also respond to changes in temperature. Strain gauge manufacturers attempt to minimize sensitivity to temperature by processing the gauge material to compensate for the thermal expansion of the specimen material for which the gauge is intended. While compensated gauges reduce the thermal sensitivity, they do not totally remove it.

By using two strain gauges in the bridge, the effect of temperature can be further minimized. For example, Figure 5 illustrates a strain gauge configuration where one gauge is active ($R_G + \Delta R$), and a second gauge is placed transverse to the applied strain. Therefore, the strain has little effect on the second gauge, called the dummy gauge. However, any changes in temperature will affect both gauges in the same way. Because the temperature changes are identical in the two gauges, the ratio of their resistance does not change, the voltage V_O does not change, and the effects of the temperature change are minimized.

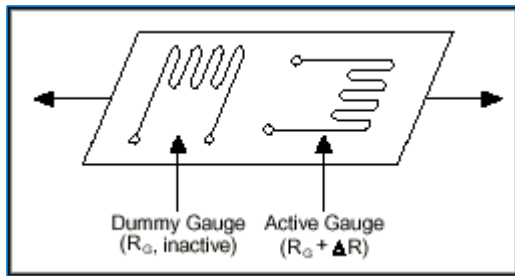


Figure 5. Use of a Dummy Gauge to Eliminate Temperature Effects

The sensitivity of the bridge to strain can be doubled by making both gauges active in a half-bridge configuration. For example, Figure 6 illustrates a bending beam application with one bridge mounted in tension ($R_G + \Delta R$) and the other mounted in compression ($R_G - \Delta R$). This half-bridge configuration, whose circuit diagram is also illustrated in Figure 6, yields an output voltage that is linear and approximately doubles the output of the quarter-bridge circuit.

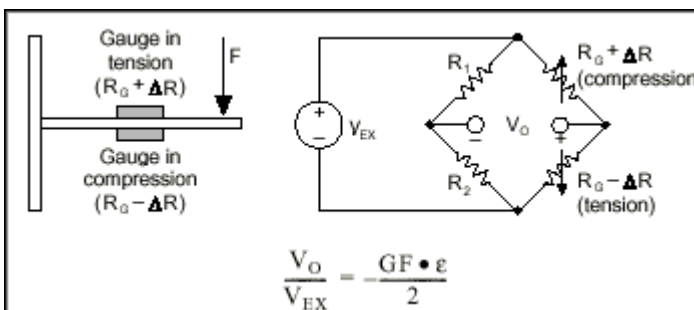


Figure 6. Half-Bridge Circuit

Finally, you can further increase the sensitivity of the circuit by making all four of the arms of the bridge active strain gauges in a full-bridge configuration. The full-bridge circuit is shown in Figure 7.

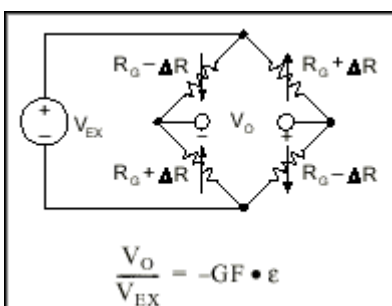


Figure 7. Full-Bridge Circuit

The equations given here for the Wheatstone bridge circuits assume an initially balanced bridge that generates zero output when no strain is applied. In practice however, resistance tolerances and strain induced by gauge application will generate some initial offset voltage. This initial offset voltage is typically handled in two ways. First, you can use a special offset-nulling, or balancing, circuit to adjust the resistance in the bridge to rebalance the bridge to zero output. Alternatively, you can measure the initial unstrained output of the circuit and compensate in software.

The equations given above for quarter, half, and full-bridge strain gauge configurations assume that the lead wire resistance is negligible. While ignoring the lead resistances may be beneficial to understanding the basics of strain gauge measurements, doing so in practice can be a major source of error. For example, consider the 2-wire connection of a strain gauge shown in Figure 8a. Suppose each lead wire connected to the strain gauge is 15 m long with lead resistance R_L equal to 1 W. Therefore, the lead resistance adds 2 W of resistance to that arm of the bridge. Besides adding an offset error, the lead resistance also desensitizes the output of the bridge.

You can compensate for this error by measuring the lead resistance R_L and accounting for it in the strain calculations. However, a more difficult problem arises from changes in the lead resistance due to temperature fluctuations. Given typical temperature coefficients for copper wire, a slight change in temperature can generate a measurement error of several μm .

Using a 3-wire connection can eliminate the effects of variable lead wire resistance because the lead resistances affect adjacent legs of the bridge. As seen in Figure 8b, changes in lead wire resistance, R_{L2} , do not change the ratio of the bridge legs R_3 and R_G . Therefore, any changes in resistance due to temperature cancel each other.

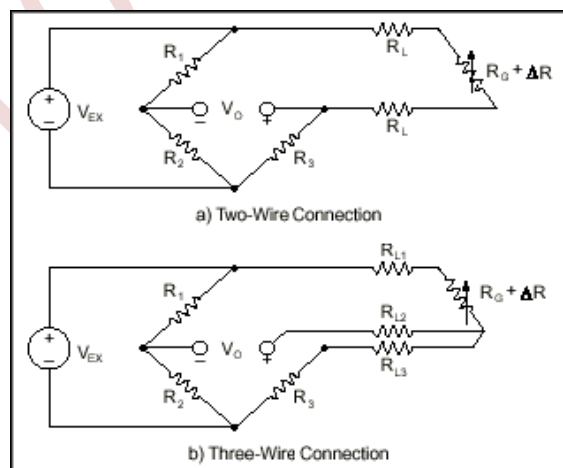


Figure 8. 2-Wire and 3-Wire Connections of Quarter-Bridge Circuit

Selection of a proper gauge:

Three primary considerations in strain gauge selection are mentioned below:

1. Operating temperature
2. Nature of the strain to be detected
3. Stability requirements

In addition, choosing the right carrier material, grid alloy, adhesive, and protective coating plays an important role in the success of the application.

Following are the chief characteristics of bonded resistance strain gauges:

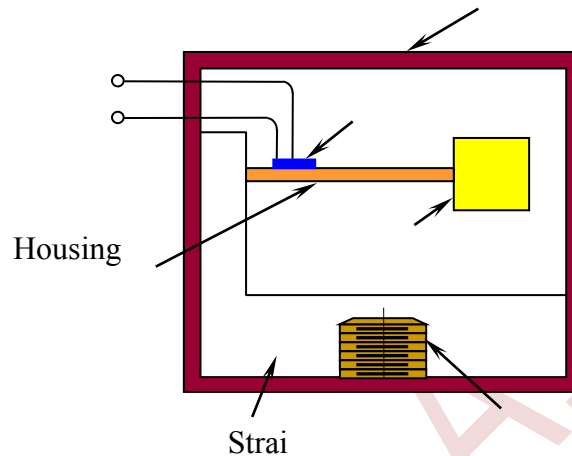
- They are reasonably inexpensive.
- They can pull off overall accuracy of better than $\pm 0.10\%$.
- They are available in a short gauge length and have small physical size.
- These strain gauges are only moderately affected by temperature changes.
- They are extremely sensitive and have low mass.
- Bonded resistance strain gages can be employed to measure both static and dynamic strain.
- These types of strain gauges are appropriate for a wide variety of environmental conditions. They can measure strain in jet engine turbines operating at very high temperatures and in cryogenic fluid applications at temperatures as low as $-452\text{ F } (-269^{\circ}\text{C})$.

Kinds of effects shape:

- Change in line length
- Change in angle
- Change in volume
- Change in Resistance

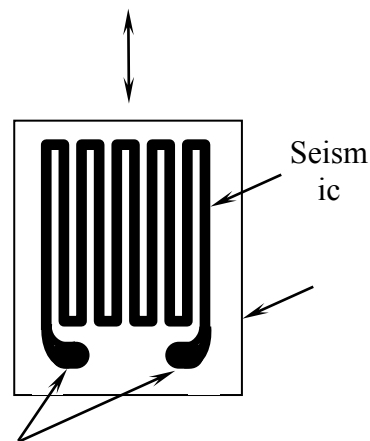
Using strain gauge to measuring pressure:

- The change in resistance is measured using an electrical circuit
- Many variables can be measured – displacement, acceleration, pressure, temperature, liquid level, stress, force and torque
- Some variables (stress, force, torque) can be determined by measuring the strain directly
- Other variables can be measured by converting the measure and into stress using a front-end device

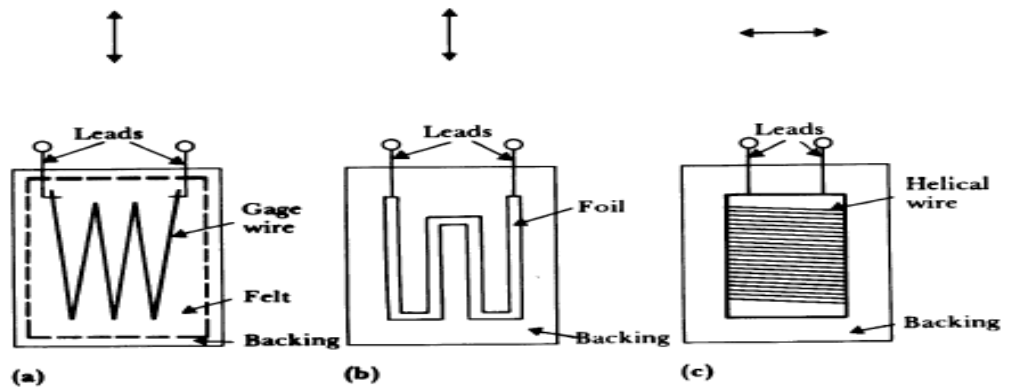
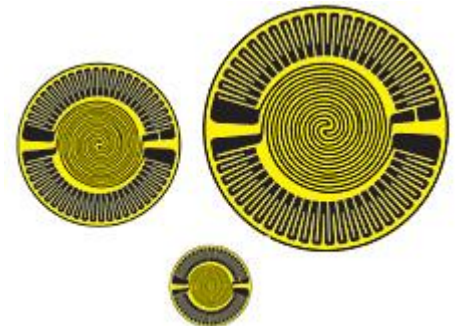
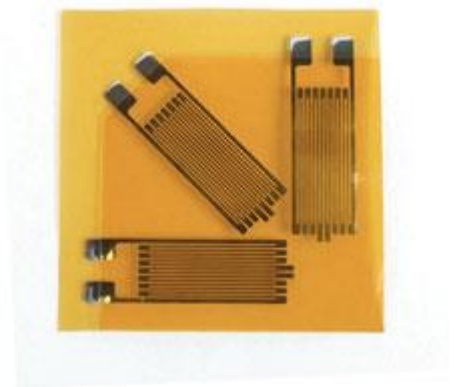
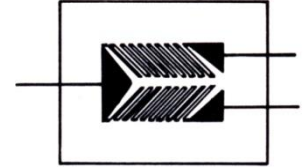
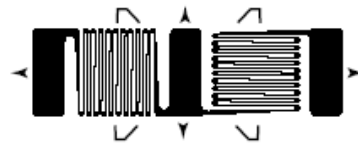
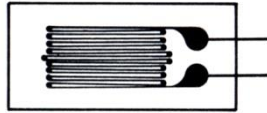


Strain gage accelerometer

Strain gages are manufactured as metallic foil (copper-nickel alloy – constantan)

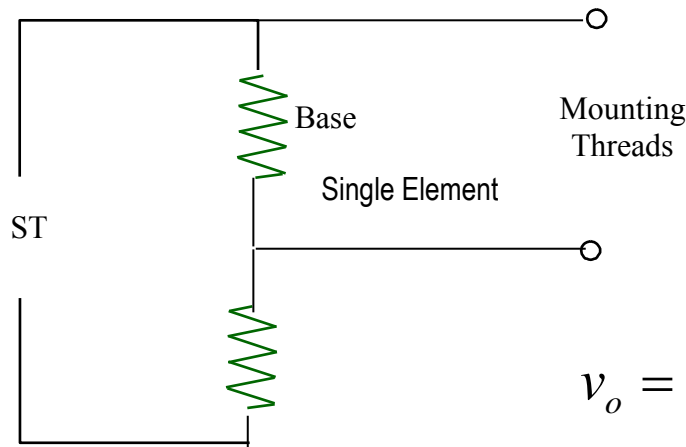


Types of strain gauges:



Types of connection for strain gauge:

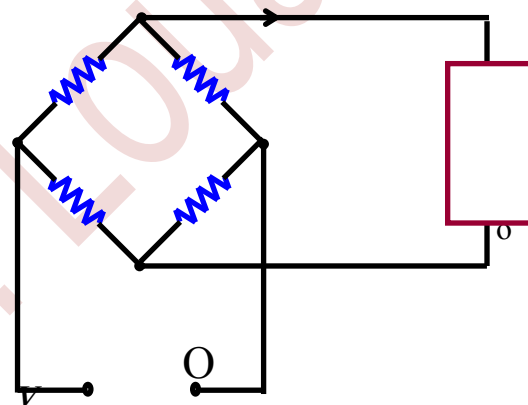
1- Potentiometer or Ballast Circuit:



$$v_o = \frac{R}{(R + R_c)} v_{\text{ref}}$$

- Ambient temperature changes will introduce error
- Variations in supply voltage will affect the output
- Electrical loading effect will be significant
- Change in voltage due to strain is a very small percentage of the output

2- Wheatstone Bridge Circuit:



$$v_o = \frac{R_1 v_{\text{ref}}}{(R_1 + R_2)} - \frac{R_3 v_{\text{ref}}}{(R_3 + R_4)} = \frac{(R_1 R_4 - R_2 R_3)}{(R_1 + R_2)(R_3 + R_4)} v_{\text{ref}}$$

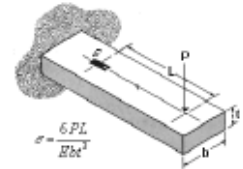
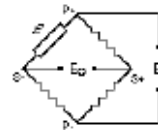
When the bridge is balanced:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Quarter Bridge Circuit - Bending

◆ Simplest

❖ Not recommended!

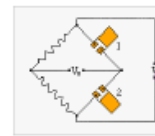
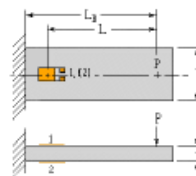


$$\frac{E_0}{E_1} = \frac{F \epsilon}{4 + 2F \epsilon}$$

Half-Bridge Circuit

◆ Bending

❖ Gages in adjacent legs

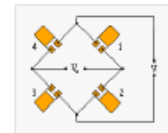
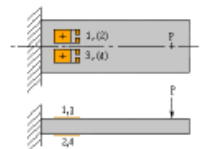


$$\frac{V_0}{V} = \frac{F \epsilon_1}{2} = \frac{F 3PL_b \left[2 - \frac{L}{L_b} \right]}{E b t^3}$$

Full-Bridge Circuit - Bending

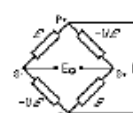
◆ Maximum sensitivity

◆ Temperature compensation

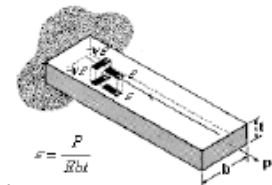


$$\frac{V_0}{V} = F \epsilon_1 = \frac{F 6PL_b \left[2 - \frac{L}{L_b} \right]}{E b t^3}$$

Full-Bridge Circuit – Axial Loading

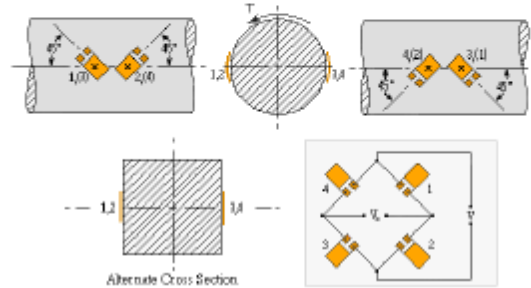


$$\frac{E_0}{E_1} = \frac{F \epsilon (1 + \nu)}{2 + F \epsilon (1 - \nu)}$$



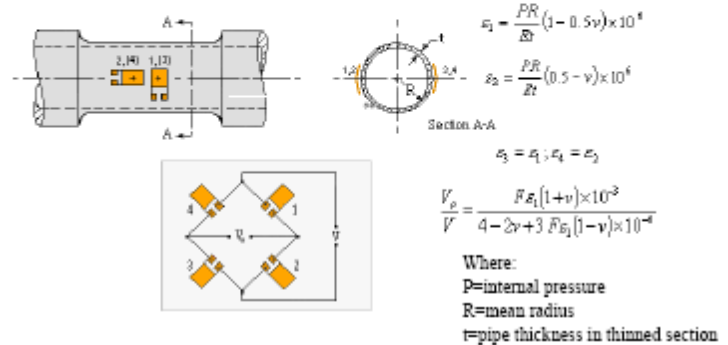
Torque Measurement with Strain Gages

◆ Torque measurement with strain gages



Pressure sensing with strain gages

◆ Pressure sensor using strain gages and thinned tube



CALIBRATION METHOD:

Null Balance Method:

- When the strain gage in the bridge deforms, the balance is upset.
- Balance is restored by changing a variable resistor
- The amount of change corresponds to the change in strain
- Time consuming – servo balancing can be used



MEASUREMENT OF SURFACE FINISH

Surface Roughness

"The irregularities in the surface texture which are inherent in the production process but excluding waviness and errors of form" British Standard 1134

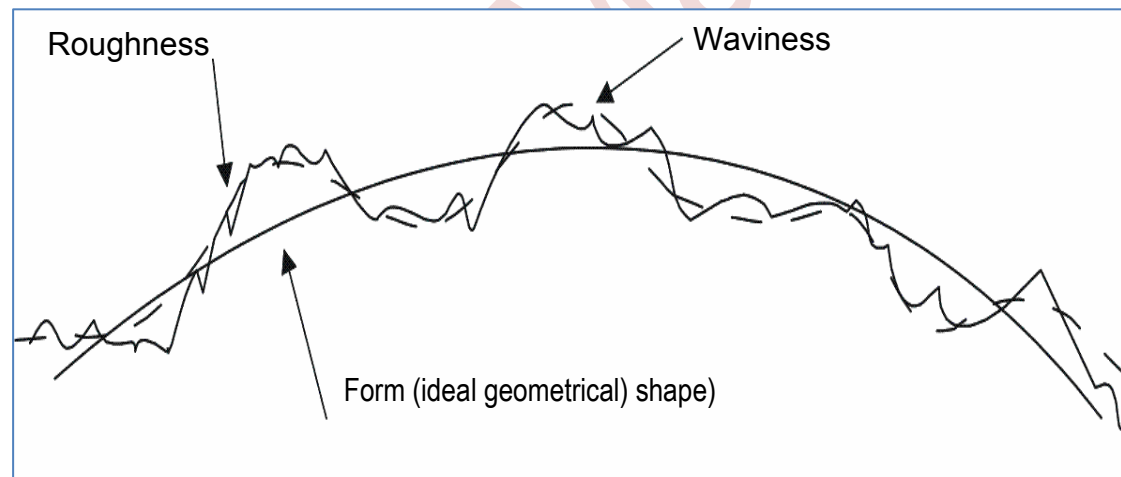
Surface properties:

Engineering surfaces never have an ideal geometrical shape, but instead include different deviations.

With regards to the level of approximation they can be considered:

- smooth and even,
- smooth and wavy,
- rough and even,
- rough and wavy.

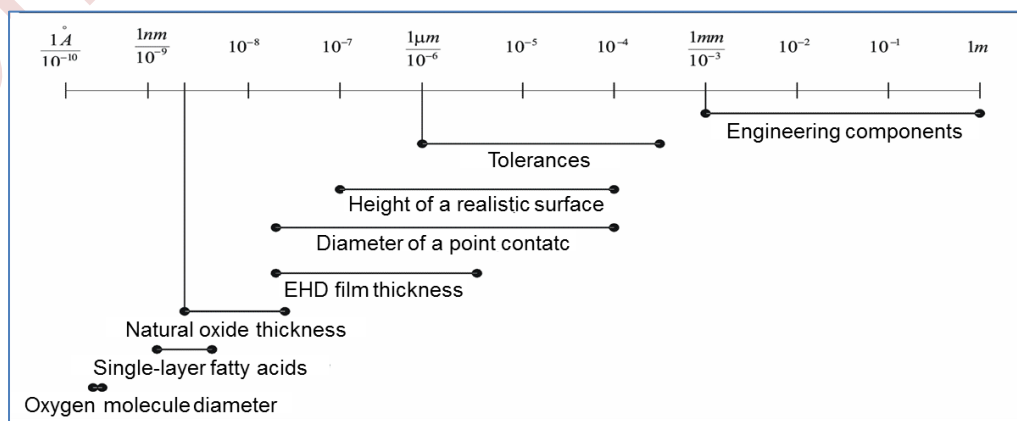
Surface
topography



Errors of form:

- Micro-geometrical deviations – roughness (*important for interaction of surfaces*)
- Macro-geometrical deviations – waviness

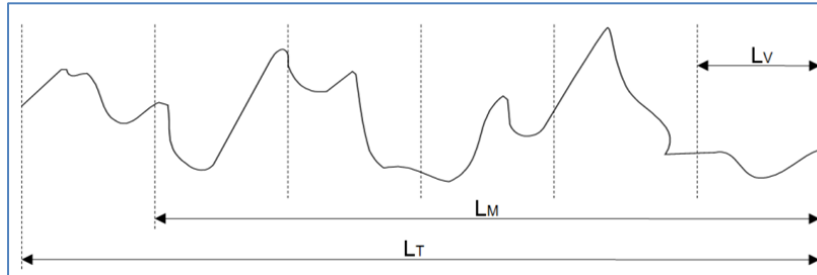
➤ What is „rough“?



Analysis of the measured surface roughness parameters:

Basic element:
surface profile.

$$(L_T = L_V + L_M = L_V + 5 \times L_V)$$



Traversing length is denoted with L_T and represents the distance that is traversed across the surface by the stylus when characterizing the surface, i.e. measurement length.

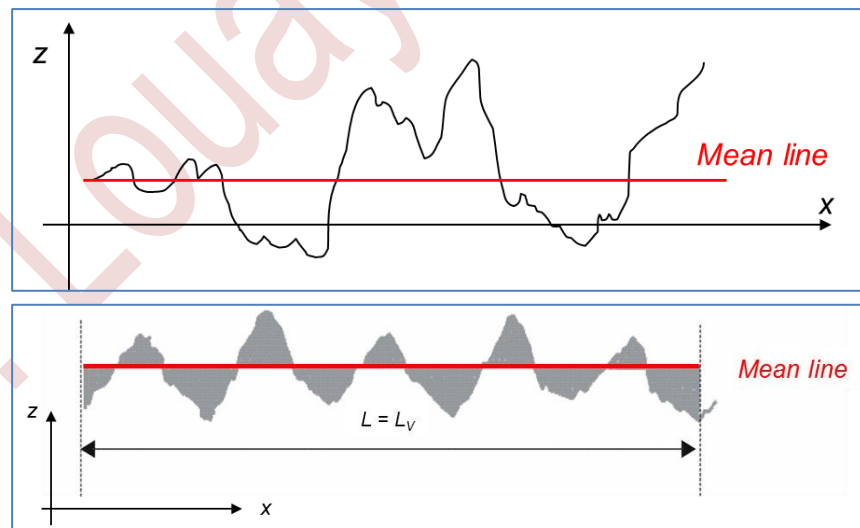
Assessment length L_M is the length over which surface data is acquired and assessed.

Sampling length (reference length) is denoted by L_V . It is a length of a section inside the assessment length and it is equivalent to wavelength of the filter, λ_c (it distinguishes the roughness from the waviness).

Standardized: important to choose the correct reference length and assessment length, so that the macro-geometrical deviations are excluded from the measurement.

Mean line of the profile is denoted by m . It is a line with a shape of geometrical profile (perfect geometric line) and it runs parallel to that profile.

The mean line of the profile is determined so that the sum of squared deviations from this line is the smallest.
...or otherwise: *Surface area above and below the mean line of the profile is the same!*



Average surface roughness produced by standard machining processes

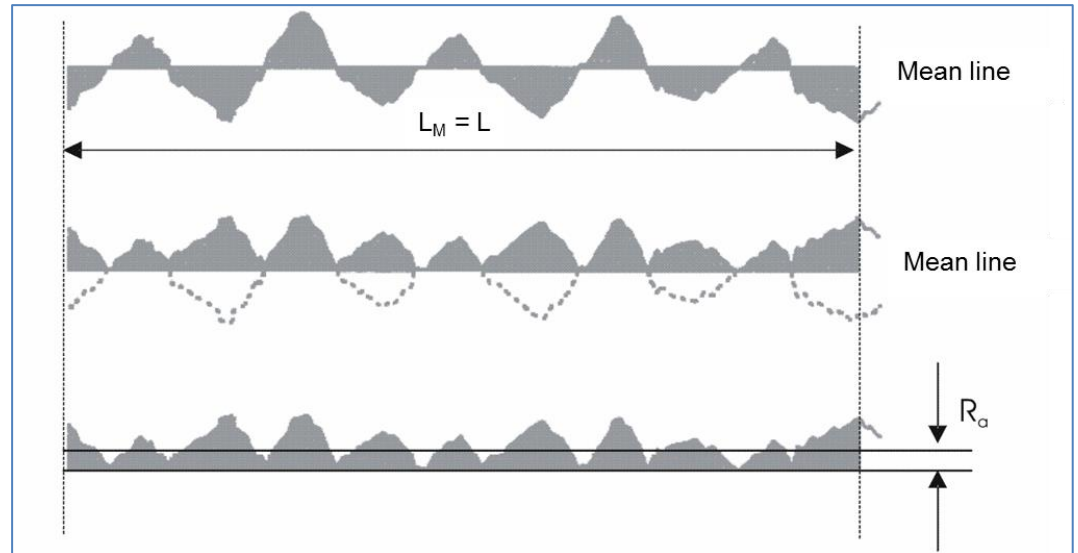
Planning (wood): 2,5 - 25 mm
Milling, drilling: 0,8 - 8 mm
Turning: 0,8 - 2,5 mm
Grinding: 0,25 - 2,5 mm
Honing, lapping: 0,25 mm

Arithmetical mean deviation, R_a :

The most widely recognized and used parameter for surface roughness characterization.

R_a is arithmetical mean deviation of all the measured values in the assessed profile (L_M) from the mean line of that profile.

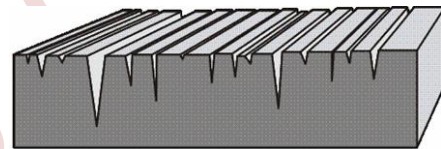
$$R_a = \frac{1}{l} \int_0^l |z(x)| dx$$



Arithmetical mean deviation of the assessed profile, R_a :

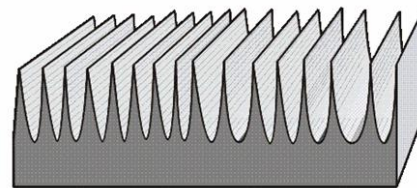
Awareging of data:

- R_a does not differentiate between profile peaks and valleys!!



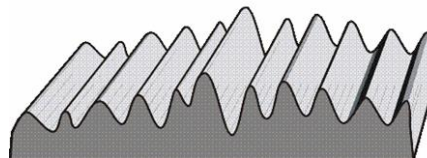
$$R_a = 2,4 \mu\text{m}$$

- R_a or any other parameter by itself: not sufficient.



$$R_a = 2,5 \mu\text{m}$$

- Additional parameters necessary: more sensitive & able to distinguish between surfaces with different shapes and/or ratios of peaks and valleys.



$$R_a = 2,4 \mu\text{m}$$

Dimensions and Tolerances:

- Factors that determine the performance of a manufactured product, other than mechanical and physical properties, include :
 - Dimensions - linear or angular sizes of a component specified on the part drawing
 - Tolerances - allowable variations from the specified part dimensions that are permitted in manufacturing

Dimensions (ANSI Y14.5M-1982):

A *dimension* is "a numerical value expressed in appropriate units of measure and indicated on a drawing and in other documents along with lines, symbols, and notes to define the size or geometric characteristic, or both, of a part or part feature"

- The dimension indicates the part size desired by the designer, if the part could be made with no errors or variations in the fabrication process

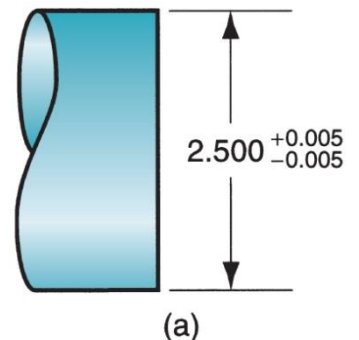
Tolerances (ANSI Y14.5M-1982):

A *tolerance* is "the total amount by which a specific dimension is permitted to vary. The tolerance is the difference between the maximum and minimum limits"

- Variations occur in any manufacturing process, which are manifested as variations in part size
- Tolerances are used to define the limits of the allowed variation.

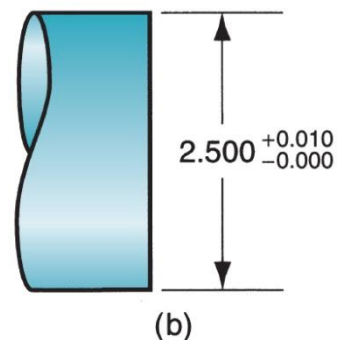
Bilateral Tolerance:

- Variation is permitted in both positive and negative directions from the nominal dimension
- Possible for a bilateral tolerance to be unbalanced
 - Ex: $2.500 +0.010, -0.005$



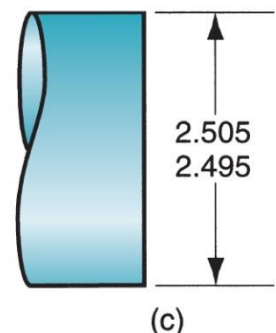
Unilateral Tolerance:

- Variation from the specified dimension is permitted in only one direction
- Either positive or negative, but not both



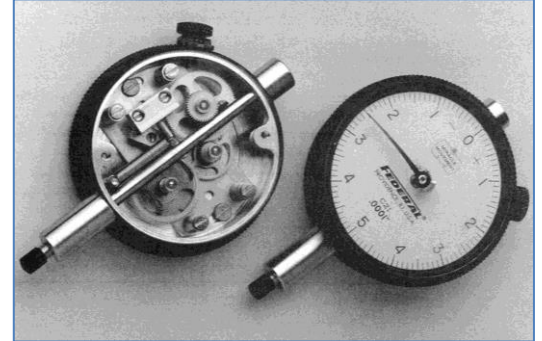
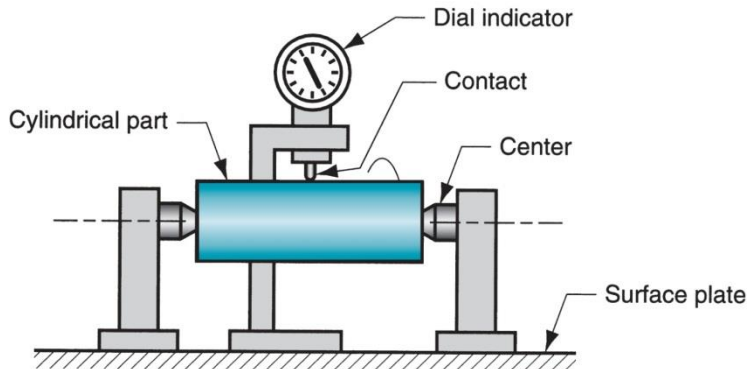
Limit Dimensions:

- Permissible variation in a part feature size consists of the maximum and minimum dimensions allowed



Dial Indicator:

- Front view shows dial and graduated face; back view shows cover plate removed.
- As part is rotated about its center, variations in outside surface relative to center are indicated on the dial



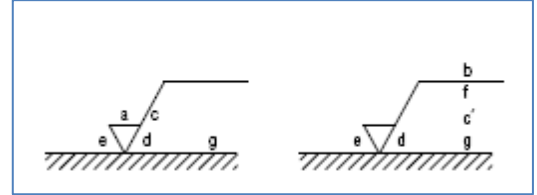
Four Elements of Surface Texture:

1. Roughness - small, finely-spaced deviations from nominal surface
 - Determined by material characteristics and processes that formed the surface
2. Waviness - deviations of much larger spacing
 - Waviness deviations occur due to work deflection, vibration, tooling, and similar factors
 - Roughness is superimposed on waviness
3. Lay - predominant direction or pattern of the surface texture
4. Flaws - irregularities that occur occasionally on the surface
 - Includes cracks, scratches, inclusions, and similar defects in the surface
 - Although some flaws relate to surface texture, they also affect surface integrity

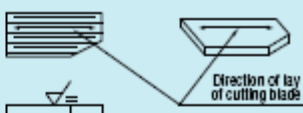
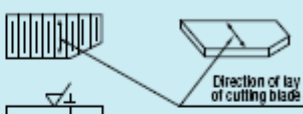
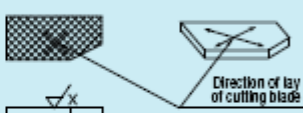
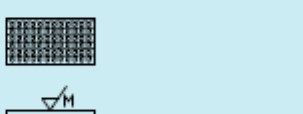
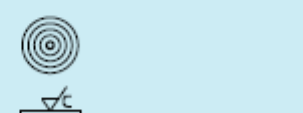
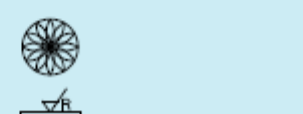
Positions of respective indicating symbols relative to indicating symbol of surface:

Each grain surface position is indicated as shown in Fig. 1. This includes surface roughness, cut-off value or reference length, processing method, symbol of direction of lay, surface waviness, etc.

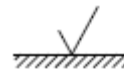
- a: Value of R_a
- b: Processing method
- c: Cutoff value. Evaluation length
- c': Reference length. Evaluation length
- d: Symbol of direction of lay
- f: Parameter other than R_a (With t_p , parameter/cutoff level)
- g: Surface waviness (according to JIS B 0610)



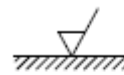
Reference: The location of lay of e in Fig. 1 is given as the finish allowance in ISO 1302.

Symbol	Meaning	Figure
=	Parallel to the projected surface on which the direction of lay of the cutting blade is indicated. (ex) Shaped surface	
⊥	Perpendicular to the projected surface on which the direction of lay of the cutting blade is indicated. (ex) Shaped surface (when viewed from the side), machined or cylindrical ground surface.	
X	Intersection of two diagonal lines on the projected surface on which the direction of lay of the cutting blade is indicated. (ex) Honing finished surface	
M	Multidirectional intersection or non-directional point on the projected surface on which the direction of lay of the cutting blade is indicated. (ex) Rapping finished surface, super finished surface, face milled or end milled surface in surfacing feed direction	
C	Concentric circles roughly centered on the same on the surface on which the direction of lay of the cutting blade is indicated. (ex) Facing surface	
R	Radiating shape roughly centered on the same point on the surface on which the direction of lay of the cutting blade is indicated.	

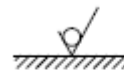
Indicating symbol of surface



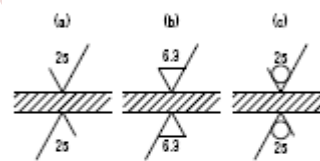
Indicating symbol of surface requiring removal press



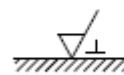
Indicating symbol of surface on which no removal process is permitted



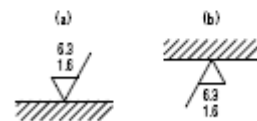
Examples indicating the upper limits of R_a



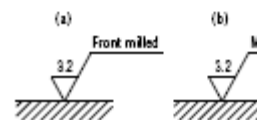
Examples indicating direction of lay



Examples indicating the upper limit and lower limit of R_a



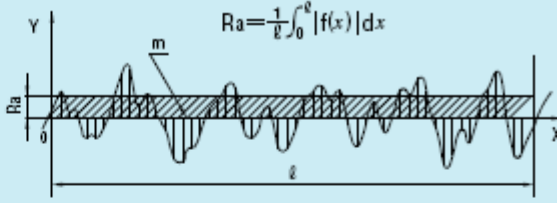
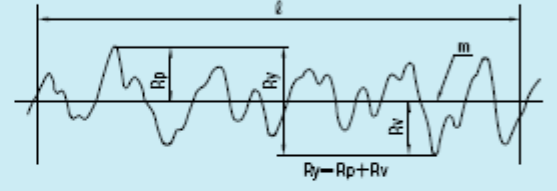
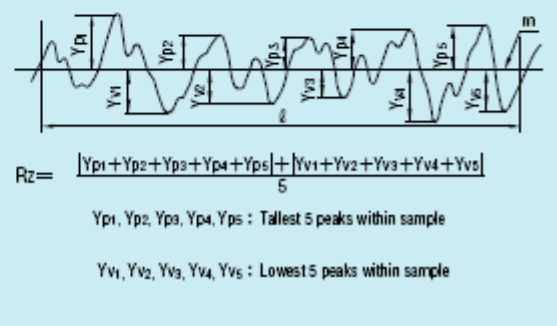
Examples indicating processing method



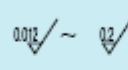

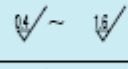

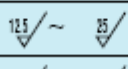

Categories of surface roughness:

Definitions and indications for surface roughness parameters (for industrial products) are specified. They are arithmetical mean roughness (R_a), maximum height (R_y), ten-point mean roughness (R_z), mean spacing of profile irregularities (S_m), mean spacing of local peaks of the profile (S) and profile bearing length ratio (t_p). Surface roughness is given as the arithmetical mean value for a randomly sampled area. [Mean center line roughness (R_a) is defined in the annexes of JIS B 0031 and JIS B 0061].

Typical ways for obtaining surface roughness:

<p>Arithmetical mean roughness (R_a)</p> <p>A section of standard length is sampled from the mean line on the roughness chart. The mean line is laid on a Cartesian coordinate system wherein the mean line runs in the direction of the x-axis and magnification is the y-axis. The value obtained with the formula on the right is expressed in micrometer (μm) when $y=f(x)$.</p>	
<p>Maximum peak (R_y)</p> <p>A section of standard length is sampled from the mean line on the roughness chart. The distance between the peaks and valleys of the sampled line is measured in the y direction. The value is expressed in micrometer (μm).</p> <p>Note: To obtain R_y, sample only the standard length. The part, where peaks and valleys are wide enough to be interpreted as scratches, should be avoided.</p>	
<p>Ten-point mean roughness (R_z)</p> <p>A section of standard length is sampled from the mean line on the roughness chart. The distance between the peaks and valleys of the sampled line is measured in the y direction.</p> <p>Then, the average peak is obtained among 5 tallest peaks (Y_p), as is the average valley between 5 lowest valleys (Y_v).</p> <p>The sum of these two values is expressed in micrometer (μm).</p>	 <p>$R_z = \frac{Y_{p1} + Y_{p2} + Y_{p3} + Y_{p4} + Y_{p5} + Y_{v1} + Y_{v2} + Y_{v3} + Y_{v4} + Y_{v5}}{5}$</p> <p>$Y_{p1}, Y_{p2}, Y_{p3}, Y_{p4}, Y_{p5}$: Tallest 5 peaks within sample</p> <p>$Y_{v1}, Y_{v2}, Y_{v3}, Y_{v4}, Y_{v5}$: Lowest 5 peaks within sample</p>

Reference: Relationship between arithmetical mean roughness (R_a) and conventional symbols:

Arithmetical mean roughness Ra			Max. height Ry	Ten-point mean roughness Rz	Standard length of Ry · Rz ℓ (mm)	Triangular Indication
Preferred number series	Cut-off value λ (mm)	Indication of surface texture on drawings	Preferred number series			
0.012 a	0.08		0.05 s	0.05 z	0.08	
0.025 a	0.25		0.1 s	0.1 z	0.25	
0.05 a			0.2 s	0.2 z		
0.1 a			0.4 s	0.4 z		
0.2 a	0.8		0.8 s	0.8 z	0.8	
0.4 a			1.6 s	1.6 z		
0.8 a			3.2 s	3.2 z		
1.6 a			6.3 s	6.3 z		
3.2 a	0.25		12.5 s	12.5 z	0.25	
6.3 a			25 s	25 z		
12.5 a	8		50 s	50 z	8	
25 a			100 s	100 z		
50 a	—		200 s	200 z	—	
100 a			400 s	400 z		

The evaluation length of R_a , R_y and R_z : Five times the cut-off value standard length respectively.

Symbols for Geometrical Characteristics:

Tolerances	Characteristics	Symbol
Form	Straightness	—
	Flatness	▭
	Roundness	○
	Cylindricity	⊘
	Profile any line	⌒
	Profile any surface	⌒
Orientation	Parallelism	//
	Perpendicularity	⊥
	Angularity	∠
	Profile any line	⌒
	Profile any surface	⌒
Location	Position	⊕
	Concentricity (for centre points)	⊙
	Coaxiality (for axes)	⊙
	Symmetry	≡
	Profile any line	⌒
	Profile any surface	⌒
Run-out	Circular run-out	↗
	Total run-out	↗

Methods of measuring surface finish:

1. SURFACE INSPECTION BY COMPARISON MEHODS: these include:

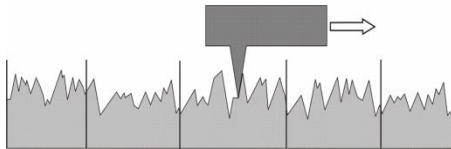
- i. Touch inspection
- ii. Visual inspection
- iii. Scratch inspection
- iv. Microscopic inspection
- v. Surface photographs
- vi. Micro interferometer
- vii. Wallace surface dynamometer
- viii. Reflected light intensity
- ix. Comparison by standard specimens

2. Direct Instrument Measurement:

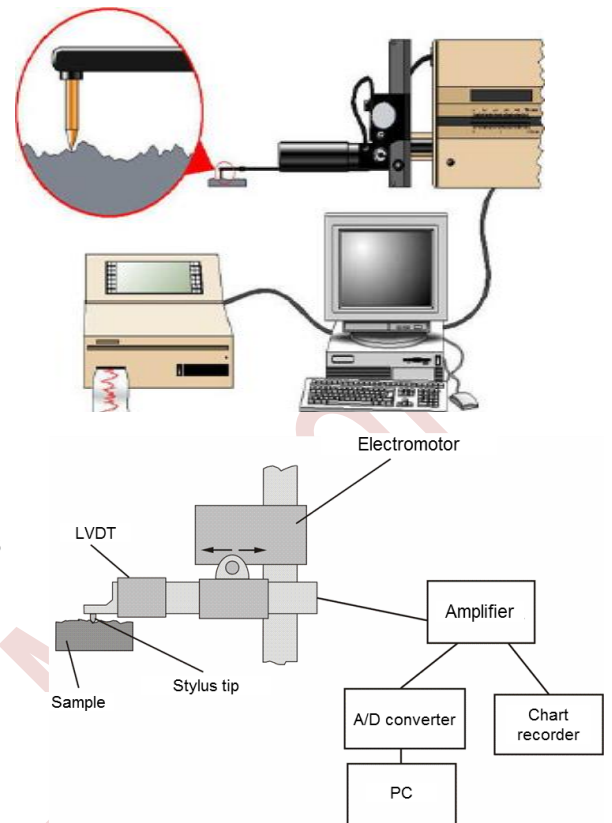
- i. Intersection method
- ii. Interference method
- iii. Stylus method

Stylus Instrument Surface scan :

- Maximal measuring area 100x100 mm
- Vertical range 1 mm
- Horizontal resolution 2µm
- Vertical resolution 8 nm



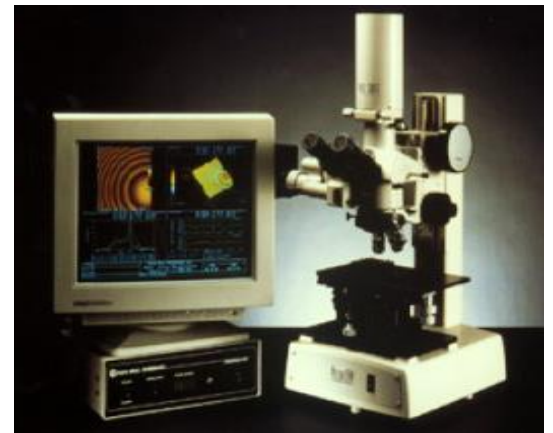
2D Profilo-meter :The resolution depends on the stylus tip and the velocity of scanning.



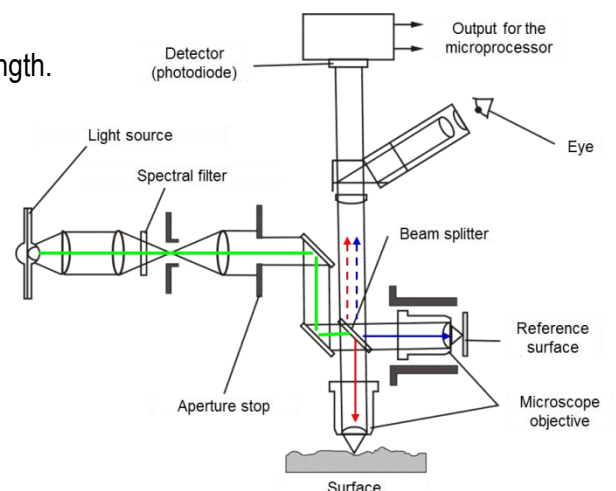
Optical Instruments(Optical interferometer)

PST Micro-Xam Interferometer:

- Maximal measuring area 1.3x1.0 mm
- Vertical range 5 mm
- Horizontal resolution 0.3 µm
- Vertical resolution 0.05 nm



3D:The resolution depends on optics and light wavelength.



Scanning Probe Microscopy SPM

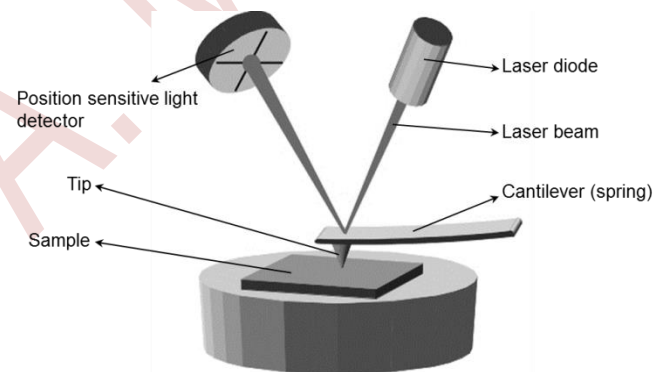
Primary forms

- I. Scanning Tunneling Microscopy STM
- II. Atomic Force Microscopy AFM

AFM Operation Modes:

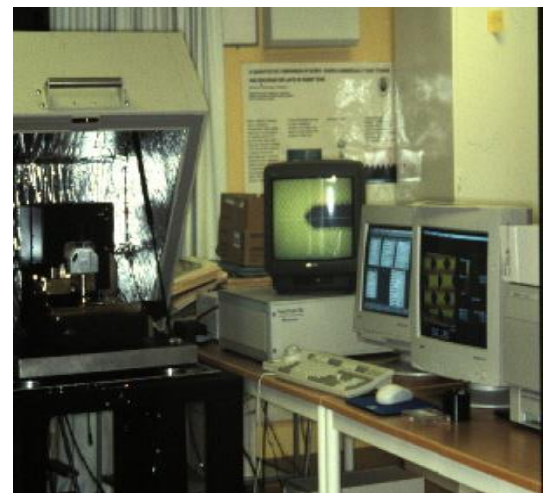
- Contact
- Tapping
- Non-Contact

3D: The resolution depends on the laser, scanner, feedback loop, software, probe (tip)...



Dimension 3000 SPM:

- Maximal measuring area $100 \times 100 \mu\text{m}$
- Vertical range $6 \mu\text{m}$
- Horizontal resolution 100 pm
- Vertical resolution 10 pm



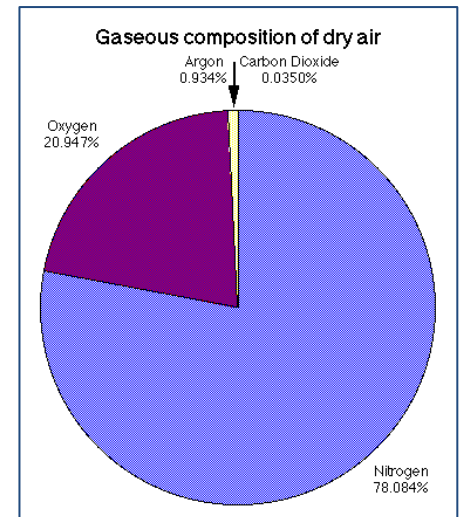
AIR POLLUTION



COMPOSITON OF AIR:

- 78% Nitrogen (N_2)
- 21% Oxygen (O_2)
- 0.9% Argon
- 0.035% (CO_2)

- **POLLUTION:** Anything that negatively affects the health, survival, or activities of humans or other living organisms. Or any visible or invisible particle or gas found in the air that is not part of the original, normal composition.



EFFECTS OF AIR POLLUTION:

Human Health acute:

- 1- short duration exposure and/or immediate effects

Examples:

Irritation of eyes, nose and throat

Upper respiratory infections (bronchitis, pneumonia)

Headaches

Nausea

Allergic reactions, etc.

- 2- chronic: long duration exposure and/or long term effects

Examples:

Lung cancer

Heart disease

Damage to brain, nerves, liver, kidneys, etc.

- 3- Aesthetic

- 4- Damage to organisms

- 5- Damage to ecosystems

- 6- Damage to property: Effects of dry deposition of sulfur dioxide, which causes the formation of gypsum.

Gypsum traps particulate matter to form heavy, black incrustation.

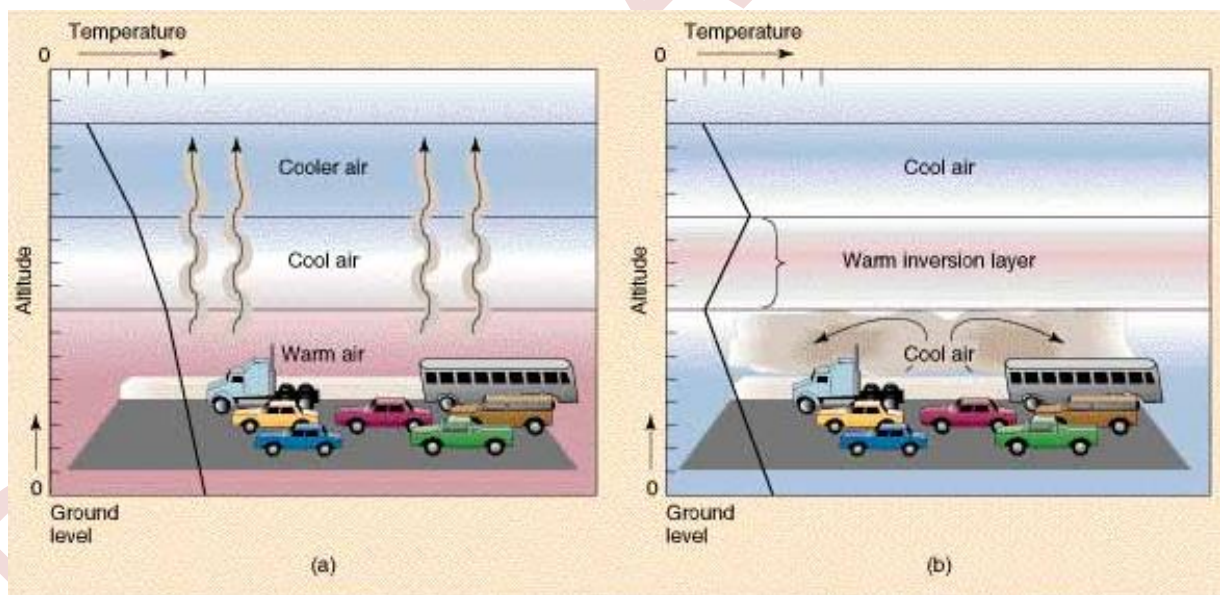
FACTORS INFLUENCING EFFECTS:

- **Chemical Nature**
 - How active and harmful
- **Concentration**
 - See Temperature Inversions
- **Persistence**
 - How long pollutant stays around

TEMPERATURE INVERSION:

- **Normally air temp decreases with increasing altitude.**
 - Ground heats up and heats air above it which rises, expands, and cools.
 - This rising air carries pollutants up and away from where humans breathe and dilute the pollutants in more air space.

Temperature Inversion: occurs at ground level when cool air is created under or slips under relatively warmer air just above it.



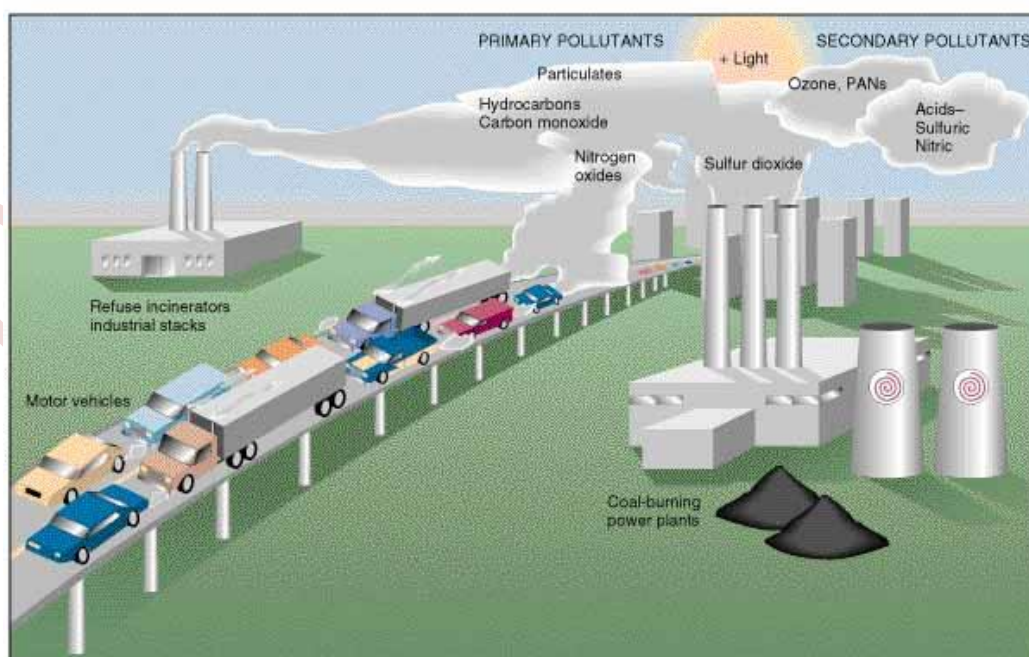
- **Temperature Inversions break when:**
 - Sun comes out and heats ground which heats air above ground to warmer than layer above it.
 - During cloudy weather, sun may not be strong enough to break up inversion for several hours or days.

- Temperature Inversions are bad because
 - Air pollution concentrates in this layer that we breathe.
 - In 1963, 300 people in NY City died due to temp inversion.

- Temperature Inversions
 - Occur almost every night
 - Occur more dramatically in cities near mountains
 - This is why we have smokestacks

SOURCES OF AIR POLLUTION:

- Natural sources include: Volcanoes, fires, dust storms...
- Human (Anthropogenic): Stationary vs. Mobile
 - Stationary: e.g., power plants
 - Mobile: e.g., transportation
- Primary vs. Secondary
 - Primary pollutants: enter air directly as pollutants – direct products of combustion and evaporation.
 - Secondary pollutants: primary pollutants that undergo further reactions in atmosphere to produce additional undesirable compounds.



TYPES OF AIR POLLUTANTS:

■ Criteria Pollutants

- Clean Air Act mandates NAAQS--national ambient air quality standards (max concentrations that can be in the air).
- SPLONC = SO₂, Particulate Matter, Lead, O₃, NO₂, CO

NAAQS: these are primary standards intended to protect human health

TABLE 9.8 National Ambient Air Quality Standards (NAAQS)

POLLUTANT	PRIMARY (HEALTH-BASED) AVERAGING TIME	STANDARD CONCENTRATION
Particulates	Annual geometric mean ^a	50 µg/m ³
	24 hours	150 µg/m ³
SO ₂	Annual arithmetic mean ^b	80 µg/m ³ (0.03 ppm)
	24 hours	120 µg/m ³ (0.14 ppm)
CO	8 hours	10 mg/m ³ (9 ppm)
	1 hour	40 mg/m ³ (35 ppm)
NO ₂	Annual arithmetic mean	80 µg/m ³ (0.05 ppm)
O ₃	Daily maximum 1-hour average	235 µg/m ³ (0.12 ppm)
Lead	Maximum quarterly average	1.5 µg/m ³

^aThe geometric mean is obtained by taking the n th root of the product of n numbers. This tends to reduce the impact of a few very large numbers in a set.

^bAn arithmetic mean is the average determined by dividing the sum of a group of data points by the number of points.

Pollutant	Primary Stds.	Averaging Times	Secondary Stds.
Carbon Monoxide	9 ppm (10 mg/m ³)	8-hour(1)	None
	35 ppm (40 mg/m ³)	1-hour(1)	None
Lead	1.5 µg/m ³	Quarterly Average	Same as Primary
Nitrogen Dioxide	0.053 ppm (100 µg/m ³)	Annual (Arithmetic Mean)	Same as Primary
Particulate Matter (PM ₁₀)	Revoked(2)	Annual(2) (Arith. Mean)	
	150 µg/m ³	24 hour(3)	
Particulate Matter (PM _{2.5})	15.0 µg/m ³	Annual(4) (Arith. Mean)	
	35 µg/m ³	24-hour(5)	Same as Primary
Ozone	0.08 ppm	8-hour(6)	Same as Primary
	0.12 ppm *	1-hour(7)	Same as Primary
Sulfur Oxides	0.03 ppm	Annual (Arith. Mean)	-----
	0.14 ppm	24-hour(1)	-----
	-----	3-hour(1)	0.5 ppm (1300 µg/m ³)

▪ Aerosols

- Particulates solid phase
 - Dust
 - Ash
 - Fumes
- Solid and liquid
 - Smoke (from combustion)
 - Coastal aerosols
- Liquid
- Aggregate gases (sulfate, nitrate)

▪ Gases

- CO_x
- SO_x
- NO_x
- PAH

SULFUR DIOXIDE: SO₂

- Sources: mostly stationary fuel combustion (esp. coal power plants)
- Main Effects:
 - Acid Deposition
 - Corrosive
 - ▼ Damages lungs
 - ▼ Damages structures
 - ▼ Damages environment

PARTICULATE MATTER:

- Examples: dust, soot, lead, arsenic
- Sources: industry (38%) and stationary (25%) and mobile (21%) fuel combustion
- Main Effects: depends on pollutant
 - Usually decreases lung function

LEAD: Sources: paints and smelting plants

- Main Effects: Affects brain and nervous system

OZONE: O₃: Sources: secondary pollutant from nitrogen dioxide, hydrocarbons, sunlight

- Mostly transportation; also stationary fuel combustion

Main Effects:

- Damages lungs; irritates eyes
- Damages plants
- Damages structures

NITROGEN DIOXIDE: NO₂: Sources: secondary pollutant from nitrogen oxide

- Mostly fuel combustion (stationary and transportation)

Main Effects:

- Acid Deposition
- Forms ozone
- Damages lungs
- Produces brown haze in air

CARBON MONOXIDE: CO: Sources: mostly transportation

Main Effects:

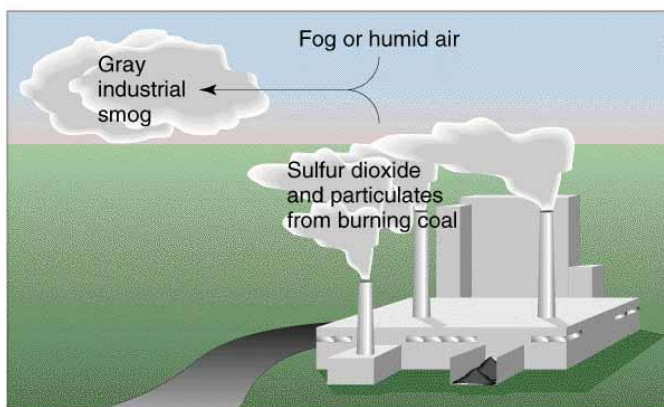
- Reduces blood's capacity to carry oxygen (headaches and worse)
- Forms ozone

TYPES OF AIR POLLUTANTS:

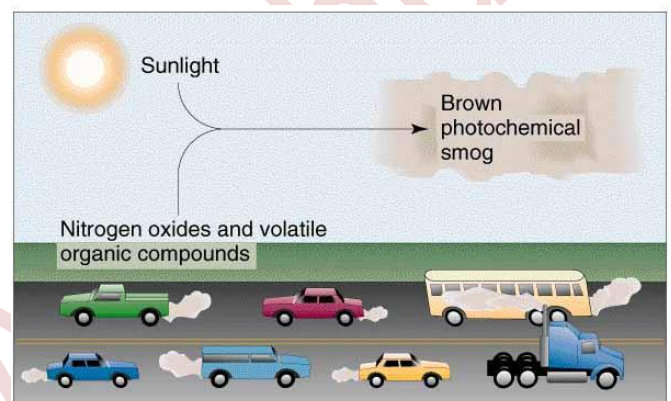
- Non criteria Pollutants
 - Clean Air Act mandates emission standards (how much can come out of the smokestack)
 - Examples: arsenic, asbestos, mercury, radioactive isotopes.

THERE ARE TWO KINDS OF SMOG:

- Industrial smog (gray smog): occurs where coal is burned and atmosphere is humid.
- Photochemical smog (brown smog): occurs where sunlight acts on vehicle pollutants.



(a) Industrial smog



(b) Photochemical smog

- **Industrial Smog:** Burning coal
 - Sulfur dioxide, sulfuric acid, suspended particles

Developed versus developing countries

- Air pollution control in the U.S. and Europe
- China, India, Ukraine, Eastern Europe

Photochemical Smog:

- Photochemical reactions
- Photochemical smog
 - Brown-air smog
 - Sources
- Climate effects
- Urban areas

WAYS TO REDUCE AIR POLLUTION:

- **DRIVE LESS** → carpool, walk or ride a bike, shop by phone or mail, ride public transit, telecommute.
- **DRIVE SMART** → obey the speed limit, combine all errands in one trip, use cruise control, keep car tuned, don't top off at the pump, replace car's air filter, keep tires properly inflated, buy clean cars.
- **BUY AIR-FRIENDLY PRODUCTS** → buy products that are water-based or are low in VOCs, buy water-based paints, paint with a brush instead of a sprayer, use a push or electric lawn mower, use propane or gas barbecue.
- **SAVE ENERGY** → turn off lights when you leave a room, use fluorescent lighting, use a programmable thermostat, insulate your home, use a fan instead of an air-conditioner, install low-flow shower heads.
- **WASTE NOT** → choose recycled products, choose products with recycled packaging, print or Xerox on both sides of the paper, reuse paper bags, recycle papers, plastics and metals.
- **DON'T CREATE DUST** → don't use fireplace on days with unhealthy air, use rake instead of leaf blower, and drive slowly on dirt roads.

Natural Factors That Reduce Air Pollution:

- Particles heavier than air
- Rain and snow
- Salty sea spray from oceans
- Winds
- Chemical reactions

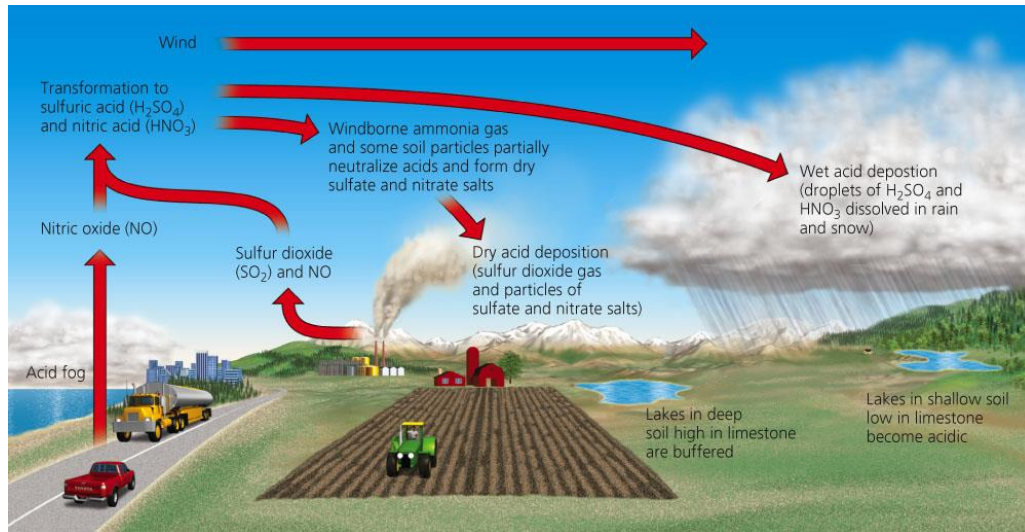
Natural Factors That Increase Air Pollution (1):

- Urban buildings
- Hills and mountains
- High temperatures
- VOC emissions from certain trees and plants
- Grasshopper effect
- Temperature inversions

Acid Deposition:

- Sulfur dioxides and nitrogen oxides
- Wet and dry deposition
- Acid rain
- Regional air pollution
 - Midwest coal-burning power plants

- Prevailing winds



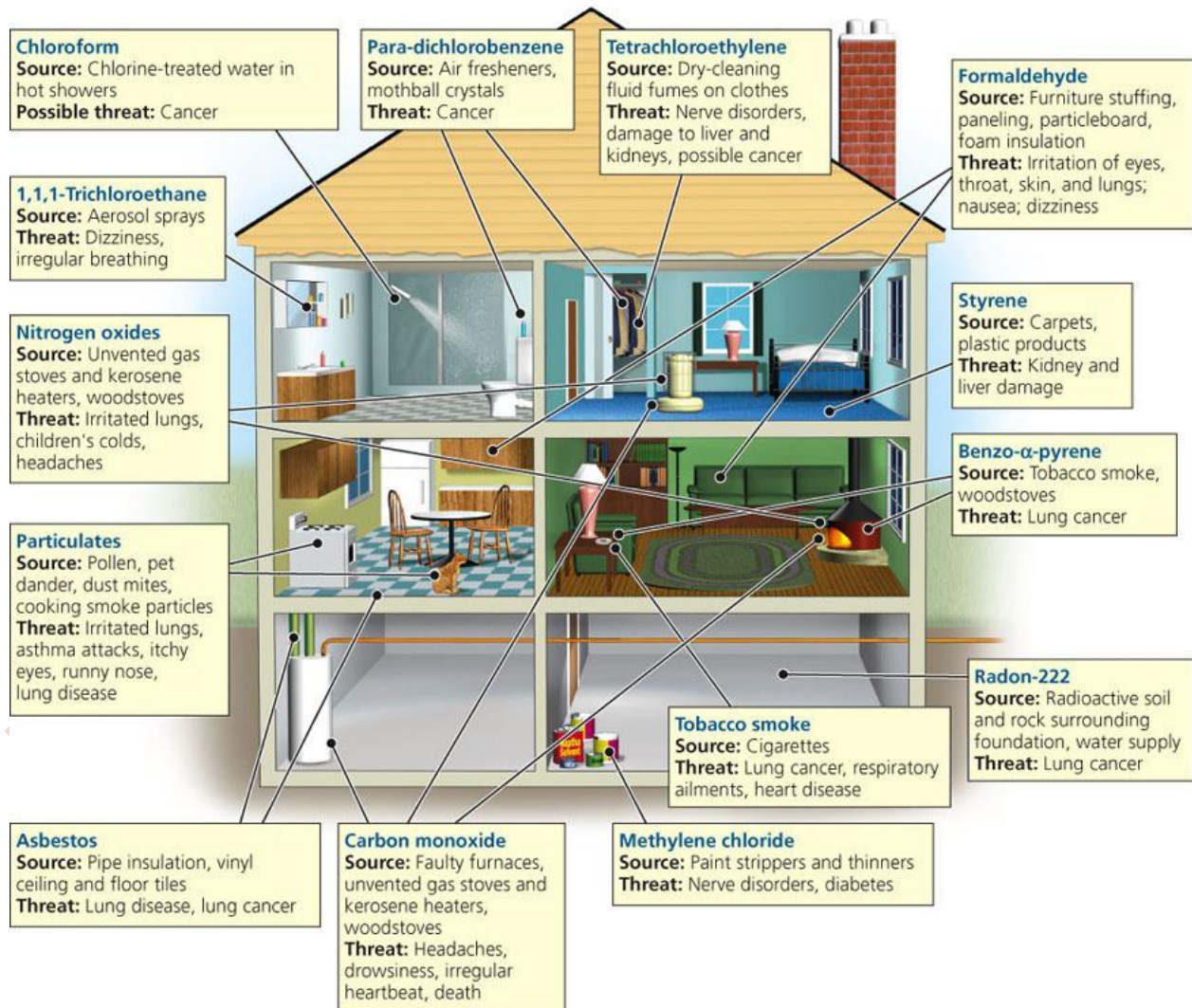
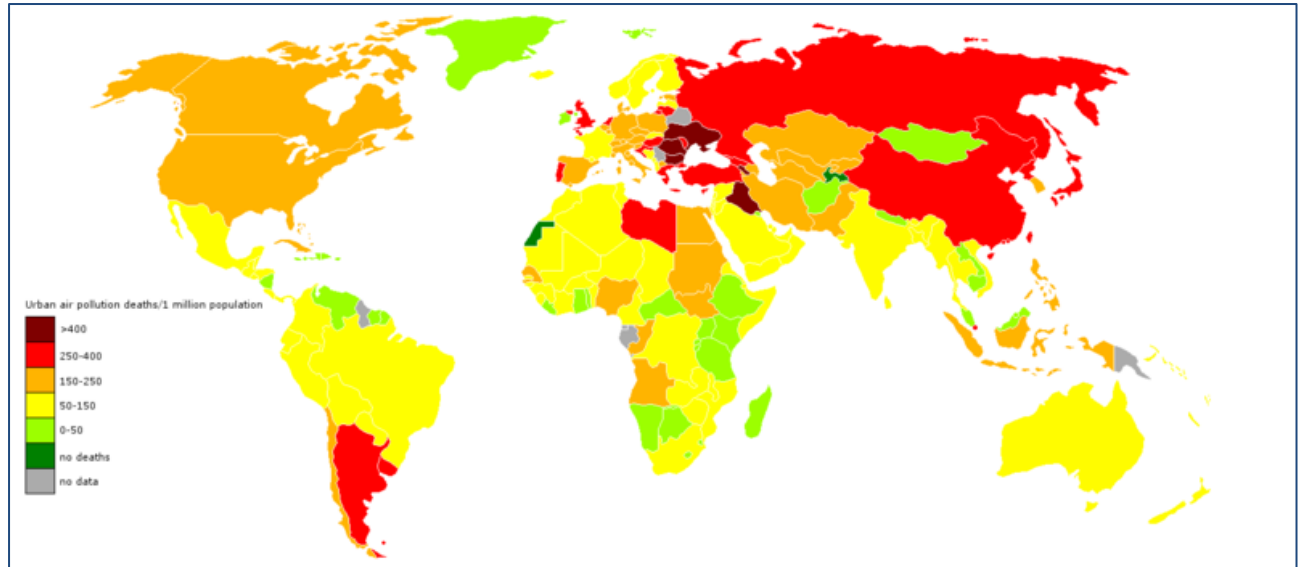
Harmful Effects of Acid Deposition (1):

- Respiratory diseases in humans
- Toxic metal leaching
- Structural damage
- Kills fish and other aquatic organisms
- Leaches plant nutrients from soil
- Acid clouds and fog at mountaintops

Indoor Air Pollution:

- Often higher concentration in buildings and cars
- Most time is spent indoors or in cars
- EPA – top cancer risk
- Sick-building syndrome (SBS)
- Developing countries
 - Indoor cooking and heating

Air Quality	Air Quality Index	Protect Your Health
Good	0-50	No health impacts are expected when air quality is in this range.
Moderate	51-100	Unusually sensitive people should consider limiting prolonged outdoor exertion.
Unhealthy for Sensitive Groups	101-150	Active children and adults, and people with respiratory disease, such as asthma, should limit prolonged outdoor exertion.
Unhealthy	151-200	Active children and adults, and people with respiratory disease, such as asthma, should limit prolonged outdoor exertion, everyone else, especially children should limit prolonged outdoor exertion.
Very Unhealthy (Alert)	201-300	Active children and adults, and people with respiratory disease, such as asthma, should limit prolonged outdoor exertion everyone else, especially children, should limit outdoor exertion.



Former Uses of CFCs:

- Coolants in air conditioners and refrigerators
- Propellants in aerosol cans
- Cleaning solutions for electronic parts
- Fumigants
- Bubbles in plastic packing foam

Ozone Thinning:

- Seasonal changes
- More severe over Antarctica than the Arctic
- Consequences

Factors that affect air pollution:

- Emissions (traffic, industrial, domestic)
- Geography (terrain)
- Weather conditions (rain, winds, humidity)
- Season
- Time of day
- Population density
- Indoor vs outdoor

Indoor pollutants:

- Non-specific symptoms
- Household vs work space
- Sick building syndrome (20% exposed)
 - Cigarette smoke, combustion products
 - Organic offgasing (glue, fabrics, furnishings)
 - Biological agents (infections, allergens)
 - Additional factors (stress, fatigue, diet, alcohol)

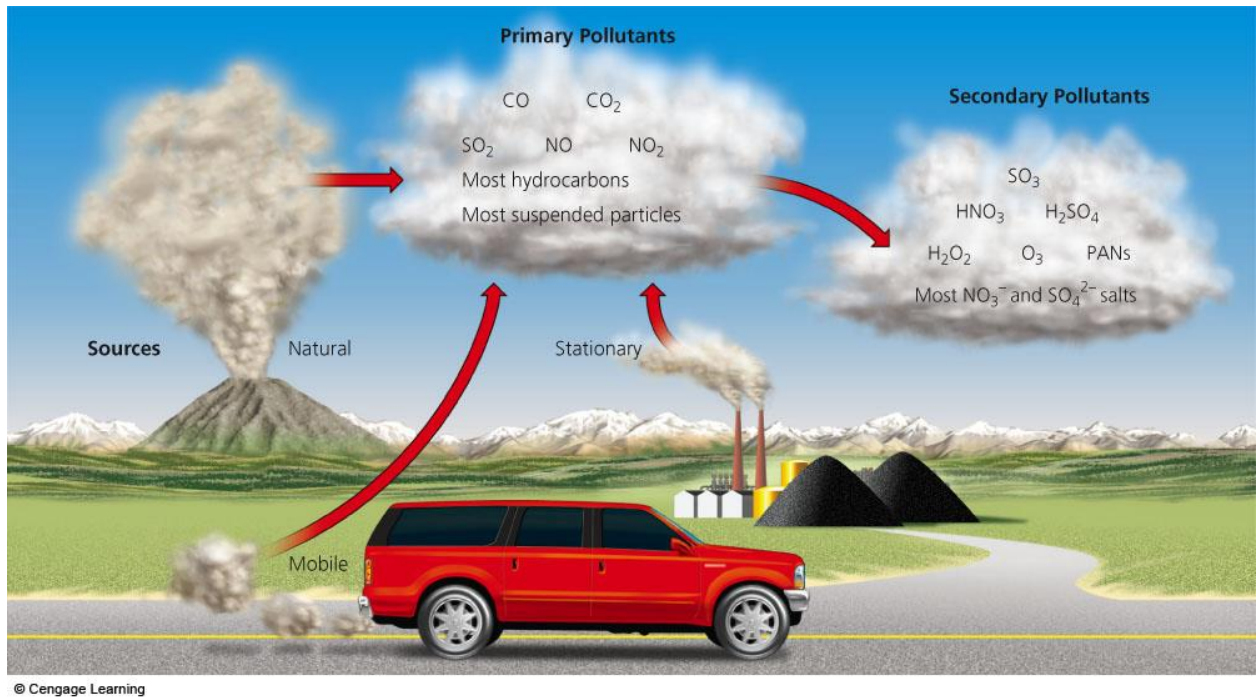
Types of Major Air Pollutants (1)

- Carbon oxides (CO)
- Nitrogen oxides and nitric acid (NO, HNO₃)
- Sulfur dioxide and sulfuric acid (SO₂, H₂SO₄)
- Particulates (SPM)
- Ozone (O₃)

Types of Major Air Pollutants (2):

- Volatile organic compounds (VOCs)
- Radioactive radon (Rn)

Sources and Types of Air Pollutants:



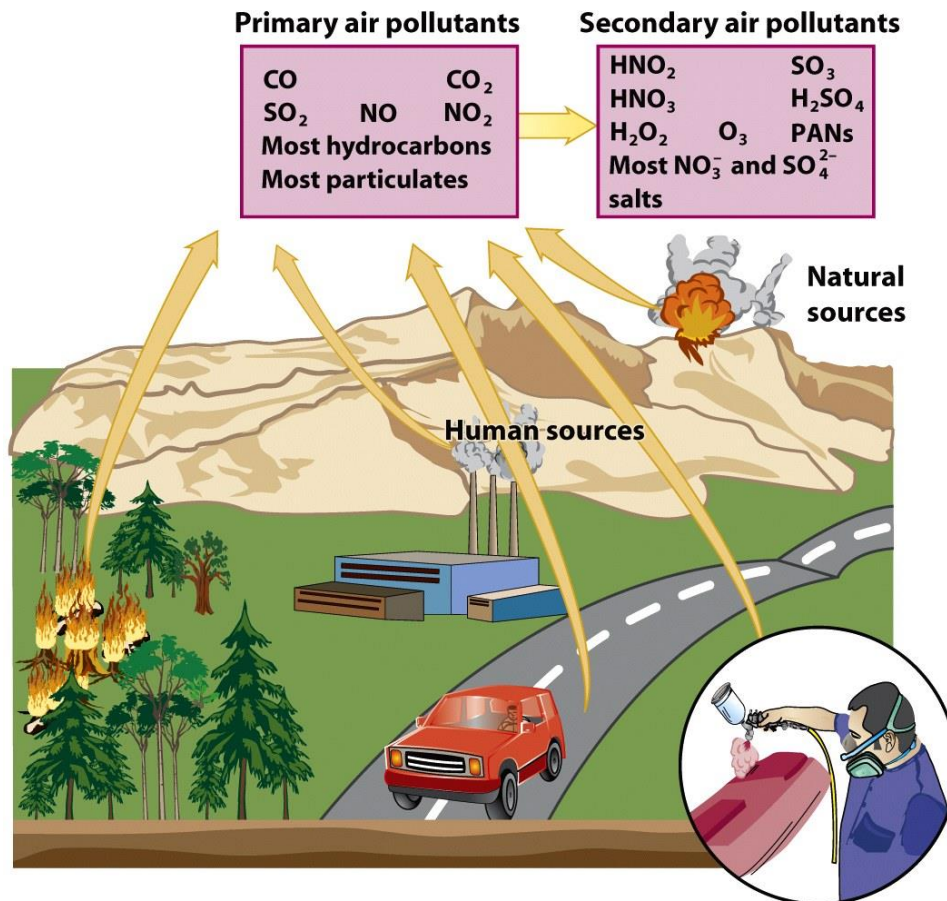
Major Air Pollutants:

Table 20.1 Major Air Pollutants

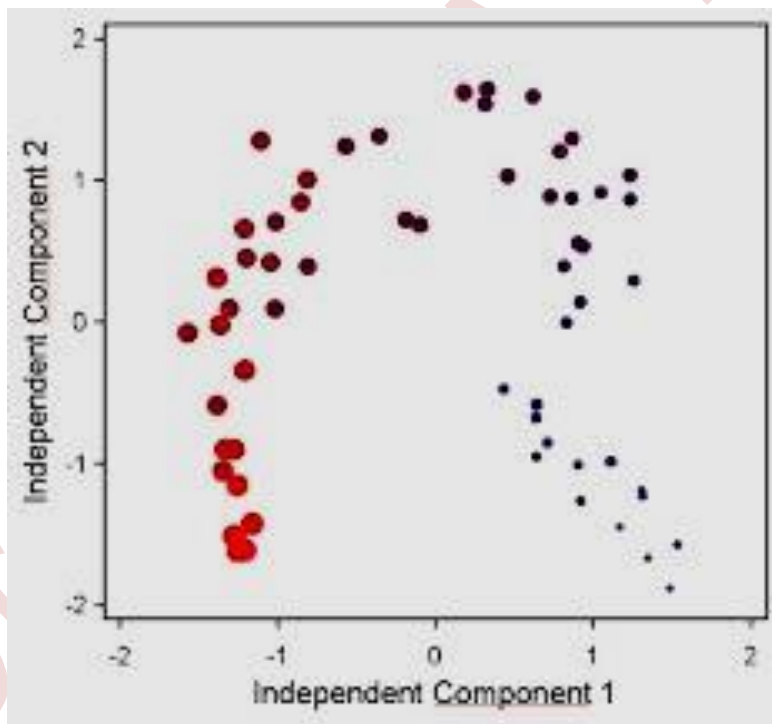
<i>Pollutant</i>	<i>Composition</i>	<i>Primary or Secondary</i>	<i>Characteristics</i>
Particulate matter			
Dust	Variable	Primary	Solid particles
Lead	Pb	Primary	Solid particles
Sulfuric acid	H ₂ SO ₄	Secondary	Liquid droplets
Nitrogen oxides			
Nitrogen dioxide	NO ₂	Primary	Reddish-brown gas
Sulfur oxides			
Sulfur dioxide	SO ₂	Primary	Colorless gas with strong odor
Carbon oxides			
Carbon monoxide	CO	Primary	Colorless, odorless gas
Carbon dioxide*	CO ₂	Primary	Colorless, odorless gas
Hydrocarbons			
Methane	CH ₄	Primary	Colorless, odorless gas
Benzene	C ₆ H ₆	Primary	Liquid with sweet smell
Ozone			
	O ₃	Secondary	Pale blue gas with acrid odor
Air toxics			
Chlorine	Cl ₂	Primary	Yellow-green gas

* Discussed in Chapter 21.

Source: Environmental Protection Agency.



ANALYSIS OF EXPERIMENTAL DATA



Note: Part of this lecture is depending on measurement lectures of Dr.Anees Abd Al-malak AlFakri (1985).

Analysis of experimental data is to determine errors, precision, and general validity of experimental measurements. Errors will creep into all experiments regardless of the care which is exerted.

Types of experimental data:

- a. Single-sample data: this type the uncertainties may not be discovered by repetition .e.g. to measure pressure with a pressure gauge and a single instrument is used for the entire set of observation.
- b. Multi-sample data: are obtained where enough experiments are performed so that the reliability of the results can be assured by statistics. e.g. To measure pressure with more than one pressure gauge for the same total set of observational.

Experimental error: the real errors in experimental data are those factors that are always vague to some extent and carry some amount of uncertainty. The aim of the experiments is to determine just how uncertain a particular observation may be and to devise a consistent way of specifying the uncertainty in analytical form. The experimental uncertainty may be defined as the possible value the error may have.

Types of errors that may cause uncertainty in experimental measurements:

1. **Inhibited errors** in apparatus or instrument construction.
2. **Fixed errors** which will cause repeated readings to be in error by roughly the same amount. (systematic errors) e.g. (sensor not reaching zone temperature in temperature measurements)
3. **Random errors** which may be caused by personal fluctuation, random electronic, fluctuations in apparatus or instrument, etc... These random errors usually follow a certain statistical distribution but not always. (Note: it is very difficult to distinguish between fixed errors and random errors)

Uncertainty Analysis: the uncertainty in a single variable is usually given as:

$$x = x \pm \Delta x$$

Where the plus or minus notation is used to describe the uncertainty e. g. Like

Temperature=70°C±0.5°C

Pressure=100 kN/m² ±1 kN/m²

And this shows (\pm) the uncertainty about the accuracy of the measurement i.e. temp may be of any value between 70.5 and 69.5 °C. The amount of uncertainty ($\pm \Delta x$) could be very much lower by careful calibration of the instrument with standard of very high precision. (Note: Δx could be considered as absolute limits of error or as statistical bound). (70.5-69.5)°C.

Combination of component errors in overall system accuracy calculation:

Consider a measurement system made up of a chain of components each of which is subject to individual accuracy. Now if the individual accuracies are known, how is the overall accuracy computed? Or inversely, if there must be certain accuracy in a computed result, what errors are allowable in the individual instruments?

Let $N = f(u_1, u_2, u_3, \dots, u_n)$

Where N is a quantity (dependent variable) which is a function of n independent variables $u_1, u_2, u_3, \dots, u_n$.

The independent variables ($u_1, u_2, u_3, \dots, u_n$) are the measured quantities (instrument or component variables) and are in error by $\pm \Delta u_1, \pm \Delta u_2, \dots, \pm \Delta u_n$. And these errors are considered in the following analysis as:

First: The absolute limits of errors:

These errors will cause an error ΔN in the computed result N .

Example: $Re = \frac{\rho U D}{\mu}$

$(Re \pm \Delta Re) = f[(\rho \pm \Delta \rho), (u \pm \Delta u), (D \pm \Delta D), (\mu \pm \Delta \mu)]$

By expanding the function f in Taylor series:

$$\begin{aligned} f(u_1 \pm \Delta u_1, u_2 \pm \Delta u_2, \dots, u_n \pm \Delta u_n) \\ = f(u_1, u_2, \dots, u_n) + \left[\Delta u_1 \frac{df}{du_1} + \Delta u_2 \frac{df}{du_2} + \dots + \Delta u_n \frac{df}{du_n} \right] \\ + \frac{1}{2!} \left[(\Delta u_1)^2 \frac{d^2 f}{du_1^2} + \dots \right] + \dots \end{aligned}$$

Where all the partial derivatives are to be evaluated at the known values of u_1, u_2, \dots, u_n ; i.e. (measured values) usually $\Delta u_1, \Delta u_2, \dots, \Delta u_n$ are small quantities and so $(\Delta u)^2$ terms, can be neglected.

The absolute value of the error (E_a):

$$E_a = \Delta N = \left| \Delta u_1 \frac{df}{du_1} \right| + \left| \Delta u_2 \frac{df}{du_2} \right| + \dots + \left| \Delta u_n \frac{df}{du_n} \right|$$

Note: that the absolute values sign, are used because some of the partial derivatives might be negative, and for positive Δu such a term would reduce the total error. Also note that the above form shows which variables (u) exerts the strongest influence on the accuracy of the overall results.

If the relative or percentage error E_r is desired the:

$$E_r = \frac{\Delta N}{N} \times 100 = \frac{100 E_a}{N}$$

And the computed result may be expressed as $N \pm E_a$ or $N \pm E_r\%$.

In carrying the above computation, questions of significant figure and rounding off will occur.

When it is required to know what component accuracies (Δu) are needed for a certain overall accuracy.

The problem is mathematically in ale terminate (infinite number of combinations of individual accuracies that could result in the same overall accuracy). To solve the problem the method of equal effects in used (each source of error will attribute equal amount to the total error.

$$\left| \frac{df}{du_1} \Delta u_1 \right| = \left| \frac{df}{du_2} \Delta u_2 \right| = \dots = \left| \frac{df}{du_n} \Delta u_n \right| = \frac{\Delta N}{n}$$

Thus :

$$\left| \frac{df}{du_i} \Delta u_i \right| = \frac{\Delta N}{n}$$

$$\text{And } \Delta u_i = \frac{\Delta N}{n \left(\frac{df}{du_i} \right)} \quad i=1,2,3,\dots,n$$

Second: when the Δu_i are not considered as absolute limits of error but rather as statistical bound:

The formula for computing overall errors must be modified and it can be shown that the proper method of combining such errors is according to the root-sum square formula:

$$E_{a(rss)} = \left[(\Delta u_1 \frac{df}{du_1})^2 + (\Delta u_2 \frac{df}{du_2})^2 + \dots + (\Delta u_n \frac{df}{du_n})^2 \right]^{1/2}$$

Note: $E_{a(rss)}$ is always $< E_a$

Also component accuracy for a certain overall accuracy is given by:

$$\Delta u_i = \Delta N / [\sqrt{n} (df/du_i)]$$

Example:

The resistance of a certain size of copper wire is given as $R=R_0[1+\alpha(T-20)]$ where;

$R_0=6\Omega \pm 0.3\%$ is the resistance at 20°C

$A=0.004/^\circ\text{C} \pm 1\%$ is the temperature coefficient

$T=30 \pm 1^\circ\text{C}$ temperature of the wire. Calculate the resistance of the wire and its uncertainty?

Solution:

Nominal resistance $R=6[1+0.004(30-20)]=6.24 \Omega$

To calculate the uncertainty:

$$dR/dR_0 = 1+\alpha(T-20)=1+0.004(30-20)=1.04$$

$$dR/d\alpha=R_0(T-20)=6(30-20)=60$$

$$dR/dT=R_0*\alpha=6*0.004=0.024$$

$$\Delta R_0=6*0.0003=0.018\Omega$$

$$\Delta\alpha=0.004*0.01=4\times 10^{-5} /^\circ\text{C}$$

$$\Delta T=1^\circ\text{C}$$

The uncertainty in the resistance is:

$$\Delta R=[(1.04)^2*(0.018)^2+(60)^2*(4\times 10^{-5})^2+(0.024)^2*(1)^2]^{1/2}$$

$$\Delta R=0.0305\Omega \text{ or } 0.49\%$$

Note: Instrument uncertainties (errors) are frequently expressed as %.

e.g. Manufacturers of pressure gauge state that it is (accurate within ± 1.0 percent).

This statement normally refers to percent of full scale.

Thus gauge with arrange of 0-100 kPa would have an uncertainty of ± 10 percent when reading a pressure of only 10 kPa.

This means that the uncertainty in the calculated result, either as an absolute value or percentage, can vary widely depending on the range of operation of instruments used to make the primary measurements.

Statistical Analysis of Experimental Data:

When a set of reading of an instrument is taken, the individual readings will vary somewhat from each other (e.g. Temperature measurement using thermocouple, pressure measurement.....)

The arithmetic mean is given by:

$$X_m = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{Where } x_i = \text{each reading and } n = \text{number of readings}$$

The deviation (d_i) for each reading is defined by:

$$d_i = x_i - x_m$$

Note that the average of the deviations of all the readings = 0, as

$$\bar{d}_i = \frac{1}{n} \sum_{i=1}^n d_i = \frac{1}{n} \sum_{i=1}^n (x_i - x_m) = x_m - \frac{1}{n} (nx_m) = 0$$

The average of the absolute values of the deviations is given by:

$$|\bar{d}_i| = \frac{1}{n} \sum_{i=1}^n |d_i| = \frac{1}{n} \sum_{i=1}^n |x_i - x_m|$$

The standard deviation (or root mean square deviation) is defined by:

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^n (x_i - x_m)^2 \right]^{1/2} \quad (\text{and } \sigma^2 \text{ is called the variance})$$

Note: usually it requires at least 20 measurements in order to obtain reliable estimates of σ .

For small sets of data, a sample standard deviation is defined by

$$\sigma = \left[\frac{\sum_{i=1}^n (x_i - x_m)^2}{n - 1} \right]^{1/2}$$

The Gaussian or Normal error distribution:

For an experimental observation it would be suspected that the observations have been subjected to many random errors. These random errors may make the final reading either too large or too small, depending on many unknown circumstances. Assuming that there are many small errors that contribute to the final error and each small error is of equal magnitude and equally likely to be positive or negative. The Gaussian(or normal error) distribution may be derived.

If the measurement is designated by x , the Gaussian distribution gives the probability that the measurement will lie between $x+dx$ and is:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-x_m)^2/2\sigma^2} \dots\dots\dots 1$$

Where x_m =mean reading and σ =standard deviation.

Note: $P(x)$ is also defined as the probability density , the units of $P(x)$ are those of $(1/x)$ as show in the plot, the most probable reading is x_m .

The standard deviation is a measure of the width of the distribution curve.

(The larger value of σ , the flatter the curve and hence the larger the expected error of all the measurements).

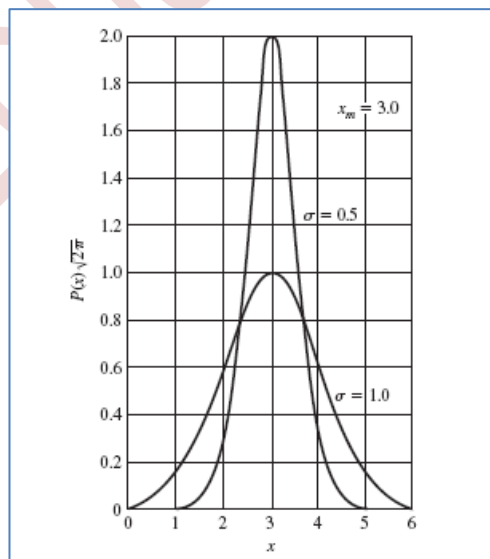


Figure 17.1 The Gaussian or normal error distribution for two values of the standard deviation.

How does the normal error distribution apply to experimental data?

For sets of data where a large number of measurements are taken, experiments indicate that the measurements do indeed follow a distribution like that known, when the experiment is under control. If an important parameter is not controlled, one gets just scatter, i.e no sensible distribution at all. Thus, as a matter of experimental verification the Gaussian distribution is believed to represent the random errors in an adequate manner for a properly controlled experiment.

From equation [1], the maximum probability occurs at $x=x_m$ and the value of this probability is :

$$P(x_m) = \frac{1}{\sigma\sqrt{2\pi}}$$

The smaller the value of σ , the larger values of the maximum probability. The probability that a measurement will fall within a certain range x_1 of the mean reading is:

$$P = \int_{x_m-x_1}^{x_m+x_1} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-x_m)^2/2\sigma^2} dx \dots\dots\dots 2$$

$$\text{Let } \zeta = \frac{x-x_m}{\sigma}$$

Then equation 2 becomes:

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\zeta_2}^{+\zeta_1} e^{-\frac{\zeta^2}{2}} d\zeta \quad \text{Where } \zeta_1 = \frac{x_1}{\sigma}$$

(Tables are available giving the value of the function $\frac{1}{\sqrt{2\pi}} e^{-\frac{\zeta^2}{2}}$ and its integral for different values of ζ .)

If we have a sufficiently large number of data points, the error for each point should follow the Gaussian distribution and we can determine the probability that certain data follow within a specified deviation from the mean value. The following table gives the enhanced for certain deviations from the mean value of the normal distribution curves.

Deviation	Chance of result falling within thin specified deviation
$\pm 0.6745 \sigma$	1-1
σ	2.15-1
2σ	21-1
3σ	369-1

For a set of data points, if some of the data points look out of place in comparison with the bulk of the data. It is therefore necessary to decide if these points are the result of some experimental blunder and hence may be neglected or if they represent some unknown type of physical phenomena either to neglect these points or not, a consistent basis for elimination must be available.

Suppose n measurements of a quantity are taken and n is large enough that we may expect the results to follow the Gaussian error distribution.

This distribution may be used to compute the probability that a given reading will deviate a certain amount from the mean. We would not expect a probability much smaller than $(1/n)$ because this would be unlikely to occur in the set of n measurements. Thus, if the probability for the observed deviation of a certain point is less than $(1/n)$, then that point may be neglected from the data.

Example:

Calculate the probability that a measurement will fall within 1, 2, and 3 standard deviation of the mean value and compare them with the values given in the table.

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\zeta_1}^{+\zeta_1} e^{-\frac{\zeta^2}{2}} d\zeta \quad \text{Where } \zeta_1 = \frac{x_1}{\sigma}$$

$$\text{Now } \int_{-\zeta_1}^{+\zeta_1} e^{-\frac{\zeta^2}{2}} d\zeta = 2 \int_0^{\zeta_1} e^{-\frac{\zeta^2}{2}} d\zeta$$

Using available tables to find the value of the integral for $\zeta_1 = 1, 2, \text{ and } 3$

$$P(1) = 2 \times (0.34134) = 0.6827 \quad \text{from table for } \zeta = 1.0$$

$$P(2) = 2 \times (0.47725) = 0.9545$$

$$P(3) = 2 \times (0.49865) = 0.9973$$

Now using the odds given in table, the probabilities are calculated as:

$$P(1) = 2.15 / (2.15 + 1) = 0.6827$$

$$P(2) = 21 / (21 + 1) = 0.9545$$

$$P(3) = 369 / (369 + 1) = 0.9973$$

Chauvenet's criterion for rejecting a reading:

A reading may be rejected (neglected) if the probability of obtaining the particular deviation from the mean is less than $(1/2n)$

The following table lists values of the ratio of deviation to standard deviation for various values of n according to Chauvenet's criterion.

n	D_{\max}/σ
Number of readings	Ratio of maximum acceptable deviation to standard deviation
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
10	1.96
15	2.13
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1000	3.48

In applying Chauvenet's criterion to eliminate dubious data points :

- Calculate the mean value and standard deviation using data points.
- The deviations of the individual points are then compared with the standard deviation in accordance with the information given in the table, and the dubious points are eliminated.

For the final data presentations a new mean value and standard deviation are computed with the dubious points eliminated from the calculation. Thus the criterion might be applied a second or third time to eliminate additional points, but only the first application.

Example: using Chauvenet's criterion, test the data points in the following test for possible inconsistency.

Eliminate the questionable points and calculate a new standard deviation for the adjusted data.

No. of reading	1	2	3	4	5	6	7	8	9	10
X cm	5.3	5.73	6.77	5.26	4.33	5.45	6.09	5.64	5.81	5.75

$$x_m = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{10} (5.3 + 5.73 + 6.77 + 5.26 + 4.33 + 5.45 + 6.09 + 5.64 + 5.81 + 5.75) =$$

$$5.613 \text{ cm}$$

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^n (x_i - x_m)^2 \right]^{1/2}$$

$$= \left[\frac{1}{10} \{ (5.3 - 5.613)^2 + (5.73 - 5.613)^2 + \dots + (5.75 - 5.613)^2 \} \right]^{1/2}$$

$$= 0.5944 \text{ cm}$$

$$\sigma^2 = 0.3533 \text{ cm}^2$$

$$|\bar{d}_i| = \frac{1}{n} \sum_{i=1}^n |d_i| = \frac{1}{n} \sum_{i=1}^n |x_i - x_m| = \frac{1}{10} (4.224) = 0.4224 \text{ cm}$$

The best estimate of standard deviation for the data ($n < 20$)

$$\sigma = \left[\frac{\sum_{i=1}^n (x_i - x_m)^2}{n-1} \right]^{1/2} = \left[\frac{1}{(10-1)} (3.536) \right]^{1/2} = 0.627 \text{ cm}$$

To check data points, calculate d_i/σ , thus $|x_i - x_m|/\sigma$

No. of reading	1	2	3	4	5	6	7	8	9	10
d_i/σ	0.499	0.187	1.845	0.563	2.046	0.260	0.761	0.043	0.314	0.219

Now according to table given for $n=10$ and $d_{max}/\sigma=1.96$, thus data point number 5 has $d_i/\sigma=2.046 > d_{max}/\sigma(1.96)$

And hence may be eliminated.

New mean value $x_m = (1/9)(51.80) = 5.756 \text{ cm}$

$$\sigma = [(1/8)(1.7044)^2]^{1/2} = 0.4615 \text{ cm}$$

Comparison of Data with Normal Distribution

We have seen that the normal error distribution offers a means for examining experimental data for statistical consistency. In particular, it enables us to eliminate questionable readings with the Chauvenet criterion and thus obtain a better estimate of the standard deviation and mean reading. If the distribution of random errors is not *normal*, then this elimination technique will not apply. It is to our advantage, therefore, to determine if the data are following a normal distribution before making too many conclusions about the mean value, variances, and so forth. Specially constructed probability graph paper is available for this purpose and may be purchased from a technical drawing shop. The paper uses the coordinate system shown in Fig. 17.2. The ordinate has the percent of readings at or below the value of the abscissa, and the abscissa is the value of a particular reading. The ordinate spacing's are arranged so that the gaussian-distribution curve will plot as a straight line on the graph. In addition, this straight line will intersect the 50 percent ordinate at an abscissa equal to the arithmetic mean of the data.

Thus, to determine if a set of data points is distributed normally, we plot the data on probability paper and see how well they match with the theoretical straight line. It is to be noted that the largest reading cannot be plotted on the graph because the ordinate does not extend to 100 percent. In assessing the validity of the data, we should not place as much reliance on the points near the upper and lower ends of the curve since they are closer to the “tails” of the probability distribution and are thus less likely to be valid.

An alternative approach is to plot the cumulative frequencies for the normal distribution using the tabular values from Table 3.2 or computer software. This plot is then displayed and compared with the actual frequency distribution to make an evaluation of the “normality” of the data.

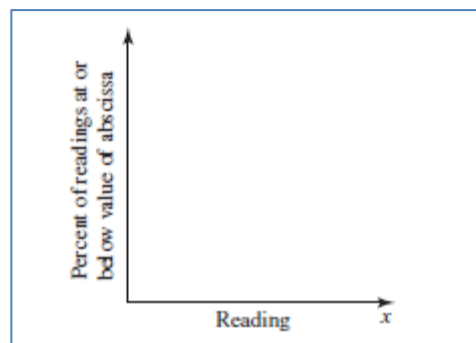


Figure 17.2 Probability graph paper.

Example : USE OF PROBABILITY GRAPH PAPER AND COMPUTER COMPARISON.

The following data are collected for a certain measurement. Plot the data on probability paper and comment on the normality of the distribution: Make the same comparison with a computer generated display.

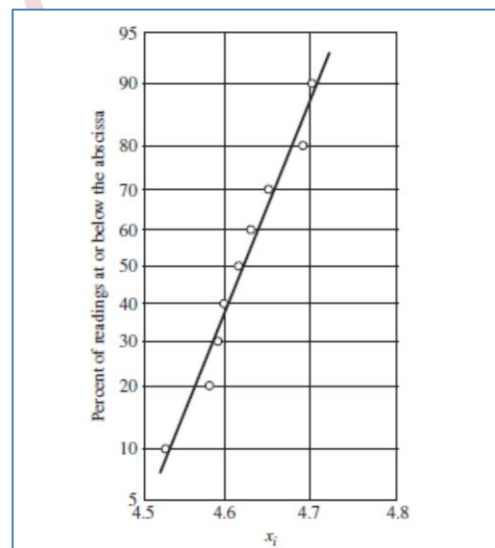
No. of reading	1	2	3	4	5	6	7	8	9	10
X cm	4.62	4.69	4.86	4.53	4.6	4.65	4.59	4.7	4.58	4.63

From these data the mean value is calculated as

$$\sum_{i=1}^{10} x_i = 46.45$$

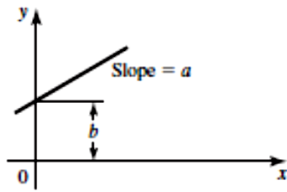
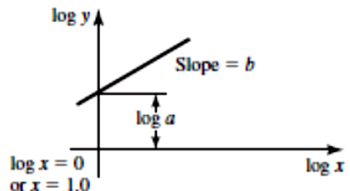
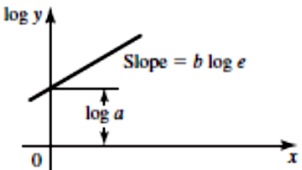
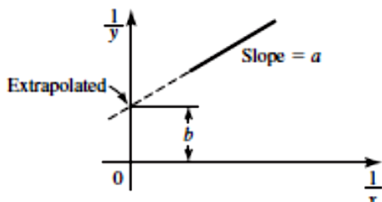
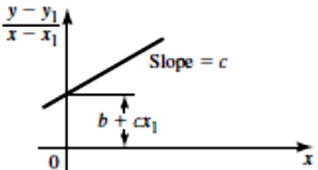
$$x_m = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{10} (46.45) = 4.645 \text{ cm}$$

The data are plotted in the example figure (a) indicating a reasonably normal distribution. It should be noted that the straight line crosses the 50 percent ordinate at a value of approximately $x = 4.62$, which is not in agreement with the calculated value of x_m . Note that point 3, $x = 4.86$, does not appear on the plot since it would represent the 100 percent ordinate.



Choice of Graph Formats

The engineer has many graph formats available for presenting experimental data or calculation results. While bar charts, column charts, pie charts, and similar types of displays have some applications, by far the most frequently used display is the x-y graph with choices of coordinates to match the situation. This basic graph has several variations in format that we shall illustrate by plotting the simple table of x-y data.

Functional Relationship	Method of Plot	Graphical Determination of Parameters
$y = ax + b$	y vs. x on linear paper	
$y = ax^b$	log y vs. log x on loglog paper	
$y = ae^{bx}$	log y vs. x on semilog paper	
$y = \frac{x}{a + bx}$	$\frac{1}{y}$ vs. $\frac{1}{x}$ on linear paper	
$y = a + bx + cx^2$	$\frac{y - y_1}{x - x_1}$ vs. x on linear paper	

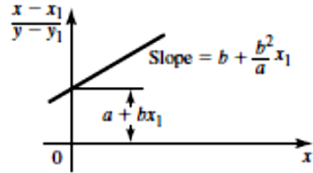
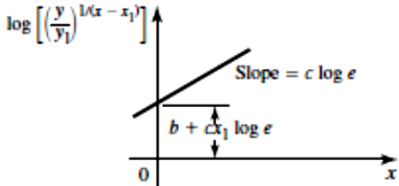
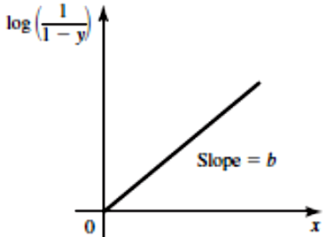
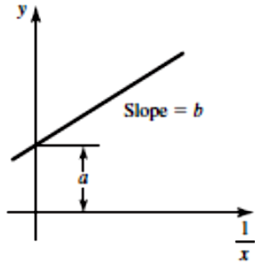
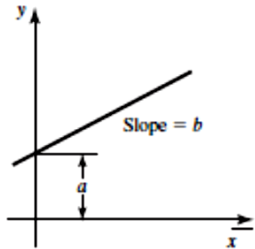
Functional Relationship	Method of Plot	Graphical Determination of Parameters
$y = \frac{x}{a + bx} + c$	$\frac{x - x_1}{y - y_1}$ vs. x on linear paper	
$y = ae^{bx+cx^2}$	$\log \left[\left(\frac{y}{y_1} \right)^{1/(x-x_1)} \right]$ vs. x on semilog paper	
$y = 1 - e^{-bx}$	$\log \left(\frac{1}{1-y} \right)$ vs. x on semilog paper	
$y = a + \frac{b}{x}$	y vs. $\frac{1}{x}$ on linear paper	
$y = a + b\sqrt{x}$	y vs. \sqrt{x} on linear paper	

Table 3.1 Values of the gaussian normal error distribution

Values of the function $(1/\sqrt{2\pi})e^{-\eta^2/2}$ for different values of the argument η . Each figure in the body of the table is preceded by a decimal point.

η	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	39894	39892	39886	39876	39862	39844	39822	39797	39767	39733
0.1	39695	39654	39608	39559	39505	39448	39387	39322	39253	39181
0.2	39104	39024	38940	38853	38762	38667	38568	38466	38361	38251
0.3	38139	38023	37903	37780	37654	37524	37391	37255	37115	36973
0.4	36827	36678	36526	36371	36213	36053	35889	35723	35553	35381
0.5	35207	35029	34849	34667	34482	34294	34105	33912	33718	33521
0.6	33322	33121	32918	32713	32506	32297	32086	31875	31659	31443
0.7	31225	31006	30785	30563	30339	30114	29887	29658	29430	29200
0.8	28969	28737	28504	28269	28034	27798	27562	27324	27086	26848
0.9	26609	26369	26129	25888	25647	25406	25164	24923	24681	24439
1.0	24197	23955	23713	23471	23230	22988	22747	22506	22265	22025
1.1	21785	21546	21307	21069	20831	20594	20357	20121	19886	19652
1.2	19419	19186	18954	18724	18494	18265	18037	17810	17585	17360
1.3	17137	16915	16694	16474	16256	16038	15822	15608	15395	15183
1.4	14973	14764	14556	14350	14146	13943	13742	13542	13344	13147
1.5	12952	12758	12566	12376	12188	12001	11816	11632	11450	11270
1.6	11092	10915	10741	10567	10396	10226	10059	09893	09728	09566
1.7	09405	09246	09089	08933	08780	08628	08478	08329	08183	08038
1.8	07895	07754	07614	07477	07341	07206	07074	06943	06814	06687
1.9	06562	06438	06316	06195	06077	05959	05844	05730	05618	05508
2.0	05399	05292	05186	05082	04980	04879	04780	04682	04586	04491
2.1	04398	04307	04217	04128	04041	03955	03871	03788	03706	03626
2.2	03547	03470	03394	03319	03246	03174	03103	03034	02965	02898
2.3	02833	02768	02705	02643	02582	02522	02463	02406	02349	02294
2.4	02239	02186	02134	02083	02033	01984	01936	01888	01842	01797
2.5	01753	01709	01667	01625	01585	01545	01506	01468	01431	01394
2.6	01358	01323	01289	01256	01223	01191	01160	01130	01100	01071
2.7	01042	01014	00987	00961	00935	00909	00885	00861	00837	00814
2.8	00792	00770	00748	00727	00707	00687	00668	00649	00631	00613
2.9	00595	00578	00562	00545	00530	00514	00499	00485	00470	00457
3.0	00443									
3.5	008727									
4.0	0001338									
4.5	0000160									
5.0	000001487									

Table 3.2 Integrals of the gaussian normal error function

Values of the integral $(1/\sqrt{2\pi}) \int_0^{\eta_1} e^{-\eta^2/2} d\eta$ are given for different values of the argument η_1 . It may be observed that

$$\frac{1}{\sqrt{2\pi}} \int_{-\eta_1}^{+\eta_1} e^{-\eta^2/2} d\eta = 2 \frac{1}{\sqrt{2\pi}} \int_0^{\eta_1} e^{-\eta^2/2} d\eta$$

The values are related to the error function since

$$\text{erf } \eta_1 = \frac{1}{\sqrt{\pi}} \int_{-\eta_1}^{+\eta_1} e^{-\eta^2} d\eta$$

so that the tabular values are equal to $\frac{1}{2} \text{erf } (\eta_1/\sqrt{2})$. Each figure in the body of the table is preceded by a decimal point.

η_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0000	00399	00798	01197	01595	01994	02392	02790	03188	03586
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15554	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20450	20884	21226	21566	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25084	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34850	35083	35313	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41308	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42786	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361
2.5	49379	49296	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861
3.0	49865									
3.5	4997674									
4.0	4999683									
4.5	4999966									
5.0	4999997133									