Subject : Fluid Mechanics (I) — موضوع : موائع 1

Weekly Hours : Theoretical: UNITS:5 — الساعات الأسبوعية : نظري : 2 الوحدات : 5

Tutorial : 1 — مناقشة : 1

Experimental : 1 — عملي : 1

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Fluid Mech. ميكانيك الموائع

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   1.2 Newton Law of viscosity
   1.3 Bulk Modulus of Elastity
   1.4 surface tension

2. Fluid static
   2.1 Definitions
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   2.4 Units and scales of pressure Mesurment
   2.5 Manometers (pressure Mesurement).
   2.6 Force on plane surface
   2.7 Force on Curved surface.
   2.8 Buoyant Force.
   2.9 Stability of Floating and submerged Bodies.
   2.10 Relative Equilibrium.

3. Fluid flow Concept and Basic Equations.
   3.1 Definitions
   3.2 Continuity equation
   3.3 Euler Equation of motion along streamline.
   3.4 Bernoulli Equation (Energy equation)
   3.5 Flow mesurement.
   Pitot tube, orifice meter, Venturi meter, Nozzle
   3.6 Resistance to Flow in open and closed Conduits
   3.7 Linear Momentum Equation and Its

Application

- 3.8 Introduction for pumping and Turbines application

4. Dimensional analysis and dynamic similitude.
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   - 4.2 Discu of Dimensionless parameters
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     Weber No., Mach No.
   - 4.3 Similitude; Model studies.

Referance.

1. Fluid Mechanics, Vector L. Streeter
   E. Benjamin Wylie.

2. Fluid Mechanics and Engineering application
   Robert L. Dogerti and Joshef B. Frinziery

-1- CH-1

Fluid mechanics

Definisions

1. Fluid; It is a substance that deforms continuosly when subjected to a shear stress. It is either gas or liquid.

2. Shear stress :- $\tau = \frac{F}{A} = \frac{\text{shear force}}{\text{surface area}}$

3- Shear force :- It is the force components tangents to a surface of liquid.

4. Viscosity :- μ اللزوجة :- It is the property of fluid by virtue of which it offers resistance to shear.

- Molasses (دبس) and tar (قير) are example for highly viscous liquids.
- water and air have very small resistance

- The viscosity of gas increase with temperature
- " " " liquid decrease " "

Units $\mu = N.s/m^2$ or $kg/m.s$

A common unit is Poise (P) =

$$1 \text{ poise } (g/cm \cdot s) = 0.1 \ N \cdot s/m^2 \quad (Pa \cdot s)$$
$$= 0.1 \ kg/m \cdot s$$
$$10P = 1 \ kg/m \cdot s.$$

5. Kinematic Viscosity :- ν : It is the ratio of Viscosity to mass density.

$$\nu = \frac{\mu}{\rho} = 1 \ m^2/s$$
$$= 1 \ cm^2/s \quad (stoke)$$

6. Density : ρ الكثافة : It is the mass per unit Volume

$$\rho = \frac{m}{V} = kg/m^3$$

كثافة الماء $\rho_{water} = 1000 \ kg/m^3$

7- Specific weight γ : (unit gravity force) The force per unit volume. It change الكثافة الوزنية with location.

الكثافة الوزنية للماء $\gamma_w = \rho g = 9.81 \times 1000 = 9810 \ N/m^3$

8- specific gravity S :- (relative density)

الوزن النوعي

$$S = \frac{\gamma_s}{\gamma_w} = \frac{\text{Specific weight of substance}}{\text{" " " water}}$$

المادة Substance : تعني الاجسام الصلبة والموائع

9. pressure : P : the normal force pushing against a plane area divided by the area.
units : N/m^2 or Pascal (P_a)

10. Vapor pressure :- The vapor molecules exert a partial pressure in the space known as vapor pressure.

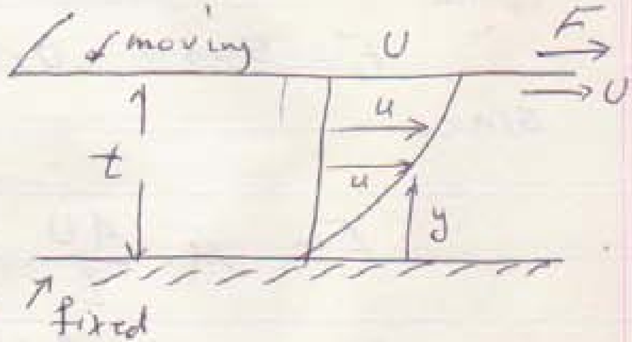
ان نمو تقلبات البخار يؤثر على المكائن الهيدروليكية

11. Perfect gas; It is a substance that satisfies the Perfect gas Law $PV_s = RT$ or $P = \rho RT$

Newtons Law of Viscosity

experimentally shown that

$$F \propto \frac{AU}{t}$$



A = the area of the moving plate m^2

السطح ، السطح العلوي المتحرك

U = steady Velocity of moving plate m/s

t = the distance between the plates m

المسافة بين الصفيحتين .

i.e $$\boxed{F = \mu \cdot \frac{AU}{t}}$$ ~~Newton Law of Viscosity~~ —— (1)

since $\tau = \frac{F}{A}$

$$\therefore \tau = \mu \cdot \frac{U}{t} \quad \text{—— (2)}$$

$\frac{U}{t}$ = the angular deformation of fluid.
التشوه الزاوي للمائع

$$= \frac{du}{dy}$$

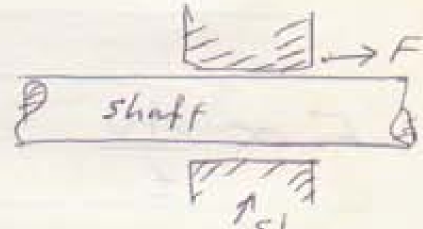
$$\boxed{\tau = \mu \frac{du}{dy}} \quad \mu = \text{اللزوجة} \quad \text{—— (3)}$$

Newtons low of viscosity

Newtonian fluid :- هو المائع الذي يتبع قانون
نيوتن للزوجة.

Q.1.5

$F_1 = 500N \quad U = 1m/s$



since

$$F = \mu \cdot \frac{AU}{t}$$

$$500 = \mu \cdot \frac{A \times 1}{t} \quad \therefore \mu = \frac{500t}{A}$$

since T = Constant درجة الحرارة ثابتة
أي ان اللزوجة ثابتة

$\therefore$

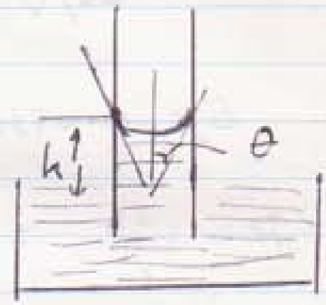
$$F_2 = \mu \cdot \frac{AU}{t} \quad , \quad 1500 = \frac{500t}{A} \times \frac{AU}{t}$$

$U = 3m/s$

12. Specific Volume : v_s : it is the reciprocal of density

$$v_s = \frac{1}{\rho} = m^3/kg$$

13. Surface tension الشد السطحي



$$\gamma h \pi r^2 = 2\pi r \sigma \cos\theta$$

σ = Surface tension Constant
معامل الشد السطحي.

$$h = \frac{2\sigma \cos\theta}{\gamma r}$$

القوة الى الاعلى (قوة الشد) = الوزن (القوة الاسفل)

h = ارتفاع عمود السائل.

pressure at droplet الضغط داخل قطرة ارتفاعة $P = \frac{2\sigma}{r}$ N/m^2

14. Bulk modulus of Elasticity :- K

$$K = -\frac{dP}{dV/V} = N/m^2$$

$$\text{or } K = -\frac{\Delta P}{\Delta V/V} = -\frac{P_2 - P_1}{\frac{V_2 - V_1}{V_1}}$$

K : The ~~Volumetric~~ Compressive stress per unit Volumetric strain.

Ex: A liquid compressed in a cylinder has a volume of 1 liter (1000 cm³) at 1 MN/m² and volume of 995 cm³ at 2 MN/m³. What is its bulk modulus of elasticity?

$$K = -\frac{\Delta P}{\Delta V / V} = \frac{(2-1)\ MN/m^2}{\frac{(995-1000)}{1000}} = 200\ MPa.$$

Q.142 $\sigma_w = 0.0736\ N/m$

$$P = \frac{2\sigma}{r} = \frac{2 \times 0.0736}{\frac{0.05}{2}} \times 1000 = 5.89\ kPa$$

gage.

1.18 $d_s = 50\ mm$ $d_c = 50.1\ mm$

$$\therefore t = \frac{50.1 - 50}{2} \quad \therefore t = 0.05\ mm$$

μ at $0°C = 1.6 \times 10^{-2}$ Pa.s

μ at $120°C = 2 \times 10^{-3}$ Pa.s

$$F_1 = \mu \frac{AU}{t} = 1.6 \times 10^{-2} \frac{AU}{0.05 \times 10^{-3}} = 320\ AU$$

$$F_2 = 2 \times 10^{-3} \times \frac{1}{0.05 \times 10^{-3}} \times AU = 40\ AU$$

$$\therefore \frac{F_1}{F_2} = 8 \quad \& \quad \frac{F_1 - F_2}{F_1} = \frac{320AU - 40AU}{320\ AU}$$

$$= 87\%$$

upper surface is in contact with air, which offers almost no resistance to the flow. Using Newton's law of viscosity, decide what the value of du/dy, y measured normal to the inclined plane, must be at the upper surface. Would a linear variation of u with y be expected?

1.4 What kinds of rheological materials are paint and grease?

1.5 A Newtonian fluid is in the clearance between a shaft and a concentric sleeve. When a force of 500 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 1 m/s. If a 1500-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

1.6 Determine the gravity force in newtons of 3 kg mass at a place where $g = 9.7$ m/s².

1.7 When standard scale masses and a balance are used, a body is found to be equivalent in force of gravity to two of the 1-kg scale masses at a location where $g = 9.7$ m/s². Calculate the gravity force on a correctly calibrated spring balance (for sea level) at this location.

1.8 Determine the unit gravity force γ for water at 25° C and $g = 9.75$ m/s².

1.9 On another planet, where gravity is 3 m/s², find the force of gravity on 400 L of material $\rho = 800$ kg/m³.

1.10 A correctly calibrated spring scale records the gravity force of a 2-kg body as 17.0 N at a location away from the earth. What is the value of g at this location?

1.11 The gravity force on a bag of flour at sea level is 20 N. What is its mass at a location where $g = 9.6$ m/s²?

1.12 What is the kinematic viscosity of liquid of viscosity 0.002 Pa·s and a relative density of 0.8?

1.13 A shear stress of 4 mPa causes a Newtonian fluid to have an angular deformation of 1 rad/s. What is its viscosity?

1.14 A plate, 0.5 mm distant from a fixed plate, moves at 0.25 m/s and requires a force per unit area of 2 Pa to maintain this speed. Determine the viscosity of the substance between the plates.

1.15 Determine the viscosity of fluid between shaft and sleeve in Fig. 1.6.

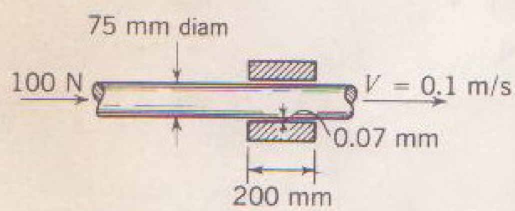


Figure 1.6 Problem 1.15.

1.16 A flywheel weighing 600 N has a radius of gyration of 300 mm. When it is rotating 600 rpm, its speed reduces 1 rpm/s owing to fluid viscosity between sleeve and shaft. The sleeve length is 50 mm; shaft diameter is 20 mm; and radial clearance is 0.05 mm. Determine the fluid viscosity.

1.17 A 25mm diameter steel cylinder 300 mm long falls, because of its own gravity force at a uniform rate of 0.1 m/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the tube and the cylinder. The temperature is 38°C. Relative density of steel = 7.85.

1.18 A piston of diameter 50.00 mm moves within a cylinder of 50.10 mm. Determine the percent decrease in force necessary to move the piston when the lubricant warms up from 0 to 120°C. Use crude-oil viscosity from Fig. C.1, Appendix C.

1.19 How much greater is the viscosity of water at 0°C than at 100°C? How much greater is its kinematic viscosity for the same temperature range?

1.20 A fluid has a viscosity of 0.6 Pa·s and a relative density of 0.7. Determine its kinematic viscosity.

1.21 A fluid has a relative density of 0.78. For a kinematic viscosity of 1.0×10^{-6} m^2/s determine the viscosity.

X **1.22** A body with gravity force of 500 N with a flat surface area of 0.2 m^2 slides down a lubricated inclined plane making a 30° angle with the horizontal. For viscosity of 0.1 Pa·s and body speed of 1 m/s determine the lubricant film thickness.

1.23 What is the viscosity of gasoline at 25°C?

1.24 Determine the kinematic viscosity of benzene at 27°C.

1.25 Calculate the value of the gas constant R for relative molecular mass of 44.

1.26 What is the specific volume of a substance of relative density 0.75?

1.27 What is the relation between specific volume and unit gravity force?

1.28 The density of a substance is 2900 kg/m^3. What is its (*a*) relative density, (*b*) specific volume, and (*c*) unit gravity force?

1.29 A force, expressed by $\mathbf{F} = 4\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$, acts upon a square area, 2 by 2 cm, in the xy plane. Resolve this force into a normal-force and a shear-force component. What are the pressure and the shear stress? Repeat the calculations for $\mathbf{F} = -4\mathbf{i} + 3\mathbf{j} - 9\mathbf{k}$.

1.30 A gas at 20°C and 0.2 MPa abs has a volume of 40 L and a gas constant $R = 210$ m·N/kg·K. Determine the density and mass of the gas.

1.31 What is the density of air at 400 kPa abs and 30°C?

1.32 What is the density of water vapor at 0.3 kPa abs and 30°C?

1.33 A gas with relative molecular mass 28 has a volume of 100 L and a pressure and temperature of 80 kPa abs and 330 K, respectively. What are its specific volume and density?

1.34 One kilogram of hydrogen is confined in a volume of 150 L at −40°C. What is the pressure?

1.35 Express the bulk modulus of elasticity in terms of density change rather than volume change.

1.36 For constant bulk modulus of elasticity, how does the density of a liquid vary with the pressure?

1.37 What is the bulk modulus of a liquid that has a density increase of 0.02 percent for a pressure increase of 0.6 MPa?

1.38 For $K = 2.2$ GPa for bulk modulus of elasticity of water what pressure is required to reduce its volume by 0.5 percent?

1.39 A steel container expands in volume 1 percent when the pressure within it is increased by 70 MPa. At standard pressure, P = 101.3 kPa it holds 450 kg water, $\rho = 1000$ kg/m^3. For $K = 2.06$ GPa when it is filled, how many kilograms mass water need be added to increase the pressure to 70 MPa?

1.40 What is the isothermal bulk modulus for air at 0.4 MPa abs?

1.41 At what pressure can cavitation be expected at the inlet of a pump that is handling water at 20°C?

1.42 What is the pressure within a droplet of water of 0.05 mm diameter at 20°C if the pressure outside the droplet is standard atmospheric pressure of 101.3 kPa?

1.43 A small circular jet of mercury 0.1 mm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C?

1.44 Determine the capillary rise for distilled water at 40°C in a circular 6 mm diameter glass tube.

1.45 What diameter of glass tube is required if the capillary effects on the water within are not to exceed 0.5 mm?

1.46 Using the data given in Fig. 1.4, estimate the capillary rise of tap water between two parallel glass plates 5 mm apart.

1.47 A method of determining the surface tension of a liquid is to find the force needed to pull a

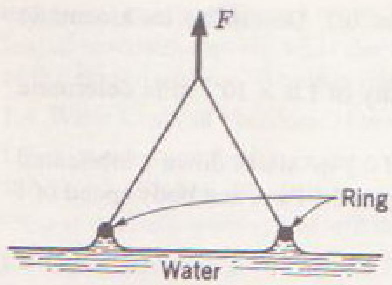
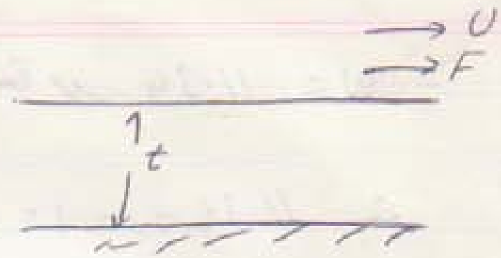


Figure 1.7 Problem 1.47.

platinum wire ring from the surface (Fig. 1.7). Estimate the force necessary to remove a 20-mm-diameter ring from the surface of water at 20°C. Why is platinum used as the material for the ring?

3

Q.1.14 $\frac{F}{A} = 2\ Pa\ (N/m^2)$



$t = 0.5\ mm$

$U = 0.25\ m/s$

since

$$F = \mu \frac{AU}{t} \quad \text{or} \quad \frac{F}{A} = \mu \frac{U}{t} = 2\ Pa$$

$$\therefore 2 = \mu \cdot \frac{0.25}{0.5 \times 10^{-3}} \qquad \therefore \mu = 0.004\ Pa.s$$

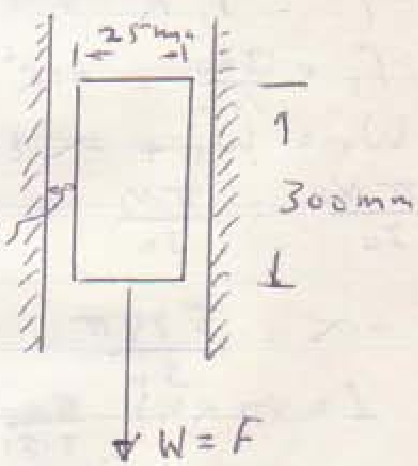
Q. 1.17.

$U = 0.1\ m/s \qquad T = 38\ ^\circ C$

Castor-oil $\therefore \mu = 3 \times 10^{-1}$

$= 0.3\ Pa.s$

$S_{steel} = 7.85 \qquad t = ?$



$$F = \mu . \frac{AU}{t} = W$$

$$A = \pi D L = \pi \times 0.025 \times 0.3$$

$$= 0.023562\ m^2$$

$$W = mg = \rho V g = \gamma_s V$$

since $S = \frac{\gamma_s}{\gamma_w} \qquad \therefore \gamma_s = 7.85 \times 9810 = 77008.5\ N/m^3$

$$V = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.025)^2 \times 0.3 = 1.47 \times 10^{-4}\ m^3$$

$\therefore W = 11.34$ N

$\therefore 11.34 = 0.3 \times \dfrac{0.023562 \times 0.1}{t}$

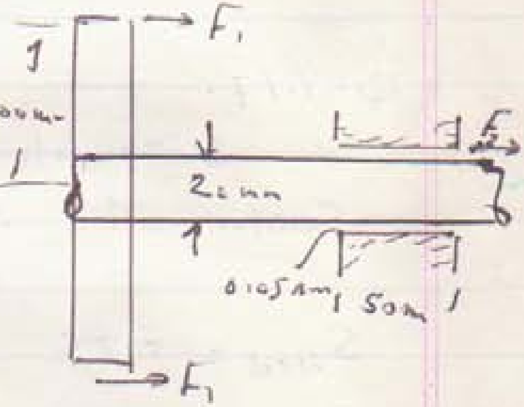
$\therefore t = \dfrac{0.3 \times 0.023562 \times 0.1}{11.34} = 6.233 \times 10^{-5}$ m

$= 0.06233$ mm.

1.16 $W = 600$ N $\quad R = 300$ mm $\quad D_s = 20$ mm

$N = 600$ rpm reduced by 1 rpm/s

find $\mu = ?$



الحل $T = F_1 R = I \alpha$

$F_1 \times \dfrac{300}{1000} = m R^2 \alpha$ ——— (1)

$\omega_2 = \omega_1 + \alpha t$

$\dfrac{\pi N_2}{30} = \dfrac{\pi N_1}{30} + \alpha \times 1$

$\therefore \alpha = \dfrac{599\pi}{30} - \dfrac{600\pi}{30} = -\dfrac{\pi}{30}$ deceleration

$I = m R^2 = \dfrac{600}{9.81} \times 0.3^2$

$\therefore F_1 = 1.92$ kN

ΣM ناخد عزوم حول shaft.

$F_1 R_1 = F_2 R_2$

$1.92 \times 0.3 = \mu \times \dfrac{\pi D N}{60} \times \dfrac{\pi \times 20 \times 50}{10^6} \times \dfrac{10^3}{0.05} \times 0.01$

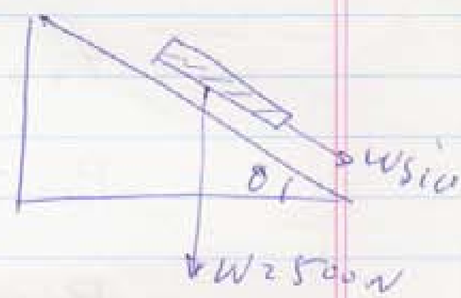
$\mu = 1.46$ N.s/m²

1.6 $W = mg = 3 \times 9.7 = 29.1\ N$

1.22 $W = 500\ N \quad A = 0.2\ m^2 \quad \theta = 30°$

$\mu = 0.1\ Pa.s \quad U = 1\ m/s$

$F = W\sin\theta = \mu \dfrac{AU}{t}$



$500 \sin 30 = 0.1 \times \dfrac{0.2 \times 1}{t}$

$t = \dfrac{0.1 \times 0.2 \times 1}{250}$

$= 0.08 \times 10^{-3}\ m$

$= 0.08\ mm$

1.15

$F = \mu \dfrac{AU}{t}$

$100 = \mu \times \dfrac{\pi \times 0.075 \times 0.2 \times 0.1}{0.07 \times 10^{-3}}$

$\mu = \dfrac{100 \times 0.07 \times 10^{-3}}{\pi \times 0.075 \times 0.2 \times 0.1} = 1.485\ Pa.s$

1.25

$R = \dfrac{8312}{M} = \dfrac{8312}{44} = 188.9\ m.N/kg.K$

1.29 a) $F = F_x + F_y + F_z$

$= 4i + 3j + 9k$

$F_z = F_k = 9\ N \quad F_s = \sqrt{F_x^2 + F_y^2} = \sqrt{16+9} = 5$ ✓

$P_N = \frac{9}{4} = 2.25\ P_a \quad P_s = \frac{5}{4} = 1.25\ P_a$

b) $F = -4i + 3j + 9k$

$F_z = F_k = -9 \qquad F_s = 5$

$P_N = \frac{-9}{4} = -2.25\ P_a$

$P_s = \frac{5}{4} = 1.25\ P_a$

1.30 $PV = mRT \qquad P = \rho RT$

$0.2 \times 10^6 \times 0.04 = m \times 210 \times 293$

$m = 0.13\ kg$

$0.2 \times 10^6 = \rho \times 210 \times 293$

$\rho = 3.25\ kg/m^3$

1.37 $k = \frac{\Delta P}{\frac{\Delta \rho}{\rho}} = \frac{0.06 \times 10^6}{0.02 \times 10^{-2}} = 3 \times 10^9\ P_a$

1.38 $k = -\frac{\Delta P}{\frac{\Delta V}{V}} = -\frac{\Delta P}{0.5 \times 10^{-2}} = 2.2 \times 10^9$

$\Delta P = -0.5 \times 2.2 \times 10^7 = -1.1 \times 10^7\ P_a$

$= 11\ MPa$

1.39 $V_2 = 1.01\ V_1 \qquad V_1 = \frac{m}{\rho} = \frac{450}{1000} = 0.45\ m^3$

$\therefore V_2 = 1.01 \times 0.45 = 0.4545$

$\Delta \rho = \rho \frac{\Delta P}{k} = 1000 \times \frac{70 \times 10^6}{2.06 \times 10^9} = 33.98\ kg/m^3$

$\therefore \rho_2 = 1000 + 33.98 = 1033.98 \ kg/m^3$

$\therefore M_2 = \rho_2 V_2 = 1033.98 \times 0.4545$

$M_2 = 469.94 \ kg$

$\therefore \Delta m = 469.94 - 450 = 19.94 \ kg$ be added.

1.40 $\quad K = -\dfrac{dP}{dV/V}$ and $PV = mRT$

$\therefore \dfrac{dP}{dV} = -\dfrac{mRT}{V^2}$

$\therefore K = +V \times +\dfrac{mRT}{V^2} = \dfrac{mRT}{V} = P$

$\therefore K = 0.4 \ MPa$

1.41 [534 of] C-1 الجدول

1.43 $\quad F_s = F_p$ or $2\sigma L = P_i D L$

$P_i = \dfrac{2\sigma}{D} = \dfrac{2 \times 0.0736}{0.1 \times 10^{-3}} = 1472 \ Pa$

[illegible] 1.44 $\quad h = \dfrac{2\sigma}{\gamma r} = \dfrac{2 \times 0.0701}{9810 \times 3 \times 10^{-3}} = 4.8 \ mm$

1.45 } fig 1.4
1.46 }

1.47 $\quad F = 2(\pi D \sigma) = 2 \times \pi \times 0.02 \times 0.0736$
$= 9.25 \times 10^{-3} \ N$

يستخدم البلاستيك لكونه لا يتآكل
Corrosion

1 CH-2 Fluid Statics

It can be divided in two parts

1. Study of pressure and its variation throughout a fluid.

دراسة الضغط وتغيره خلال المائع .

2. Study of pressure forces on a finite surface

دراسة قوة الضغط على سطوح غير محددة .

عندما يكون السائل ساكن فان القوة الوحيدة المؤثرة على السائل هي قوة الضغط على السطح وان اجهاد القص (Shear stress) يساوي صفر .

Pressure at a point

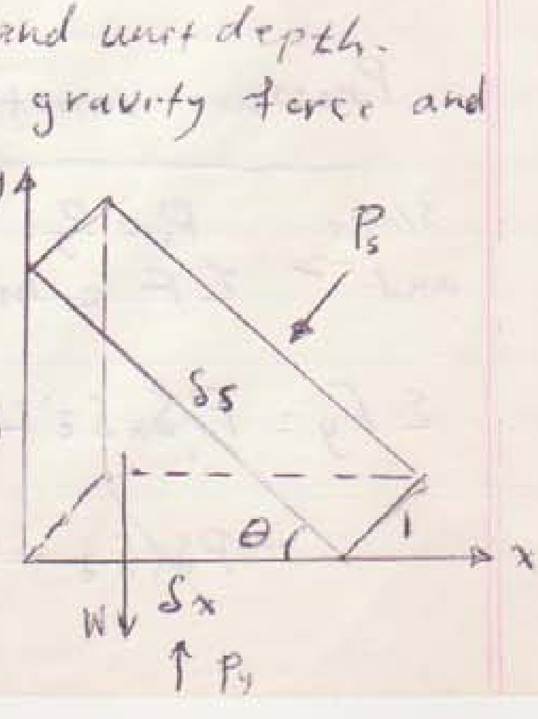
At a point a fluid at rest has the same pressure in all direction.

To determine this assume an element its coordenats dx, dy, ds, and unit depth.

since there is only only gravity force and pressure force.

i.e $\Sigma F = ma = 0$

(static fluid)



$\Sigma F_x = P_x \delta y \times 1 - P_s \delta s \sin\theta = \rho \frac{\delta x \delta y}{2} \times 1 \; a_x = 0$

$\Sigma F_y = P_y \delta_x \times 1 - P_s \delta_s \cos\theta - \gamma \frac{\delta_x \delta_y \times 1}{2} = \rho \frac{\delta_x \delta_y \times 1}{2} \times a_y = 0$

Since P_x, P_y & P_s is the average pressure on each phase., a_x & $a_y = 0$ static fluid -
and

$$\delta s \sin\theta = \delta y \quad , \quad \delta s \cos\theta = \delta x$$

$$\therefore \quad \Sigma F_x = P_x \delta y - P_s \delta y = 0 \qquad \therefore \boxed{P_x = P_s}$$

also

$$\Sigma F_y = P_y \delta x - P_s \delta x - \gamma \frac{\delta x \delta y}{2} = 0$$

since

$$\delta x \delta y \longrightarrow 0 \text{ with respect to } \delta x \text{ or } \delta y$$

$\therefore$

$$P_y = P_s \qquad \text{and} \qquad \boxed{P_x = P_y = P_s = P}$$

Pressure Variation in static fluid

since $P_x = P_y = P_z = P$
and $\Sigma F = ma = 0$ static fluid

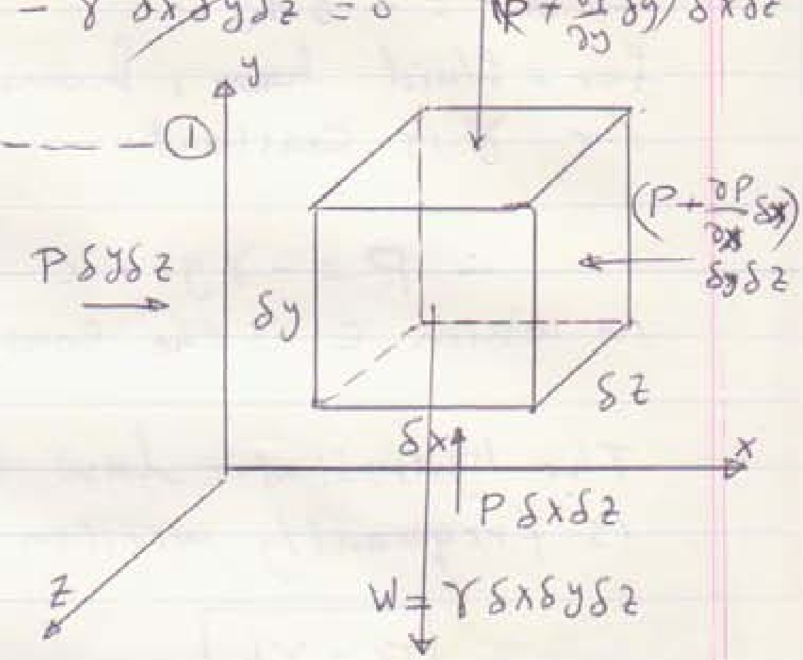
$$\Sigma F_y = P \delta_x \delta z - \left(P + \frac{\partial P}{\partial y} \delta y \right) \delta x \delta z - \gamma \delta_x \delta_y \delta_z = 0$$

$$= P \delta_x \delta z - P \delta_x \delta z - \frac{\partial P}{\partial y} \delta y \delta x \delta z - \gamma \delta x \delta y \delta z = 0$$

2

$$= -\frac{\partial P}{\partial y}\,\delta y\,\delta x\,\delta z - \gamma\,\delta x\,\delta y\,\delta z = 0$$

$$\frac{\partial P}{\partial y} = -\gamma \qquad \text{------ (1)}$$



$$\Sigma F_x = P\,\delta y\,\delta z - \left(P + \frac{\partial P}{\delta x}\delta x\right)\delta y\,\delta z = 0$$

$$= -\frac{\partial P}{\partial x}\,\delta x\,\delta y\,\delta z = 0$$

since $\delta x\,\delta y\,\delta z \neq 0$ (Volume حجم التريحز)

$$\therefore \frac{\partial P}{\partial x} = 0 \qquad \text{------ (2)} \qquad \therefore P = c \text{ ثابت}$$

also

$$\Sigma F_z = -\frac{\partial P}{\partial z}\,\delta x\,\delta y\,\delta z \qquad \text{i.e}$$

i.e

$$\frac{\partial P}{\partial z} = 0 \qquad \text{--- (3)} \qquad \therefore P = c \text{ ثابت}$$

$\therefore$ P in x & z direction is constant and P is a function of y in y-dir. only.

$\therefore \; dp = -\gamma dy$

for a fluid ~~h~~, homogeneous and incompressible i.e γ is constant.

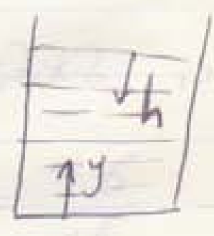
$$\therefore \; P = -\gamma y + c$$

in which c is the constant of integration

The hydrostatic law of pressure variation is frequantly written in the form:

$$\boxed{P = \gamma h} \qquad \text{when} \quad h = -y$$

h = the depth of fluid from the surface to a certain point.



3.

Pressure Variation in a compressible fluid:

When the fluid is a perfact gas at rest and Constant temprature:

i.e $P = \rho R T$, T = Const. + R = Const.

or $\frac{P}{\rho}$ = Const. i.e $\frac{P}{\rho} = \frac{P_0}{\rho_0} \Rightarrow \rho = \rho_0 \frac{P}{P_0}$ -- (1)

and $dP = -\gamma dy = -\rho g dy$ -- (2)

from equ. 1, 2

$$dP = -\rho_0 \frac{P}{P_0} g\, dy$$

$$\therefore \int_{y_0}^{y} dy = -\frac{P_0}{\rho_0 g} \int_{P_0}^{P} \frac{dP}{P}$$

$$\therefore y - y_0 = -\frac{P_0}{\rho_0 g} \ln \frac{P}{P_0}$$

$$\therefore P = P_0 \exp\left(-\frac{(y - y_0)}{P_0/\rho_0 g}\right) \quad \text{-- (3)}$$

which is the equation of variation of pressure for isothermal gas, T = const.

since the atmospheric frequently is assumd