to have a temperture gradiant

$$T = T_0 + \beta y \qquad \beta = -0.00651 \ ^\circ C/m$$

$$\therefore \rho = \frac{P}{RT} = \frac{P}{R(T_0 + \beta y)}$$

$$\therefore \boxed{dP = -\frac{P}{R(T_0 + \beta y)} g \, dy} \quad \text{--- (4)}$$

B) Units and Scales of pressure measurment :

Pressure may be expressed with respect to any aribitrary datum - The usual datum are absolute zero and local atmospheric pressure.

absolute pressure = pressure gage + barometer reading

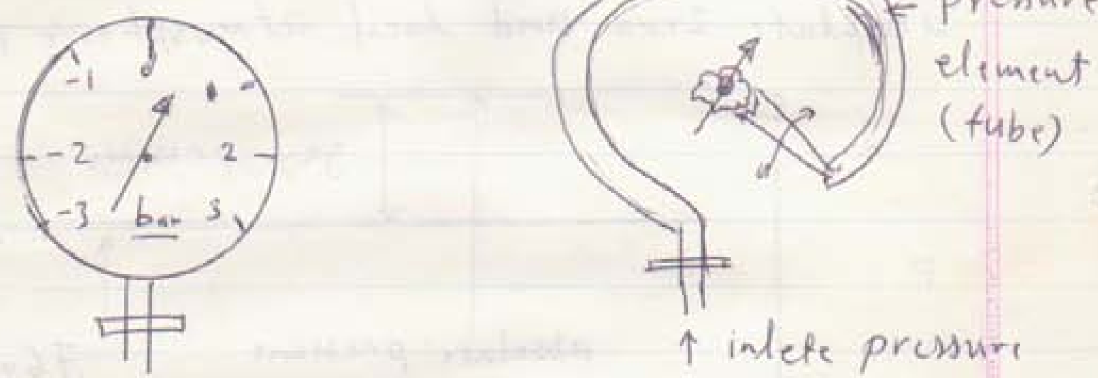
قراءة البارومتر + ضغط العيار = الضغط المطلق
(الضغط الجوي)

علماً ان الضغط الجوي يقاس بواسطة البارومتر ويتغير من منطقة الى آخر ويسمى (local barometer P) .

$$\therefore P_{abs} = P_g + P_{atm}$$

1- Bourdon gage مقياس بوردون

Typical devices used for measuring gage pressure.

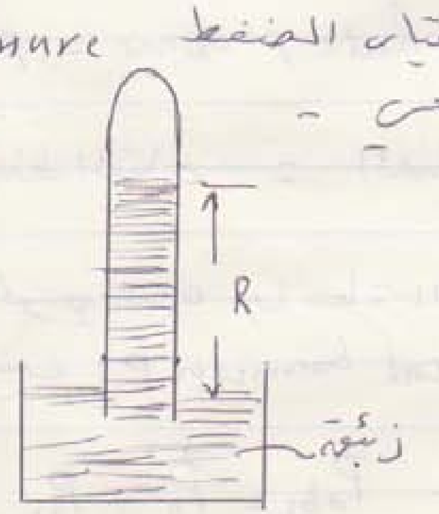


مبدأ عمله يعتمد على تمدد الأنبوب حيث يتكون من معدن ذو قابلية تمدد وبالتالي يتحرك المؤشر.

2- Barometer بارومتر

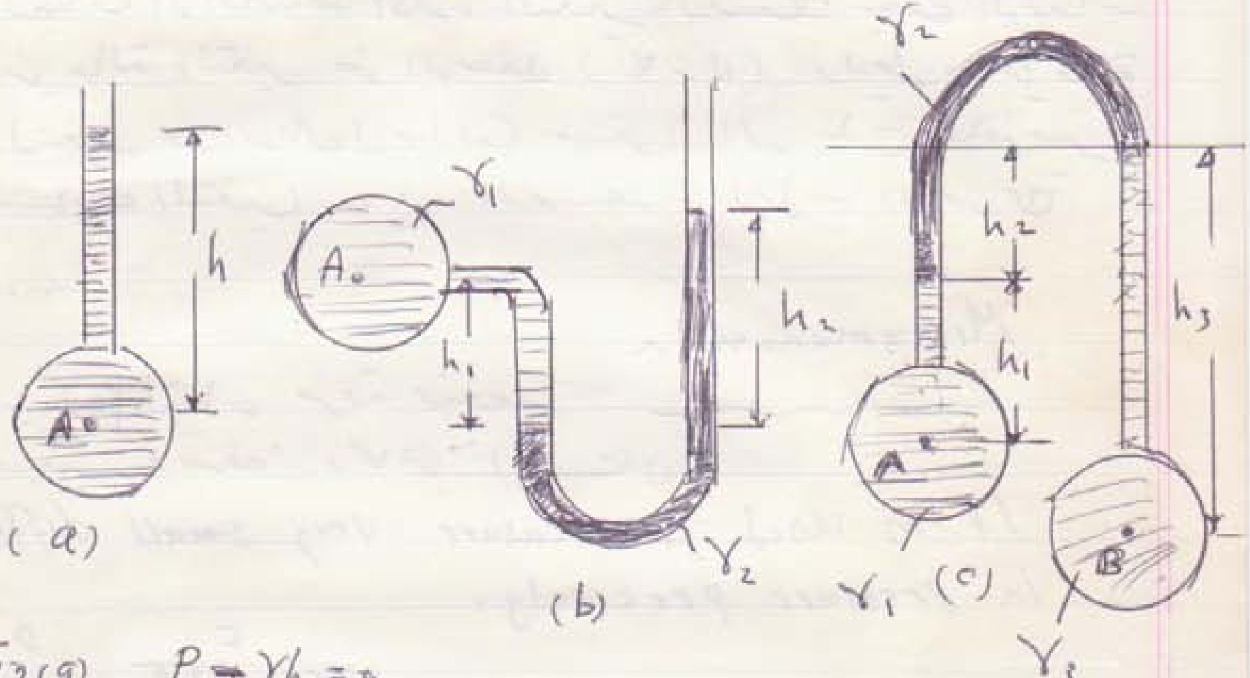
devices used to measure the local atmospheric pressure جهاز يستخدم لقياس الضغط الجوي الموقعي.

$$P = \gamma h = \gamma R$$



3- Manometers :-

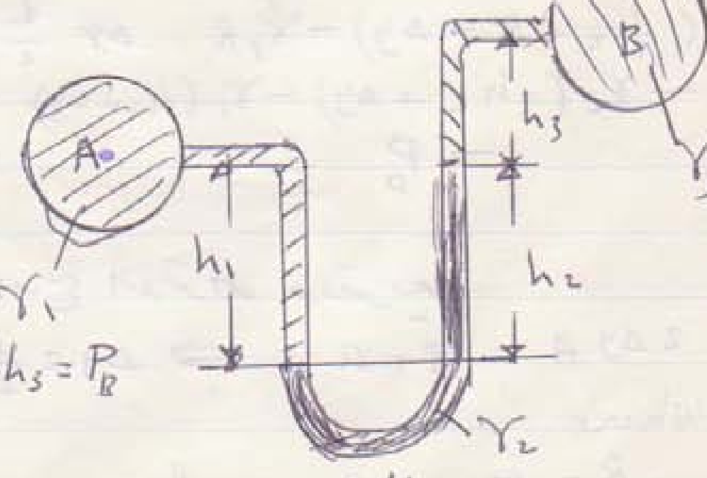
It is used to determine the differance in pressure



(a) (b) (c)

Fig (a) $P_A - \gamma h = 0$

$\therefore P_A = \gamma h$



(d)

Fig (b)

$P_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$

Fig (c)

$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_3 h_3 = P_B$

Fig (d)

$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$

يمكن حساب الضغط في جميع المانوميترات وفقاً لما يلي

١. نبدأ بضغط احدى جهتي المانوميتر مثلاً P_A على انه تلاحظ الوحدات.

٢. نضيف الى الضغط اعلاه التغير بالضغط بنفس الوحدات في حالة التغير نحو الاسفل (نازل) ونطرح في حالة التغير نحو الاعلى (صاعد) حتى الى نهاية المانوميتر

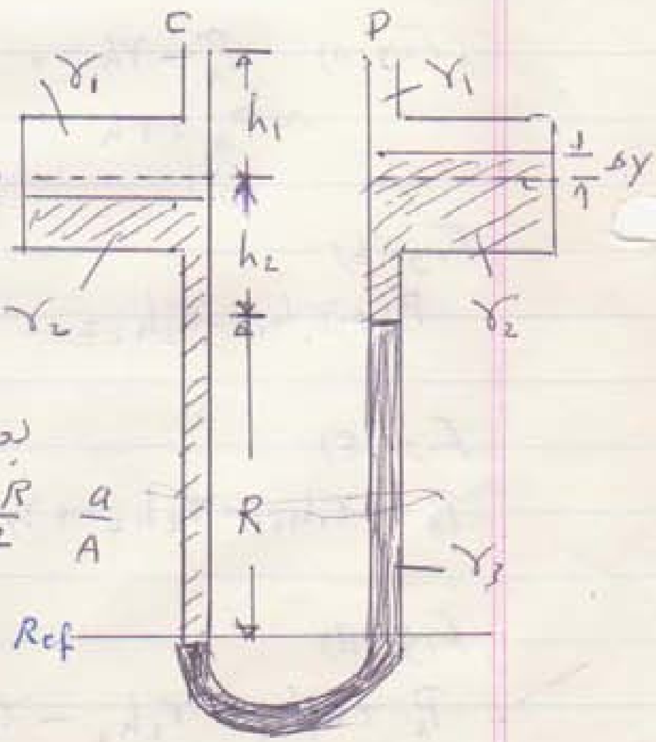
٣. تساوي المتغيرات مع الضغط في الجهة الاخرى.

Micromanometer

يستخدم لقياس فرق ضغط صغير وبدقة وكما في الشكل.

It is used to measure very small differeance in pressure precisely.

$$P_C + \gamma_1 (h_1 + \Delta y) + \gamma_2 (h_2 + R - \Delta y) - \gamma_3 R - \gamma_2 (h_2 + \Delta y) - \gamma_1 (h_1 - \Delta y) = P_D$$



بعد نقل الاحجام وتعويض

$$2 \Delta y A = R \cdot a \Rightarrow \Delta y = \frac{R}{2} \frac{a}{A}$$

Where

R = manometer reading

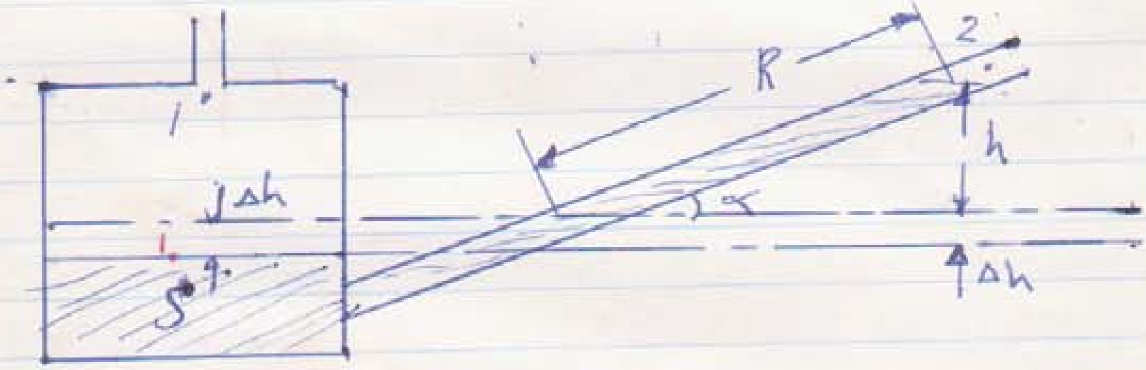
a = مساحة مقطع الانبوب

A = مساحة المقطع الكبير

$$\boxed{P_C - P_D = R \left\{ \gamma_3 - \gamma_2 \left(1 - \frac{a}{A}\right) - \gamma_1 \frac{a}{A} \right\}}$$

Inclined Manometer

يستخدم لقراءة ضغط الغاز المحصور في الخزانات. يتم التصفير عندما تكون الفتحات A ، B مفتوحة، ويحدد ارتفاع السائل في الأنبوب المائل حيث يرتفع بسبب فرق المساحة، ثم نغلق A فيتحرك السائل في الأنبوب ويقرأ الارتفاع الجديد



$$P_1 - S\gamma_w (h + \Delta h) = P_2$$

$$P_1 - P_2 = S\gamma_w (h + \Delta h) \quad \text{—— (1)}$$

Since

$$h = R \sin\alpha \quad \text{and} \quad \Delta h\, A = R \cdot a \quad \text{—— (2)}$$

$a =$ مساحة مقطع الأنبوب m^2

$A =$ " " الخزان

$R =$ قراءة المانومتر

$\alpha =$ زاوية الميل

$$\therefore P_1 - P_2 = S\gamma_w \left(R\sin\alpha + \frac{a}{A}R\right)$$

or

$$P_1 - P_2 = S\gamma_w \left(\sin\alpha + \frac{a}{A}\right) R \quad \text{—— (3)}$$

or

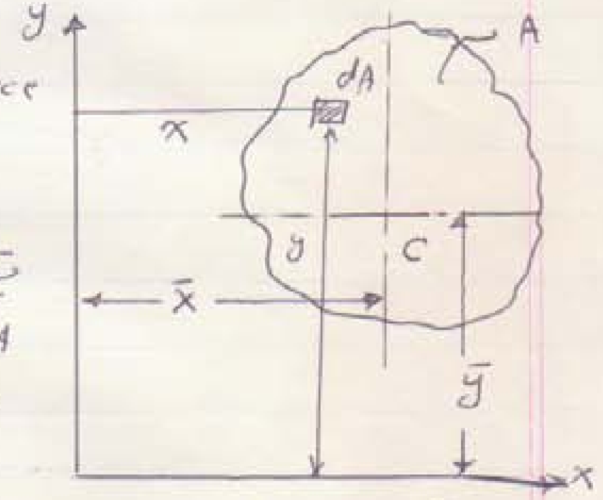
$$\boxed{P_1 - P_2 = C\,R} \quad \text{—— (4)} \qquad \text{Where } C = S\gamma_w \left(\sin\alpha + \frac{a}{A}\right)$$

Force on plane area

:- The distributed forces resulting from the action of fluid on finite area may be conveniently replaced by a resultant force

ان توزيع القوة على السطوح يمكن ان يتحول الى محصلة قوى

1- Horizontal surface سطح افقي



The magnitude of the force acking on one side of the surface is

قيمة القوة مؤثرة على سطح افقي

$$F = \int P\,dA = P\int dA = PA$$

حيث P ثابت على السطح

$$\therefore \boxed{F = PA}$$

To find the action of the resultant force take an element and take the moment of the distribution force about any axis say xy plane

ان نقطة او مركز تأثير القوة في مركز المساحة وابعاده $\bar{x}$ ، $\bar{y}$

The moment about y-axis

$$PA\bar{x} = \int_A xP\,dA \qquad \text{since } P = \text{constant}$$

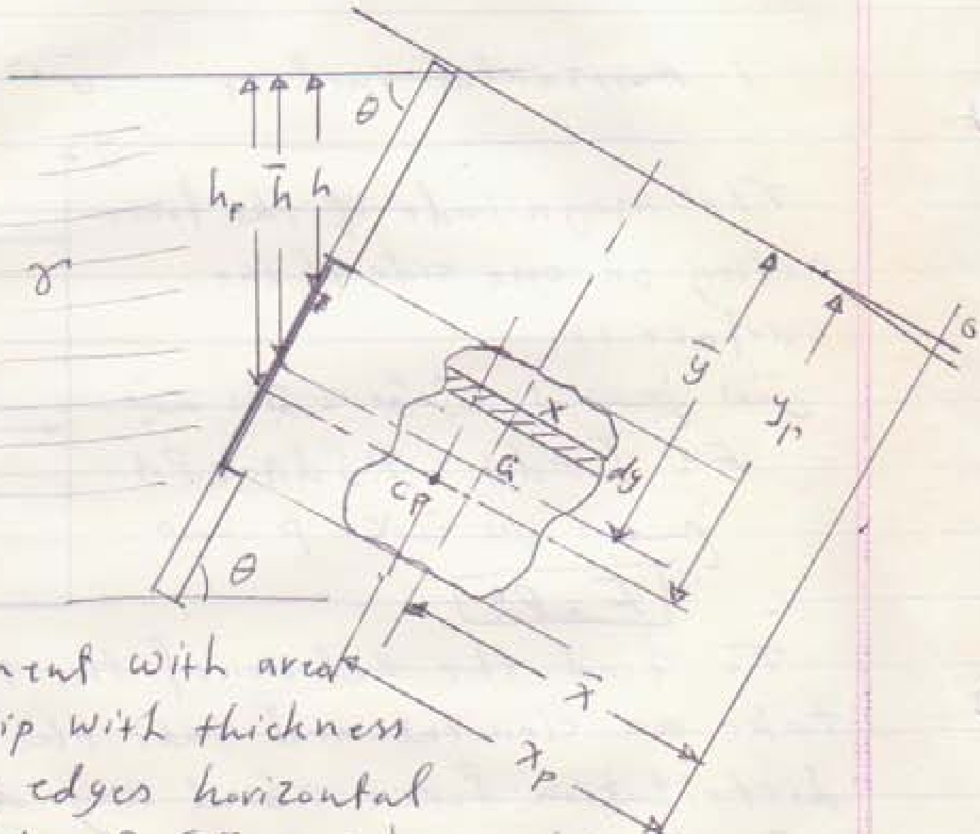
$$\therefore \bar{x} = \frac{1}{A}\int_A x\,dA$$

$\bar{x}$ = the distance from the y-axis to the Centroid of the area بعد مركز المساحة عن المحور y

also $\bar{y} = \frac{1}{A}\int_A y\,dA$

2- Inclined surface سطح مائل

$\bar{y}$ = بعد مركز المساحة المائل من مستوى سطح السائل



For an element with area δA as A a strip with thickness dy with long edges horizontal the magnitude of δF acting on it is

$$\delta F = P\,\delta A = \gamma h\,\delta A = \gamma y \sin\theta\ \delta A$$

since all elemental forces are parallel

$$\therefore\ F = \int P\,dA = \gamma \sin\theta \int y\,dA$$

from appendix (A) $\int y\,dA = \bar{y}A$

$\therefore F = \gamma \sin\theta \, \bar{y} A = \gamma \bar{h} A = \boxed{P_G A = F}$

$P_G =$ الضغط في مركز المساحة المغمورة

$\therefore F =$ pressure at the centroid $\times$ area

القوة = الضغط في مركز المساحة المغمورة × المساحة المغمورة من السطح

Center of Pressure

ان مركز تأثير القوة في نقطة ~~~~~~

تسمى مركز الضغط واحداثياتها x_p , y_p

To find the pressure Center : take a moment about the axises i.e

$$x_p F = \int x \, p \, dA \quad , \quad y_p F = \int_A y \, p \, dA$$

$$\therefore \quad x_p = \frac{1}{F} \int_A x \, p \, dA \quad , \quad y_p = \frac{1}{F} \int_A y \, p \, dA$$

$\therefore$ from fig

$$x_p = \frac{1}{\gamma \bar{y} A \sin\theta} \int_A x \gamma y \sin\theta \, dA$$

$$= \frac{1}{\bar{y} A} \int_A x y \, dA$$

from Appendix A $\int_A x y \, dA = I_{xy} =$ عزم القصور الذاتي حول المحاور

$$\therefore x_p = \frac{I_{xy}}{\bar{y} A} \qquad \text{also} \quad I_{xy} = \bar{I}_{yx} + \bar{x} \bar{y} A$$

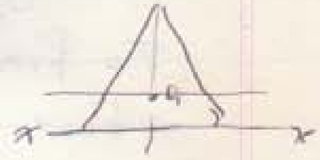
$$\therefore x_p = \frac{\bar{x} \bar{y} A + \bar{I}_{xy}}{\bar{y} A} = \boxed{\therefore x_p = \bar{x} + \frac{\bar{I}_{xy}}{\bar{y} A}}$$

also $$y_p = \frac{1}{\bar{y}A}\int_A y^2\,dA$$
and
$$\left.\begin{aligned} \int y^2\,dA &= I_x \\ \& \; I_x &= I_G + \bar{y}^2 A \end{aligned}\right\} \text{(Appendix A)}$$

$$\boxed{\therefore \; y_p = \bar{y} + \frac{\bar{I}_G}{\bar{y}A}}$$

$\bar{I}_x =$ عزم القصور الذاتي حول المحور $x-x$

$\bar{I}_G =$ " " " حول المركز.



Force components on curved surface:-

1- Horizontal Component: المركبة الأفقية

~~Take~~ It is equal to the pressure force exerted on a projection of the curved surface - The vertical plane of projection is normal to the direction of the component.

$$\delta F_x = P\,\delta A \cos\theta$$

$$F_x = \int_A P \cos\theta \, dA$$

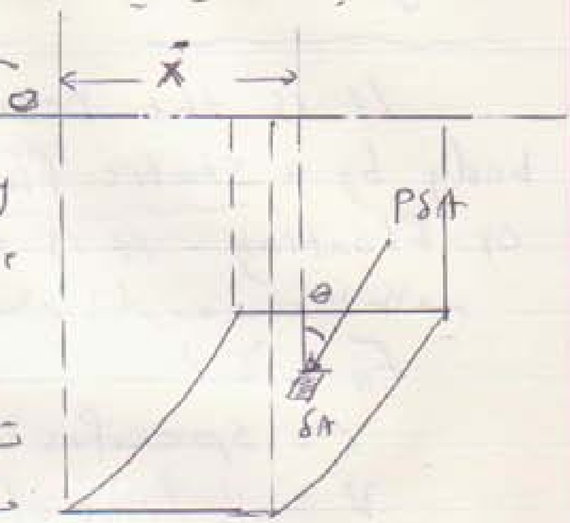
$\cos\theta\,\delta A$ مسقط مساحة الشريحة

δA على المحور العمودي على المحور $x-x$.

$F_H =$ الضغط في مركز المساحة × مسقط المساحة المعزول من الجسم على المحور العمودي.

$$\boxed{F_H = P_G \times A}$$

2- Vertical Component المركبة العمودية



It is equal to the weight of liquid vertically above the curved surface and extending up to the free surface.

تساوي الوزن السائل فوق المنحني مستمرة الى سطح السائل .

$$F_v = \int_A p\cos\theta\, dA \qquad p = \gamma h$$

$$\therefore F_v = \gamma \int_A h\cos\theta\, dA$$

$\cos\theta\,\delta A =$ مسقط الشريحة δA على المحور $x-x$

$$\therefore F_v = \gamma \int_V dV \qquad \text{where } h\delta A = V$$

حجم السائل فوق الشريحة

$$\boxed{\therefore F_v = \gamma V}$$

$\gamma =$ معامل الكثافة الوزنية للسائل

$V =$ حجم السائل فوق المنحني

$$\bar{x} = \frac{1}{V}\int_V x\, dV$$

لايجاد بعد احداثيات الحجم

= The distance from O to the line of action

δV = the volume of the prism of heigh h and base $\cos\theta\,\delta A$ or the volume of liquid vertically above the area element

Buoyant Force   القوة الرافعة

It is the resultant force exerted on a body by a Static fluid in which it is Submerged or Floating. It is always acts vertically upward.

هي محصلة القوة المؤثرة على الاجسام المغمورة او الطافية وتكون اتجاهها دائماً الى الاعلى

$$F_B = \gamma V$$

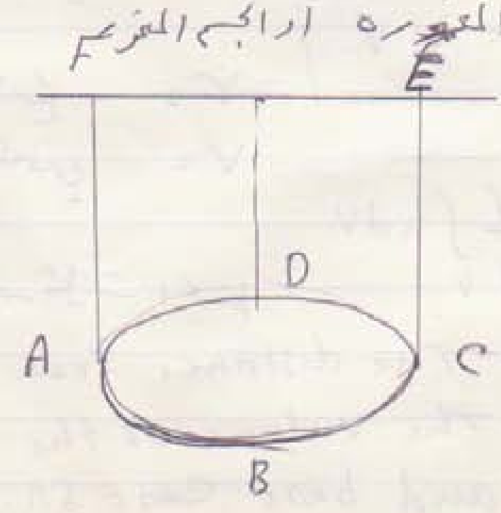
γ = specific weight of the liquid N/m^3

V = Volume of fluid displaced

$$\bar{x} = \frac{1}{V}\int_V x\,dV$$

$$F_B = F_{V_{ABCEFA}} - F_{V_{ADCEFA}}$$

The buoyant force acts through the Centroid of the displaced Volume of fluid. القوة الرافعة تؤثر في مركز المساحة المغمورة او الجسم المغمور

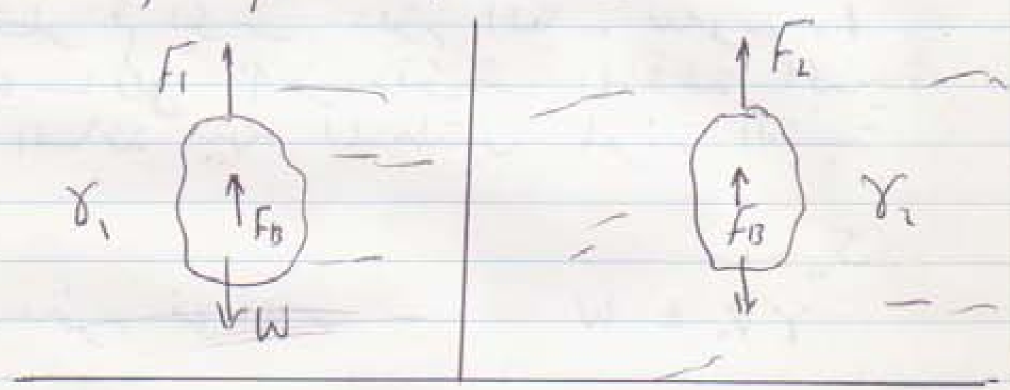


In solving a static problem involving submerged or floating objects. weighing an odd-~~sta~~ shaped object syspended in two different fluids yields sufficent data, To determine its weight, volume, unit gravity force and relative density as shown in fig.

F_1 = قوة شد الخيط

W = gravity force.

V = volume of liquid dispaced



The equilbrium equation are written

$$F_1 + \gamma_1 V = W \qquad F_2 + \gamma_2 V = W$$

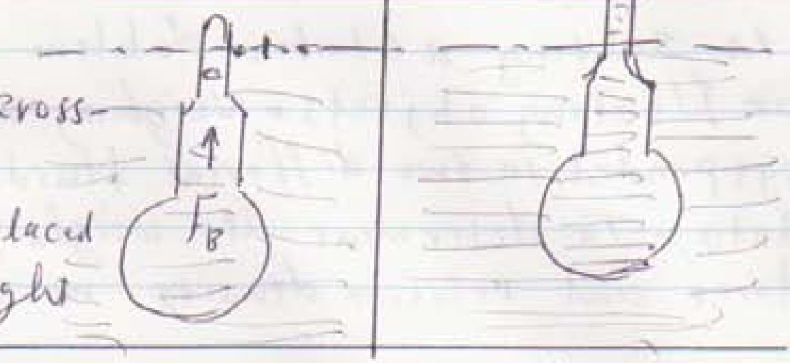
$$\therefore V = \frac{F_1 - F_2}{\gamma_2 - \gamma_1} \quad \text{and} \quad W = \frac{F_1 \gamma_2 - F_2 \gamma_1}{\gamma_2 - \gamma_1}$$

من المعادلات اعلاه. يمكن استنتاج طريقة تدريج مقياس الوزن النوعي للسوائل.

Hydrometer

It is uses the principle of bouyant force to determine the relative density of liquids.

assume
a = the upper cross-section
V_0 = volume displaced
γ = specific weight of destial water



S = relative density of liquid to be determined.

Hydrometer ~~النوعي~~ نفس الطريقة اعلاه لغمر مقياس الوزن
في ماء مقطر ثم نؤشر علامة المقياس بعلامة 1.0 .
ثم نغمر في سائل آخر ونؤشر النقطة الجديدة
وتقسم المسافة بين النقطتين لمعرفة النسبة .

$$S_w = 1$$

$$\gamma V_0 = W$$ المقياس مغمور في الماء

$S = ?$

$$(V_0 - \Delta V) \cdot S\gamma = W \quad \text{--- (2)}$$

and

$$\Delta V = a \, \Delta h \quad \text{--- (3)}$$ المسافة بين النقطتين $= \Delta h$

subsituting (3) in (2)

$$\therefore \gamma V_0 = (V_0 - a\,\Delta h)\,\gamma \cdot S$$

$$\therefore V_0 = S V_0 - a S \Delta h$$

$$\boxed{\therefore \Delta h = \frac{V_0}{a}\left(\frac{S-1}{S}\right)}$$

Ex. 2.12

$W = 1.5\ N$ وزن الجسم في الهواء

قوة شد الخيط $F = 1.1\ N$ 〃 〃 الماء

$\gamma V =$ وزن حجم السائل المزاح

$\therefore W = F + \gamma V$

$1.5 = 1.1 + 9806\ V \qquad \therefore V = 0.0000408\ m^3$

$\therefore V = 40.8\ cm^3$

$W = S \gamma_w V \qquad \therefore S = \dfrac{1.5}{9806 \times 0.0000408} = 3.75$

Stability of floating and submerged bodies

A body floating in a static liquid has vertical stability

Stable Unstable Neutral.

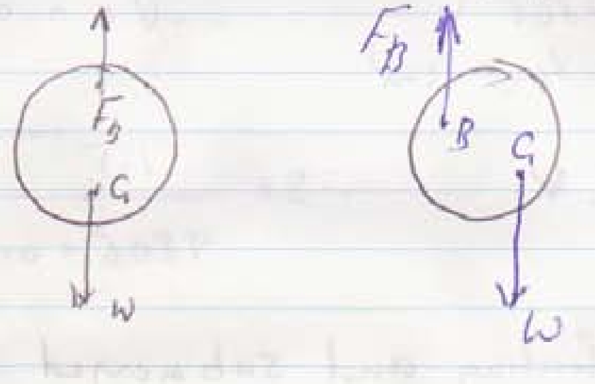
A body has linear stability when a small linear displacement in any direction sets up restoring force tending to return it to its original position.

يكون الجسم مستقر عند امالته بتأثير عليه بعزم ويعود الى حالته الاصلية
اما في حالة عدم العودة الى حالته الاصلية يعتبر الجسم غير مستقر، اما
اذا دار حول نفسه فيسمى (neutral)

Determination of Rotational stability of floating objects :-

Any floating object with center of gravity below its center of buoyancy (center of displaced volum) floats in stable equilibrium.

اي جسم يقع مركز ثقله اسفل مركز تأثير القوة الدافعة (مركز الجزء المغمور) يكون الجسم مستقر.



Stable Unstable

When a body submerged or floating in a liquid as shown below

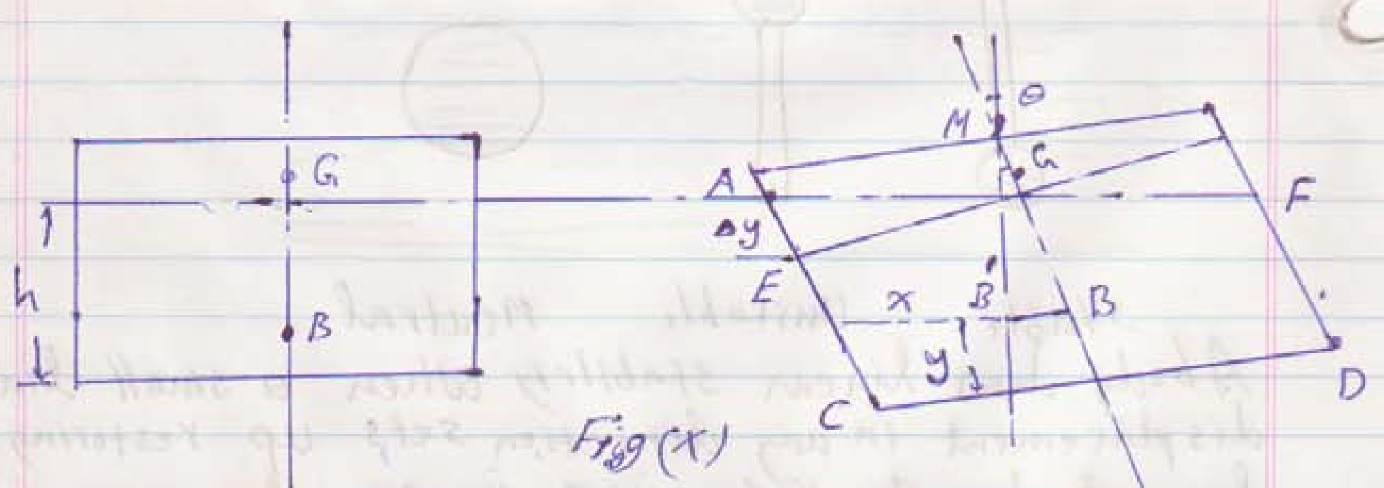


Fig (X)

B' the center of buoyant force acts upward and G the center of gravity of the body

The Intersection of buoyant force and the Centerline is called the metacentric, designated M.

M above G the body is stable
M at G " " " neutral
M below G " " " unstable

The distance MG is called the metacentric height.
Then the restoring Coupling is $= W\, MG \sin\theta$

in which θ is the angular displacement and ~~th~~ W the weight of the body.

Ex: As shown in fig(x) above a block 6m wide and 20 m long has a gross mass of 200 Mg. Its Center of gravity is 30 cm above the water surface. Find the Metacentric hight and restoring Couple when $\Delta y = 30$ cm.

The depth of submegence in water is

$$F_B = W$$
$$\gamma V = 200\,000 \times 9.81$$
$$9810 \times 6 \times 20 \times h = 200\,000 \times 9.81$$
$$h = 1.667\ m$$

The centroid in the tipped position is located with moment about AC and CD

$$\bar{x} = \frac{1.367 \times 6 \times 3 + 0.6 \times 6 \times \frac{1}{2} \times 2}{1.667 \times 6} = 2.82 \text{ m}$$

$$\bar{y} = \frac{1.367 \times 6 \times \frac{1.367}{2} + 0.6 \times 6 \times \frac{1}{2}(0.2 + 1.367)}{1.667 \times 6} = 0.842$$

By similar triangle AEO and B'BM

$$\frac{\Delta y}{\frac{b}{2}} = \frac{B'B}{MB} \qquad \Delta y = 0.3 \qquad \frac{b}{2} = 3 \text{ m}$$

$$B'P = 3 - 2.82 = 0.18 \text{ m}$$

$\therefore MB = 1.8$ m

G = 1.967 from the bottom (CD)

$\therefore GB = 1.967 - 0.842 = 1.125$ m

$$\therefore MG = MB - GB = 1.8 - 1.125$$
$$= 0.675 \text{ m}$$

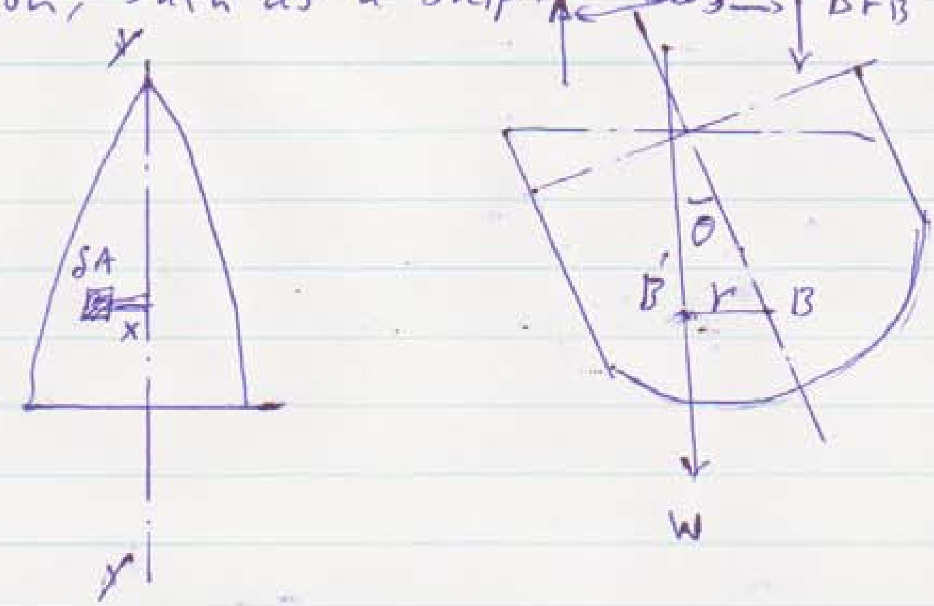
$\therefore$ the body is stable where MG is positive.

The restoring coupling $= W\, MG \sin\theta$

$$= 200\,000 \times 9.806 \times 0.675 \times \frac{0.3}{\sqrt{3^2 + 0.3^2}}$$
$$\therefore = 131 \text{ kN.m.}$$

Nonprismatic Cross-section body

For a floating object of variable cross-section, such as a ship:



The restoring Coupling $F_B \cdot S$ should be equal to $W r$ where W is the weight of the body and r the distance shift.

$$\Delta F_B \cdot S = W \cdot r$$

if we take an element δA on horizontal section through the body at liquid surface.

The volume $x \theta \delta A$

Force $= \gamma V = \gamma x \theta \delta A$

moment about $O = \gamma x^2 \theta \delta A$ for small θ

$$\therefore \Delta F_B \cdot S = \gamma \theta \int_A x^2 \, dA = \gamma \theta I$$

where $\int_A x^2 \, dA = I$ (moment of Inertia)

$\therefore \gamma \theta I = Wr = \gamma' V r$
wher V is the total volume of liquid displaced
since θ is very small

$$\therefore MB \sin\theta = MB\,\theta = r \quad \text{or} \quad MB = \frac{r}{\theta} = \frac{I}{V}$$

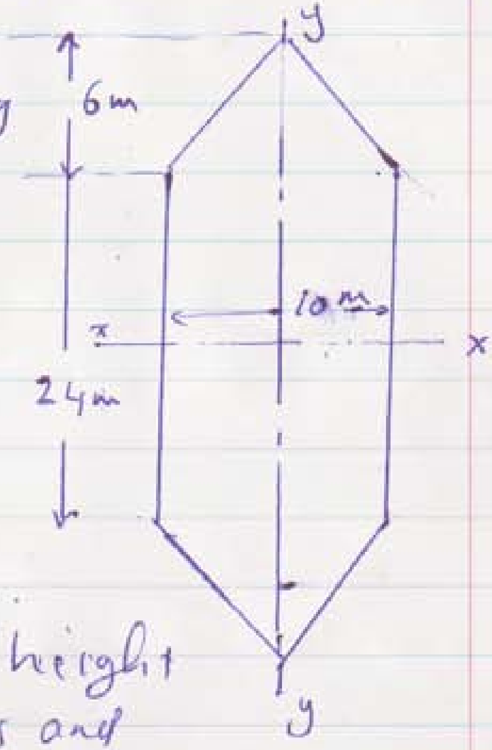
The metacentric ~~moment~~ height is then

$$MG = MB \mp GB$$

$$\text{or} \quad MG = \frac{I}{V} \mp GB$$

The minus sign is used if G is above B −
and the plus sign when G is below B +

Ex. A body displacing 1 Gg has the horizontal cross-section at the waterline shown in fig. Its center of buoyancy is 2 m below the water surface and its center of gravity is 0.5 m below the water surface. Determine the metacentric height for rolling about ~~y-y~~ axis and pitching about ~~x-x~~ axis.



mass displaced = 1 Gg = 1 000 000 kg

Center of buoyancy = 2 m below water surface
" " gravity = 0.5 m " " "

$\therefore GB = 2 - 0.5 = 1.5$ m
as for rolling about y-y axis's

$$V = \frac{\text{mass displaced}}{\text{density}} = \frac{1000\,000}{1000} = 1000\ m^3$$

$$I_{yy} = \frac{bh^3}{12} + 4\left(\frac{1}{12}bh^3\right) = \frac{1}{12} \times 24 \times 10^3 + 4 \times \frac{1}{12} \times 6 \times 5^3$$

$$= 2250\ m^4$$

$$I_{xx} = \frac{1}{12} \times 10 \times 24^3 + 2 \times \frac{1}{36} \times 10 \times 6^3$$

$$= 23400\ m^4$$

For rolling $MG = \frac{I}{V} - GB = \frac{2250}{1000} - 1.5 = 0.75$ m

For pitching $MG = \frac{23400}{1000} - 1.5 = 21.9$ m

$\therefore$ the body is stable.

Relative Equilibrium

Fluid masses in Relative equilibrium-

for steady flow masses in motion no shear stress will occure if there is no relative motion between adjacent Layer of the fluid.

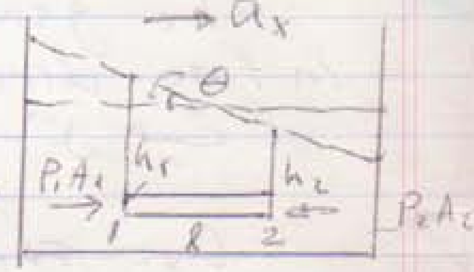
1- Uniform linear acceleration:

a. horizontal acceleration:

$$\Sigma F = ma$$

$$P_1\,dA - P_2\,dA_2 = \frac{\gamma}{g}\,l\,dA\,a_x$$

or $$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{l\,a_x}{g}$$ dA [illegible] γ



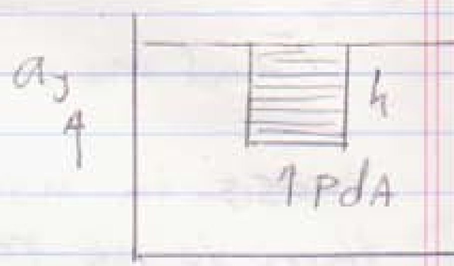
$$h_1 - h_2 = \frac{l\,a_x}{g}$$

or $$\frac{h_1 - h_2}{l} = \frac{a_x}{g}$$

from fig- the left side is the slope

$$\boxed{\therefore \tan\theta = \frac{a_x}{g}}$$

b- Vertical acceleration :-



$$\Sigma F_y = m\, a_y$$

$$\therefore PdA - \gamma h\, dA = \frac{\gamma h\, dA}{g}\, a_y$$

$$\therefore P = \gamma h\left(1 + \frac{a_y}{g}\right) \qquad \text{upward}$$

$$P = \gamma h\left(1 - \frac{a_y}{g}\right) \qquad \text{down ward.}$$

The general equation for a tank moved in two direction x & y

عندما يتحرك جسم او خزان على سطح مائل حيث

a_x = The acceleration in x-dir.
a_y = 〃 〃 〃 y-dir.
P_o = The intial pressure and equal to atmospheric pressure when the tank is opend.

$$\boxed{P = P_o - \gamma \frac{a_x}{g} x - \gamma\left(1 + \frac{a_y}{g}\right) y} \quad \text{—— (1)}$$

and $$\boxed{\tan\theta = -\frac{a_x}{a_y + g}}$$ ميل السطح —— (2)

(−) لتعني الزاوية في الربع الثاني.

2- Uniform Rotational Vortex flow

Consider liquid rotating about the central axises with angular velocity (ω) rad/sec.

The slope of water caused by normal acceleration (a_n) and the gravitational acceleration (g).

$$\text{slope} = \frac{dh}{dr} = \frac{a_n}{g}$$

$$\therefore dh = \frac{a_n}{g}\,dr$$

since $a_n = \omega^2 r$

$$\therefore dh = \frac{\omega^2 r^2}{g}\,dr$$

or $h = \dfrac{\omega^2 r^2}{2g} + c$ at $r = 0$ $h = 0$

$\therefore c = 0$

$$\therefore \boxed{h = \frac{\omega^2 r^2}{2g}} \quad \text{—— (1)}$$

$$\boxed{P = P_0 + \gamma \frac{\omega^2 r^2}{2g} - \gamma y} \quad \text{—— (2)}$$

$\gamma \dfrac{\omega^2 r^2}{2g}$ تمثل التغير بالضغط تبعاً لـ r

التغير بالضغط $= \gamma y$ تبعاً لـ y-dir

$y = -h$