الجامعة التكنولوجية
قسم هندسة المكائن و المعدات

Fluid Mechanics II
ميكانيك الموائع(2)
(ملخص)

3^{rd} year Mechanical Engineering
المرحلة الثالثة- عام/تكييف/سيارات

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مدرسو المادة

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أستاذ مساعد	أستاذ مساعد

Syllabus

مفردات المادة (فرع الميكانيك العام)

Subject No.: ME\653	**رقم الموضوع: همك/ 653**
Subject: Fluid Mechanics II	**الموضوع: ميكانيك الموائع II**
Units:5	**الوحدات :5**
Weekly Hours: Theoretical:2	**الساعات الإسبوعية: نظري:2**
Tutorial:1	**مناقشة:1**
Practical:1	**عملي:1**

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3	- Hydrodynamic lubrication	- التزييت الهيدروديناميكي	3
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Syllabus

مفردات المادة (فرعي التكييف و السيارات)

Subject No.: ME\663 **Subject: Fluid Mechanics II** **Units:5** **Weekly Hours: Theoretical:2** **Tutorial:1** **Practical:1**	**رقم الموضوع: همك/ 663** **الموضوع: ميكانيك الموائع II** **الوحدات :5** **الساعات الإسبوعية: نظري:2** **مناقشة:1** **عملي:1**

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18	**Pumps** - Specific speed - Dimensional analysis and similarity laws for pumps	**المضخات** - السرعة النوعية - التحليل البعدي و قوانين التشابه للمضخات	18
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1- Fluid Mechanics By Streeter & Wylie

2- Fluid Mechanics with Engineering Applications
By Daugherty & Franzini

3- Introduction to Fluid Mechanics
By Fox & McDonald

4- Engineering Fluid Mechanics
By Roberson & Crowe

ملحوظة:
هذه المحاضرات تتضمن ملخص لمادة ميكانيك الموائع II للمرحلة الثالثة وهي لا تغني الطالب عن حضور الساعات النظرية المقررة في جدول الدروس الأسبوعي.

Chapter One
Navier – Stokes Equations

Contents

1- Navier-Stokes equations.
2- Steady laminar flow between parallel flat plates.
3- Hydrodynamic lubrication.
4- Laminar flow between concentric rotating cylinders.
5- Examples.
6- Problems; sheet No. 1

1- Navier-Stokes equations:

The general equations of motion for viscous incompressible, Newtonian fluids may be written in the following form:

x- direction:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \text{------(1)}$$

y- direction:

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=\rho g_y-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right] \text{------(2)}$$

Equations (1) and (2) are called: Navier Stokes equations.

2- Steady laminar flow between parallel flat plates:

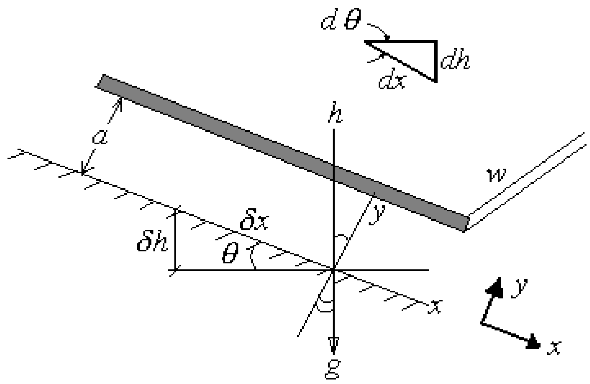


Fig.25

The fluid moves in the x- direction without acceleration.

v= 0 , w= 0 , $\frac{\partial}{\partial t}=0$

the Navier-Stokes equation in the x- direction (eq. 1) reduces to:

$$-\rho g_x + \frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \text{ ---------(3)}$$

$$g_x = g.\sin\theta = -g\frac{dh}{dx}$$

eq. 3 will be

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx} \text{ ---------(4)}$$

Integration of eq. 4:

$$\frac{du}{dy} = \frac{1}{\mu}\frac{d(p+\gamma h)}{dx}y + A$$

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}y^2 + Ay + B \text{ -----------(5)}$$

<u>B.C</u> (Two fixed parallel plates)

$$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$$

$$y = a \qquad u = 0 \quad \Rightarrow \quad A = \frac{-a}{2\mu}\frac{d(p+\gamma h)}{dx}$$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}\left(y^2 - ay\right) \text{ -----------(6)}$$

<u>B.C</u> (One plate is fixed and the other plate moves with a constant velocity U) (Couette flow)

$$y = 0 \qquad u = 0 \quad \Rightarrow \quad B = 0$$

$$y = a \qquad u = U \quad \Rightarrow \quad A = \frac{U}{a} - \frac{a}{2\mu}\frac{d(p+\gamma h)}{dx}$$

eq. 5 will be

$$u = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}\left(y^2 - ay\right) + \frac{\mathrm{Uy}}{\mathrm{a}} \text{ -----------(7)}$$

For the case of horizontal parallel plates:

$$\frac{dh}{dx} = 0$$

eq. 7 will be

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - ay\right) + \frac{\mathrm{Uy}}{\mathrm{a}} \text{ -----------(8)}$$

The location of maximum velocity u_{max} may be found by evaluating $\frac{du}{dy}$ and setting it to zero.
The volume flow rate is

$$Q = w\int_0^a u.dy \text{ ------------(9)}$$

3- Hydrodynamic lubrication:

Sliding bearing

Large forces are developed in small clearance when the surfaces are slightly inclined and one is in motion so that fluid is wedged into the decreasing space. Usually the oils employed for lubrication are highly viscous and the flow is of laminar nature.

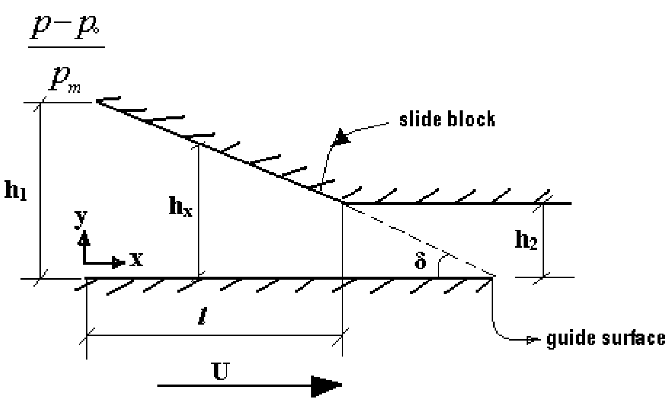


Fig.32

Assumptions:
The acceleration is zero.
The body force is small and can be neglected.
Also $\frac{\partial^2 u}{\partial y^2} \rangle\rangle \frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2} \rangle\rangle \frac{\partial^2 u}{\partial z^2}$
The Navier-Stokes equation in the x-direction (eq. 1) reduces to:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

Integration:

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + Ay + B$$

B.C

$$y = 0 \qquad u = U \quad \Rightarrow \quad B = U$$

$$y = h_x \qquad u = 0 \quad \Rightarrow \quad A = \frac{-h_x}{2\mu}\frac{dp}{dx} - \frac{U}{h_x}$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h_x y\right) + U\left(1 - \frac{y}{h_x}\right)$$

The volume flow rate in every section will be constant.

$$Q = w\int_0^{h_x} u.dy \qquad \text{assume w} = 1$$

$$\therefore\ Q = \frac{Uh_x}{2} - \frac{h_x^3}{12\mu}\frac{dp}{dx} \quad \text{--------(*)}$$

** For a constant taper bearing:

$$\delta = \frac{h_1 - h_2}{l}$$

$$\therefore\ h_x = \left(h_1 - \delta x\right)$$

Sub in eq.(*) and solving for $\frac{dp}{dx}$ produces:

$$\frac{dp}{dx} = \frac{6\mu U}{\left(h_1 - \delta x\right)^2} - \frac{12\mu Q}{\left(h_1 - \delta x\right)^3}$$

Integration gives:

$$p(x) = \frac{6\mu U}{\delta\left(h_1 - \delta x\right)} - \frac{6\mu Q}{\delta\left(h_1 - \delta x\right)^2} + C \quad \text{---------(**)}$$

B.C

$$x = 0 \qquad p = p_o = 0$$

$$x = l \qquad p = p_o = 0$$

$$\Rightarrow\ Q = \frac{Uh_1h_2}{h_1 + h_2} \qquad \text{and} \qquad C = \frac{-6\mu U}{\delta\left(h_1 + h_2\right)}$$

With these values inserted in eq.(**) we obtain the pressure distribution inside the bearing.

$$p(x) = \frac{6\mu Ux\left(h_x - h_2\right)}{h_x^2\left(h_1 + h_2\right)}$$

The load that the bearing will support per unit width is:

$$F = \int_0^l p(x).dx$$

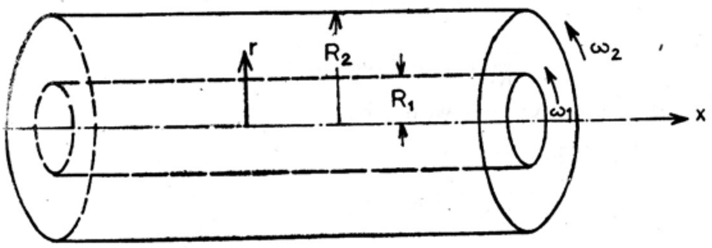
$$F = \frac{6\mu Ul^2}{\left(h_1 - h_2\right)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]$$

where

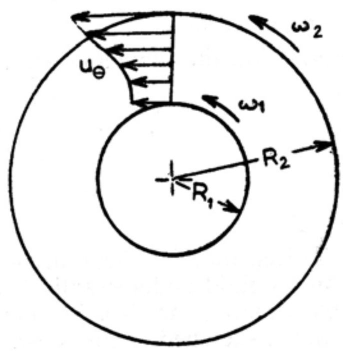
$$k = \frac{h_1}{h_2}$$

4- Laminar flow between concentric rotating cylinders:

Consider the purely circulatory flow of a fluid contained between two long concentric rotating cylinders of radius R_1 and R_2 at angular velocities ω_1 and ω_2.



(a) PICTORIAL REPRESENTATION



(b) VELOCITY PROFILE

In this case the Navier-Stokes equations in cylindrical coordinates are used.

r- direction:

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+w\frac{\partial u_r}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+g_r$$

θ- direction:

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+w\frac{\partial u_\theta}{\partial z}=\frac{-1}{\rho r}\frac{\partial p}{\partial \theta}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+g_\theta$$

In the above equations:

$u_r = 0$

$w = 0$

$\frac{\partial}{\partial t}=0 \quad , \quad \frac{\partial u_\theta}{\partial \theta}=0 \quad , \quad \frac{\partial p}{\partial \theta}=0$

body force = 0

The equation in θ- direction reduces to:

$$\frac{d^2 u_\theta}{dr^2}+\frac{d}{dr}\left(\frac{u_\theta}{r}\right)=0$$

Integration:

$$\frac{1}{r}\frac{d}{dr}\left(ru_\theta\right)=A$$

$$u_\theta = Ar+\frac{B}{r} \quad \text{---------(i)}$$

B.C

$r = R_1 \qquad u_\theta = R_1\omega_1$

$r = R_2 \qquad u_\theta = R_2\omega_2$

$$\Rightarrow \quad A=\omega_1+\frac{R_2^2}{R_2^2-R_1^2}\left(\omega_2-\omega_1\right)$$

$$B=-\frac{R_1^2R_2^2}{R_2^2-R_1^2}\left(\omega_2-\omega_1\right)$$

Sub. in eq.(i) yields:

$$u_\theta=\frac{1}{R_2^2-R_1^2}\left[\left(\omega_2R_2^2-\omega_1R_1^2\right)r-\frac{R_1^2R_2^2}{r}\left(\omega_2-\omega_1\right)\right] \quad \text{-------(ii)}$$

The shear stress may be evaluated by the equation:

$$\tau=\mu\left[r\frac{d}{dr}\left(\frac{u_\theta}{r}\right)\right]$$

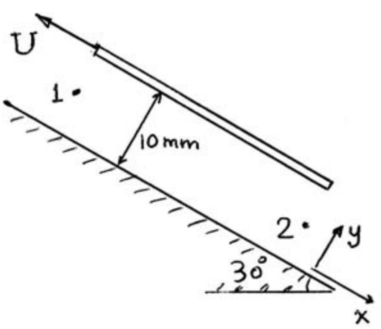
By using eq.(ii):

$$\tau=\frac{2\mu}{R_2^2-R_1^2}\frac{R_1^2R_2^2}{r^2}\left(\omega_2-\omega_1\right)$$

5- Examples:

1- Using the Navier-Stokes equation in the flow direction, calculate the power required to pull (1m × 1m) flat plate at speed (1 m/s) over an inclined surface. The oil between the surfaces has (ρ = 900 kg/m^3 , μ = 0.06 Pa.s).The pressure difference between points 1 and 2 is (100 kN/m^2) .

Solution:



The Navier-Stokes equation in x- direction

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

We have: Acceleration =0 , v=0 , w=0 , $\frac{\partial^2 u}{\partial x^2}=0$, $\frac{\partial^2 u}{\partial z^2}$

The equation reduces to:

$$\frac{d^2u}{dy^2}=\frac{1}{\mu}\frac{dp}{dx}-\frac{\rho}{\mu}g_x$$

Integration

$$\frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y+A$$

$$u=\frac{1}{2\mu}\frac{dp}{dx}y^2-\frac{\rho}{2\mu}g_x y^2+Ay+B$$

B.C (b=10 mm)

$$y=0 \qquad u=0 \quad \Rightarrow \quad B=0$$

$$y=b \qquad u=-U \quad \Rightarrow \quad A=\frac{-U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$$

$$\therefore \quad \frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y-\frac{U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$$

The shearing force on the moving plate:

$$F=\tau_o\times area$$

$$F=\mu\cdot\left.\frac{du}{dy}\right|_{y=b}\times area$$

area=1 m^2

$$F=-\frac{\mu U}{b}+\frac{b}{2}\frac{dp}{dx}-\frac{b}{2}\rho g_x$$

We have $g_x=g\cdot\sin\theta$, $\frac{dp}{dx}=\frac{-\Delta p}{l}$

$$F=\frac{-0.06\times 1}{0.01}-\frac{0.01}{2}\left(\frac{100\times 10^3}{1}\right)-\frac{0.01}{2}\times 900\times 9.81\times\sin 30$$

$$F=-528\,N$$

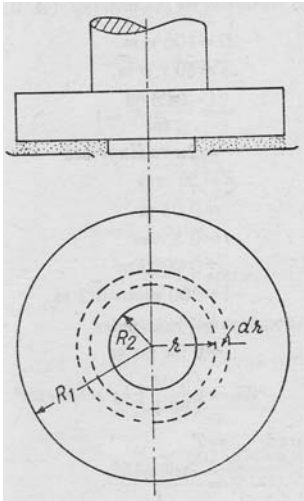
$$Power=F\cdot U$$

$$Power=528\times 1=528\,W \qquad \text{(Ans)}$$

2- The external and internal diameters of a **collar bearing** are (20 cm and 15 cm) respectively. An oil film (0.3 mm) thick is maintained between the collar surface and the bearing. Find the power lost in overcoming the viscous resistance of the oil when the shaft is running at (240 rpm). Take μ=0.1 Pa.s .

Solution:

The figure shows two views of a collar bearing.



Consider an elemental angular circular strip of the bearing surface of radius r and width dr. Circumferential resistance offered by the oil film over the strip

$=\tau \times area$

$$=\mu \frac{\omega r}{h} \times 2\pi r dr = \frac{2\pi\mu\omega r^2 dr}{h}$$

Torque required to overcome this resistance:

$$T = \frac{2\pi\mu\omega r^2 dr}{h} \times r = \frac{2\pi\mu\omega r^3 dr}{h}$$

Total torque required $T = \int_{R_2}^{R_1} \frac{2\pi\mu\omega r^3 dr}{h} = \frac{\pi\mu\omega}{2h}\left(R_1^4 - R_2^4\right)$

Power absorbed by the bearing$=T\omega = \frac{\pi\mu\omega^2}{2h}\left(R_1^4 - R_2^4\right)$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi \text{ rad/s}$$

$$\text{Power} = \frac{\pi \times 0.1(8\pi)^2}{2 \times 0.0003}\left(0.1^4 - 0.075^4\right) = 22.6 \text{ W} \quad \text{(Ans)}$$

University of Technology	Sheet No. 1
Mechanical Engineering Dep.	Navier-Stokes Equations
Fluid Mechanics II (3 rd year)	2011/2012

1- Using the Navier-Stokes equations, determine the pressure gradient along flow, the average velocity, and the discharge for an oil of viscosity 0.02 $N.s/m^2$ flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s. [-3200 N/m^2 per m ; 1.33 m/s ; 0.0133 m^3/s]

2- An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as shown in figure. The two plates move in opposite directions with constant velocities U_1 and U_2. The pressure gradient in the x-direction is zero. Use the Navier-Stokes equations to derive expression for the velocity distribution between the plates. Assume laminar flow.

[$u = \frac{y}{b}(U_1 + U_2) - U_2$]

3- Two parallel plates are spaced 2 mm apart, and oil ($\mu = 0.1$ $N.s/m^2$, S = 0.8) flows at a rate of 24×10^{-4} m^3/s per m of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60^o with the horizontal and if the flow is downward between the plates? [-353.2 kPa/m]

4- Using the Navier-Stokes equations, find the velocity profile for fully developed flow of water ($\mu = 1.14\times10^{-3}$ Pa.s) between parallel plates with the upper plate moving as shown in figure. Assume the volume flow rate per unit depth for zero pressure gradient between the plates is 3.75×10^{-3} m^3/s. Determine:

a- the velocity of the moving plate.
b- the shear stress on the lower plate.
c- the pressure gradient that will give zero shear stress at y = 0.25b. (b = 2.5 mm)
d- the adverse pressure gradient that will give zero volume flow rate between the plates.

[3 m/s ; 1.37 N/m^2 ; 2.19 kN/m^2 per m ; -3.28 kN/m^2 per m]

5- A vertical shaft passes through a bearing and is lubricated with an oil ($\mu = 0.2$ Pa.s) as shown in figure. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rpm. (Hint: The flow between the shaft and bearing can be treated as laminar flow between two flat plates with zero pressure gradient). [0.355 N.m]

6- Determine the force on the piston of the figure due to shear, and the leakage from the pressure chamber for U = 0. [295.1 N ; 1.636×10^{-8} m^3/s]

7- A layer of viscous liquid of thickness b flows steadily down an inclined plane. Show that, by using the Navier-Stokes equations that velocity distribution is: $u = \frac{\gamma}{2\mu}\left(2by - y^2\right)\sin\theta$ and that the discharge per unit width is: $Q = \frac{\gamma}{3\mu} b^3 \sin\theta$

8- A wide moving belt passes through a container of a viscous liquid. The belt moving vertically upward with a constant velocity V_o, as illustrated in figure. Because of viscous forces the belt picks up a film of fluid of thickness h. Gravity tends to make the fluid drain down the belt. Use the Navier-Stokes equations to determine an expression for the average velocity v_{av} of the fluid film as it is dragged up the belt. Assume the flow is laminar, steady, and uniform. [$v_{av} = V_o - \frac{\gamma h^2}{3\mu}$]

9- Determine the formulas for shear stress on each plate and for the velocity distribution for flow in the figure when an adverse pressure gradient exists such that Q = 0.

[$\tau_{y=0} = \frac{-2\mu U}{b}$; $\tau_{y=b} = \frac{4\mu U}{b}$; $u = 3U\frac{y^2}{b^2} - 2U\frac{y}{b}$]

10- A plate 2 mm thick and 1 m wide is pulled between the walls shown in figure at speed of 0.4 m/s. The space over and below the plate is filled with glycerin (μ = 0.62 N.s/m^2). The plate is positioned midway between the walls. Using the Navier-Stokes equations, determine the force required to pull the plate at the speed given for zero pressure gradient; and the pressure gradient that will give zero volume flow rate.

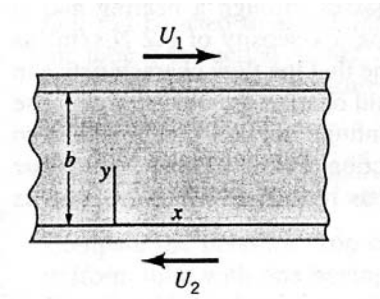
[496 N ; 372 kN/m^2.m]

11- A slider plate 0.5 m wide constitutes a bearing as shown in figure. Estimate:
a- the load carrying capacity.
b- the drag.
c- the power lost in the bearing.
d- the maximum pressure in the oil and its location.

[739.6 kN ; 348.6 N ; 348.6 W ; 12500 kN/m^2 ; 150 mm]

12- Consider a shaft that turns inside a stationary cylinder, with a lubricating fluid in the annular region. Using the Navier-Stokes equation in θ-direction, show that the torque per unit length acting on the shaft is given by: $T = \frac{4\pi\mu\omega R_1^2}{\left(\frac{R_1}{R_2}\right)^2 - 1}$

Where: ω = angular velocity of the shaft.
R_1 = radius of the shaft.
R_2 = radius of the cylinder.



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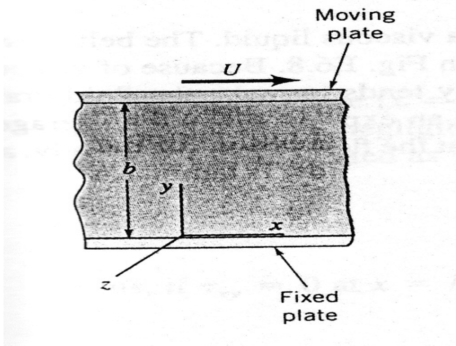

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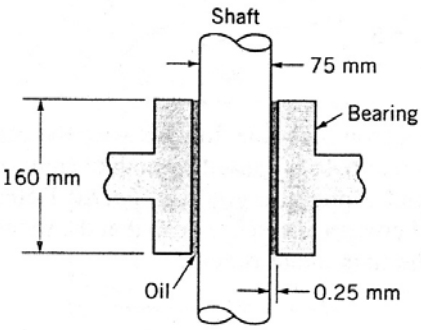

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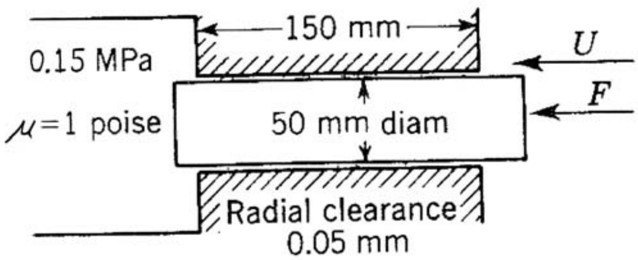

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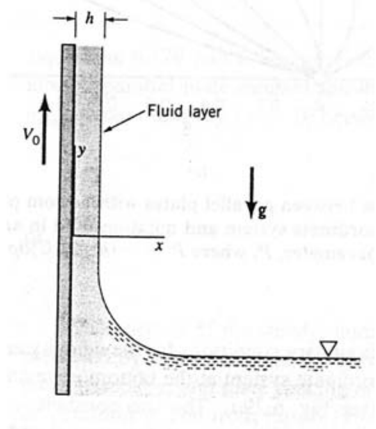

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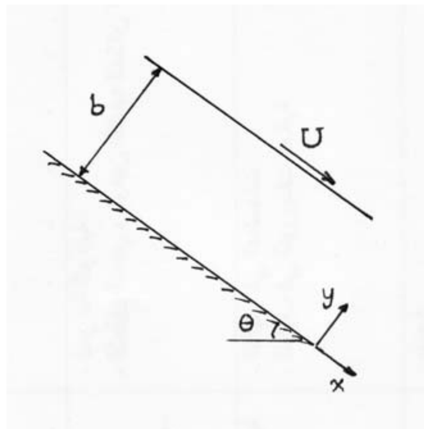

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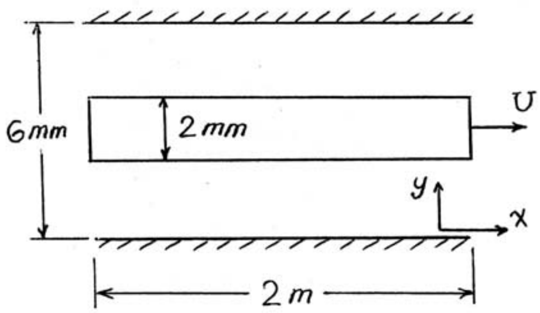
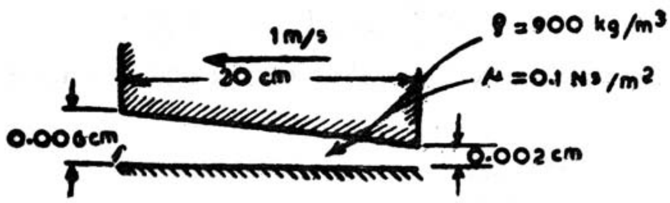


Problem No. 10



Problem No. 11