Chapter Three
Two-Phase Flow

Contents

1- Introduction:

Background

Two-phase flow is widely seen in the nature: Rain, snow, smog, dust-flow, etc.

Two-phase Flow:
Is a flow of two-phase mixture of a substance with the same chemical composition.   (e.g., Water-steam flow, water-ice flow, etc.)

Two-Component Flow:
Is a flow of two-component mixture with different chemical composition. (e.g., air-water flow, oil-water flow, air-dust flow, etc.)

Two- phase flows are related to the phase-change Phenomena.
(e.g., boiling, condensation, freezing, melting, solidification, crystallization).

2- Notation and relations:

Subscripts

f = fluid
g= gas
l= liquid
h= enthalpy

Void fraction α :

$$\alpha = {A_g}/{A}$$

Thus

$$1 - \alpha = {A_f}/{A}$$

And

$$A = A_f + A_g$$

Mass flow rate W_g, W_f**:**

$$W_g = u_g \rho_g A_g$$
$$W_f = u_f \rho_f A_f$$

Total mass flow rate *W*:

$$W = W_g + W_f$$

Volumetric flow rate Q_g, Q_f:

$$Q_f = u_f A_f = {W_f}/{\rho_f}$$

$$Q_g = u_g A_g = \frac{W_g}{\rho_g}$$

Total volumetric flow rate Q:

$$Q = Q_g + Q_f$$

Quality (or mass quality) *x*:

$$x = {W_g}/{W}$$

And

$$1 - x = {W_f}/{W}$$

And in a thermodynamic equilibrium,

$$x = {(i - i_f)}/{i_{fg}}$$

Mass flux (or superficial velocity) *j*:

$G = W/_A$

And

$G_g = \dfrac{W_g}{A}$

$G_f = \dfrac{W_f}{A}$

Therefore

$G = G_f + G_g$

Volumetric flux (or superficial velocity) *j*:

$j = Q/_A$

$j_g = Q_g/_A \quad , \quad j_f = Q_f/_A$

Thus

$j = j_g + j_f$

Volumetric quality β:

$\beta = Q_g/_Q$

$1 - \beta = Q_f/_Q$

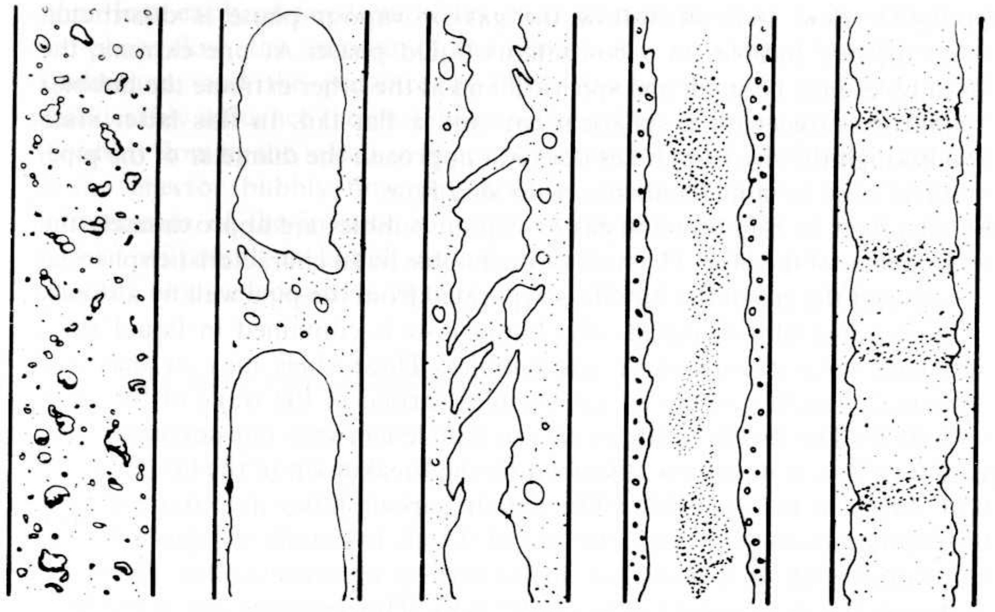
Relative velocity u_{gf}, u_{fg}:

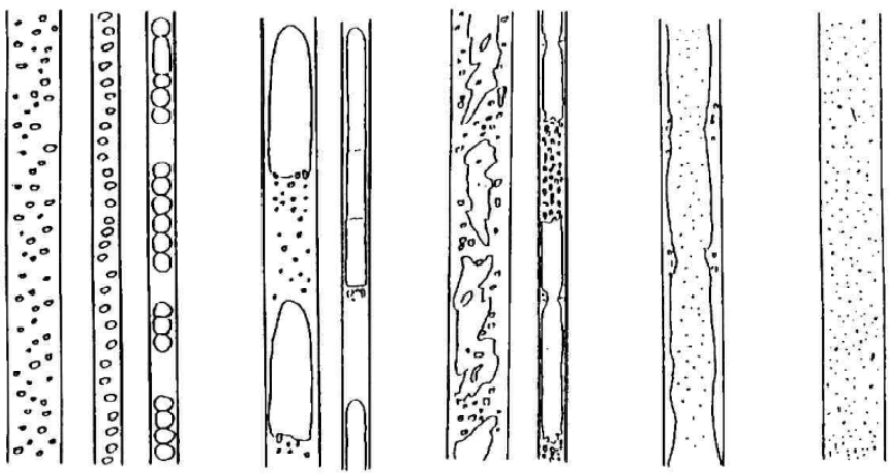
$u_{gf} = u_g - u_f$

$u_{fg} = u_f - u_g$

3- Flow patterns:

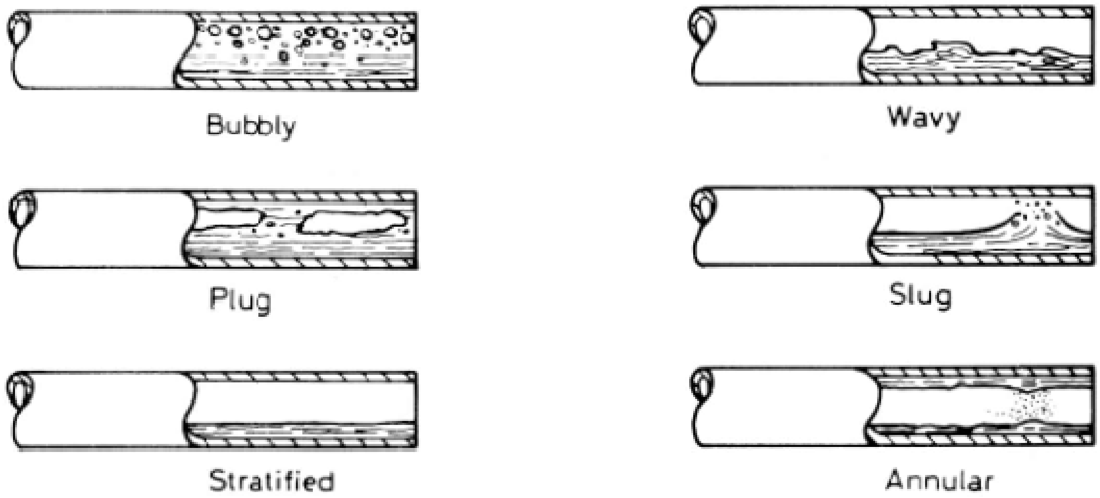
Vertical flow patterns





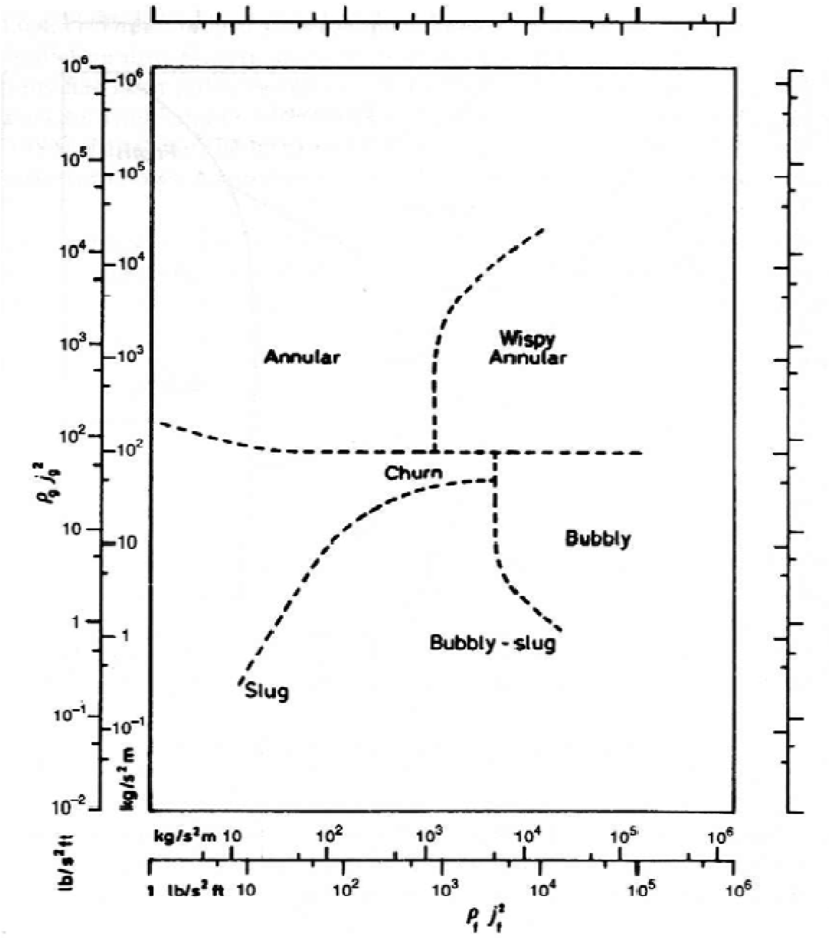
(a), (d), (f), (h) and (i): Flow patterns frequently observed in large diameter tubes.
(b), (c), (e) and (g): Flow patterns specially appear in capillary tubes.

Horizontal flow patterns



Flow pattern maps

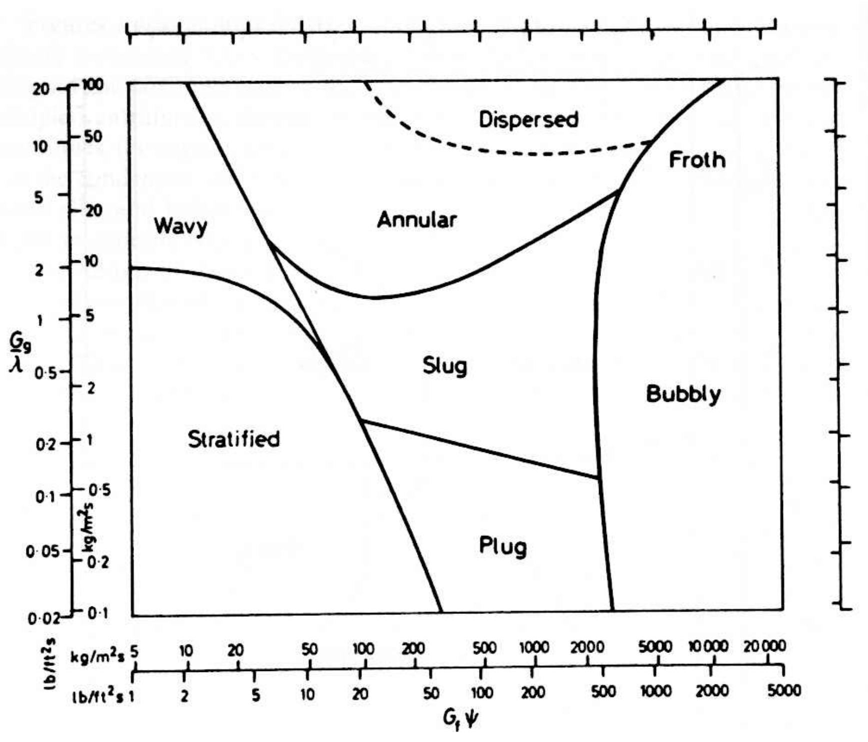
(1) Vertical flow [Hewitt & Roberts, 1969]



$$\rho_f j_f^2 = \frac{\left(G(1-x)\right)^2}{\rho_f}$$

$$\rho_g j_{fg}^2 = \frac{\left(G(x)\right)^2}{\rho_g}$$

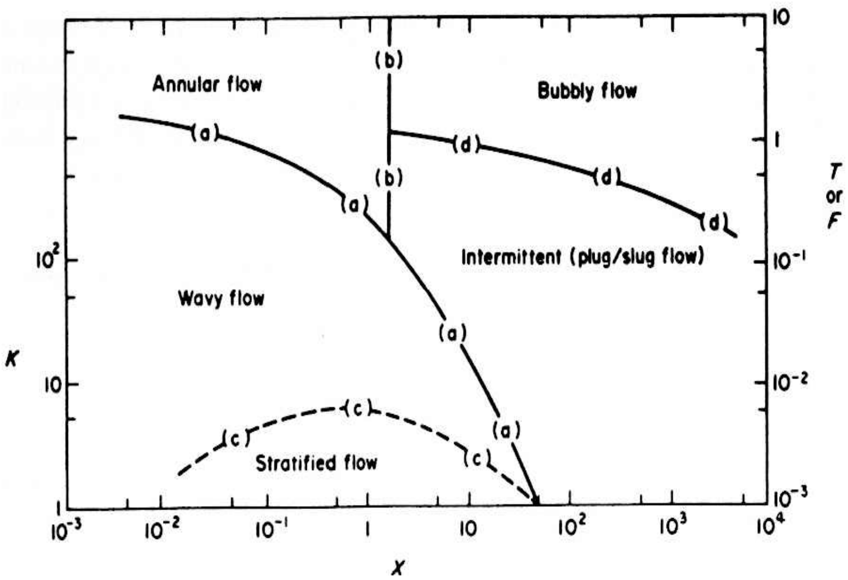
(2) Horizontal flow [Baker, 1954]



$$\lambda = \left[\left(\frac{\rho_g}{\rho_A}\right)\left(\frac{\rho_f}{\rho_w}\right)\right]^{\frac{1}{2}}$$

$$\psi = \frac{\sigma_w}{\sigma}\left[\left(\frac{\mu_f}{\mu_w}\right)\left(\frac{\rho_w}{\rho_f}\right)^2\right]^{\frac{1}{3}}$$

(3) Generalized map [Taitel & Dukler, 1976]



a+b : F vs. X

c : K vs. X

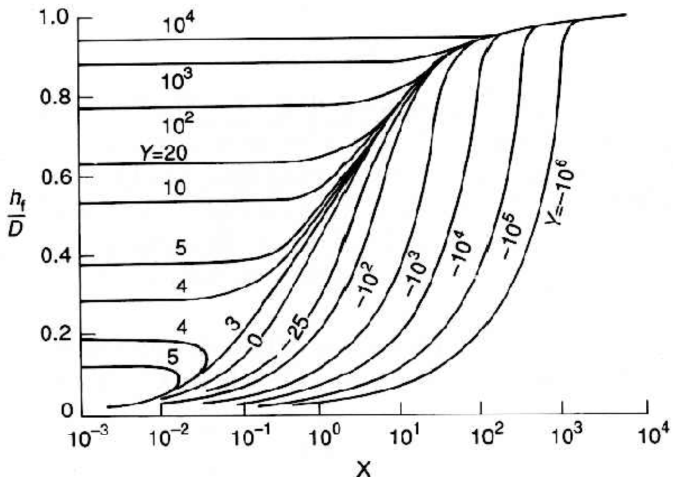
d : T vs. X

$$X = \left(\frac{dp}{dz}\right)_f \Big/ \left(\frac{dp}{dz}\right)_g$$

$$K = \left[\frac{\rho_g j_g^2 j_f}{(\rho_f - \rho_g) g cos\theta}\right]^{0.5}$$

$$T = \left[\frac{\left(\frac{dp}{dz}\right)_f}{(\rho_f - \rho_g) g cos\theta}\right]^{0.5}$$

$$F = \left(\frac{\rho_g}{\rho_f - \rho_g}\right)^{0.5} \frac{j_g}{(D_g cos\theta)^{0.5}}$$



$$Y = \frac{(\rho_f - \rho_g) g \sin\theta}{\left(\frac{dp}{dz}\right)_F}$$

4- Homogeneous flow model:

Continuity equation:

$$A\rho_h u_h = W = const$$

Momentum Equation:

$$-\frac{dp}{dz} = -\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_Z - \left(\frac{dp}{dz}\right)_a$$

$$-\frac{dp}{dz} = \frac{1}{A}\frac{dF}{dz} + \frac{g \sin\theta}{v_h} + \frac{W}{A}\frac{du_h}{dz}$$

Energy Equation:

$$\frac{di}{dz} + \frac{1}{2}\frac{d}{dz}\left(u_h^2\right) + g \sin\theta = \frac{dq}{dz} - \frac{dw}{dz}$$

For dw=0, the energy equation becomes:

$$-\frac{dp}{dz} = \frac{1}{v_h}\left(\frac{dE}{dZ} + \frac{1}{2}\frac{d}{dz}\left(u_h^2\right) + g \sin\theta\right)$$

$$dF = \tau_w p dZ$$

τ_w : wall shear stress ,p:tube perimeter

$$\tau_w = f_h\left(\frac{1}{2}\rho_h u_h^2\right)$$

f_h: friction factor

For a tube of diameter, D

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}\frac{dF}{dz}$$

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}(\tau_w p) = \frac{p}{A} f_h \left(\frac{1}{2}\rho_h u_h^2\right)$$

$$= \frac{2 f_h G^2 v_h}{D}$$

Get $f_{\mathbf{h}}$

$$Re_{\mathbf{h}} = \frac{GD}{\mu_{\mathbf{h}}}$$

McAdams

$$\frac{1}{\mu_h} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$$

Cicchitti

$$\mu_h = x\mu_g + (1-x)\mu_f$$

Dukler

$$\mu_h = \frac{j_f}{j}\mu_f + \frac{j_g}{f}u_g = \rho_h\left[x\upsilon_g\mu_g + (1-x)v_f\mu_f\right]$$

For laminar flow:

$$f_h = \frac{16}{\mathrm{Re}_h}$$

For turbulent flow (Blasius correlation):

$$f_{\mathbf{h}} = \mathbf{0.079}\, Re_{\mathbf{h}}^{-1/4}$$

When using the McAdams Correlations,

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{fo}\left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-1} \quad \text{Laminar flow}$$

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{fo}\left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-\frac{1}{4}} \quad \text{Turbulent Flow}$$

Thus

$$\phi_{fo}^2 = \left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-1} \quad \text{(Laminar Flow)}$$

$$\phi_{fo}^2 = \left[1+x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1+x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-\frac{1}{4}} \quad \text{(Turbulent flow)}$$

Acceleration Pressure Drop

$$-\left(\frac{dp}{dz}\right)_a = G\left(\frac{du_h}{dz}\right) = G\frac{d}{dz}\left(\frac{W}{A}v_h\right)$$

$$= G^2\frac{dv_h}{dz} + Gv_hW\frac{d}{dz}\left(\frac{1}{A}\right)$$

Where

$$\frac{dv_h}{dz} = \frac{d}{dz}\left[xv_g + (1-x)v_f\right]$$

$$= \frac{dp}{dz}\left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right] + v_{gf}\frac{dx}{dz}$$

And

$$-\left(\frac{dp}{dz}\right)_a = G^2\left\{v_{gf}\frac{dx}{dz} + \frac{dp}{dz}\left[x\frac{dv_g}{dp} + (1-x)\frac{dv_f}{dp}\right] - \left(v_f + xv_{gf}\right)\frac{1}{A}\frac{dA}{dz}\right\}$$

Gravitational Pressure Drop

$$-\left(\frac{dp}{dz}\right)_Z = \frac{g\sin\theta}{v_f + xv_{fg}}$$

Finally

$$-\left(\frac{dp}{dz}\right)_a = \frac{\frac{2f_hG^2}{D}\left(v_f + xv_{fg}\right) + G^2v_{gf}\frac{dx}{dz} - G^2\left(v_f + xv_{fg}\right)\frac{1}{A}\frac{dA}{dz} + \frac{g\sin\theta}{v_f + xv_{fg}}}{1 + G^2\left[x\frac{dv_g}{dp} + \left(1-x\right)\frac{dv_f}{dp}\right]}$$

5- Separated flow model:

Pressure Drop Correlations:-

$$-\frac{dp}{dz} = -\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_Z - \left(\frac{dp}{dz}\right)_a$$

Where:

$$-\left(\frac{dp}{dz}\right)_F = \frac{1}{A}\frac{dF}{dz}$$

$$-\left(\frac{dp}{dz}\right)_Z = \left[\left(1-\alpha\right)\rho_f + \alpha\rho_g\right]g\sin\theta$$

$$-\left(\frac{dp}{dz}\right)_a = \frac{W^2}{A}\frac{d}{dz}\left[\frac{1}{A}\left\{\frac{x^2v_g}{\alpha} + \frac{\left(1-x\right)^2v_f}{1-\alpha}\right\}\right]$$

For a circular tube of diameter, D

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{fo}\phi_{fo}^2 = \frac{2f_{fo}G^2v_f}{D}\phi_{fo}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{go}\phi_{go}^2 = \frac{2f_{go}G^2v_g}{D}\phi_{go}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{f}\phi_{f}^2 = \frac{2f_{f}G^2(1-x)^2v_f}{D}\phi_{f}^2$$

$$-\left(\frac{dp}{dz}\right)_F = -\left((\frac{dp}{dz})_F\right)_{g}\phi_{g}^2 = \frac{2f_{g}G^2x^2v_g}{D}\phi_{g}^2$$

Where $\phi_{fo}, \phi_{go}, \phi_f, \phi_g$ are the two-phase frictional multipliers related to each other as follows:

$$\phi_{fo}^2 = \phi_f^2(1-x)^2\frac{f_f}{f_{fo}} = \phi_g^2x^2\frac{v_g f_g}{v_f f_{fo}} = \phi_{go}^2\frac{v_g f_{go}}{v_f f_{fo}}$$

$$-\left(\frac{dp}{dz}\right)_a = \frac{W^2}{A}\frac{d}{dz}\left[\frac{1}{A}\left\{\frac{x^2v_g}{\alpha} + \frac{(1-x)^2v_f}{1-\alpha}\right\}\right]$$

$$= G^2\frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_p \cdot \left\{\frac{(1-x)^2v_f}{(1-\alpha)^2} - \frac{x^2v_g}{\alpha^2}\right\}\right]$$

$$+G^2\frac{dp}{dz}\left[\left\{\frac{x^2dv_g}{\alpha dp} + \frac{(1-x)^2}{1-\alpha}\frac{dv_f}{dp}\right\} + \left(\frac{\partial\alpha}{\partial p}\right)_x \cdot \left\{\frac{(1-x)^2v_f}{(1-\alpha)^2} - \frac{x^2v_g}{\alpha^2}\right\}\right]$$

$$-\frac{G^2}{A}\frac{dA}{dz}\left[\left\{\frac{x^2v_g}{\alpha} + \frac{(1-x)^2v_f}{1-\alpha}\right\}\right]$$

Finally,

$$-\frac{dp}{dz}=\frac{H_1}{H_2}$$

Where

$$H_1=\frac{2f_{fo}G^2v_f}{D}\phi_{fo}^2+G^2\frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha}-\frac{2(1-x)v_f}{1-\alpha}\right\}+\left(\frac{d\alpha}{dx}\right).\left\{\frac{(1-x)^2v_f}{(1-\alpha)^2}-\frac{x^2v_g}{\alpha^2}\right\}\right]$$

$$-\frac{G^2}{A}\frac{dA}{dz}\left[\left\{\frac{x^2v_g}{\alpha}+\frac{(1-x)^2v_f}{1-\alpha}\right\}\right]+\left(\rho_g\alpha+(1-\alpha)\rho_f\right)g\sin\theta$$

and

$$H_2=1+G^2\left[\left\{\frac{x^2dv_g}{\alpha dp}+\frac{(1-x)^2}{1-\alpha}\frac{dv_f}{dp}\right\}+\left(\frac{d\alpha}{dp}\right).\left\{\frac{(1-x)^2v_f}{(1-\alpha)^2}-\frac{x^2v_g}{\alpha^2}\right\}\right]$$

6- Examples:

1- Using the homogeneous flow pressure drop method, calculate the two phase pressure drop for up flow in vertical tube of 10mm internal diameter that is 2m long .The flow is adiabatic ,the mass flow rate is 0.02 kg/s and the vapor quality is 0.05 .The fluid is R-123 at a saturation temperature of 3^{o}C and saturation pressure of 0.37 bar ,whose physical properties are :
ρ_l= 1518 kg/m^3 , ρ_g= 2.6 kg/m^3 , μ_g= 12.6×10^{-6} Pa.s , μ_l= 58.56×10^{-5} Pa.s

Solution:

$$\alpha_h = \frac{1}{1+\frac{(1-x)\rho_g}{x\rho_l}} = 0.9685$$

$$\rho_h = \rho_l(1-\alpha_h) + \rho_{g\alpha_h} = 50.3\frac{kg}{m^3}$$

$$\Delta p_z = \rho_h L g \, sin\theta = 50.3 * 2 * 9.81 * sin90$$
$$\Delta p_z = 987 \quad {}^{N}\!/_{m^2}$$

$$\frac{1}{\mu_h} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l} = 0.000557 \, Pa.s$$

$$Re = \frac{GD}{\mu_h} = 4571$$

$$f = \frac{0.079}{Re^{0.25}} = 0.00961$$

$$\Delta p_F = \frac{2 * f * L * G^2}{D * \rho_h} = 4953 \quad {}^{N}\!/_{m^2}$$

Total pressure drop $= \Delta p_F + \Delta p_z = 5.94 \, kPa.$

University of Technology	Sheet No. 3
Mechanical Engineering Dep.	Two-Phase Flow
Fluid Mechanics II (3 rd year)	2011/2012

1- Find the flow pattern when 4 kg/s of steam –water mixture of quality 20% at 20 bar flows in a circular tube of internal diameter 0.1 m:

a) When the flow is vertically upward.

b) When the flow is horizontal.

The physical properties required are:

$\rho_l = 850\ kg/m^3$, $\rho_g = 10\ kg/m^3$, $\mu_l = 128*10^{-6}$ Pa.s , $\mu_g = 16*10^{-6}$ Pa.s

2- A steam –water mixture of quality 0.1 at 5 bar flow through a smooth vertical round tube of diameter 0.05m.The total flow rate is 0.6 kg/s .Calculate:

a) The homogeneous void fraction.

b) Homogeneous gravitational pressure gradient.

c) Homogeneous frictional pressure gradient.

The physical properties required are:

$\rho_l = 915\ kg/m^3$, $\rho_g = 2.67\ kg/m^3$, $\mu_l = 180*10^{-6}$ Pa.s , $\mu_g = 14*10^{-6}$ Pa.s

[0.97 ; 255 N/m^3 ; 547 N/m^3]

3- Consider mixture of water (W_l=0.42 kg/s) and air (W_g=0.01 kg/s) flowing upward in a vertical pipe (D=25mm ,L=45cm). Given friction factor of ($f=0.079/Re^{0.25}$).Find the total pressure drop ,the volumetric flow rates ,void fraction ,and mean water and air velocity. Using homogeneous model.

the physical properties required are:

$\rho_l = 915\ kg/m^3$, $\rho_g = 1.2\ kg/m^3$,$\mu_l = 1*10^{-3}$ Pa.s , $\mu_g = 1.8*10^{-5}$ Pa.s

[3169 N/m^2 ; 0.00833m^3/s ; 0.0042m^3/s ; 0.952 ; 17.65m/s]