Heat Transfer

Arranged by

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CHAPTER

1 Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference.

1-1 | CONDUCTION HEAT TRANSFER

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. We say that the energy is transferred by conduction and that the heat-transfer rate per unit area is proportional to the normal temperature gradient:

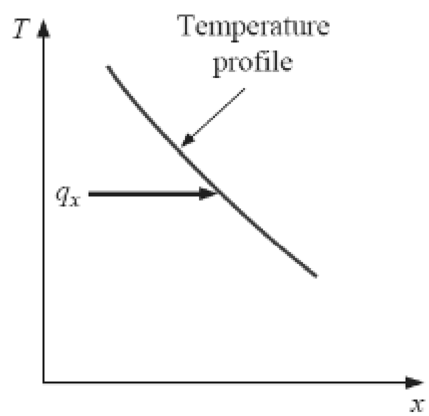
$$\frac{q_x}{A} \sim \frac{\partial T}{\partial x}$$

When the proportionality constant is inserted,

$$q_x = -kA\,\frac{\partial T}{\partial x} \tag{1-1}$$

where q_x is the heat-transfer rate and $\partial T/\partial x$ is the temperature gradient in the direction of the heat flow. The positive constant k is called the *thermal conductivity* of the material, and the minus sign is inserted so that the second principle of thermodynamics will be satisfied; i.e., heat must flow downhill on the temperature scale, as indicated in the coordinate system of Figure 1-1. Equation (1-1) is called Fourier's law of heat conduction after the French mathematical physicist Joseph Fourier, who made very significant contributions to the analytical treatment of conduction heat transfer. It is important to note that Equation (1-1) is the defining equation for the thermal conductivity and that k has the units of watts per meter per Celsius degree in a typical system of units in which the heat flow is expressed in watts.

Figure 1-1 | Sketch showing direction of heat flow.



Make energy balance on the conducting element shown in Fig.1-2

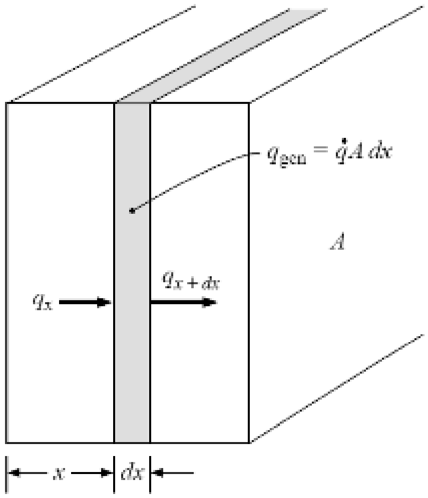
Energy conducted in left face + heat generated within element
= change in internal energy + energy conducted out right face

These energy quantities are given as follows:

$$\text{Energy in left face} = q_x = -kA\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$

Figure 1-2 | Elemental volume for one-dimensional heat-conduction analysis.



$$\text{Change in internal energy} = \rho c A \frac{\partial T}{\partial \tau} dx$$

$$\text{Energy out right face} = q_{x+dx} = -kA \left.\frac{\partial T}{\partial x}\right]_{x+dx}$$

$$= -A\left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) dx\right]$$

where

$\dot{q}$ = energy generated per unit volume, W/m^3
c = specific heat of material, J/kg · °C
ρ = density, kg/m^3

Combining the relations above gives

$$-kA \frac{\partial T}{\partial x} + \dot{q} A\, dx = \rho c A \frac{\partial T}{\partial \tau} dx - A\left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) dx\right]$$

or

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \dot{q} = \rho c \frac{\partial T}{\partial \tau} \qquad \textbf{[1-2]}$$

This is the one-dimensional heat-conduction equation. To treat more than one-dimensional heat flow, we need consider only the heat conducted in and out of a unit volume in all three coordinate directions, as shown in Figure 1-3a. The energy balance yields

$$q_x + q_y + q_z + q_{\text{gen}} = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{dE}{d\tau}$$

and the energy quantities are given by

$$q_x = -k\,dy\,dz\,\frac{\partial T}{\partial x}$$

$$q_{x+dx} = -\left[k\,\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right)dx\right]dy\,dz$$

$$q_y = -k\,dx\,dz\,\frac{\partial T}{\partial y}$$

$$q_{y+dy} = -\left[k\,\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right)dy\right]dx\,dz$$

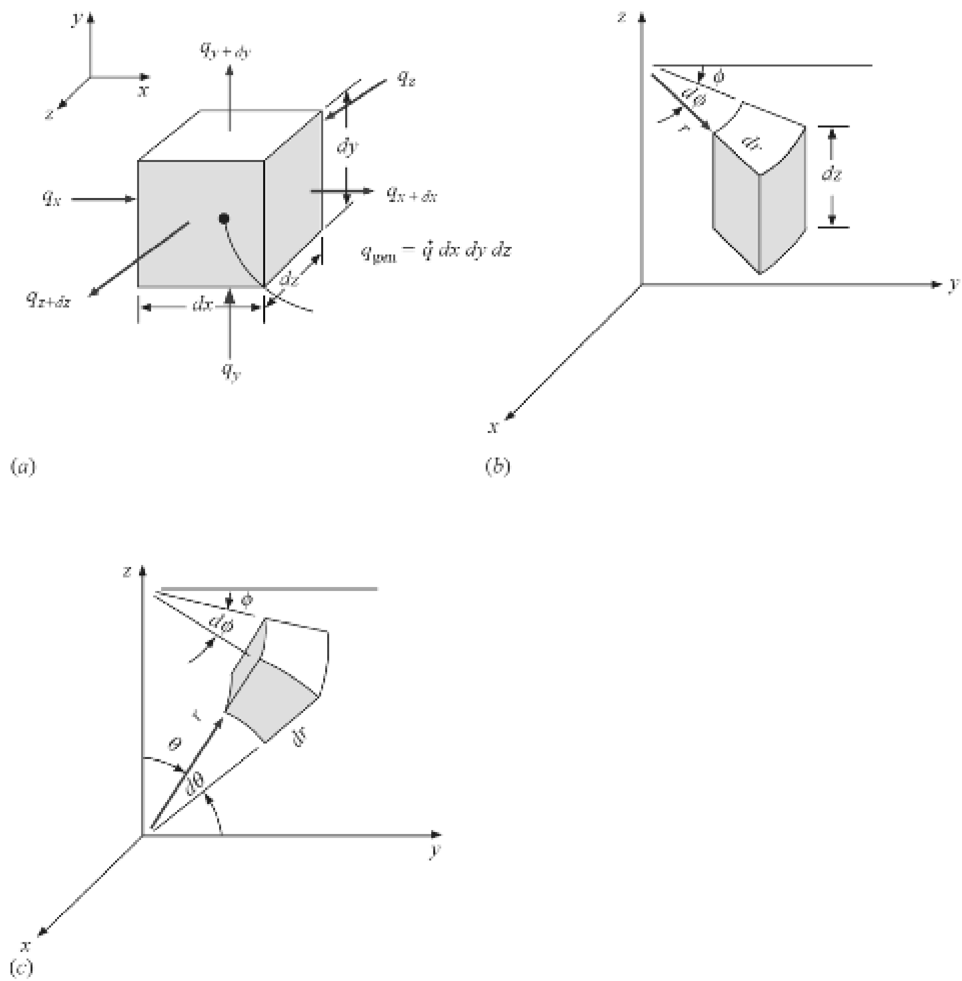
$$q_z = -k\,dx\,dy\,\frac{\partial T}{\partial z}$$

$$q_{z+dz} = -\left[k\,\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right)dz\right]dx\,dy$$

$$q_{\text{gen}} = \dot{q}\,dx\,dy\,dz$$

$$\frac{dE}{d\tau} = \rho c\,dx\,dy\,dz\,\frac{\partial T}{\partial \tau}$$

Figure 1-3 | Elemental volume for three-dimensional heat-conduction analysis: (*a*) cartesian coordinates; (*b*) cylindrical coordinates; (*c*) spherical coordinates.



so that the general three-dimensional heat-conduction equation is

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial \tau} \qquad \textbf{[1-3]}$$

For constant thermal conductivity, Equation (1-3) is written

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad \textbf{[1-3}\boldsymbol{a}\textbf{]}$$

where the quantity $\alpha = k/\rho c$ is called the *thermal diffusivity* of the material. The larger the value of α, the faster heat will diffuse through the material. This may be seen by examining the quantities that make up α. A high value of α could result either from a high value of thermal conductivity, which would indicate a rapid energy-transfer rate, or from a low value of the thermal heat capacity ρc. A low value of the heat capacity would mean that less of the energy moving through the material would be absorbed and used to raise the temperature of

the material; thus more energy would be available for further transfer. Thermal diffusivity α has units of square meters per second.

In the derivations above, the expression for the derivative at $x + dx$ has been written in the form of a Taylor-series expansion with only the first two terms of the series employed for the development.

Equation (1-3a) may be transformed into either cylindrical or spherical coordinates by standard calculus techniques. The results are as follows:

Cylindrical coordinates:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \textbf{[1-3}\boldsymbol{b}\textbf{]}$$

Spherical coordinates:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \textbf{[1-3}\boldsymbol{c}\textbf{]}$$

The coordinate systems for use with Equations (1-3b) and (1-3c) are indicated in Figure 1-3b and c, respectively.

Steady-state one-dimensional heat flow (no heat generation):

$$\frac{d^2 T}{dx^2} = 0 \quad \textbf{[1-4]}$$

Note that this equation is the same as Equation (1-1) when $q = \text{constant}$.

Steady-state one-dimensional heat flow in cylindrical coordinates (no heat generation):

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0 \quad \textbf{[1-5]}$$

Steady-state one-dimensional heat flow with heat sources:

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \textbf{[1-6]}$$

Two-dimensional steady-state conduction without heat sources:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \textbf{[1-7]}$$

1-2 | THERMAL CONDUCTIVITY

Equation (1-1) is the defining equation for thermal conductivity. On the basis of this definition, experimental measurements may be made to determine the thermal conductivity of different materials. For gases at moderately low temperatures, analytical treatments in the kinetic theory of gases may be used to predict accurately the experimentally observed values. In some cases, theories are available for the prediction of thermal conductivities in

liquids and solids, but in general, many open questions and concepts still need clarification where liquids and solids are concerned.

Table 1-1 lists typical values of the thermal conductivities for several materials to indicate the relative orders of magnitude to be expected in practice. More complete tabular information is given in Appendix A. In general, the thermal conductivity is strongly temperature-dependent.

Table 1-1 | Thermal conductivity of various materials at 0°C.

	Thermal conductivity k	
Material	**W/m · °C**	**Btu/h · ft · °F**
Metals:		
Silver (pure)	410	237
Copper (pure)	385	223
Aluminum (pure)	202	117
Nickel (pure)	93	54
Iron (pure)	73	42
Carbon steel, 1% C	43	25
Lead (pure)	35	20.3
Chrome-nickel steel (18% Cr, 8% Ni)	16.3	9.4
Nonmetallic solids:		
Diamond	2300	1329
Quartz, parallel to axis	41.6	24
Magnesite	4.15	2.4
Marble	2.08–2.94	1.2–1.7
Sandstone	1.83	1.06
Glass, window	0.78	0.45
Maple or oak	0.17	0.096
Hard rubber	0.15	0.087
Polyvinyl chloride	0.09	0.052
Styrofoam	0.033	0.019
Sawdust	0.059	0.034
Glass wool	0.038	0.022
Ice	2.22	1.28
Liquids:		
Mercury	8.21	4.74
Water	0.556	0.327
Ammonia	0.540	0.312
Lubricating oil, SAE 50	0.147	0.085
Freon 12, CCl_2F_2	0.073	0.042
Gases:		
Hydrogen	0.175	0.101
Helium	0.141	0.081
Air	0.024	0.0139
Water vapor (saturated)	0.0206	0.0119
Carbon dioxide	0.0146	0.00844

Figure 1-4 | Thermal conductivities of some typical gases [1 W/m · °C = 0.5779 Btu/h · ft · °F].

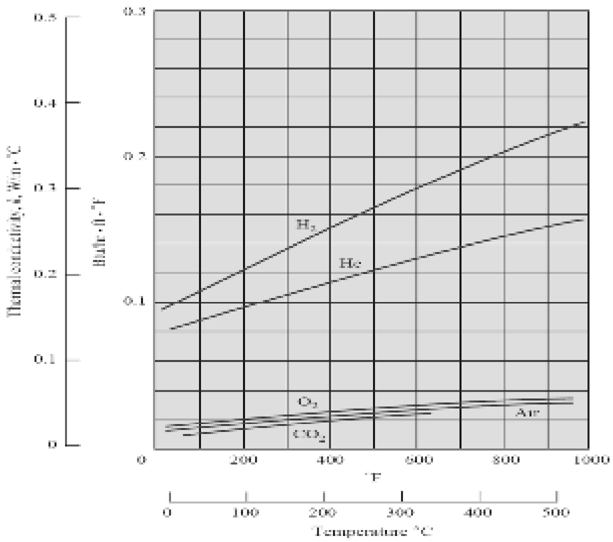


Figure 1-5 | Thermal conductivities of some typical liquids.

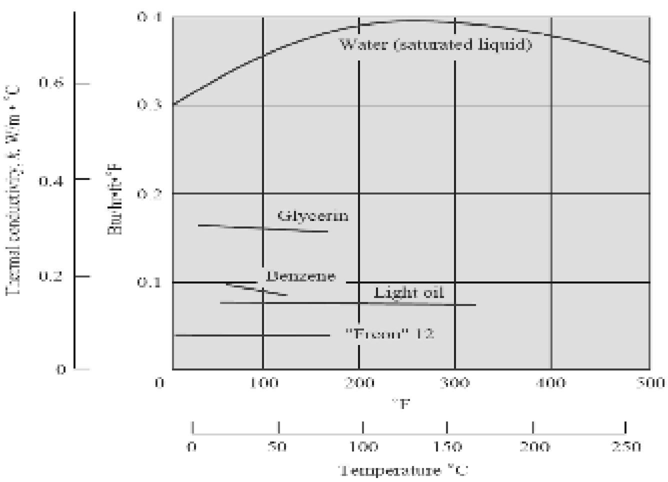
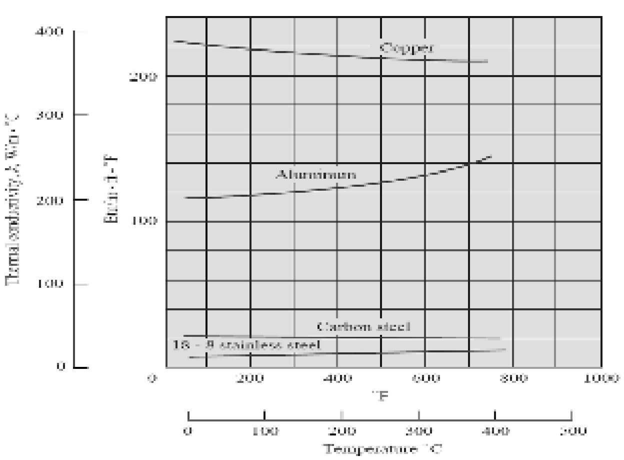


Figure 1-6 Thermal conductivities of some typical solids.



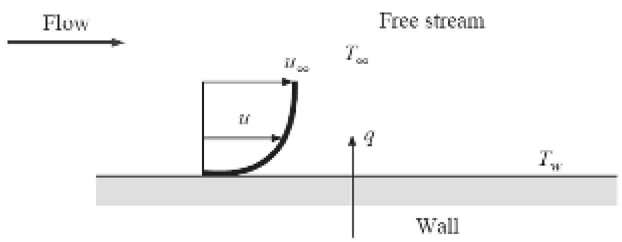
1-3 | CONVECTION HEAT TRANSFER

It is well known that a hot plate of metal will cool faster when placed in front of a fan than when exposed to still air. We say that the heat is convected away, and we call the process *convection heat transfer*. The term *convection* provides the reader with an intuitive notion concerning the heat-transfer process; however, this intuitive notion must be expanded to enable one to arrive at anything like an adequate analytical treatment of the problem. For example, we know that the velocity at which the air blows over the hot plate obviously influences the heat-transfer rate. But does it influence the cooling in a linear way; i.e., if the velocity is doubled, will the heat-transfer rate double? We should suspect that the heat-transfer rate might be different if we cooled the plate with water instead of air, but, again, how much difference would there be? These questions may be answered with the aid of some rather basic analyses presented in later chapters. For now, we sketch the physical mechanism of convection heat transfer and show its relation to the conduction process.

To express the overall effect of convection, we use Newton's law of cooling:

$$q = hA\,(T_w - T_\infty) \qquad \textbf{[1-8]}$$

Figure 1-7 | Convection heat transfer from a plate.



If a heated plate were exposed to ambient room air without an external source of motion, a movement of the air would be experienced as a result of the density gradients near the plate. We call this *natural*, or *free*, convection as opposed to *forced* convection, which is experienced in the case of the fan blowing air over a plate. Boiling and condensation phenomena are also grouped under the general subject of convection heat transfer. The approximate ranges of convection heat-transfer coefficients are indicated in Table 1-3.

1-4 | RADIATION HEAT TRANSFER

In contrast to the mechanisms of conduction and convection, where energy transfer through a material medium is involved, heat may also be transferred through regions where a perfect vacuum exists. The mechanism in this case is electromagnetic radiation. We shall limit our discussion to electromagnetic radiation that is propagated as a result of a temperature difference; this is called *thermal radiation*.

Thermodynamic considerations show* that an ideal thermal radiator, or *blackbody*, will emit energy at a rate proportional to the fourth power of the absolute temperature of the

body and directly proportional to its surface area. Thus

$$q_{\text{emitted}} = \sigma A T^4 \qquad \textbf{[1-9]}$$

where σ is the proportionality constant and is called the Stefan-Boltzmann constant with the value of 5.669×10^{-8} W/m$^2 \cdot$ K^4. Equation (1-9) is called the Stefan-Boltzmann law of thermal radiation, and it applies only to blackbodies. It is important to note that this equation is valid only for thermal radiation; other types of electromagnetic radiation may not be treated so simply.

Equation (1-9) governs only radiation *emitted* by a blackbody. The net radiant *exchange* between two surfaces will be proportional to the difference in absolute temperatures to the fourth power; i.e.,

$$\frac{q_{\text{net exchange}}}{A} \propto \sigma(T_1^4 - T_2^4) \qquad \textbf{[1-10]}$$

EXAMPLE 1-1 Conduction Through Copper Plate

One face of a copper plate 3 cm thick is maintained at 400°C, and the other face is maintained at 100°C. How much heat is transferred through the plate?

■ **Solution**

From Appendix A, the thermal conductivity for copper is 370 W/m · °C at 250°C. From Fourier's law

$$\frac{q}{A} = -k\frac{dT}{dx}$$

Integrating gives

$$\frac{q}{A} = -k\frac{\Delta T}{\Delta x} = \frac{-(370)(100-400)}{3 \times 10^{-2}} = 3.7 \text{ MW/m}^2 \quad [1.173 \times 10^6 \text{ Btu/h} \cdot \text{ft}^2]$$

Convection Calculation **EXAMPLE 1-2**

Air at 20°C blows over a hot plate 50 by 75 cm maintained at 250°C. The convection heat-transfer coefficient is 25 W/m^2 · °C. Calculate the heat transfer.

■ **Solution**

From Newton's law of cooling

$$\begin{aligned} q &= hA\,(T_w - T_\infty) \\ &= (25)(0.50)(0.75)(250-20) \\ &= 2.156 \text{ kW} \quad [7356 \text{ Btu/h}] \end{aligned}$$

Multimode Heat Transfer

EXAMPLE 1-3

Assuming that the plate in Example 1-2 is made of carbon steel (1%) 2 cm thick and that 300 W is lost from the plate surface by radiation, calculate the inside plate temperature.

■ **Solution**

The heat conducted through the plate must be equal to the sum of convection and radiation heat losses:

$$q_{\text{cond}} = q_{\text{conv}} + q_{\text{rad}}$$

$$-kA\,\frac{\Delta T}{\Delta x} = 2.156 + 0.3 = 2.456 \text{ kW}$$

$$\Delta T = \frac{(-2456)(0.02)}{(0.5)(0.75)(43)} = -3.05^\circ\text{C} \quad [-5.49^\circ\text{F}]$$

where the value of k is taken from Table 1-1. The inside plate temperature is therefore

$$T_i = 250 + 3.05 = 253.05^\circ\text{C}$$

Heat Source and Convection

EXAMPLE 1-4

An electric current is passed through a wire 1 mm in diameter and 10 cm long. The wire is submerged in liquid water at atmospheric pressure, and the current is increased until the water boils. For this situation $h = 5000$ W/m$^2 \cdot$ °C, and the water temperature will be 100°C. How much electric power must be supplied to the wire to maintain the wire surface at 114°C?

■ **Solution**

The total convection loss is given by Equation (1-8):

$$q = hA\,(T_w - T_\infty)$$

For this problem the surface area of the wire is

$$A = \pi dL = \pi(1 \times 10^{-3})(10 \times 10^{-2}) = 3.142 \times 10^{-4} \text{ m}^2$$

The heat transfer is therefore

$$q = (5000 \text{ W/m}^2 \cdot {}^\circ\text{C})(3.142 \times 10^{-4} \text{ m}^2)(114 - 100) = 21.99 \text{ W} \quad [75.03 \text{ Btu/h}]$$

and this is equal to the electric power that must be applied.

EXAMPLE 1-5

Radiation Heat Transfer

Two infinite black plates at 800°C and 300°C exchange heat by radiation. Calculate the heat transfer per unit area.

■ **Solution**

Equation (1-10) may be employed for this problem, so we find immediately

$$\begin{aligned} q/A &= \sigma(T_1^4 - T_2^4) \\ &= (5.669 \times 10^{-8})(1073^4 - 573^4) \\ &= 69.03 \text{ kW/m}^2 \quad [21{,}884 \text{ Btu/h} \cdot \text{ft}^2] \end{aligned}$$

EXAMPLE 1-6 Total Heat Loss by Convection and Radiation

A horizontal steel pipe having a diameter of 5 cm is maintained at a temperature of 50°C in a large room where the air and wall temperature are at 20°C. The surface emissivity of the steel may be taken as 0.8. Using the data of Table 1-3, calculate the total heat lost by the pipe per unit length.

■ Solution

The total heat loss is the sum of convection and radiation. From Table 1-3 we see that an estimate for the heat-transfer coefficient for *free* convection with this geometry and air is $h = 6.5\ \mathrm{W/m^2 \cdot °C}$. The surface area is πdL, so the convection loss per unit length is

$$\begin{aligned} q/L]_{\text{conv}} &= h(\pi d)(T_w - T_\infty) \\ &= (6.5)(\pi)(0.05)(50 - 20) = 30.63\ \mathrm{W/m} \end{aligned}$$

The pipe is a body surrounded by a large enclosure so the radiation heat transfer can be calculated from Equation (1-12). With $T_1 = 50°\mathrm{C} = 323°\mathrm{K}$ and $T_2 = 20°\mathrm{C} = 293°\mathrm{K}$, we have

$$\begin{aligned} q/L]_{\text{rad}} &= \epsilon_1(\pi d_1)\sigma(T_1^4 - T_2^4) \\ &= (0.8)(\pi)(0.05)(5.669 \times 10^{-8})(323^4 - 293^4) \\ &= 25.04\ \mathrm{W/m} \end{aligned}$$

The total heat loss is therefore

$$\begin{aligned} q/L]_{\text{tot}} &= q/L]_{\text{conv}} + q/L]_{\text{rad}} \\ &= 30.63 + 25.04 = 55.67\ \mathrm{W/m} \end{aligned}$$

In this example we see that the convection and radiation are about the same. To neglect either would be a serious mistake.

PROBLEMS

1-1 If 3 kW is conducted through a section of insulating material 0.6 m^2 in cross section and 2.5 cm thick and the thermal conductivity may be taken as 0.2 W/m · °C, compute the temperature difference across the material.

1-2 A temperature difference of 85°C is impressed across a fiberglass layer of 13 cm thickness. The thermal conductivity of the fiberglass is 0.035 W/m · °C. Compute the heat transferred through the material per hour per unit area.

1-3 A truncated cone 30 cm high is constructed of aluminum. The diameter at the top is 7.5 cm, and the diameter at the bottom is 12.5 cm. The lower surface is maintained at 93°C; the upper surface, at 540°C. The other surface is insulated. Assuming one-dimensional heat flow, what is the rate of heat transfer in watts?

1-4 The temperatures on the faces of a plane wall 15 cm thick are 375 and 85°C. The wall is constructed of a special glass with the following properties: $k = 0.78$ W/m · °C, $\rho = 2700$ kg/m^3, $c_p = 0.84$ kJ/kg · °C. What is the heat flow through the wall at steady-state conditions?

1-5 A certain superinsulation material having a thermal conductivity of 2×10^{-4} W/m · °C is used to insulate a tank of liquid nitrogen that is maintained at −196°C; 199 kJ is required to vaporize each kilogram mass of nitrogen at this temperature. Assuming that the tank is a sphere having an inner diameter (ID) of 0.52 m, estimate the amount of nitrogen vaporized per day for an insulation thickness of 2.5 cm and an ambient temperature of 21°C. Assume that the outer temperature of the insulation is 21°C.

1-6 Rank the following materials in order of (*a*) transient response and (*b*) steady-state conduction. Taking the material with the highest rank, give the other materials as a percentage of the maximum: aluminum, copper, silver, iron, lead, chrome steel (18% Cr, 8% Ni), magnesium. What do you conclude from this ranking?

1-7 A 50-cm-diameter pipeline in the Arctic carries hot oil at 30°C and is exposed to a surrounding temperature of −20°C. A special powder insulation 5 cm thick surrounds the pipe and has a thermal conductivity of 7 mW/m · °C. The convection heat-transfer coefficient on the outside of the pipe is 9 W/m^2 · °C. Estimate the energy loss from the pipe per meter of length.

1-8 Some people might recall being told to be sure to put on a hat when outside in cold weather because "you lose all the heat out the top of your head." Comment on the validity of this statement.

1-9 A 5-cm layer of loosely packed asbestos is placed between two plates at 100 and 200°C. Calculate the heat transfer across the layer.

1-10 A certain insulation has a thermal conductivity of 10 W/m · °C. What thickness is necessary to effect a temperature drop of 500°C for a heat flow of 400 W/m^2?