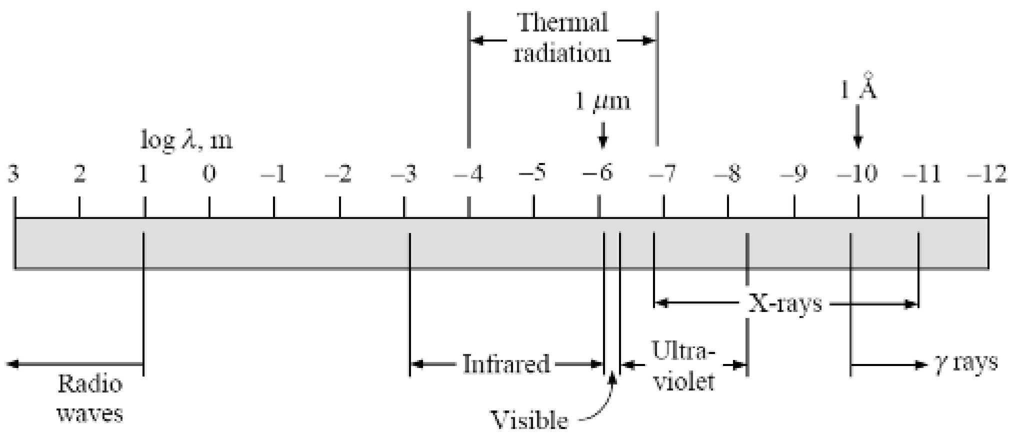
CHAPTER

8 Radiation Heat Transfer

Thermal radiation is that electromagnetic radiation emitted by a body as a result of its temperature.

A portion of the electromagnetic spectrum is shown in **Figure 8-1**. Thermal radiation lies in the range from about **0.1 to 100 µm**, while the visible-light portion of the spectrum is very narrow, extending from about **0.35 to 0.75 µm**.

Figure 8-1 | Electromagnetic spectrum.



When the energy density is integrated over all wavelengths, the total energy emitted is proportional to absolute temperature to the fourth power:

$$E_b = \sigma T^4 \qquad \text{.......... [8-1]}$$

Equation (8-1) is called the Stefan-Boltzmann law, $\mathbf{E_b}$ is the energy radiated per unit time and per unit area by the ideal radiator, and $\boldsymbol{\sigma}$ is the Stefan-Boltzmann constant, which has the value

$$\sigma = 5.669 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$$

Where $\mathbf{E_b}$ is in watts per square meter and **T** is in degrees **Kelvin**. In the thermodynamic analysis the energy density is related to the energy radiated from a surface per unit time and per unit area. Thus the heated interior surface of an enclosure produces a certain energy density of thermal radiation in the enclosure.

We are interested in radiant exchange with surfaces, hence the reason for the expression of radiation from a surface in terms of its temperature. The subscript b in Equation [8-1] denotes that this is the radiation from a blackbody.

We call this *blackbody radiation* because materials that obey this law appear black to the eye; they appear black because they do not reflect any radiation. Thus a blackbody is also considered as one that absorbs all radiation incident upon it. $\mathbf{E_b}$ is called the *emissive power* of a blackbody.

It is important to note at this point that the "blackness" of a surface to thermal radiation can be quite deceiving in so far as visual observations are concerned. A surface coated with lampblack appears black to the eye and turns out to be black for the thermal-radiation spectrum. On the other hand, snow and ice appear quite bright to the eye but are essentially "black" for long-wavelength thermal radiation. Many white paints are also essentially black for long-wavelength radiation.

RADIATION PROPERTIES

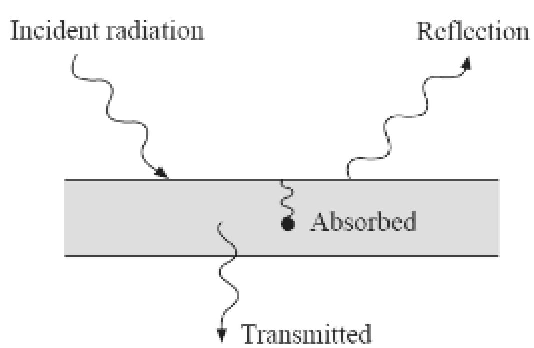
When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed, and part is transmitted, as shown in **Figure 8-2**. We define the reflectivity $\boldsymbol{\rho}$ as the fraction reflected, the absorptivity $\boldsymbol{\alpha}$ as the fraction absorbed, and the transmissivity $\boldsymbol{\tau}$ as the fraction transmitted. Thus

$$\rho + \alpha + \tau = 1 \qquad \text{...... [8-2]}$$

Most solid bodies do not transmit thermal radiation, so that for many applied problems the transmissivity may be taken as zero. Then

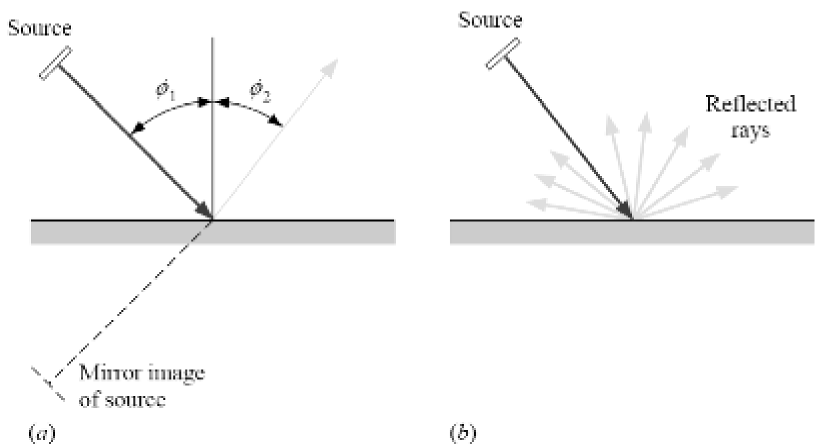
$$\rho + \alpha = 1 \qquad \text{....... [8-3]}$$

Figure 8-2 | Sketch showing effects of incident radiation.



Two types of reflection phenomena may be observed when radiation strikes a surface. If the angle of incidence is equal to the angle of reflection, the reflection is called *specular*. On the other hand, when an incident beam is distributed uniformly in all directions after reflection, the reflection is called *diffuse*. These two types of reflection are depicted in **Figure 8-3**. Note that a specular reflection presents a mirror image of the source to the observer. No real surface is either specular or diffuse. An ordinary mirror is quite specular for visible light, but would not necessarily be specular over the entire wavelength range of thermal radiation. Ordinarily, a rough surface exhibits diffuse behavior better than a highly polished surface. Similarly, a polished surface is more specular than a rough surface.

Figure 8-3 | (*a*) Specular ($\phi_1 = \phi_2$) and (*b*) diffuse reflection.



Kirchhoff's identity

The emissive power of a body **E** is defined as the energy emitted by the body per unit area and per unit time. One may perform a thought experiment to establish a relation between the emissive power of a body and the material properties defined above. Assume that a perfectly black enclosure is available, i.e., one that absorbs all the incident radiation falling upon it, as shown schematically in **Figure 8-4**. This enclosure will also emit radiation according to the $\mathbf{T^4}$ law. Let the radiant flux arriving at some area in the enclosure be $\mathbf{q_i}$ W/m^2. Now suppose that a body is placed inside the enclosure and allowed to come into temperature equilibrium with it. At equilibrium the energy absorbed by the body must be equal to the energy emitted; otherwise there would be an energy flow into or out of the body that would raise or lower its temperature. At equilibrium we may write

$$EA = q_i A\alpha \quad \text{….. [8-4]}$$

If we now replace the body in the enclosure with a blackbody of the same size and shape and allow it to come to equilibrium with the enclosure *at the same temperature,*

$$E_b A = q_i A(1) \qquad \text{..... [8-5]}$$

Since the absorptivity of a blackbody is unity. If Equation [8-4] is divided by Equation [8-5],

$$\frac{E}{E_b} = \alpha \qquad \text{..... [8-6]}$$

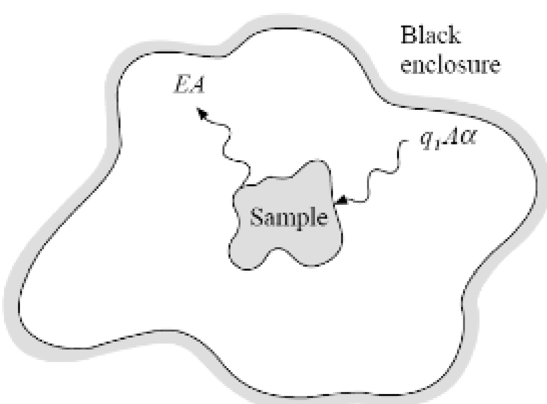
and we find that the ratio of the emissive power of a body to the emissive power of a blackbody *at the same temperature* is equal to the absorptivity of the body. This ratio is defined as the *emissivity* (**ε**) of the body,

$$\epsilon = \frac{E}{E_b} \qquad \text{...... [8-7]}$$

So that

$$\epsilon = \alpha \qquad \text{...... [8-8]}$$

Figure 8-4 | Sketch showing model used for deriving Kirchhoff's law.



RADIATION SHAPE FACTOR

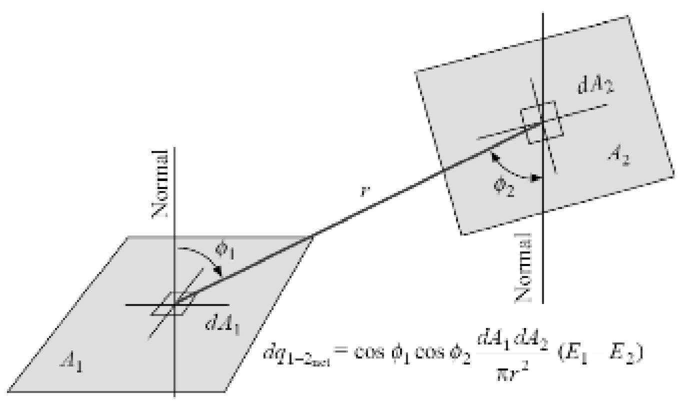
Consider two black surfaces **A_1** and **A_2**, as shown in **Figure 8-8**. We wish to obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The problem becomes essentially one of determining the amount of energy that leaves one surface and reaches the other. To solve this problem the *radiation shape factors* are defined as

F_{1-2} = fraction of energy leaving surface 1 that reaches surface 2

F_{2-1} = fraction of energy leaving surface 2 that reaches surface 1

F_{i-j} = fraction of energy leaving surface i that reaches surface j

Figure 8-8 | Sketch showing area elements used in deriving radiation shape factor.



Other names for the radiation shape factor are *view factor, angle factor,* and *configuration factor.* The energy leaving surface **1** and arriving at surface **2** is

$$E_{b1} A_1 F_{12} \qquad \text{.... [8-9]}$$

and the energy leaving surface **2** and arriving at surface **1** is

$$E_{b2} A_2 F_{21} \qquad \text{.... [8-10]}$$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

$$E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21} = Q_{1-2} \qquad \text{.... [8-11]}$$

If both surfaces are at the same temperature, there can be no heat exchange, that is, $\mathbf{Q_{1-2} = 0}$.
Also, for $\mathbf{T_1 = T_2}$

$$E_{b1} = E_{b2}$$

So that

$$A_1 F_{12} = A_2 F_{21} \qquad \text{.... [8-12]}$$

The net heat exchange is therefore

$$Q_{1-2} = A_1 F_{12}(E_{b1} - E_{b2}) = A_2 F_{21}(E_{b1} - E_{b2}) \qquad \text{.... [8-13]}$$

Equation (8-18) is known as a reciprocity relation, and it applies in a general way for any two surfaces ***i*** and ***j***:

$$A_i F_{ij} = A_j F_{ji} \qquad \text{.... [8-13]}$$

Although the relation is derived for black surfaces, it holds for other surfaces also as long as diffuse radiation is involved.

RELATIONS BETWEEN SHAPE FACTORS

Some useful relations between shape factors may be obtained by considering the system shown in **Figure 8-19**. Suppose that the shape factor for radiation from $\mathbf{A_3}$ to the combined area $\mathbf{A_{1,2}}$ is desired. This shape factor must be given very simply as

$$F_{3-1,2} = F_{3-1} + F_{3-2} \qquad \text{.... [8-14]}$$

That is, the total shape factor is the sum of its parts. We could also write Equation [8-13] as

$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2} \qquad \text{.... [8-15]}$$

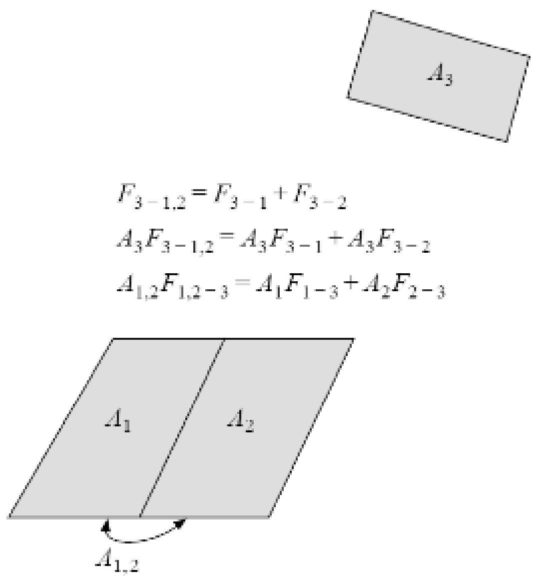
and making use of the reciprocity relations

$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$
$$A_3 F_{3-1} = A_1 F_{1-3}$$
$$A_3 F_{3-2} = A_2 F_{2-3} \qquad \text{.... [8-16]}$$

The expression could be rewritten

$$A_{1,2}F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3} \quad \text{…. [8-17]}$$

Figure 8-19 | Sketch showing some relations between shape factors.



In the foregoing discussion the tacit assumption has been made that the various bodies do not see themselves, that is,

$$F_{11} = F_{22} = F_{33} = 0 \cdots \quad \text{…. [8-18]}$$

To be perfectly general, we must include the possibility of concave curved surfaces, which may then see themselves. The general relation is therefore

$$\sum_{j=1}^{n} F_{ij} = 1.0 \quad \text{…. [8-19]}$$

Where, $\boldsymbol{F_{ij}}$ is the fraction of the total energy leaving surface $\boldsymbol{i}$ that arrives at surface $\boldsymbol{j}$. Thus for a three-surface enclosure we would write

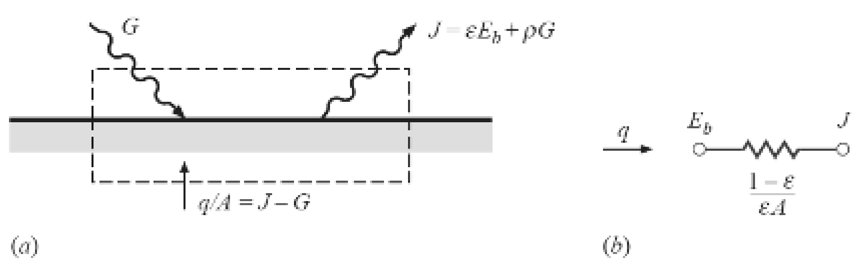
$$F_{11} + F_{12} + F_{13} = 1.0 \quad \text{…. [8-20]}$$

and F_{11} represents the fraction of energy leaving surface **1** that strikes surface **1**. A certain amount of care is required in analyzing radiation exchange between curved surfaces.

HEAT EXCHANGE BETWEEN NONBLACK BODIES

The calculation of the radiation heat transfer between black surfaces is relatively easy because all the radiant energy that strikes a surface is absorbed. The main problem is one of determining the geometric shape factor, but once this is accomplished, the calculation of the heat exchange is very simple. When nonblack bodies are involved, the situation is much more complex, for all the energy striking a surface will not be absorbed; part will be reflected back to another heat-transfer surface, and part may be reflected out of the system entirely. The problem can become complicated because the radiant energy can be reflected back and forth between the heat-transfer surfaces several times. The analysis of the problem must take into consideration these multiple reflections if correct conclusions are to be drawn. We shall assume that all surfaces considered in our analysis are diffuse, gray, and uniform in temperature and that the reflective and emissive properties are constant over the entire surface. Two new terms may be defined:

Figure 8-24 | (*a*) Surface energy balance for opaque material; (*b*) element representing "surface resistance" in the radiation-network method.



$$G = \text{irradiation}$$
$$= \text{total radiation incident upon a surface per unit time and per unit area}$$
$$J = \text{radiosity}$$
$$= \text{total radiation that leaves a surface per unit time and per unit area}$$

As shown in Figure 8-24, the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

$$J = \epsilon E_b + \rho G \qquad \text{.... [8-21]}$$

Where **ε** is the emissivity and $\boldsymbol{E_b}$ is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon \qquad \text{.... [8-22]}$$

So that

$$J = \epsilon E_b + (1 - \epsilon)G \qquad \text{.... [8-23]}$$

The net energy leaving the surface is the difference between the radiosity and the irradiation:

$$\frac{q}{A} = J - G = \epsilon E_b + (1 - \epsilon)G - G \qquad \text{.... [8-24]}$$

Solving for *G* in terms of *J* from Equation [8-24],

$$q = \frac{\epsilon A}{1 - \epsilon}(E_b - J) \qquad \text{.... [8-25]}$$

Or

$$q = \frac{E_b - J}{(1 - \epsilon)/\epsilon A} \qquad \text{.... [8-26]}$$

At this point we introduce a very useful interpretation for Equation [8-26]. If the denominator of the right side is considered as the surface resistance to radiation heat transfer, the numerator as a potential difference, and the heat flow as the "current," then a network element could be drawn as in **Figure 8-24(*b*)** to represent the physical situation.

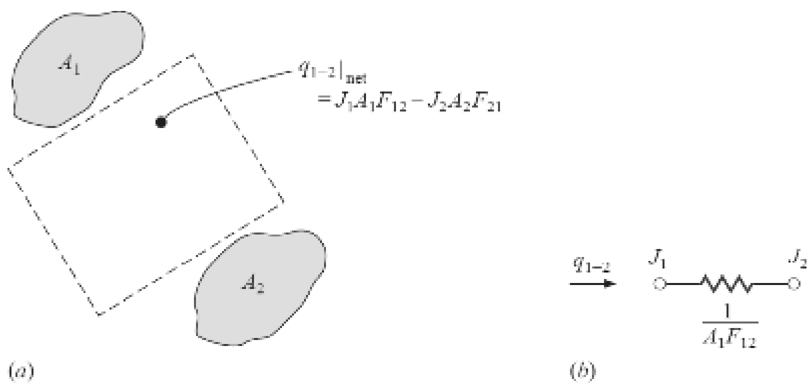
Now consider the exchange of radiant energy by two surfaces, $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$, shown in **Figure 8-25**. Of that total radiation leaving surface **1**, the amount that reaches surface **2** is

$$J_1 A_1 F_{12} \qquad \text{.... [8-27]}$$

and of that total energy leaving surface **2**, the amount that reaches surface **1** is

$$J_2 A_2 F_{21} \qquad \text{.... [8-28]}$$

Figure 8-25 | (*a*) Spatial energy exchange between two surfaces; (*b*) element representing "space resistance" in the radiation-network method.



The net interchange between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21} \quad \text{.... [8-29]}$$

But

$$A_1 F_{12} = A_2 F_{21} \quad \text{.... [8-30]}$$

So that

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21} \quad \text{.... [8-31]}$$

Or

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \quad \text{.... [8-32]}$$

We may thus construct a network element that represents Equation [8-32], as shown in **Figure 8-25*b***. The two network elements shown in **Figures 8-24** and **8-25** represent the essentials of the radiation-network method. To construct a network for a particular radiation heat-transfer problem we need only connect a "surface resistance" **(1− ε)/ εA** to each surface and a "space resistance" **$1/A_iF_{ij}$** between the radiosity potentials. For example, two surfaces that exchange heat with each other *and nothing else* would be represented by the network shown in Figure 8-26. In this case the net heat transfer would be the overall potential difference divided by the sum of the resistances:

$$q_{net} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} = \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} \qquad \text{.... [8-33]}$$

A network for a three-body problem is shown in **Figure 8-27**. In this case, each of the bodies exchanges heat with the other two. The heat exchange between body **1** and body **2** would be

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \qquad \text{.... [8-34]}$$

and that between body **1** and body **3**,

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1 F_{13}} \qquad \text{.... [8-35]}$$

To determine the heat flows in a problem of this type, the values of the radiosities must be calculated. This may be accomplished by performing standard methods of analysis used in dc circuit theory. The most convenient method is an application of Kirchhoff's current law to the circuit, which states that the sum of the currents entering a node is zero.

Figure 8-26 | Radiation network for two surfaces that see each other and nothing else.

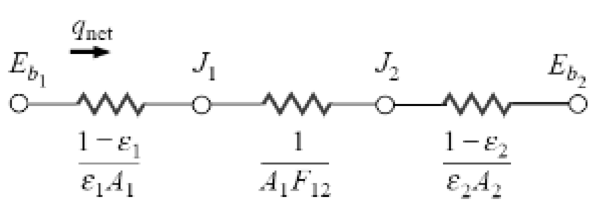
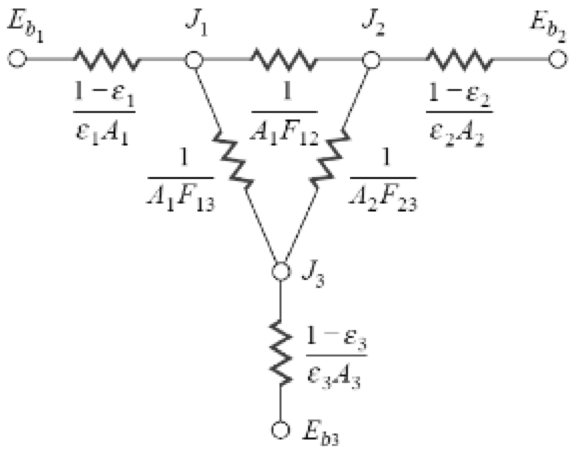


Figure 8-27 | Radiation network for three surfaces that see each other and nothing else.



Insulated Surfaces and Surfaces with Large Areas

As we have seen, $(E_b - J)$ represents the potential difference for heat flow through the surface resistance $(1-\epsilon)/\epsilon A$. If a surface is perfectly insulated, or re-radiates all the energy incident upon it, it has zero heat flow and the potential difference across the surface resistance is zero, resulting in $J = E_b$. *But, the insulated surface does not have zero surface resistance.* In effect, the J node in the network is *floating,* that is, it does not draw any current. On the other hand, a surface with a very large area $(A \to \infty)$ has a surface resistance approaching zero, which makes it behave like a blackbody with $\epsilon = 1.0$. It, too, will have $J = E_b$ because of the zero surface resistance. Thus, these two cases—insulated surface and surface with a large area—both have $J = E_b$, but for entirely different reasons. We will make use of these special cases in several examples.

A problem that may be easily solved with the network method is that of two flat surfaces exchanging heat with one another but connected by a third surface that does not exchange heat, i.e., one that is perfectly insulated. This third surface nevertheless influences the heat-transfer process because it absorbs and re-radiates energy to the other two surfaces that exchange heat. The network for this system is shown in Figure 8-28. Notice that node J_3 is not connected to a radiation surface resistance because surface 3 does not exchange energy. A surface resistance $(1-\epsilon)/\epsilon A$ exists, but because there is no heat current flow there is no potential difference, and $\boldsymbol{J_3} = \boldsymbol{E_{b3}}$. Notice also that the values for the space resistances have been written

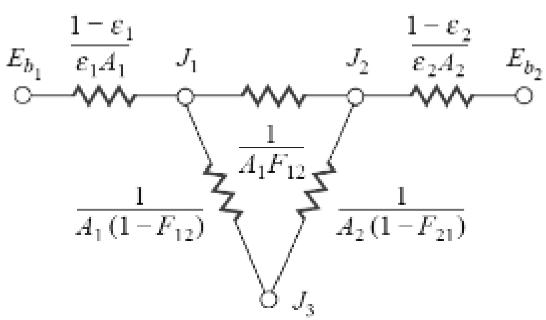
$$F_{13} = 1 - F_{12}$$

$$F_{23} = 1 - F_{21} \qquad \text{.... [8-36]}$$

since surface **3** completely surrounds the other two surfaces. For the *special case where surfaces **1** and **2** are convex*, that is, they do not see themselves and $\boldsymbol{F_{11} = F_{22} = 0}$, **Figure 8-28** is a simple series-parallel network that may be solved for the heat flow as

$$q_{\text{net}} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{A_1 + A_2 - 2A_1 F_{12}}{A_2 - A_1 (F_{12})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)} \quad \text{.... [8-37]}$$

Figure 8-28 | Radiation network for two plane or convex surfaces enclosed by a third surface that is nonconducting but re-radiating (insulated).



Where the reciprocity relation

$$A_1 F_{12} = A_2 F_{21}$$

has been used to simplify the expression. *It is to be noted again that Equation (8-37) applies only to surfaces that do not see themselves; that is,* $F_{11} = F_{22} = 0$. If these conditions do not apply, one must determine the respective shape factors and solve the network accordingly. Example 8-7 gives an appropriate illustration of a problem involving an insulated surface.

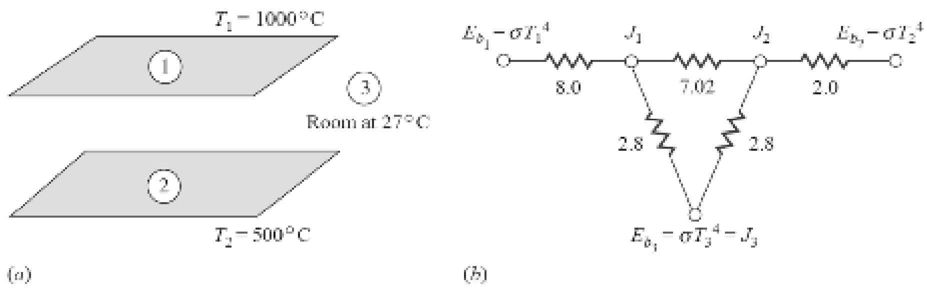
This network, and others that follow, assume that the only heat exchange is by radiation. Conduction and convection are neglected for now.

EXAMPLE 8-6 Hot Plates Enclosed by a Room

Two parallel plates **0.5** by **1.0** m are shown in Figure Example 8-6. One plate is maintained at **1000°C** and the other at **500°C**. The emissivities of the plates are **0.2** and **0.5**, respectively. The plates are located in a very large room, the walls of which are maintained at **27°C**. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.

Given $F_{12} = 0.285 = F_{21}$

Figure Example 8-6 | (*a*) Schematic. (*b*) Network.



Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown in Figure 8-27. From the data of the problem

$T_1 = 1000°C = 1273$ K	$A_1 = A_2 = 0.5$ m^2
$T_2 = 500°C = 773$ K	$\epsilon_1 = 0.2$
$T_3 = 27°C = 300$ K	$\epsilon_2 = 0.5$

Because the area of the room A_3 is very large, the resistance $(1 - \epsilon_3)/\epsilon_3 A_3$ may be taken as zero

and we obtain $E_{b_3} = J_3$

$$F_{12} = 0.285 = F_{21}$$
$$F_{13} = 1 - F_{12} = 0.715$$
$$F_{23} = 1 - F_{21} = 0.715$$

The resistances in the network are calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$

$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1:

$$\frac{E_{b_1}-J_1}{8.0}+\frac{J_2-J_1}{7.018}+\frac{E_{b_3}-J_1}{2.797}=0 \qquad \textbf{[a]}$$

node J_2:

$$\frac{J_1-J_2}{7.018}+\frac{E_{b_3}-J_2}{2.797}+\frac{E_{b_2}-J_2}{2.0}=0 \qquad \textbf{[b]}$$

Now

$$E_{b_1}=\sigma T_1^4=148.87\ \mathrm{kW/m^2} \quad [47{,}190\ \mathrm{Btu/h\cdot ft^2}]$$

$$E_{b_2}=\sigma T_2^4=20.241\ \mathrm{kW/m^2} \quad [6416\ \mathrm{Btu/h\cdot ft^2}]$$

$$E_{b_3}=\sigma T_3^4=0.4592\ \mathrm{kW/m^2} \quad [145.6\ \mathrm{Btu/h\cdot ft^2}]$$

Inserting the values of E_{b_1}, E_{b_2} and E_{b_3} into Equations (a) and (b), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$$J_1=33.469\ \mathrm{kW/m^2} \qquad J_2=15.054\ \mathrm{kW/m^2}$$

The total heat lost by plate 1 is

$$q_1=\frac{E_{b_1}-J_1}{(1-\epsilon_1)/\epsilon_1 A_1}=\frac{148.87-33.469}{8.0}=14.425\ \mathrm{kW}$$

and the total heat lost by plate 2 is

$$q_2=\frac{E_{b_2}-J_2}{(1-\epsilon_2)/\epsilon_2 A_2}=\frac{20.241-15.054}{2.0}=2.594\ \mathrm{kW}$$

The total heat received by the room is

$$q_3=\frac{J_1-J_3}{1/A_1F_{13}}+\frac{J_2-J_3}{1/A_2F_{23}}$$
$$=\frac{33.469-0.4592}{2.797}+\frac{15.054-0.4592}{2.797}=17.020\ \mathrm{kW} \quad [58{,}070\ \mathrm{Btu/h}]$$

From an overall-balance standpoint we must have

$$q_3=q_1+q_2$$

because the net energy lost by both plates must be absorbed by the room.

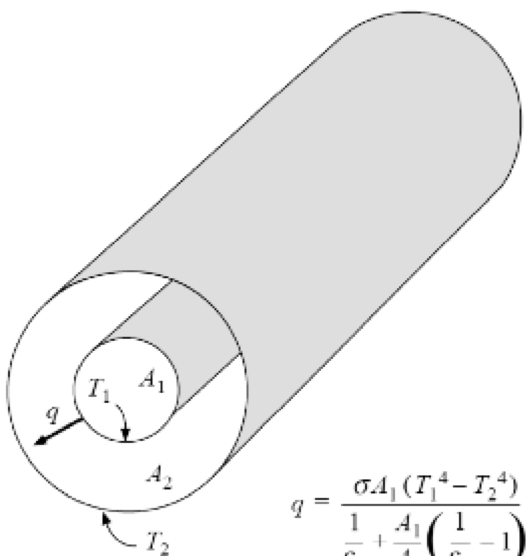
INFINITE PARALLEL SURFACES

When two infinite parallel planes are considered, A_1 and A_2 are equal; and the radiation shape factor is unity since all the radiation leaving one plane reaches the other. The network is the same as in **Figure 8-26**, and the heat flow per unit area may be obtained from Equation (8-33) by letting $A_1 = A_2$ and $F_{12} = 1.0$. Thus

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} \qquad \text{.... [8-38]}$$

When two long concentric cylinders as shown in **Figure 8-29** exchange heat we may again apply Equation (8-33). Rewriting the equation and noting that $F_{12} = 1.0$,

Figure 8-29 | Radiation exchange between two cylindrical surfaces.



$$q = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$q = \frac{\sigma A_1(T_1^4 - T_2^4)}{1/\epsilon_1 + (A_1/A_2)(1/\epsilon_2 - 1)} \qquad \text{.... [8-39]}$$

The area ratio A_1/A_2 may be replaced by the diameter ratio d_1/d_2 when cylindrical bodies are concerned.

Convex Object in Large Enclosure

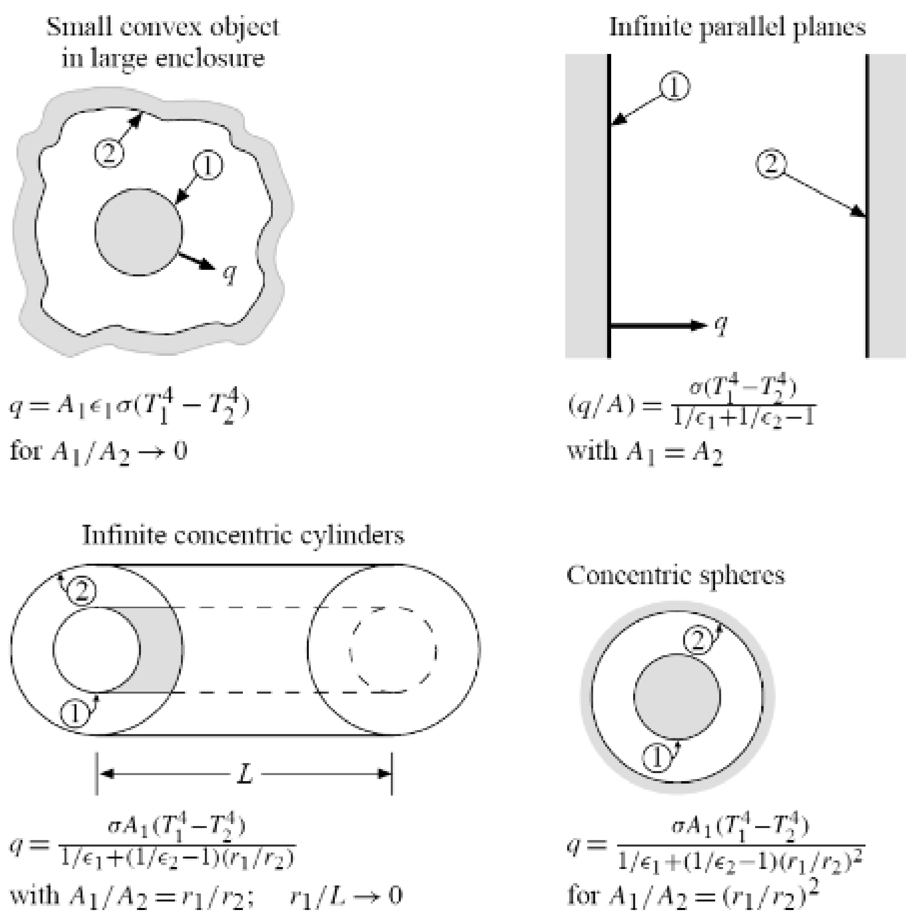
Equation (8-39) is particularly important when applied to the limiting case of a convex object completely enclosed by a very large concave surface. In this instance $A_1/A_2 \rightarrow 0$ and the following simple relation results:

$$q = \sigma A_1 \epsilon_1 (T_1^4 - T_2^4) \qquad \text{.... [8-40]}$$

This equation is readily applied to calculate the radiation-energy loss from a hot object in a large room.

Some of the radiation heat-transfer cases for simple two-body problems are summarized in Figure **8-30**. In this figure, both surfaces are assumed to be gray and diffuse.

Figure 8-30 | Radiation heat transfer between simple two-body diffuse, gray surfaces. In all cases $F_{12} = 1.0$.

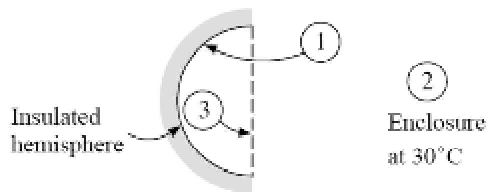


Open Hemisphere in Large Room

EXAMPLE 8-8

The 30-cm-diameter hemisphere in Figure Example 8-8 is maintained at a constant temperature of 500°C and insulated on its back side. The surface emissivity is 0.4. The opening exchanges radiant energy with a large enclosure at 30°C. Calculate the net radiant exchange.

Figure Example 8-8



■ Solution

This is an object completely surrounded by a large enclosure but the inside surface of the sphere is *not convex;* that is, it sees itself, and therefore we are *not* permitted to use Equation (8-43a). In the figure we take the inside of the sphere as surface 1 and the enclosure as surface 2. We also create an imaginary surface 3 covering the opening. We actually have a two-surface problem (surfaces 1 and 2) and therefore may use Equation (8-40) to calculate the heat transfer. Thus,

$$\begin{aligned} E_{b_1} &= \sigma T_1^4 = \sigma(773)^4 = 20{,}241 \text{ W/m}^2 \\ E_{b_2} &= \sigma T_2^4 = \sigma(303)^4 = 478 \text{ W/m}^2 \\ A_1 &= 2\pi r^2 = (2)\pi(0.15)^2 = 0.1414 \text{ m}^2 \\ \frac{1-\epsilon_1}{\epsilon_1 A_1} &= \frac{0.6}{(0.4)(0.1414)} = 10.61 \\ A_2 &\to \infty \end{aligned}$$

so that

$$\frac{1-\epsilon_2}{\epsilon_2 A_2} \to 0$$

Now, at this point we recognize that all of the radiation leaving surface 1 that will eventually arrive at enclosure 2 will also hit the imaginary surface 3 (i.e., $F_{12} = F_{13}$). We also recognize that

$$A_1 F_{13} = A_3 F_{31}$$

But, $F_{31} = 1.0$ so that

$$F_{13} = F_{12} = \frac{A_3}{A_1} = \frac{\pi r^2}{2\pi r^2} = 0.5$$

Then $1/A_1 F_{12} = 1/(0.1414)(0.5) = 14.14$ and we can calculate the heat transfer by inserting the quantities in Equation (8-40):

$$q = \frac{20{,}241 - 478}{10.61 + 14.14 + 0} = 799 \text{ W}$$

RADIATION SHIELDS

One way of reducing radiant heat transfer between two particular surfaces is to use materials that are highly reflective. An alternative method is to use radiation shields between the heat-exchange surfaces. These shields do not deliver or remove any heat from the overall system; they only place another resistance in the heat-flow path so that the overall heat transfer is retarded.

Consider the two parallel infinite planes shown in **Figure 8-33*a***. We have shown that the heat exchange between these surfaces may be calculated with Equation (8-38). Now consider the same two planes, but with a radiation shield placed between them, as in **Figure 8-33*b***. The heat transfer will be calculated for this latter case and compared with the heat transfer without the shield.

Since the shield does not deliver or remove heat from the system, the heat transfer between plate 1 and the shield must be precisely the same as that between the shield and plate 2, and this is the overall heat transfer. Thus

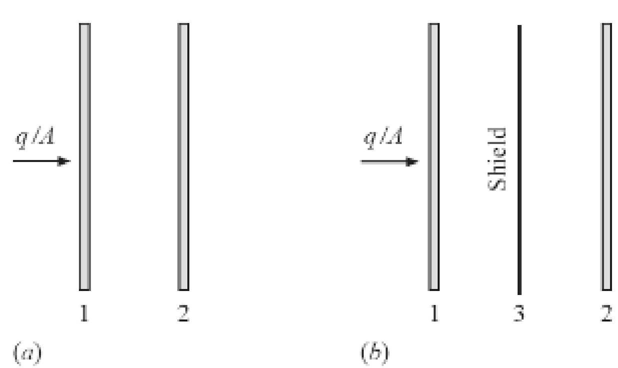
$$\left(\frac{q}{A}\right)_{1-3} = \left(\frac{q}{A}\right)_{3-2} = \frac{q}{A}$$

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_3^4)}{1/\epsilon_1 + 1/\epsilon_3 - 1} = \frac{\sigma(T_3^4 - T_2^4)}{1/\epsilon_3 + 1/\epsilon_2 - 1} \quad \text{.... [8-41]}$$

The only unknown in Equation (8-41) is the temperature of the shield T_3. Once this temperature is obtained, the heat transfer is easily calculated. If the emissivities of all three surfaces are equal, that is, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, we obtain the simple relation

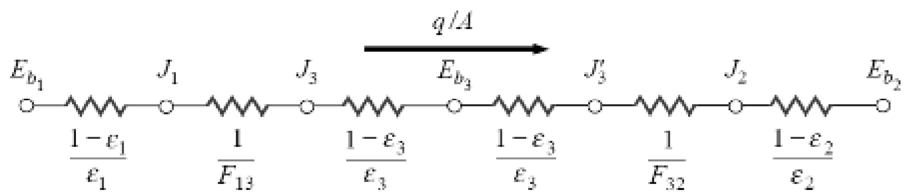
$$T_3^4 = \frac{1}{2}(T_1^4 + T_2^4) \quad \text{.... [8-42]}$$

Figure 8-33 | Radiation between parallel infinite planes with and without a radiation shield.



But since $\epsilon_3 = \epsilon_2$, we observe that this heat flow is just one-half of that which would be experienced if there were no shield present. The radiation network corresponding to the situation in Figure 8-33*b* is given in Figure 8-34.

Figure 8-34 | Radiation network for two parallel planes separated by one radiation shield.



And the heat transfer is

$$\frac{q}{A} = \frac{\frac{1}{2}\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_3 - 1} \qquad \text{.... [8-43]}$$

Multiple-radiation-shield problems may be treated in the same manner as that outlined above. When the emissivities of all surfaces are different, the overall heat transfer may be calculated most easily by using a series radiation network with the appropriate number of elements, similar to the one in **Figure 8-34**. If the emissivities of all surfaces are equal, a rather simple relation may be derived for the heat transfer when the surfaces may be considered as infinite parallel planes. Let the number of shields be ***n***. considering the radiation network for the system, all the "surface resistances" would be the same since the emissivities are equal. There would be two of these resistances for each shield and one for each heat-transfer surface. There would be ***n*+1** "space resistances," and these would all be unity since the radiation shape factors are unity for the infinite parallel planes. The total resistance in the network would thus be

$$R(n \text{ shields}) = (2n+2)\frac{1-\epsilon}{\epsilon} + (n+1)(1) = (n+1)\left(\frac{2}{\epsilon} - 1\right) \qquad \text{.... [8-44]}$$

The resistance when no shield is present is

$$R(\text{no shield}) = \frac{1}{\epsilon} + \frac{1}{\epsilon} - 1 = \frac{2}{\epsilon} - 1 \qquad \text{.... [8-45]}$$

We note that the resistance with the shields in place is ***n*+1** times as large as when the shields are absent. Thus

$$\left(\frac{q}{A}\right)_{\substack{\text{with}\\\text{shields}}} = \frac{1}{n+1}\left(\frac{q}{A}\right)_{\substack{\text{without}\\\text{shields}}} \qquad \text{.... [8-46]}$$

if the temperatures of the heat-transfer surfaces are maintained the same in both cases. The radiation-network method may also be applied to shield problems involving cylindrical systems. In these cases the proper area relations must be used in formulating the resistance elements.

Notice that the analyses above, dealing with infinite parallel planes, have been carried out on a per-unit-area basis because all areas are the same.

EXAMPLE 8-10 Heat-Transfer Reduction with Parallel-Plate Shield

Two very large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction in heat transfer when a polished-aluminum radiation shield ($\epsilon = 0.04$) is placed between them.

■ Solution

The heat transfer without the shield is given by

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} = 0.279\sigma(T_1^4 - T_2^4)$$

The radiation network for the problem with the shield in place is shown in Figure 8-34. The resistances are

$$\frac{1-\epsilon_1}{\epsilon_1} = \frac{1-0.3}{0.3} = 2.333$$

$$\frac{1-\epsilon_3}{\epsilon_3} = \frac{1-0.04}{0.04} = 24.0$$

$$\frac{1-\epsilon_2}{\epsilon_2} = \frac{1-0.8}{0.8} = 0.25$$

The total resistance with the shield is

$$2.333 + (2)(24.0) + (2)(1) + 0.25 = 52.583$$

and the heat transfer is

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{52.583} = 0.01902\sigma(T_1^4 - T_2^4)$$

so that the heat transfer is *reduced* by 93.2 percent.