CHAPTER

10 Heat Exchangers

The application of the principles of heat transfer to the design of equipment to accomplish a certain engineering objective is of extreme importance, for in applying the principles to design, the individual is working toward the important goal of product development for economic gain. Eventually, economics plays a key role in the design and selection of heat-exchange equipment, and the engineer should bear this in mind when embarking on any new heat-transfer design problem. The weight and size of heat exchangers used in space or aeronautical applications are very important parameters, and in these cases cost considerations are frequently subordinated insofar as material and heat-exchanger construction costs are concerned; however, the weight and size are important cost factors in the overall application in these fields and thus may still be considered as economic variables.

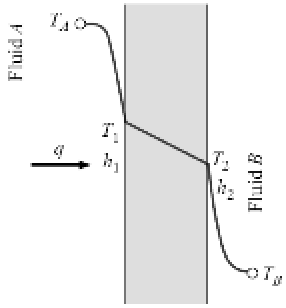
Our discussion of heat exchangers will take the form of technical analysis; that is, the methods of predicting heat-exchanger performance will be outlined, along with a discussion of the methods that may be used to estimate the heat exchanger size and type necessary to accomplish a particular task. In this respect, we limit our discussion to heat exchangers where the primary modes of heat transfer are conduction and convection. This is not to imply that radiation is not important in heat-exchanger design, for in many space applications it is the predominant means available for affecting an energy transfer.

THE OVERALL HEAT-TRANSFER COEFFICIENT

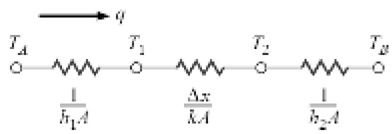
We have already discussed the overall heat-transfer coefficient in **Section 2-4** with the heat transfer through the plane wall of **Figure 10-1** expressed as

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} \qquad \text{…….. [10-1]}$$

Figure 10-1 | Overall heat transfer through a plane wall.



(a)



(b)

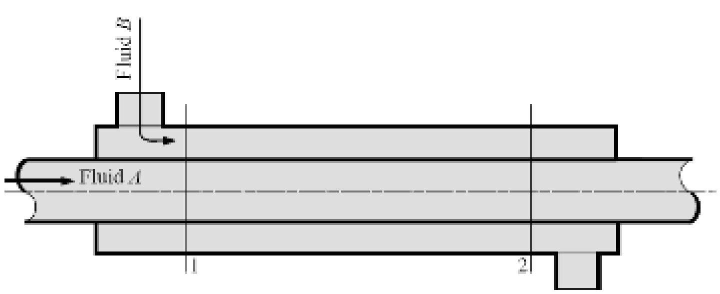
where T_A and T_B are the fluid temperatures on each side of the wall. The overall heat-transfer coefficient U is defined by the relation

$$q = UA\,\Delta T_{\text{overall}} \qquad \text{........ [10-2]}$$

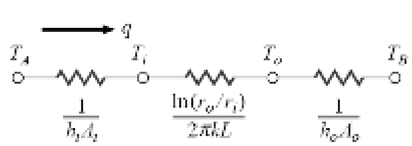
From the standpoint of heat-exchanger design, the plane wall is of infrequent application; a more important case for consideration would be that of a double-pipe heat exchanger, as shown in **Figure 10-2**. In this application one fluid flows on the inside of the smaller tube while the other fluid flows in the annular space between the two tubes. The convection coefficients are calculated by the methods described in previous chapters, and the overall heat transfer is obtained from the thermal network of **Figure 10-2*b*** as

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}} \qquad \text{........ [10-3]}$$

Figure 10-2 | Double-pipe heat exchange: (a) schematic;
(b) thermal-resistance network for overall heat transfer.



(a)



(b)

where the subscripts *i* and *o* pertain to the inside and outside of the smaller inner tube. The overall heat-transfer coefficient may be based on either the inside or outside area of the tube at the discretion of the designer. Accordingly,

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi k L} + \frac{A_i}{A_o}\frac{1}{h_o}} \qquad \text{........ [10-4a]}$$

$$U_o = \frac{1}{\frac{A_o}{A_i}\frac{1}{h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o}} \qquad \text{........ [10-4b]}$$

We should remark that the value of ***U*** is governed in many cases by only one of the convection heat-transfer coefficients. In most practical problems the conduction resistance is small compared with the convection resistances. Then, if one value of ***h*** is markedly lower than the other value, it will tend to dominate the equation for ***U***. Examples 10-1 and 10-2 illustrate this concept.

Overall Heat-Transfer Coefficient for Pipe in Air — EXAMPLE 10-1

Hot water at 98°C flows through a 2-in schedule 40 horizontal steel pipe [$k = 54$ W/m · °C] and is exposed to atmospheric air at 20°C. The water velocity is 25 cm/s. Calculate the overall heat-transfer coefficient for this situation, based on the outer area of pipe.

Solution

From Appendix A the dimensions of 2-in schedule 40 pipe are

$$\text{ID} = 2.067 \text{ in} = 0.0525 \text{ m}$$
$$\text{OD} = 2.375 \text{ in} = 0.06033 \text{ m}$$

The heat-transfer coefficient for the water flow on the inside of the pipe is determined from the flow conditions with properties evaluated at the bulk temperature. The free-convection heat-transfer coefficient on the outside of the pipe depends on the temperature difference between the surface and ambient air. This temperature difference depends on the overall energy balance. First, we evaluate h_i and then formulate an iterative procedure to determine h_o.

The properties of water at 98°C are

$$\rho = 960 \text{ kg/m}^3 \qquad \mu = 2.82 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$
$$k = 0.68 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 1.76$$

The Reynolds number is

$$\text{Re} = \frac{\rho u d}{\mu} = \frac{(960)(0.25)(0.0525)}{2.82 \times 10^{-4}} = 44{,}680 \qquad [a]$$

and since turbulent flow is encountered, we may use Equation (6-4):

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$$
$$= (0.023)(44{,}680)^{0.8}(1.76)^{0.4} = 151.4$$
$$h_i = \mathrm{Nu}\frac{k}{d} = \frac{(151.4)(0.68)}{0.0525} = 1961\ \mathrm{W/m^2\cdot{}^\circ C} \quad [345\ \mathrm{Btu/h\cdot ft^2\cdot{}^\circ F}] \qquad \textbf{[b]}$$

For unit length of the pipe the thermal resistance of the steel is

$$R_s = \frac{\ln(r_o/r_i)}{2\pi k} = \frac{\ln(0.06033/0.0525)}{2\pi(54)} = 4.097\times 10^{-4} \qquad \textbf{[c]}$$

Again, on a unit-length basis the thermal resistance on the inside is

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i 2\pi r_i} = \frac{1}{(1961)\pi(0.0525)} = 3.092\times 10^{-3} \qquad \textbf{[d]}$$

The thermal resistance for the outer surface is as yet unknown but is written, for unit lengths,

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_o} \qquad \textbf{[e]}$$

From Table 7-2, for laminar flow, the simplified relation for h_o is

$$h_o = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} = 1.32\left(\frac{T_o - T_\infty}{d}\right)^{1/4} \qquad \textbf{[f]}$$

where T_o is the unknown outside pipe surface temperature. We designate the inner pipe surface as T_i and the water temperature as T_w; then the energy balance requires

$$\frac{T_w - T_i}{R_i} = \frac{T_i - T_o}{R_s} = \frac{T_o - T_\infty}{R_o} \qquad \textbf{[g]}$$

Combining Equations (e) and (f) gives

$$\frac{T_o - T_\infty}{R_o} = 2\pi r_o \frac{1.32}{d^{1/4}}(T_o - T_\infty)^{5/4} \qquad \textbf{[h]}$$

This relation may be introduced into Equation (g) to yield two equations with the two unknowns T_i and T_o:

$$\frac{98 - T_i}{3.092\times 10^{-3}} = \frac{T_i - T_o}{4.097\times 10^{-4}}$$

$$\frac{T_i - T_o}{4.097\times 10^{-4}} = \frac{(\pi)(0.06033)(1.32)(T_o - 20)^{5/4}}{(0.06033)^{1/4}}$$

This is a nonlinear set that may be solved by iteration to give

$$T_o = 97.6^\circ\mathrm{C} \qquad T_i = 97.65^\circ\mathrm{C}$$

As a result, the outside heat-transfer coefficient and thermal resistance are

$$h_o = \frac{(1.32)(97.6 - 20)^{1/4}}{(0.06033)^{1/4}} = 7.91\ \mathrm{W/m^2\cdot{}^\circ C} \quad [1.39\ \mathrm{Btu/h\cdot ft^2\cdot{}^\circ F}]$$
$$R_o = \frac{1}{(0.06033)(7.91)\pi} = 0.667$$

The calculation clearly illustrates the fact that the free convection controls the overall heat-transfer because R_o is much larger than R_i or R_s. The overall heat-transfer coefficient based on the outer area is written in terms of these resistances as

$$U_o = \frac{1}{A_o(R_i + R_s + R_o)} \qquad [i]$$

With numerical values inserted,

$$U_o = \frac{1}{\pi(0.06033)(3.092 \times 10^{-3} + 4.097 \times 10^{-4} + 0.667)}$$
$$= 7.87 \text{ W/Area} \cdot {}^\circ\text{C}$$

In this calculation we used the outside area for a 1.0-m length as

$$A_o = \pi(0.06033) = 0.1895 \text{ m}^2/\text{m}$$
$$U_o = 7.87 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Thus, we find that the overall heat-transfer coefficient is almost completely controlled by the value of h_o. We might have expected this result strictly on the basis of our experience with the relative magnitude of convection coefficients; free-convection values for air are very low compared with forced convection with liquids.

FOULING FACTORS

After a period of operation the heat-transfer surfaces for a heat exchanger may become coated with various deposits present in the flow systems, or the surfaces may become corroded as a result of the interaction between the fluids and the material used for construction of the heat exchanger. In either event, this coating represents an additional resistance to the heat flow, and thus results in decreased performance. The overall effect is usually represented by a ***fouling factor****,* or fouling resistance, $\boldsymbol{R_f}$, which must be included along with the other thermal resistances making up the overall heat-transfer coefficient.

Fouling factors must be obtained experimentally by determining the values of $\boldsymbol{U}$ for both clean and dirty conditions in the heat exchanger. The fouling factor is thus defined as

$$R_f = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}$$

TYPES OF HEAT EXCHANGERS

One type of heat exchanger has already been mentioned, that of a double-pipe arrangement as shown in **Figure 10-2**. Either counter flow or parallel flow may be used in this type of exchanger, with either, the hot or cold fluid occupying the annular space and the other fluid occupying the inside of the inner pipe.

A type of heat exchanger widely used in the chemical-process industries is that of the shell-and-tube arrangement shown in **Figure 10-3**. One fluid flows on the inside of the tubes, while the other fluid is forced through the shell and over the outside of the tubes. To ensure that the shell-side fluid will flow across the tubes

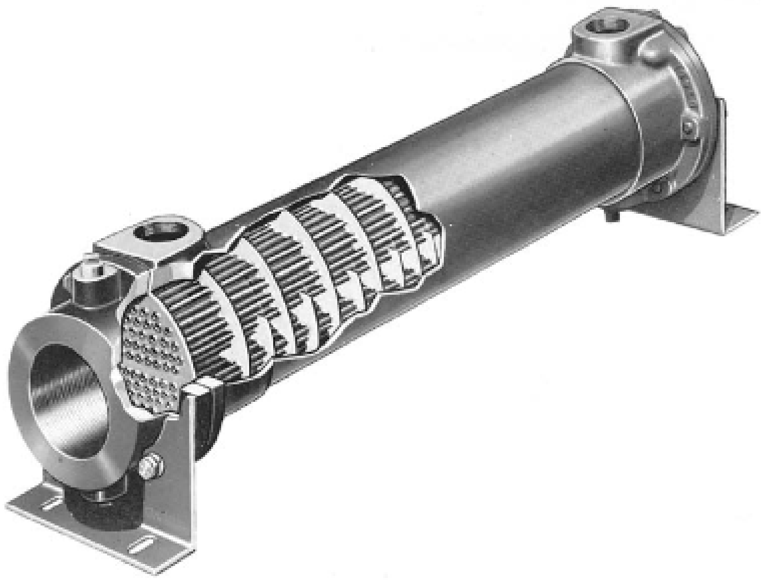
and thus induce higher heat transfer, baffles are placed in the shell as shown in the figure. Depending on the head arrangement at the ends of the exchanger, one or more tube passes may be utilized. In Figure **10-3*a*** one tube pass is used, and the head arrangement for two tube passes is shown in Figure **10-3*b***.

Shell and tube exchangers may also be employed in miniature form for specialized applications in biotechnology fields. Such exchanger with one shell pass and one tube pass is illustrated in **Figure 10-3*c*** and the internal tube construction in **Figure 10-3*d***. Small double pipe or tube-in-tube exchangers may also be constructed in a coiled configuration as shown in **Figure 10-3*e*** with an enlarged view of the inlet-outlet flow connections shown in **Figure 10-3*f***. Cross-flow exchangers are commonly used in air or gas heating and cooling applications. An example of such exchanger is shown in **Figure 10-4**, where a gas may be forced across a tube bundle, while another fluid is used inside the tubes for heating or cooling purposes. In this exchanger the gas flowing across the tubes is said to be a *mixed* stream, while the fluid in the tubes is said to be *unmixed.* The gas is mixed because it can move about freely in the exchanger as it exchanges heat. The other fluid is confined in separate tubular channels while in the exchanger so that it cannot mix with itself during the heat-transfer process.

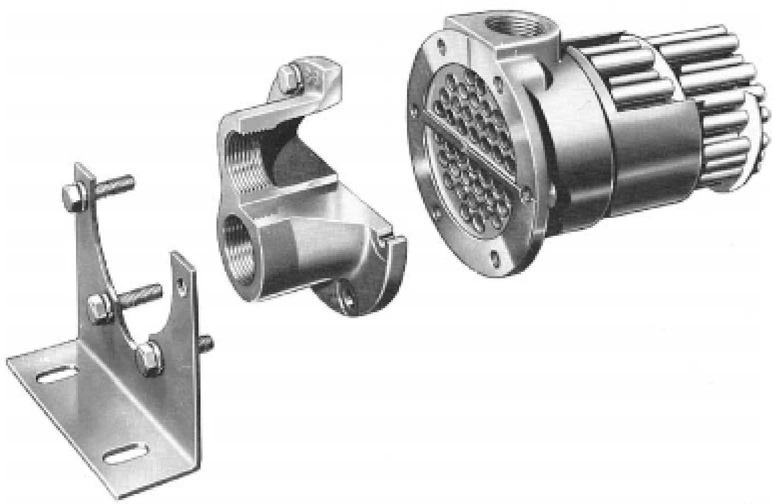
A different type of cross-flow exchanger is shown in **Figure 10-5**. In this case the gas flows across finned-tube bundles and thus is *unmixed* since it is confined in separate channels between the fins as it passes through the exchanger. This exchanger is typical of the types used in air-conditioning applications. If a fluid is unmixed, there can be a temperature gradient both parallel and normal to the flow direction, whereas when the fluid is mixed; there will be a tendency for the fluid temperature to equalize in the direction normal to the flow as a result of the mixing.

An approximate temperature profile for the gas flowing in the exchanger of **Figure 10-5** is indicated in **Figure 10-6**, assuming that the gas is being heated as it passes through the exchanger. The fact that a fluid is mixed or unmixed influences the overall heat transfer in the exchanger because this heat transfer is dependent on the temperature difference between the hot and cold fluids.

Figure 10-3 | Photos of commercial heat exchangers. (*a*) Shell-and-tube heat exchanger with one tube pass; (*b*) head arrangement for exchanger with two tube passes; (*c*) miniature shell-and-tube exchanger with one shell pass and one tube pass; (*d*) internal construction of miniature exchanger; (*e*) miniature coiled tube-in-tube exchanger; (*f*) detail of inlet–outlet fluid connections for miniature tube-in-tube exchanger.

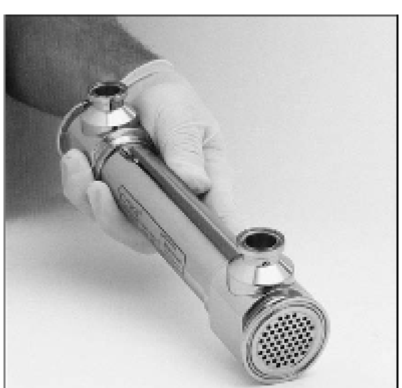


(*a*)

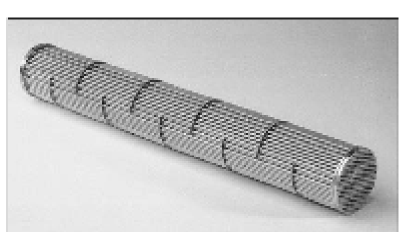


(*b*)

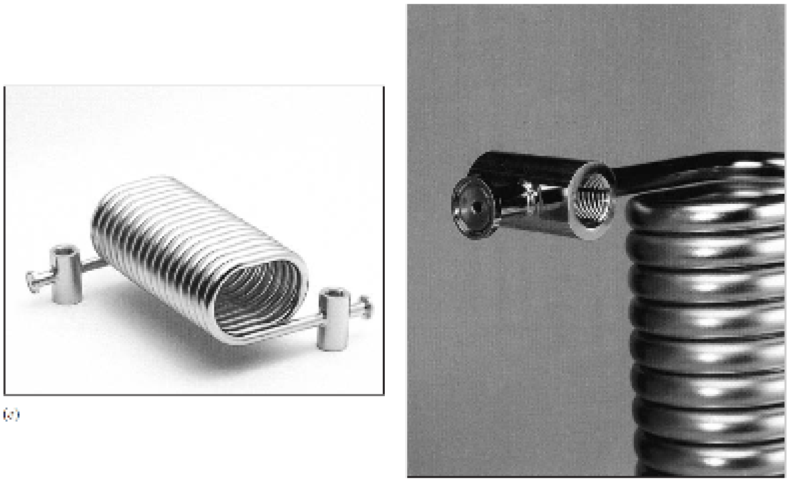
Figure 10-3 | (*Continued*).



(*c*)



(*d*)



(e) (f)

Figure 10-4 | Cross-flow heat exchanger, one fluid mixed and one unmixed.

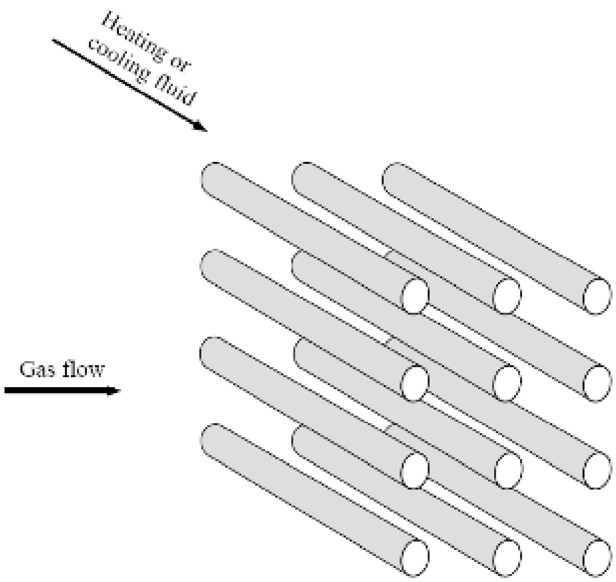
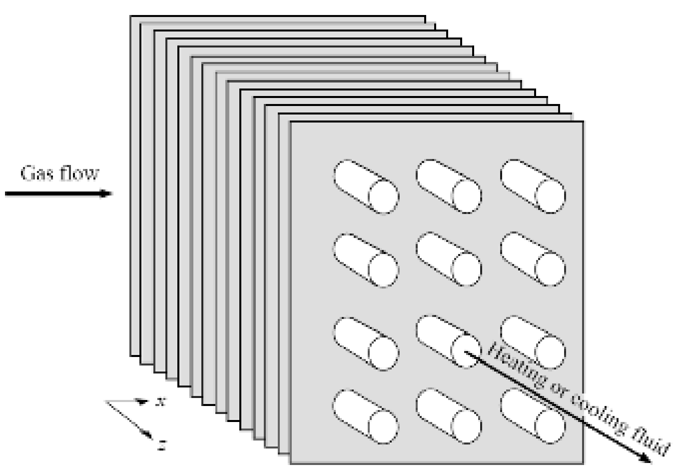


Figure 10-5 | Cross-flow heat exchanger, both fluids unmixed.



THE LOG MEAN TEMPERATURE DIFFERENCE

Consider the double-pipe heat exchanger shown in Figure 10-2. The fluids may flow in either parallel flow or counter flow, and the temperature profiles for these two cases are indicated in Figure 10-7. We propose to calculate the heat transfer in this double-pipe arrangement with

$$q = UA\,\Delta T_m \qquad \text{…………. [10-5]}$$

where
U = overall heat-transfer coefficient
A = surface area for heat transfer consistent with definition of U
ΔTm = suitable mean temperature difference across heat exchanger

Figure 10-6 | Typical temperature profile for cross-flow heat exchanger of Figure 10-5.

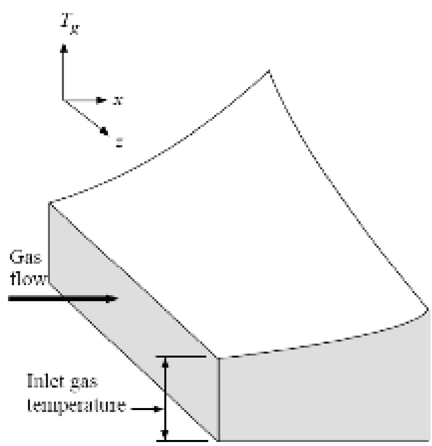
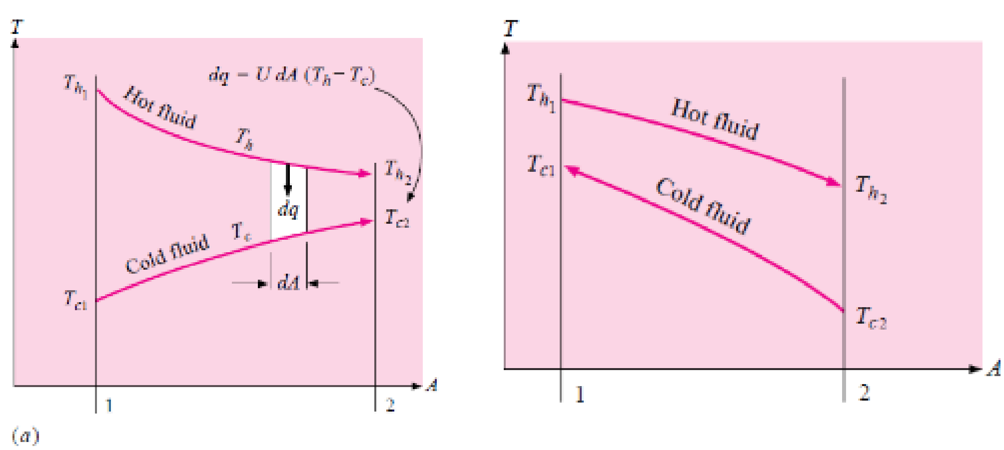


Figure 10-7 | Temperature profiles for (*a*) parallel flow and (*b*) counterflow in double-pipe heat exchanger.



An inspection of **Figure 10-7** shows that the temperature difference between the hot and cold fluids varies between inlet and outlet, and we must determine the average value for use in Equation (10-5). For the parallel-flow heat exchanger shown in **Figure 10-7**, the heat transferred through an element of area ***dA*** may be written

$$dq = -\dot{m}_h c_h dT_h = \dot{m}_c c_c dT_c \qquad \text{............ [10-6]}$$

where the subscripts ***h*** and ***c*** designate the hot and cold fluids, respectively. The heat transfer could also be expressed

$$dq = U(T_h - T_c)dA \qquad \text{............ [10-7]}$$

From Equation (10-6)

$$dT_h = \frac{-dq}{\dot{m}_h c_h}$$

$$dT_c = \frac{dq}{\dot{m}_c c_c}$$

where $\dot{m}$ represents the mass-flow rate and ***c*** is the specific heat of the fluid. Thus

$$dT_h - dT_c = d(T_h - T_c) = -dq\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) \qquad \text{............ [10-8]}$$

Solving for ***dq*** from Equation (10-7) and substituting into Equation (10-8) gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)dA \qquad \text{............ [10-9]}$$

This differential equation may now be integrated between conditions **1** and **2** as indicated in **Figure 10-7**. The result is

$$\ln\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = -UA\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) \qquad \text{............ [10-10]}$$

Returning to Equation (10-6), the products $\dot{m}_c c_c$ and $\dot{m}_h c_h$ may be expressed in terms of the total heat transfer ***q*** and the overall temperature differences of the hot and cold fluids. Thus

$$\dot{m}_h c_h = \frac{q}{T_{h1} - T_{h2}}$$

$$\dot{m}_c c_c = \frac{q}{T_{c2} - T_{c1}}$$

Substituting these relations into Equation (10-10) gives

$$q = UA\frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln[(T_{h2} - T_{c2})/(T_{h1} - T_{c1})]} \qquad \text{............ [10-11]}$$

Comparing Equation (10-11) with Equation (10-5), we find that the mean temperature difference is the grouping of terms in the brackets. Thus

$$\Delta T_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln[(T_{h2} - T_{c2})/(T_{h1} - T_{c1})]} \qquad \text{............ [10-12]}$$

This temperature difference is called the ***log mean temperature difference*** (**LMTD**). Stated verbally, it is the temperature difference at one end of the heat exchanger less the temperature difference at the other end of the exchanger divided by the natural logarithm of the ratio of these two temperature differences. It is left as an exercise for the reader to show that this relation may also be used to calculate the **LMTD**s for counter flow conditions.

The above derivation for **LMTD** involves two important assumptions: (**1**) the fluid specific heats do not vary with temperature, and (**2**) the convection heat-transfer coefficients are constant throughout the heat exchanger. The second assumption is usually the more serious one because of entrance effects, fluid viscosity, and thermal-conductivity changes, etc.

If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTD ***for a counter flow double-pipe arrangement with the same hot and cold fluid temperatures.*** The heat-transfer equation then takes the form

$$q = UAF\Delta T_m \qquad \text{............ [10-13]}$$

EXAMPLE 10-4 Calculation of Heat-Exchanger Size from Known Temperatures

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg · °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m^2 · °C. Calculate the heat-exchanger area.

■ **Solution**

The total heat transfer is determined from the energy absorbed by the water:

$$q = \dot{m}_w c_w \Delta T_w = (68)(4180)(75 - 35) = 11.37 \text{ MJ/min} \qquad [a]$$

$$= 189.5 \text{ kW} \qquad [6.47 \times 10^5 \text{ Btu/h}]$$

Since all the fluid temperatures are known, the LMTD can be calculated by using the temperature scheme in Figure 10-7*b*:

$$\Delta T_m = \frac{(110-75)-(75-35)}{\ln[(110-75)/(75-35)]} = 37.44°\text{C} \qquad \textbf{[b]}$$

Then, since $q = UA\,\Delta T_m$,

$$A = \frac{1.895\times 10^5}{(320)(37.44)} = 15.82\ \text{m}^2 \quad [170\ \text{ft}^2]$$

Design of Shell-and-Tube Heat Exchanger

EXAMPLE 10-6

Water at the rate of 30,000 lb_m/h [3.783 kg/s] is heated from 100 to 130°F [37.78 to 54.44°C] in a shell-and-tube heat exchanger. On the shell side one pass is used with water as the heating fluid, 15,000 lb_m/h [1.892 kg/s], entering the exchanger at 200°F [93.33°C]. The overall heat-transfer coefficient is 250 Btu/h · ft^2 · °F [1419 W/m^2 · °C], and the average water velocity in the $\frac{3}{4}$-in [1.905-cm] diameter tubes is 1.2 ft/s [0.366 m/s]. Because of space limitations, the tube length must not be longer than 8 ft [2.438 m]. Calculate the number of tube passes, the number of tubes per pass, and the length of the tubes, consistent with this restriction.

■ **Solution**

We first assume one tube pass and check to see if it satisfies the conditions of this problem. The exit temperature of the hot water is calculated from

$$q = \dot{m}_c c_c\,\Delta T_c = \dot{m}_h c_h\,\Delta T_h$$

$$\Delta T_h = \frac{(30{,}000)(1)(130-100)}{(15{,}000)(1)} = 60°\text{F} = 33.33°\text{C} \qquad \textbf{[a]}$$

so

$$T_{h,\text{exit}} = 93.33 - 33.33 = 60°\text{C}$$

The total required heat transfer is obtained from Equation (*a*) for the cold fluid:

$$q = (3.783)(4182)(54.44-37.78) = 263.6\ \text{kW} \quad [8.08\times 10^5\ \text{Btu/h}]$$

For a counterflow exchanger, with the required temperature

$$\text{LMTD} = \Delta T_m = \frac{(93.33-54.44)-(60-37.78)}{\ln[(93.33-54.44)/(60-37.78)]} = 29.78°\text{C}$$

$$q = UA\,\Delta T_m$$

$$A = \frac{2.636\times 10^5}{(1419)(29.78)} = 6.238\ \text{m}^2 \quad [67.1\ \text{ft}^2] \qquad \textbf{[b]}$$

Using the average water velocity in the tubes and the flow rate, we calculate the total flow area with

$$\dot{m}_c = \rho A u$$

$$A = \frac{3.783}{(1000)(0.366)} = 0.01034\ \text{m}^2 \qquad \textbf{[c]}$$

This area is the product of the number of tubes and the flow area per tube:

$$0.01034 = n\frac{\pi d^2}{4}$$

$$n = \frac{(0.01034)(4)}{\pi(0.01905)^2} = 36.3$$

or $n = 36$ tubes. The surface area per tube per meter of length is

$$\pi d = \pi(0.01905) = 0.0598 \text{ m}^2/\text{tube} \cdot \text{m}$$

We recall that the total surface area required for a one-tube-pass exchanger was calculated in Equation (b) as 6.238 m^2. We may thus compute the length of tube for this type of exchanger from

$$n\pi d\, L = 6.238$$

$$L = \frac{6.238}{(36)(0.0598)} = 2.898 \text{ m}$$

This length is greater than the allowable 2.438 m, so we must use more than one tube pass. When we increase the number of passes, we correspondingly increase the total surface area required because of the reduction in LMTD caused by the correction factor F. We next try two tube passes. From Figure 10-8, $F = 0.88$, and thus

$$A_{\text{total}} = \frac{q}{UF\Delta T_m} = \frac{2.636 \times 10^5}{(1419)(0.88)(29.78)} = 7.089 \text{ m}^2$$

The number of tubes per pass is still 36 because of the velocity requirement. For the two-tube-pass exchanger the total surface area is now related to the length by

$$A_{\text{total}} = 2n\pi d\, L$$

so that

$$L = \frac{7.089}{(2)(36)(0.0598)} = 1.646 \text{ m} \quad [5.4 \text{ ft}]$$

This length is within the 2.438-m requirement, so the final design choice is

Number of tubes per pass = 36

Number of passes = 2

Length of tube per pass = 1.646 m [5.4 ft]

EFFECTIVENESS-NTU METHOD

The LMTD approach to heat-exchanger analysis is useful when the inlet and outlet temperatures are known or are easily determined. The LMTD is then easily calculated, and the heat flow, surface area, or overall heat-transfer coefficient may be determined. When the inlet or exit temperatures are to be evaluated for a given heat exchanger, the analysis frequently involves an iterative procedure because of

the logarithmic function in the LMTD. In these cases the analysis is performed more easily by utilizing a method based on the effectiveness of the heat exchanger in transferring a given amount of heat. The effectiveness method also offers many advantages for analysis of problems in which a comparison between various types of heat exchangers must be made for purposes of selecting the type best suited to accomplish a particular heat-transfer objective.
We define the heat-exchanger effectiveness as

$$\text{Effectiveness} = \epsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid. Consider the parallel-flow and counter flow heat exchangers shown in **Figure 10-7**. For the parallel-flow exchanger

$$q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m}_c c_c (T_{c_2} - T_{c_1}) \qquad \text{............ [10-14]}$$

and for the counter flow exchanger

$$q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m}_c c_c (T_{c_1} - T_{c_2}) \qquad \text{............ [10-15]}$$

To determine the maximum possible heat transfer for the exchanger, we first recognize that this maximum value could be attained if one of the fluids were to undergo a temperature change equal to the maximum temperature difference present in the exchanger, which is the difference in the entering temperatures for the hot and cold fluids. The fluid that might undergo this maximum temperature difference is the one having the ***minimum*** value of $\dot{m}c$ because the energy balance requires that the energy received by one fluid be equal to that given up by the other fluid; if we let the fluid with the larger value of $\dot{m}c$ go through the maximum temperature difference, this would require that the other fluid undergo a temperature difference greater than the maximum, and this is impossible. So, maximum possible heat transfer is expressed as

$$q_{\max} = (\dot{m}c)_{\min} (T_{h_{\text{inlet}}} - T_{c_{\text{inlet}}}) \qquad \text{............ [10-16]}$$

The minimum fluid may be either the hot or cold fluid, depending on the mass flow rates and specific heats. For the parallel-flow exchanger

$$\epsilon_h = \frac{\dot{m}_h c_h (T_{h_1} - T_{h_2})}{\dot{m}_h c_h (T_{h_1} - T_{c_1})} = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_1}} \qquad \text{............ [10-17]}$$

$$\epsilon_c = \frac{\dot{m}_c c_c (T_{c_2} - T_{c_1})}{\dot{m}_c c_c (T_{h_1} - T_{c_1})} = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}} \qquad \text{............ [10-18]}$$

The subscripts on the effectiveness symbols designate the fluid that has the minimum value of $\dot{m}c$.
For the counter flow exchanger:

$$\epsilon_h = \frac{\dot{m}_h c_h (T_{h_1} - T_{h_2})}{\dot{m}_h c_h (T_{h_1} - T_{c_2})} = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_2}} \qquad \text{............ [10-19]}$$

$$\epsilon_c = \frac{\dot{m}_c c_c (T_{c_1} - T_{c_2})}{\dot{m}_c c_c (T_{h_1} - T_{c_2})} = \frac{T_{c_1} - T_{c_2}}{T_{h_1} - T_{c_2}} \qquad \text{............ [10-20]}$$

In a general way the effectiveness is expressed as

$$\epsilon = \frac{\Delta T(\text{minimum fluid})}{\text{Maximum temperature difference in heat exchanger}} \qquad \text{............ [10-21]}$$

The minimum fluid is always the one experiencing the larger temperature difference in the heat exchanger, and the maximum temperature difference in the heat exchanger is always the difference in inlet temperatures of the hot and cold fluids. We may derive an expression for the effectiveness in **parallel flow** double-pipe as follows. Rewriting Equation (10-10), we have

$$\ln \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = -UA \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c} \right) = \frac{-UA}{\dot{m}_c c_c} \left(1 + \frac{\dot{m}_c c_c}{\dot{m}_h c_h} \right) \qquad \text{......... [10-22]}$$

Or

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \exp\left[\frac{-UA}{\dot{m}_c c_c} \left(1 + \frac{\dot{m}_c c_c}{\dot{m}_h c_h} \right) \right] \qquad \text{............ [10-23]}$$

If the cold fluid is the minimum fluid,

$$\epsilon = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}}$$

Rewriting the temperature ratio in Equation (10-23) gives

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \frac{T_{h_1} + (\dot{m}_c c_c / \dot{m}_h c_h)(T_{c_1} - T_{c_2}) - T_{c_2}}{T_{h_1} - T_{c_1}} \qquad \text{............ [10-24]}$$

when the substitution

$$T_{h_2} = T_{h_1} + \frac{\dot{m}_c c_c}{\dot{m}_h c_h} (T_{c_1} - T_{c_2})$$

is made from Equation (10-6). Equation (10-24) may now be rewritten

$$\frac{(T_{h_1}-T_{c_1})+(\dot{m}_c c_c/\dot{m}_h c_h)(T_{c_1}-T_{c_2})+(T_{c_1}-T_{c_2})}{T_{h_1}-T_{c_1}}=1-\left(1+\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)\epsilon$$

Inserting this relation back in Equation (10-23) gives for the effectiveness

$$\epsilon=\frac{1-\exp[(-UA/\dot{m}_c c_c)(1+\dot{m}_c c_c/\dot{m}_h c_h)]}{1+\dot{m}_c c_c/\dot{m}_h c_h} \quad \text{............ [10-25]}$$

It may be shown that the same expression results for the effectiveness when the hot fluid is the minimum fluid, except that $\dot{m}_c c_c$ and $\dot{m}_h c_h$ are interchanged. As a consequence, the effectiveness is usually written

$$\epsilon=\frac{1-\exp[(-UA/C_{\min})(1+C_{\min}/C_{\max})]}{1+C_{\min}/C_{\max}} \quad \text{............ [10-26]}$$

where $C=\dot{m}c$ is defined as the capacity rate.
A similar analysis may be applied to the counter flow case, and the following relation for effectiveness results:

$$\epsilon=\frac{1-\exp[(-UA/C_{\min})(1-C_{\min}/C_{\max})]}{1-(C_{\min}/C_{\max})\exp[(-UA/C_{\min})(1-C_{\min}/C_{\max})]} \quad \text{............ [10-27]}$$

The grouping of terms ***UA/C_{min}*** is called the ***number of transfer units*** (**NTU**) since it is indicative of the size of the heat exchanger.

Off-Design Calculation of Exchanger in Example 10-4

EXAMPLE 10-10

The heat exchanger of Example 10-4 is used for heating water as described in the example. Using the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg/min of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under these new conditions.

Solution

The flow rate of oil is calculated from the energy balance for the original problem:

$$\dot{m}_h c_h\,\Delta T_h=\dot{m}_c c_c\,\Delta T_c \qquad [a]$$

$$\dot{m}_h=\frac{(68)(4180)(75-35)}{(1900)(110-75)}=170.97\ \text{kg/min}$$

The capacity rates for the new conditions are now calculated as

$$\dot{m}_h c_h=\frac{170.97}{60}(1900)=5414\ \text{W/}^\circ\text{C}$$

$$\dot{m}_c c_c=\frac{40}{60}(4180)=2787\ \text{W/}^\circ\text{C}$$

so that the water (cold fluid) is the minimum fluid, and

$$\frac{C_{\min}}{C_{\max}} = \frac{2787}{5414} = 0.515$$

$$\mathrm{NTU}_{\max} = \frac{UA}{C_{\min}} = \frac{(320)(15.82)}{2787} = 1.816 \qquad \textbf{[}\boldsymbol{b}\textbf{]}$$

where the area of 15.82 m^2 is taken from Example 10-4. From Figure 10-13 or Table 10-3 the effectiveness is

$$\epsilon = 0.744$$

and because the cold fluid is the minimum, we can write

$$\epsilon = \frac{\Delta T_{\text{cold}}}{\Delta T_{\max}} = \frac{\Delta T_{\text{cold}}}{110 - 35} = 0.744 \qquad \textbf{[}\boldsymbol{c}\textbf{]}$$

$$\Delta T_{\text{cold}} = 55.8^\circ\text{C}$$

and the exit water temperature is

$$T_{w,\text{exit}} = 35 + 55.8 = 90.8^\circ\text{C}$$

The total heat transfer under the new flow conditions is calculated as

$$q = \dot{m}_c c_c \Delta T_c = \frac{40}{60}(4180)(55.8) = 155.5\ \text{kW} \qquad [5.29 \times 10^5\ \text{Btu/h}] \qquad \textbf{[}\boldsymbol{d}\textbf{]}$$

Notice that although the flow rate has been reduced by 41 percent (68 to 40 kg/min), the heat transfer is reduced by only 18 percent (189.5 to 155.5 kW) because the exchanger is more effective at the lower flow rate.