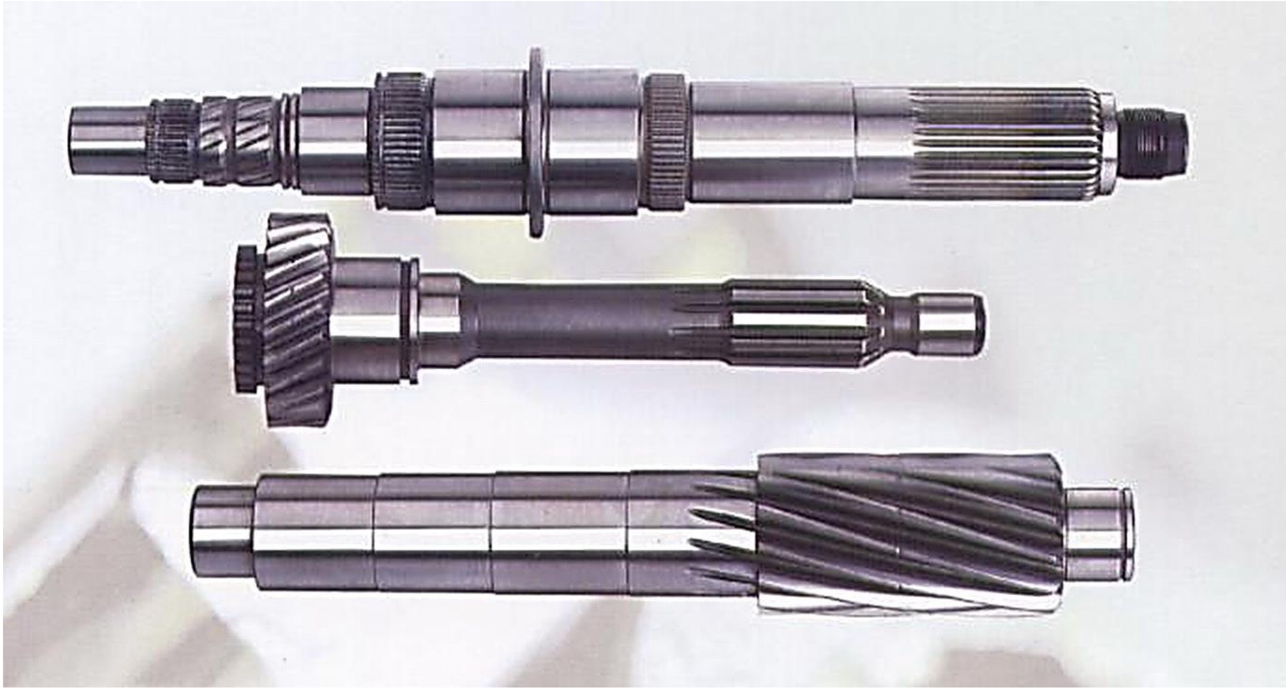
LECTURES ELEVEN, TWELVE &THIRTEEN

DESIGN OF SHAFTS



References:

Machine Elements in Mechanical Design by Robert L. Mott, P.E. (Chapter 12)

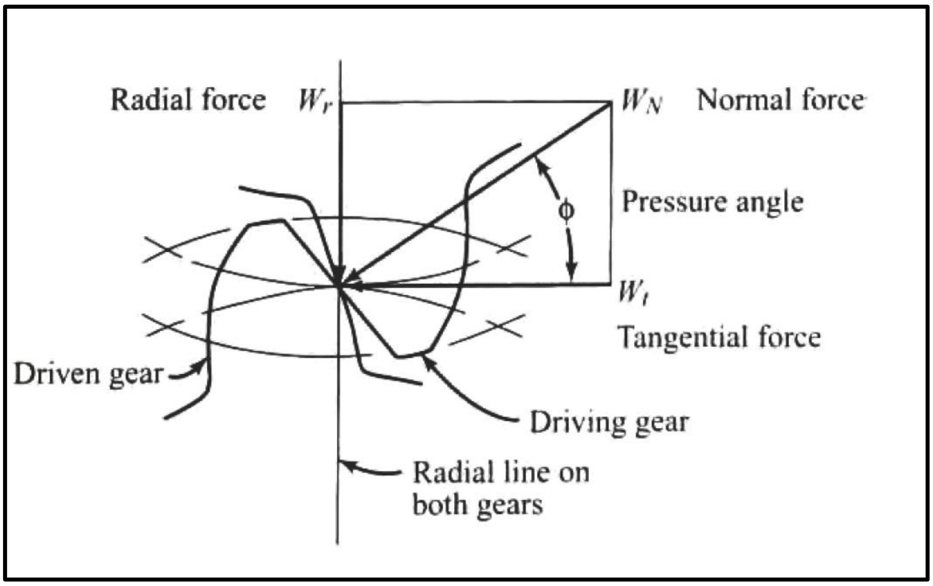
Note: Read section (12-1) objective of this chapter (Page 532)

Shaft Design Procedure (Sec. 12-2, Page 532)

1. Determine rpm of shaft
2. Determine power or torque
3. Determine part mounted on shaft (gears, belts…)
4. Determine location of bearing
5. Determine location of (key, fillets …)
6. Draw torque diagram
7. Determine forces on shaft
8. Resolve components of forces in horizontal & vertical plan
9. Solve the reaction on bearing
10. Produce S.F. Diagram & B.M. diagram
11. Select material and specify its condition (Suggested material is : AISI 1040, 4140, 4340, 4640, 5150, 6150, and 8650). Then determine S_Y & S_{ut}
12. Determine appropriate design stress, considering the manner of loading (smooth, shock…)
13. Analyze each critical point of shaft then decide which point is safer
14. Specify the final dimensions

Forces exerted on shafts (Sec. 12-3 Page 535)

1. Spur Gears:



$$Power = \frac{2\pi\, n\, T}{60} \quad \ldots (S.I.) \quad or \quad Power = \frac{T\, n}{63000} \quad \ldots . (British)$$

$$W_t = \frac{2\,T}{D} = Tangentional\ Force \quad \ldots\ldots . (12-2)$$

$$W_r = \ W_t\, tan\emptyset \ = Radial\ load \quad \ldots\ldots . (12-3)$$

$where$: $P = power\ being\ transmitted$

n = Rotational speed

T = Torque

D = pitch diameter of gear = m N

m = Module

N = No. of teeth

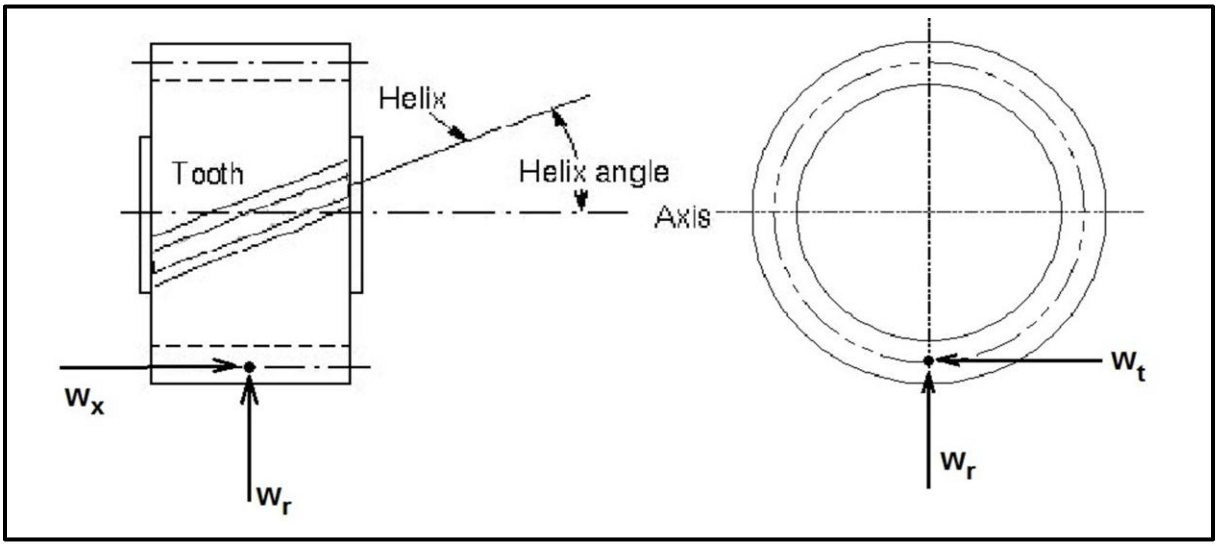
$\emptyset = pressure\ angle$

2. <u>Helical Gears:</u>

$$W_r = \frac{W_t * tan\emptyset_n}{cos\varphi} \quad \ldots\ldots(12-4)$$

$$W_x = axial\ load = W_t\ tan\varphi \quad \ldots\ldots(12-5)$$

Where $\varphi = Helix\ angle$



3. Bevel Gears:

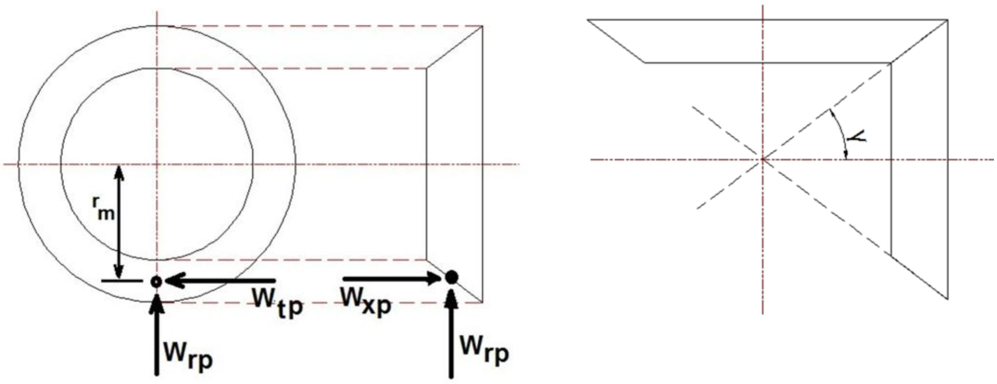
$$W_{t\,p} = W_{t\,G}$$

$$W_{x\,p} = W_{r\,G}$$

$$W_{r\,p} = W_{x\,G}$$

$$r_m = \frac{d}{2} - \frac{F}{2}\sin\gamma$$

$Where\ F = Face\ width$

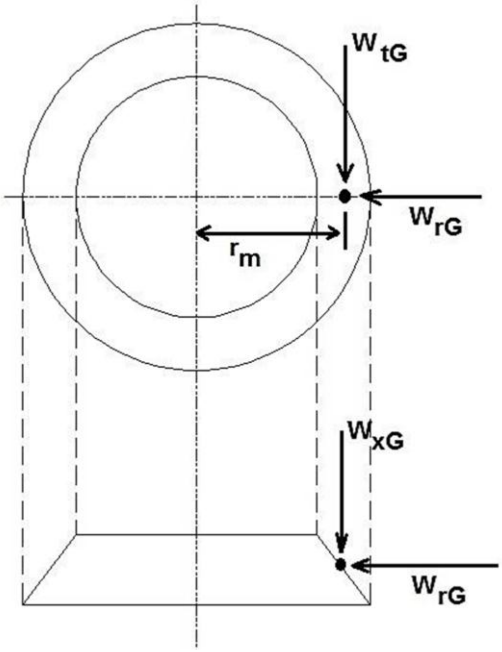


$$W_{r\,p} = W_t \tan\emptyset \cos\gamma$$

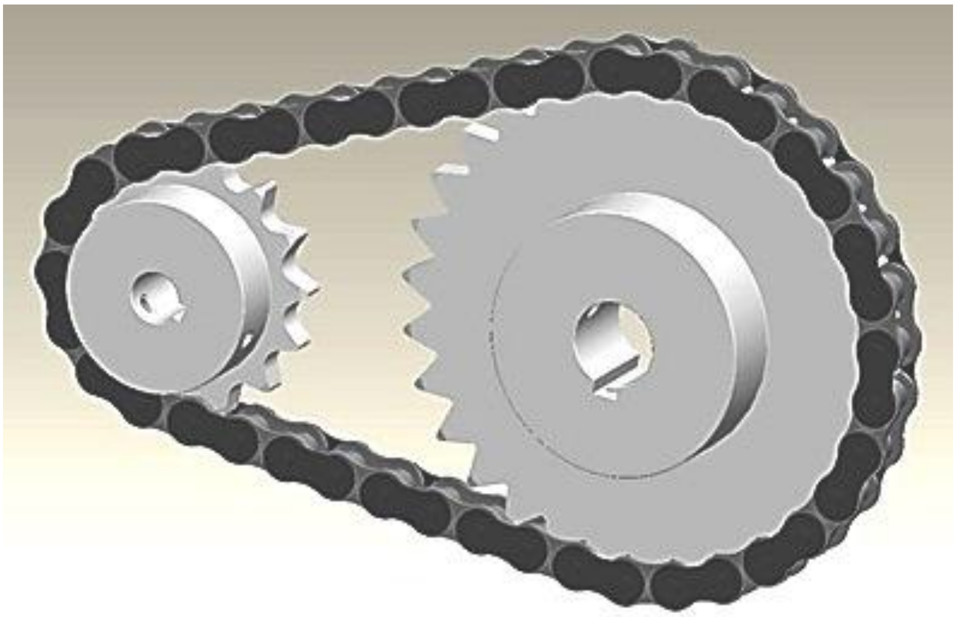
$$W_{x\,p} = W_t \tan\emptyset \sin\gamma$$

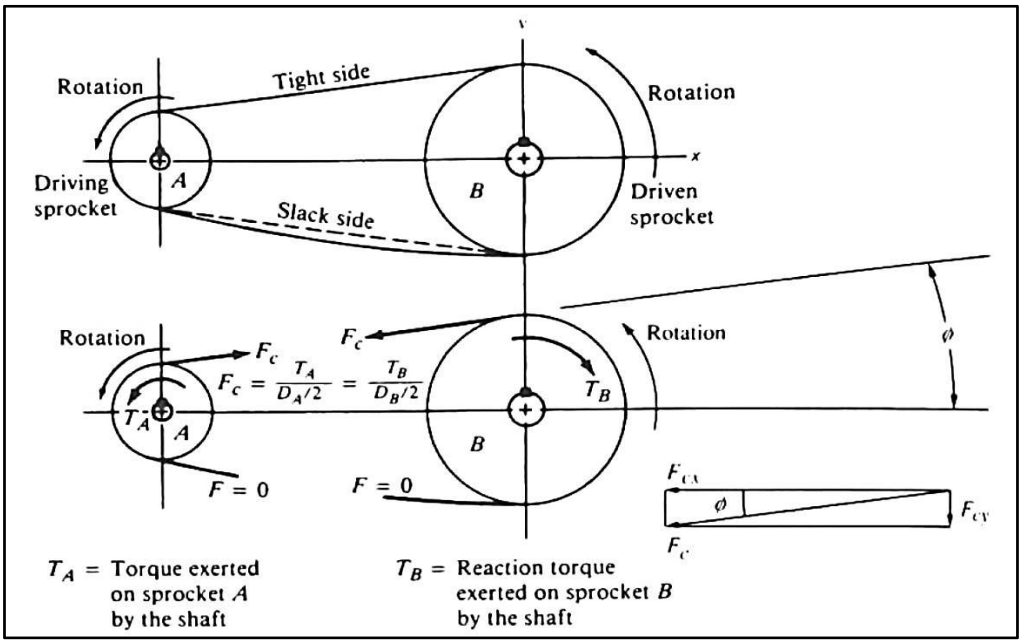
$$W_{t\,p} = \frac{T}{r_m} \quad (r_m = mean\ raduis\ of\ pinion)$$

$$\gamma = Pitch\ cone\ angle\ for\ pinion = \tan^{-1}\left(\frac{N_p}{N_G}\right)$$



4. <u>Chain Sprocket:</u>

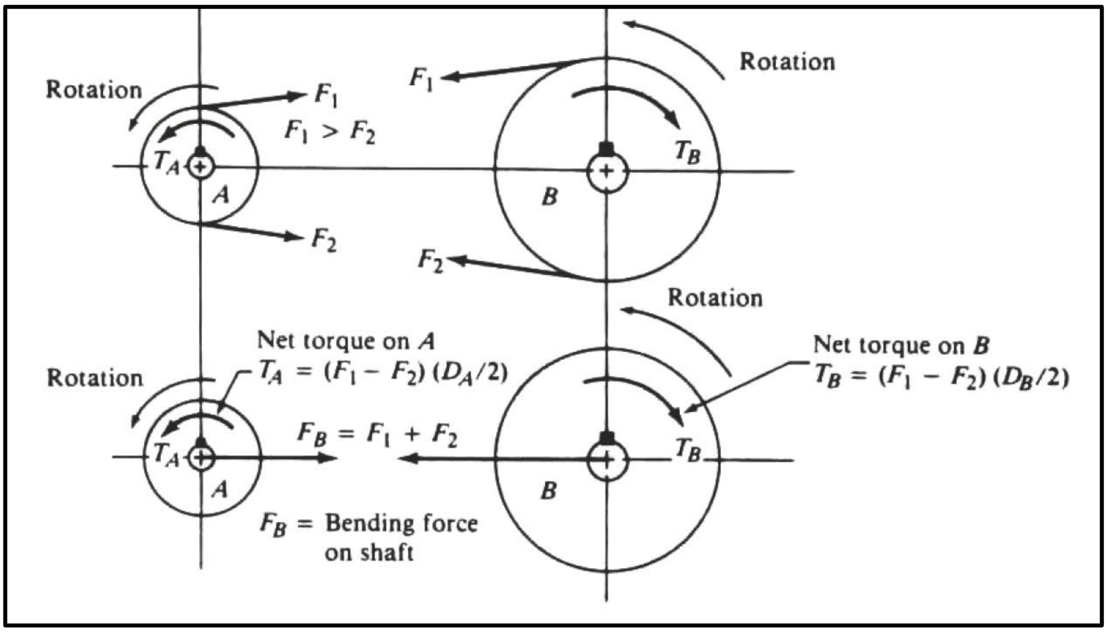


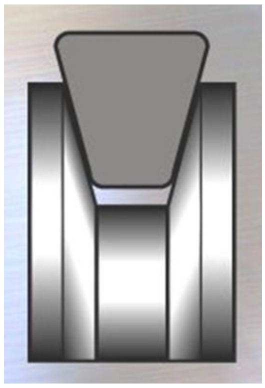


$F_c = \dfrac{2\,T}{D}$ ……(12 − 6) ; *where* $D = pitch\ diameter\ of\ sprocket$

If $\emptyset = small = 0$ $\therefore\ F_{cx} = F_c$ & $F_{cy} = 0$

5. V-Belt sheaves:





$$F_1 * \frac{D}{2} - F_2 * \frac{D}{2} = T_B \qquad \Rightarrow \qquad F_1 - F_2 = \frac{2\,T_B}{D} \quad \ldots\ldots(1)$$

Assume a ratio of $\left(\frac{F_1}{F_2}\right) = 5\ (If\ not\ given)\quad \ldots\ldots(2)$

From eq. (1) & eq. (2) Find F_1 & F_2 $then\ load\ on\ shaft = F_1 + F_2$

6. Flat-Belt sheaves:

Same as in V-Belt but assume $\left(\frac{F_1}{F_2}\right) = 3$ (if not given) then load on shaft $= F_1 + F_2$

Stress Concentration in Shafts (sec. 12-4, Page 540):

- Keyseats $\longrightarrow$ $K_t = 2$ $for\ profile\ keyseats$
  $\longrightarrow$ $K_t = 1.6$ $for\ sled\ runner\ keyseats$
- Shoulder fillets $\longrightarrow$ $K_t = 2.5$ $for\ sharp\ fillet$
  $\longrightarrow$ $K_t = 1.5$ $for\ well-round\ fillet$
- Retaining ring grooves $\longrightarrow$ $K_t = 3$ $or\ increase\ diameter\ by\ 6\%$

When shaft designed according to strength the following cases can be considered:

a) Shaft subjected to bending only (Axle)

$$\sigma_x = \frac{M*C}{I} = \frac{M*{}^{d}/_{2}}{\pi d^4/64} = \frac{32M}{\pi d^3}$$

b) Shaft subjected to torsion only

$$\tau_{xy} = \frac{T*C}{J} = \frac{T*{}^{d}/_{2}}{\pi d^4/_{32}} = \frac{16\,T}{\pi\,d^3}$$

c) Shaft subjected to torsion & bending

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \qquad (Max.\,Shear\ stress\ theory)$$

$$\sigma_{max} = \frac{32}{\pi d^3}\sqrt{M^2 + \frac{3}{4}T^2} \qquad (Von - Mises\ theory)$$

$$\sigma_{max} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right) \qquad (max.\,normal\ stress\ theory)$$

d) Shaft subjected to torsion, bending and axial load

$$\sigma_x = \frac{M*C}{I} + \frac{F_a}{A} = \frac{32M}{\pi d^3} + \frac{4F_a}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F_a*d}{8}\right)$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + T^2} \quad \ldots\ldots Max.\,Shear\ stress\ theory$$

$$\sigma_{max} = \frac{32}{\pi d^3}\sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + \frac{3}{4}T^2} \quad \ldots\ldots Von - Mises\ theory$$

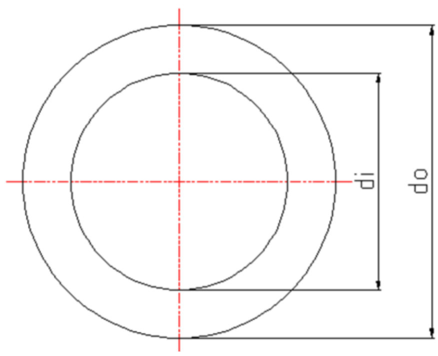
$$\sigma_{max} = \frac{16}{\pi d^3}\left[\left(M + \frac{F_a*d}{8}\right) + \sqrt{\left(M + \frac{F_a*d}{8}\right)^2 + T^2}\right] \ldots Max.\,Normal\ stress\ theory$$

- **For Hollow shaft and combined factors:**

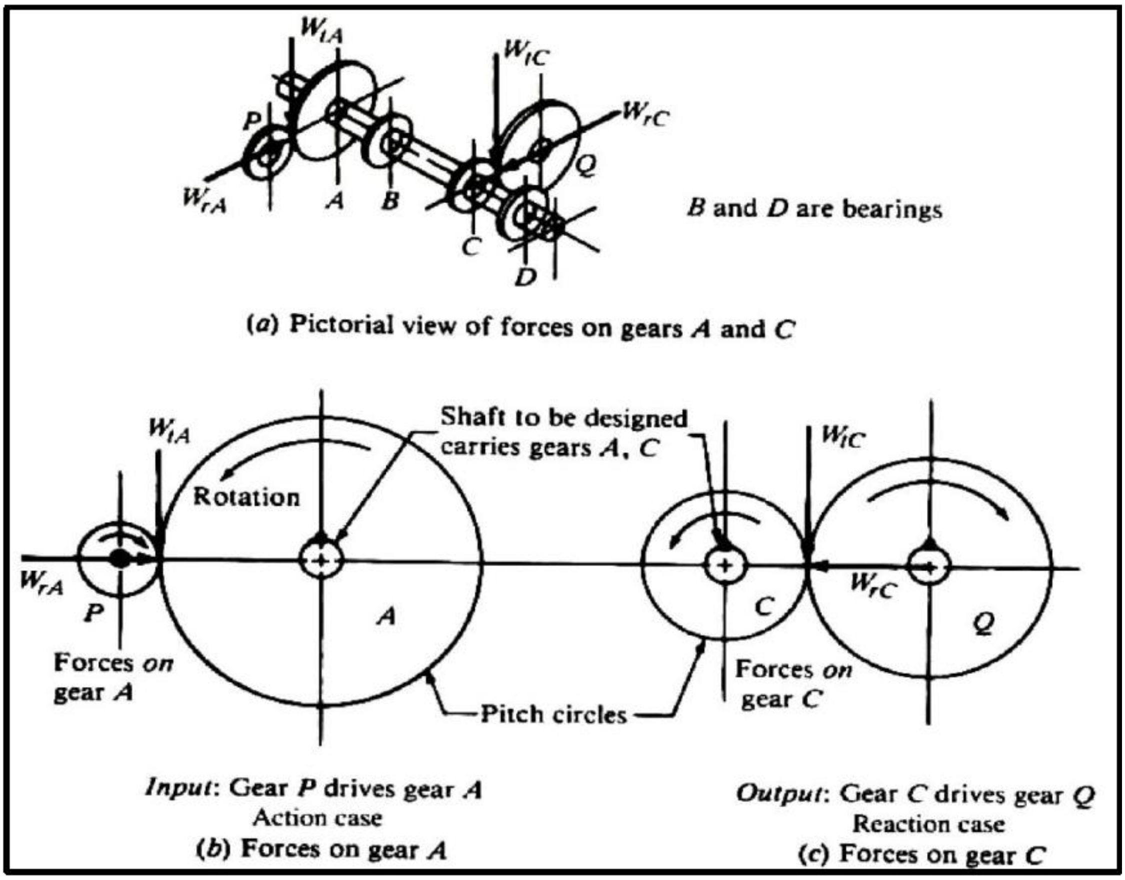
$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi d_o^2}{4}\left(1 - K^2\right)\ ;\ where\ K = \frac{d_i}{d_o}$$

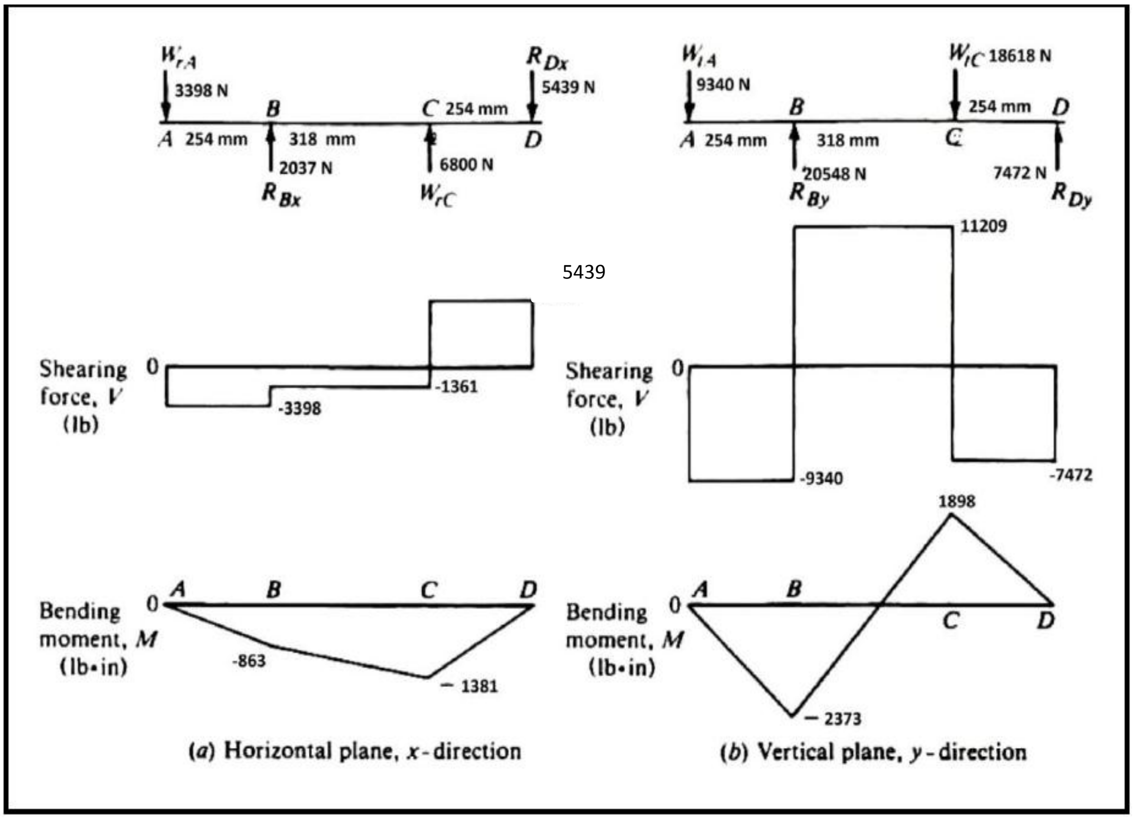
$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi d_o^4}{64}\left(1 - K^4\right)$$

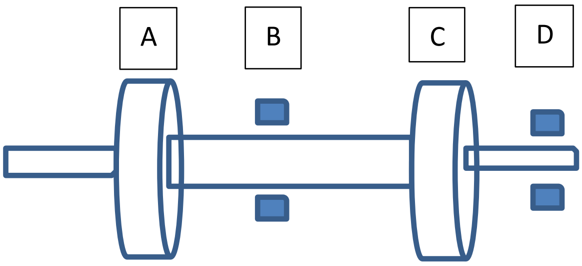
$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi d_o^4}{32}\left(1 - K^4\right)$$



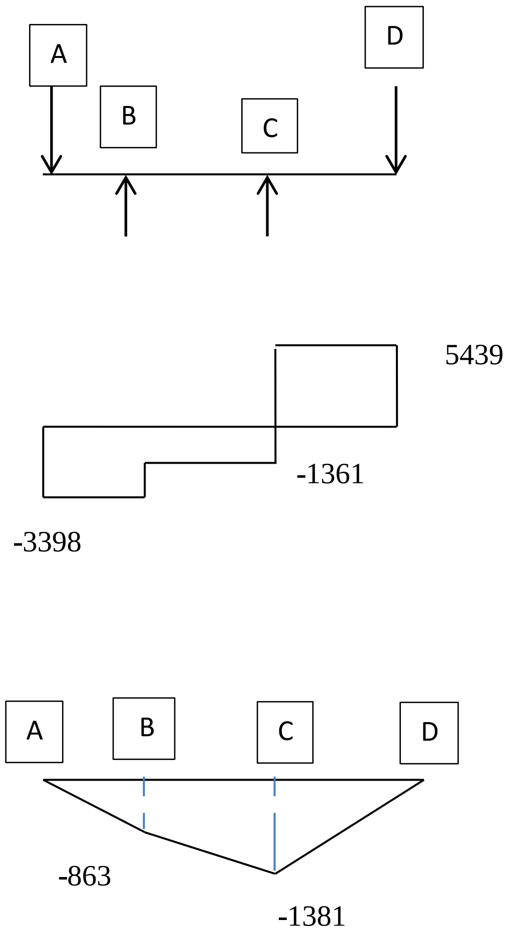
Example: Design the shaft shown in the figure below, if the diameter of shaft at points A, B, C and D is: (A=42mm, B=84mm, C=90mm and D=28mm).



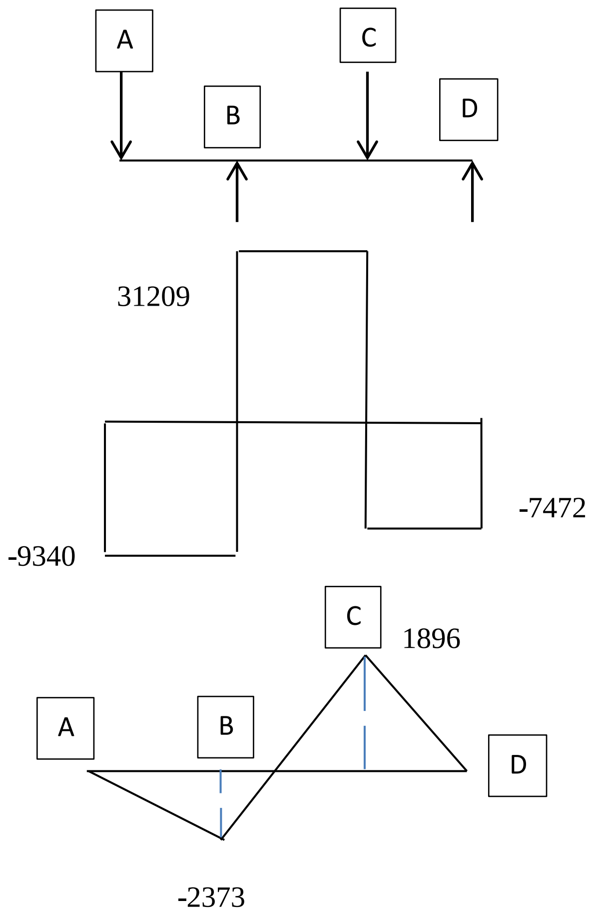




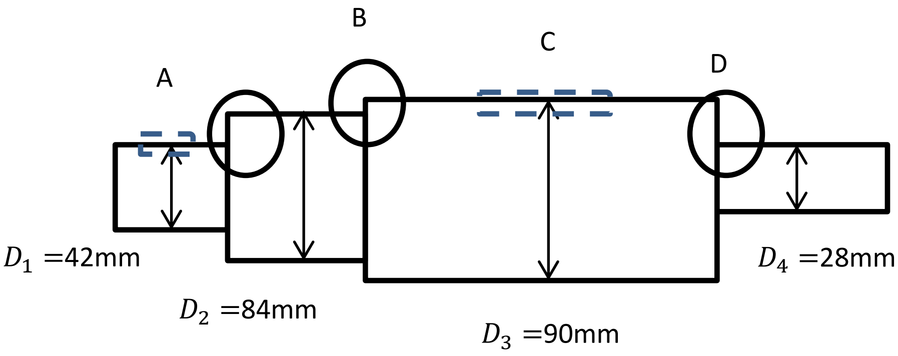
Horizontal plane



Vertical plane



K_t For key way =1.6



At point (D): assume well rounded fillet $K_t)_{bend} = 1.5$

$K_t)_{tor} = 1.5$

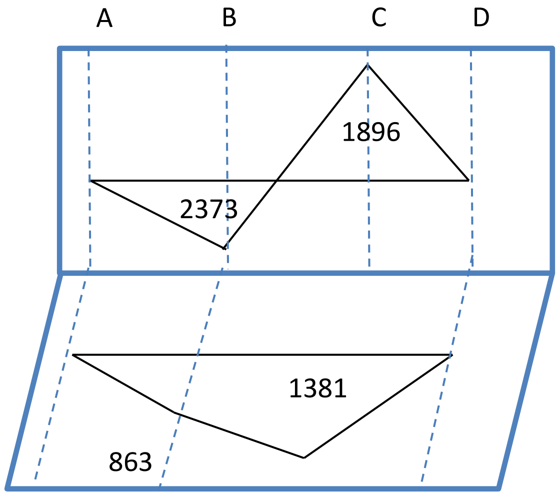
At points (A&C): assume sled-runner key way $K_t)_{key\ way} = 1.6$

At points (B&C): assume well rounded fillet $K_t)_{bend} = 1.5$

$K_t)_{tor} = 1.5$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{-------- (1)}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{-------- (2)}$$



At point A:

At this point $M = 0$ & $T = 2373$ N.m

$\sigma_x = 0$ & $\tau_{XY} = \frac{T\,c}{J} = (2373 * 1000 * 2)/(\pi*(42)^3/32) = 163.2\ N/mm^2$

$\sigma_1 = 0 + \sqrt{0 + (163.2)^2} = 163.2\ MPa$

At point B:

At this point $M_B = \sqrt{(M_{bx})^2 + \left(M_{by}\right)^2} = \sqrt{(863)^2 + (2373)^2} = 2525.5\ N.m$

Also, $T = 2373\ N.m$

$\sigma_x = \frac{M_b\,C}{I} = \frac{2525.5*1000*32}{\pi*(84)3} = 43.40\ MPa$

$\tau_{xy} = \frac{T\,r}{J} = \frac{2373*1000*16}{\pi*(84)3} = 20.40\ MPa$

$\sigma_1 = \frac{43.2}{2} + \sqrt{\left(\frac{43.2}{2}\right)^2 + (20.4)^2} = 51.5\ MPa$

$\tau_{max} = \sqrt{\left(\frac{43.2}{2}\right)^2 + (20.4)^2} = 29.8\ MPa$

At point C:

At this point $M_C = \sqrt{(M_{Cx})^2 + \left(M_{Cy}\right)^2} = \sqrt{(1381)^2 + (1896)^2} = 2348\ N.m$

Also, $T = 2373\ N.m$

$\sigma_x = \frac{M_C\,C}{I} = \frac{2348*1000*32}{\pi*(90)3} = 32.8\ MPa$

$\tau_{xy} = \frac{T\,r}{J} = \frac{2373*1000*16}{\pi*(90)3} = 16.5\ MPa$

$\sigma_1 = \frac{32.8}{2} + \sqrt{\left(\frac{32.8}{2}\right)^2 + (16.5)^2} = 39.7\ MPa$

$\tau_{max} = \sqrt{\left(\frac{32.8}{2}\right)^2 + (16.5)^2} = 23.3\ MPa$

At point D:

At this point $M_D = 0$

Also, $T = 0$

There is a vertical shearing force in x-direction (horizontal plane) and y-direction (vertical plane).

$R_D = \sqrt{(5440)^2 + (7473)^2} = 9243\ N$

$\sigma_x = 0$

$\tau_{xy} = \frac{4V}{3A} = \frac{(4*9243)}{3*(\pi*\frac{d^2}{4})} = \frac{(4*9243)}{3*(\pi*\frac{28^2}{4})} = 62.9\ MPa$

$\sigma_1 = 0 + \sqrt{0 + (62.9)^2} = 62.9\ MPa$

$\tau_{max} = \sqrt{0 + (62.9)^2} = 62.9\ MPa$

So you can see from the above results, that the worst case at point A

$\tau_{max} = 163.2\ N/mm^2$

If we assume that $K_t = 1.6$

$\tau_{design} = K_t\ \tau_{max} = 1.6*163.2 = 261.12\ MPa = S_{sy}/N$

$S_{sy} = 522.24\ MPa$, *if we assume* $(N=2)$ *and* $S_{sy} = 0.5S_y$

$S_y = 1044.48\ MPa$

From (appendix-3) use carbon and alloy steel (P. A6) AISI 4140 0QT 1000

$S_y = 1050$ MPa

Or any material that you see advisable

So you can apply

$K_t\ \sigma \le \sigma_d = S_{ut}/N$

Or

$K_t\ \tau_{max} \le \tau_d = S_{sy}/N$

If we apply Von-Mises theory for the previous example:

$\sigma` = \sqrt{\sigma_x{}^2 + 3\tau_{xy}^2}$

At point A:

$\sigma_x = 0 \quad \tau_{xy} = 163.2\ MPa$

$\sigma` = \sqrt{0 + 3 * (163.2)^2} = 282.7\ MPa$

At point B:

$\sigma` = \sqrt{(43.4)^2 + 3 * (20.4)^2} = 56\ MPa$

At point C:

$\sigma` = \sqrt{(32.8)^2 + 3 * (16.5)^2} = 43.5\ MPa$

At Point D:

$\sigma` = \sqrt{0 + 3 * (62.9)^2} = 108.25\ MPa$

Also the worst case is at point A

$\sigma = 282.7\ MPa$

$\sigma_d = K_t\,\sigma = 1.6*282.7 = 452.32\ MPa = S_{ut}/N$

$S_{ut} = 1357\ MPa \quad at\ N = 3$

From (appendix 3), P. A6

Say material chooses in AISI 4150 OQT 1000

Example (12-1), p.548:

Design the shaft shown in the figures 12-1, 12-2, 12-11 and 12-12. It is to be machined from AISI 1144 OQT 1000 steel. Gear A receives 150 KW from gear P. Gear C delivers power to gear Q. The shaft rotates at 62.8 rad/s.

Notes:

1- Use the bending moments and shear forces diagrams as shown in the previous example.

2- If you want to find the forces on the gears, you can do the followings:

$T=P/\omega$ = 150 KW*1000/62.8 rad/s = 2373 N.m

$W_{tA}=T_A/(D_A/2)$ = 2373*2/0.508 = 9341 N $\downarrow$

$W_{rA}= W_{tA} \tan\phi$ = 9341 tan 20 = 3398 N $\longrightarrow$

$W_{tC}=T_C/(D_C/2)$ = 2373*2/0.254 = 18680 N $\downarrow$

$W_{rC}= W_{tC} \tan\phi$ = 18580 tan 20 = 6800 N $\longleftarrow$

Now, solve the previous example by using fatigue equation:

Solution:

From A4-2 $\longrightarrow$ S_y= 572.28 MPa & S_u=813.61 MPa

From fig. 5-8 $\longrightarrow$ S_n = 289.6 MPa & $S'_n = S_n\,(C_m)\,(C_{st})(C_R)(C_s)$

C_m=1 *(material factor =1 for wrought steel)*

C_{st}= 1 *(type of stress factor =1 for bending stresses)*

C_R= 0.81 *(for design reliability of 0.99)*

C_s = 0.75 *(assumed from fig. 5-9 because the size of shaft is not available in this stage)*

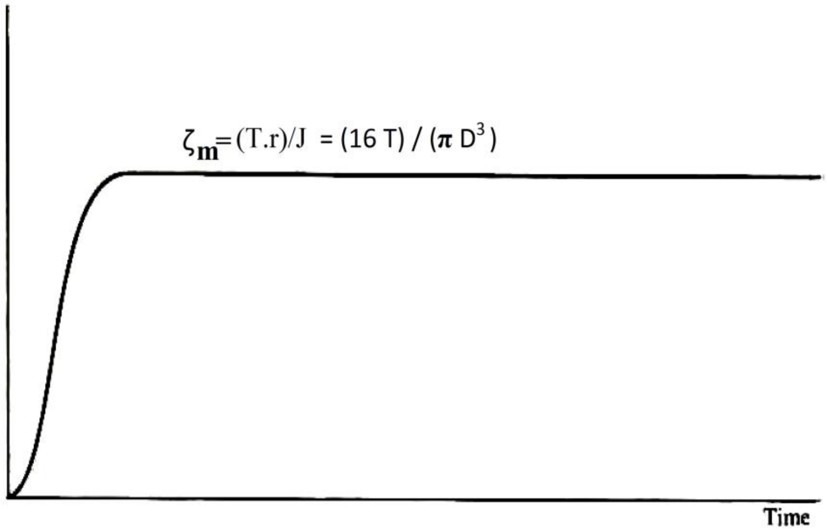
$S'_n = 289.6 * 0.75 * 0.81 = 175.82\ MPa$

$\sigma' = Von\ Miseses\ stress = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$

1. Point A:

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{0 + 3\tau_{xym}^2} = \sqrt{3}\ \frac{T.r}{J} = \sqrt{3}\ \frac{16T}{\pi D_1^3}$$

For static loading $\sigma'_m = \frac{S_y}{N} = \frac{572.3}{2} = \sqrt{3} * \frac{16*2372}{\pi D_1^3} \rightarrow D_1 = 41.9\ mm$



2. **Point B:**

$$\sigma'_m = \sqrt{\sigma^2_{xm} + 3\tau^2_{xym}} = \sqrt{0 + 3(\frac{16T}{\pi D^3})^2} = \sqrt{\frac{3}{4}(\frac{32T}{\pi D^3})^2}$$

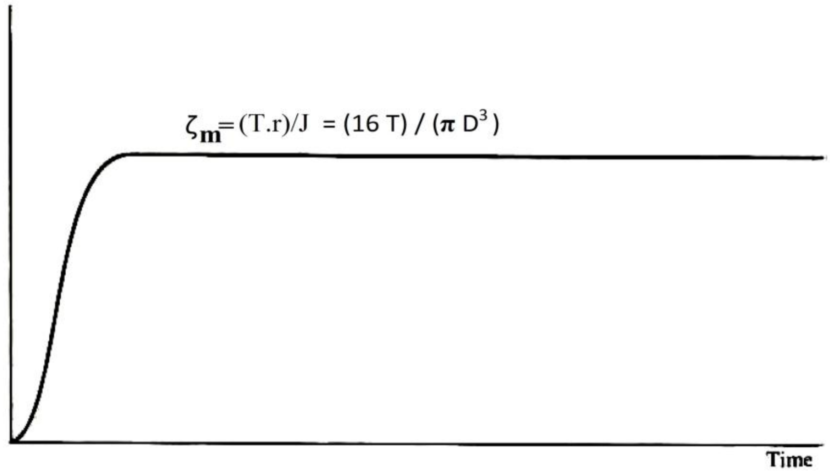
$$\sigma'_a = \sqrt{\sigma^2_{xa} + 3\tau^2_{xya}} = \sqrt{(\frac{32M}{\pi D^3})^2 + 0} = \sqrt{(\frac{32M}{\pi D^3})^2}$$

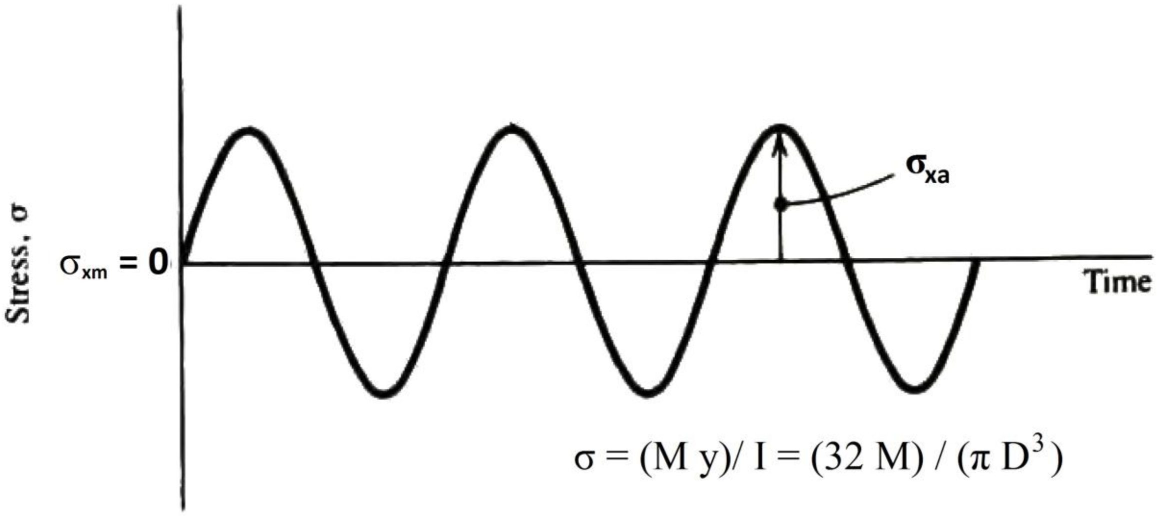
$$\frac{1}{N} = \frac{\sigma'_m}{S_y} + \frac{K_t\,\sigma'_a}{S'_n} \quad \text{(Soderberg equation)}$$

To solve the equation above for the shaft diameter (D), use this equation:

$$D = \left[\frac{32N}{\pi}\sqrt{\left[\frac{K_t\,M}{S'_n}\right]^2 + \frac{3}{4}\left[\frac{T}{S_y}\right]^2}\right]^{\frac{1}{3}}$$

$$D_2 = \left[\frac{32*2}{\pi}\sqrt{\left[\frac{1.5*2525.5}{175.8*10^3}\right]^2 + \frac{3}{4}\left[\frac{2373}{572*10^3}\right]^2}\right]^{\frac{1}{3}} = 83.82 \text{ mm}$$



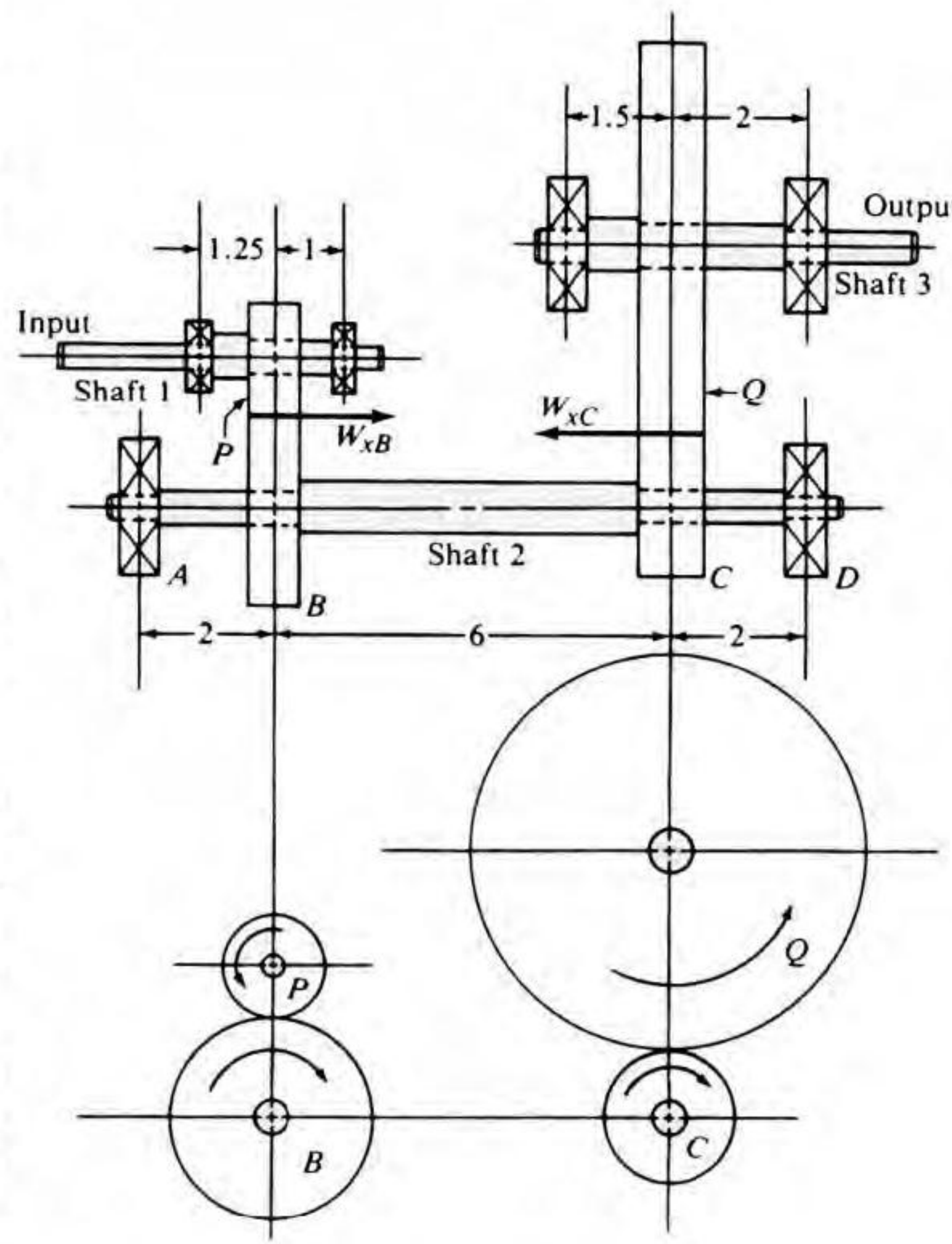


3. **Point C:** (you can use the same equation above with different value of stress concentration K_t & bending moment with same torque).

Exercises

1- Solve problems (Page 571), [Q35 and Q37] and draw complete construction for each of the above two questions.

<u>Q35:</u> The double-reduction, helical gear reducer shown in Figure PI 2-35 transmits 5.0 hp. Shaft 1 is the input, rotating at 1800 rpm and receiving power directly from an electric motor through a flexible coupling. Shaft 2 rotates at 900 rpm. Shaft 3 is the output, rotating at 300 rpm. A chain sprocket is mounted on the output shaft as shown and delivers the power upward. The data for the gears are given in Table below.



Gear	Module mm (m)	Pitch Diameter (m N) mm	No.of Teeth (N)	Face Width (F) mm
P	3	36	12	19
B	3	72	24	19
C	4	48	12	25
Q	4	144	36	25

Power= 5hp = 3728.5 watt.

Shaft (1) n_1=1800 rpm $\omega_1 = 188.5\ \frac{rad}{sec}$

Shaft (2) n_2=900 rpm $\omega_2 = 94.25\ \frac{rad}{sec}$

Shaft (3) n_3=300 rpm $\omega_3 = 31.4\ \frac{rad}{sec}$

$\phi = 14.5^o$ $\Psi = 45^o$ $\emptyset_n = 20^o$

Material AISI 4140 OQT 1200

a. **Determine the magnitude of torque in shafts at all points.**

Shaft (1) $T_1 = \frac{power}{\omega} = \frac{3728.5}{188.5} = 19.8\ N.m$

Shaft (2) $T_2 = \frac{power}{\omega} = \frac{3728.5}{94.25} = 39.6\ N.m$ $(Assuming\ \eta = 100\%)$

Shaft (3) $T_3 = \frac{power}{\omega} = \frac{3728.5}{31.4} = 118.79\ N.m$ $(Assuming\ \eta = 100\%)$

b. **Compute the forces on shafts and on bearings**

Shaft No.1

$$W_{tp} = \frac{2T_1}{D_p} = \frac{2 * 19.8 * 1000}{36} = 1100\ N$$

$$W_{rp} = W_t \frac{tan\emptyset}{cos\varphi} = 1100 * \frac{tan14.5}{cos45} = 402\ N$$

$$W_{xp} = W_t tan\varphi = 1100\ N.$$

$$W_{E)V} * 57 - W_{xp} * 18 - W_{rp} * 25 = 0 \quad (M_F = 0)$$

$$\therefore W_{E)V} = \frac{1100 * 18 + 402 * 25}{57} = 523.7\ N$$

$$\therefore W_{F)V} = 402 - 523.7 = -121.7N$$

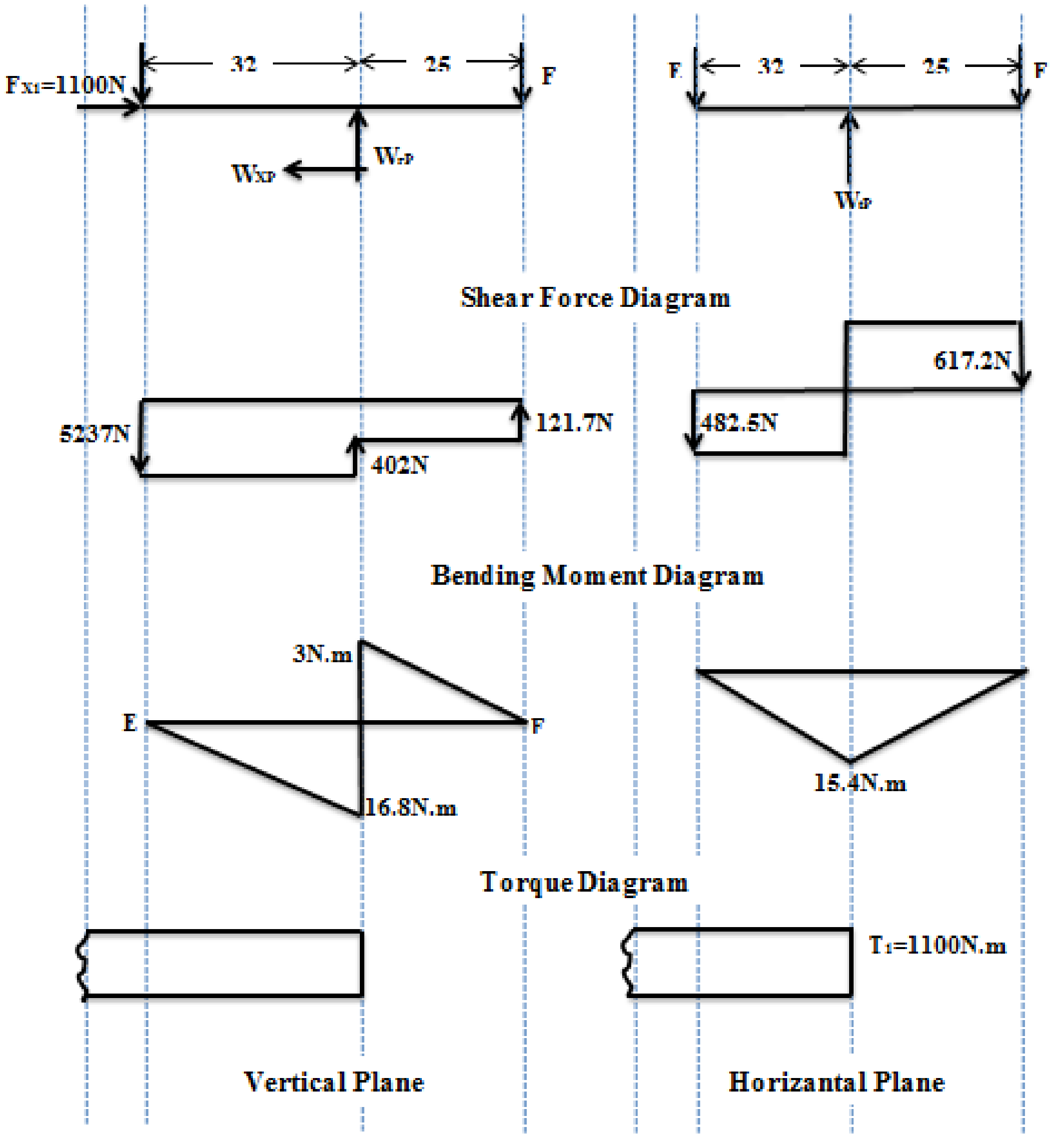
$$W_{E)H} * 57 - W_{tp} * 25 = 0 \quad (M_F = 0)$$

$$\therefore W_{E)H} = \frac{1100 * 25}{57} = 482.5\ N$$

$$\therefore W_{F)H} = 1100 - 482.5 = 617.5N$$

$$M_{P)V} = -W_{E)V} * 32 = 523.7 * 32 = 16.8\ N.m$$

$$M_{P)H} = -W_{E)H} * 32 = 482.5 * 32 = 15.4\ N.m$$



Shaft No.2

$$W_{tc} = \frac{2T_2}{D_c} = \frac{2*39.6*1000}{48} = 1650\ N$$

$$W_{rc} = W_{tc}\frac{tan\emptyset}{cos\varphi} = 1650*\frac{tan14.5}{cos45} = 603.5\ N$$

$$W_{xc} = W_{tc}tan\varphi = 1650\ N.$$

M_D=0 [Note : (W_{rp}= W_{rB})(W_{xp}= W_{xB})]

$$W_{A)V}*250 + 1100*36 - 402*200 - 1650*24 - 603.2*50 = 0$$

$$\therefore W_{A)V} = 442\ N$$

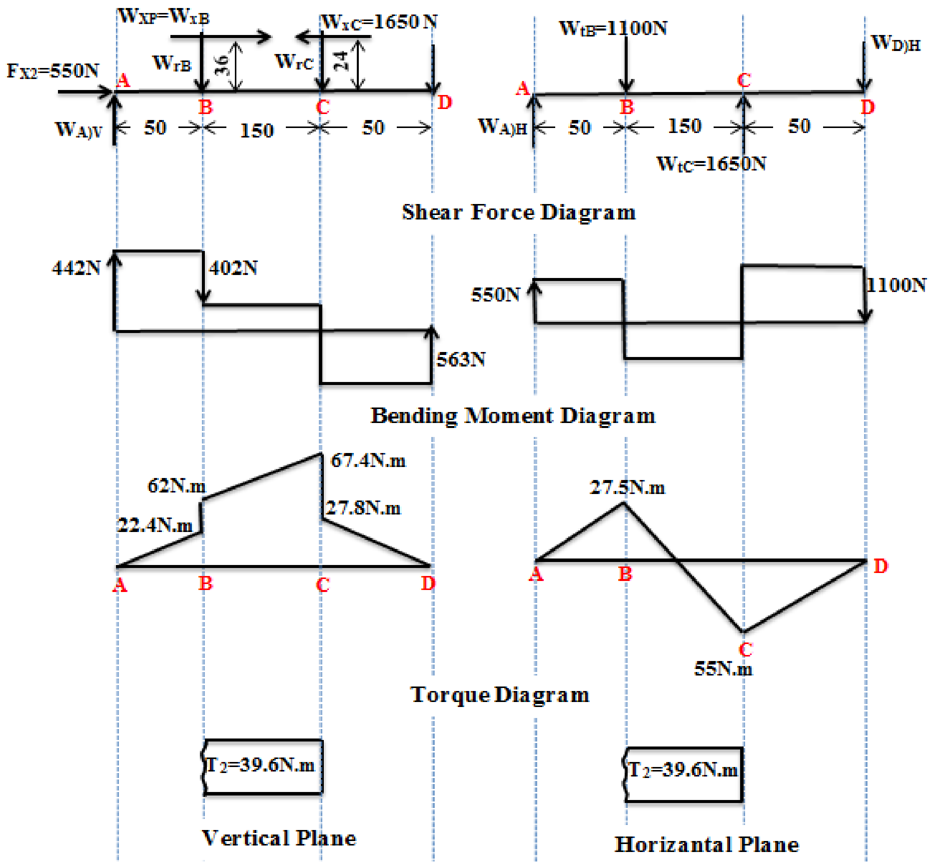
$$\therefore W_{D)V} = 402 + 603.5 - 442 = 563N$$

M_D=0 [Note : (W_{tp}= W_{tB})]

$$W_{A)H}*250 - 1100*200 + 1650*50 = 0$$

$$\therefore W_{A)H} = 550\ N$$

$$\therefore W_{D)H} = 550 - 1100 = 1650 = 1100N$$



Shaft No.3

$$W_{tQ} = \frac{2T_3}{D_Q} = \frac{2 * 118.5 * 1000}{144} = 1646\ N$$

$$W_{rQ} = W_{tQ}\frac{tan\emptyset}{cos\varphi} = 1646 * \frac{tan14.5}{cos45} = 603.2\ N$$

$$W_{xQ} = W_{tQ}tan\varphi = 1646\ N.$$

Assume Pitch dia. Of sprocket =288 mm

$$F_{chain} = \frac{2T_3}{D_{chain}} = \frac{2 * 118.5 * 1000}{288} = 823\ N$$

M_G=0 [Note : (W_{rc}= W_{rQ})(W_{xc}= W_{xQ})]

$$\therefore 823 * 138 - W_{s)V} * 88 + W_{rQ} * 38 + W_{xQ} * 72 = 0$$

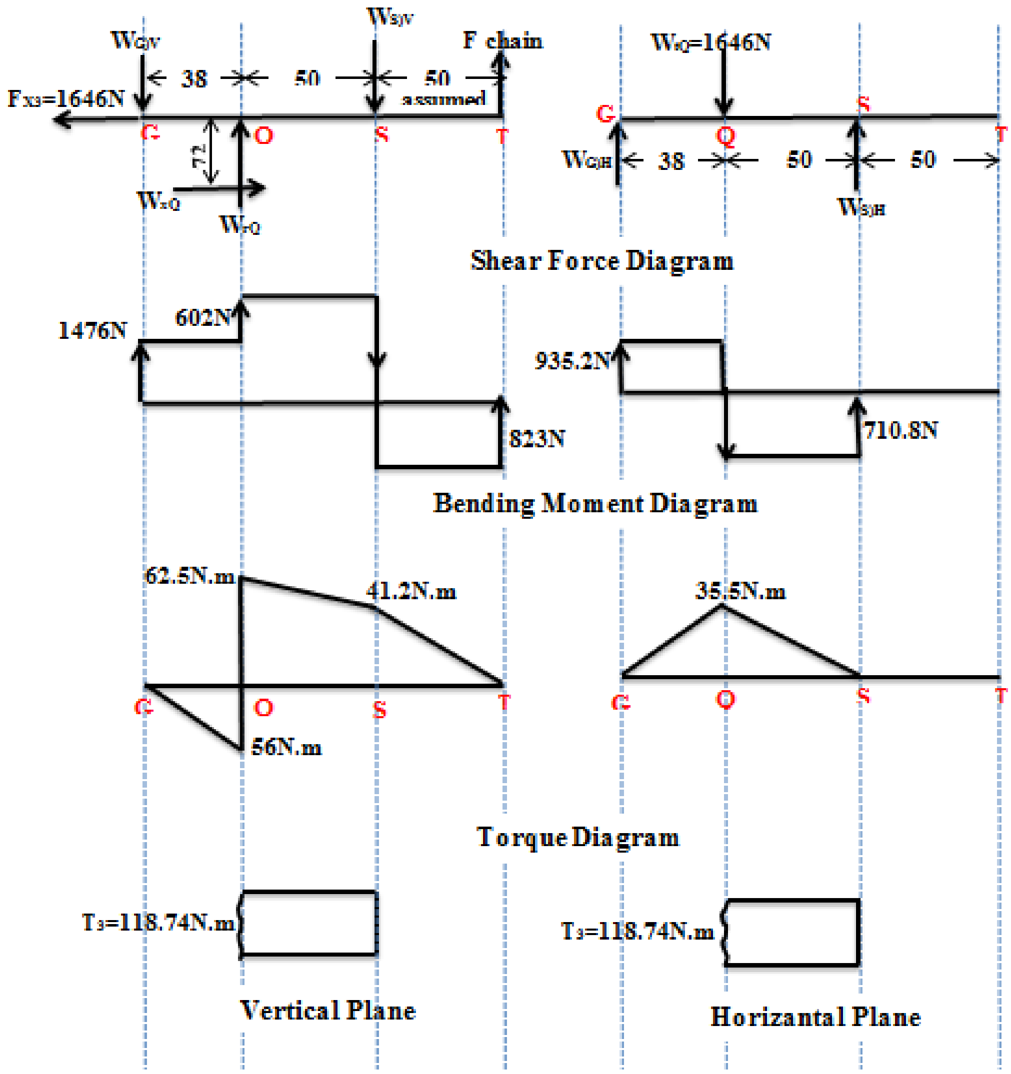
$$\therefore W_{s)V} = \frac{823 * 138 + 603.2 * 38 + 1650 * 72}{88} = 2901\ N$$

$$\therefore W_{G)V} = -1476\ N$$

M_S=0 [Note : (W_{tc}= W_{tQ})]

$$\therefore W_{G)H} = \frac{1646 * 50}{88} = 935.2\ N$$

$$\therefore W_{S)H} = 1646 - 935.2 = 710.8N$$



WG)V
WS)V
F chain
Fx3=1646N
38
50
50
assumed
G
O
S
T
72
WxQ
WrQ
WtQ=1646N
G
Q
S
T
WG)H
38
50
50
WS)H
Shear Force Diagram
1476N
602N
823N
935.2N
710.8N
Bending Moment Diagram
62.5N.m
41.2N.m
56N.m
35.5N.m
G
O
S
T
Torque Diagram
T3=118.74N.m
T3=118.74N.m
Vertical Plane
Horizantal Plane

c. Material of all shafts:

AISI 4140 OQT 1200

From figure (A4-4) P.A-9 for AISI 4140 oil quenched and tempered at 1200 F ∴ S_y=785 Mpa and S_u=896 Mpa.

From figure (5-8) Sn=335 Mpa.

$$\acute{S_n} = S_n * C_m * C_{st} * C_R * C_S$$

=335*1*1*0.81 (for design reliability of 0.99)*0.75(assumed because the size of shaft not available).

=203.5 Mpa.

d. Design Factor (N)

There are many factors influence the design factor (N) discussed before so choose a nominal value of N in our case =2 for general machine design.

e. Minimum allowable shaft diameters

The min. allowable shaft diameter is now computed at several sections along the shaft. Table below summarizes the data necessary for computed diameter of shaft.

Points	Torque(N.m)	Shearing Forces		Bending Moments		Axial Force (N)	Stress Concentration factor Kt	Loading Condition
		Vy (N)	Vx (N)	My (N.m)	Mx (N.m)			
E	19.8	524	483	0	0	1100	Fillet Kt=1.5	Static torsion static axial load
P	19.8	402	1100	16.8	15.4	1100	Key Kt=1.6	Static torsion reversed B.M static axial load
F	0	122	617	0	0	0	Fillet Kt=1.5	static shear load
A	0	442	550	0	0	550	Fillet Kt=1.5	static axial load
B	39.6	402	1100	62	27.2	(550) (1650)	Key Kt=1.6	Static torsion reversed B.M static axial load
C	39.6	603	1650	67.2	55	1650	Key Kt=1.6	Static torsion reversed B.M static axial load
D	0	563	1100	0	0	0	Fillet Kt=1.5	static shear load
G	0	1476	935	0	0	1646	Fillet Kt=1.5	static axial load
Q	118.74	603	1646	62.5	35.5	1646	Key Kt=1.6	Static torsion reversed B.M static axial load
S	118.74	2901	711	41.2	0	0	Fillet Kt=1.5	Static torsion reversed B.M static axial load
T	118.74	823	0	0	0	0	Key Kt=1.6	Static torsion

Now from table above you can see the complete analysis for each point on shafts. So to take all factors and stress will make the design complicated, then the following notes will help us for finding the shaft diameters.

Notes:

1- Neglect the stress concentration for static loading and the material is ductile.
2- Neglect the axial load and shear loads on shaft if loading is torsion only, bending moment only or combined bending and torsion.
3- When you neglect certain items, increase the shaft diameter then find design factor N for checking or find new allowable stress and should be less or equal the actual allowable stress.

Examples:

In point E (F_a*d_E/8) was neglected. Then d_E=7.64mm.

This value was checked $\boldsymbol{\sigma_{max} = 342\ Mpa\ < 392.5\ Mpa}$

In point P also (F_a*d_E/8) was neglected. Then d_p=15.9mm and increasd to 17mm, in this case check N=2.4>2.

4- Use appendix 2 page A-3 to choose the preferred basic sizes.

EXAMPLE:

In point E instead of d_E=7.64mm_choose the preferred basic size =8mm or you should choose at this point suitable bearing then from page 607, the smaller diameter of shaft=10mm. So at last the minimum diameter d_E=10mm. which_give you more safety.

1- Point E

$$\boldsymbol{\sigma_{all} = \frac{S_y}{N} = \frac{785}{2} = 392.5\ MPa}$$

$$\boldsymbol{\sigma_{max.} = \frac{32}{\pi d_E^3}\sqrt{\left(M + \frac{F_a * d_E}{8}\right)^2 + \frac{3}{4}T^2}}$$

M=0 and neglect (F_a*d_E/8) temporarily.

$$\boldsymbol{\therefore 392.5 = \frac{32}{\pi d_E^3} * \frac{\sqrt{3}}{2} T_1^2}$$

$$\therefore d_E^3 = \frac{32}{\pi(392.5)} * \frac{\sqrt{3}}{2} * (19800)^2$$

∴d_E=7.64mm

Now say d_E=8mm and check for stress.

$$\sigma_{max.} = \frac{32}{\pi 8^3}\sqrt{\left(\frac{1100 * 8}{8}\right)^2 + \frac{3}{4}(19800)^2} = 342\ MPa\ < 392.5\ MPa$$

∴d_E=8mm   is O.K.

2- Point P

$$\sigma_{max.} = \frac{32}{\pi d_P^3}\sqrt{\left(K_t M + \frac{F_a * d_P}{8}\right)^2 + \frac{3}{4}T^2}$$

$$\sigma_m = \frac{32}{\pi d_P^3}\sqrt{\left(\frac{1100 * d_P}{8}\right)^2 + \frac{3}{4}(19800)^2}$$

Neglect (F_a*d_E/8)

$$\therefore \sigma_m = \frac{32}{\pi d_P^3} * \frac{\sqrt{3}}{4} * (19800) \quad = \frac{151.338}{d_P^3}$$

$$\sigma_a = \frac{32}{\pi d_P^3}\sqrt{(K_t M)^2} = \frac{32*1.6}{\pi d_P^3}\sqrt{(16.8)^2 + (15.4)^2} * 1000 = \frac{371.613}{d_P^3}$$

$$\frac{1}{N} = \frac{\sigma_a}{\acute{S}_n} + \frac{\sigma_m}{S_y}$$

$$\therefore \frac{1}{2} = \frac{371.613}{d_P^3 * 203.5} + \frac{151.338}{d_P^3 * 785}$$

$$\therefore d_P^3 = 3652.2 + 385.6$$

∴d_P=15.9mm   Say d_P=17mm

Then, to check N

$$\sigma_m = \frac{32}{\pi(17)^3}\sqrt{\left(\frac{1100 * (17)}{8}\right)^2 + \frac{3}{4}(19800)^2} = 35.9MPa$$

$$\therefore \sigma_a = \frac{371613}{(17)^3} = 75.6\ MPa.$$

$$\therefore \frac{1}{N} = \frac{75.6}{203.5} + \frac{35.9}{785} \qquad \therefore N = 2.4 > 2 \qquad \therefore \text{diameter is O.K.}$$

3- Point F

At this point there is only vertical shearing force $=\sqrt{(121.7)^2 + (617.2)^2} =$ **629 N**

$$\therefore \tau_{max.} = \frac{4 * V}{3 * A} = \frac{4 * 629 * 4}{3 * \pi * d_F^2}$$

$$\tau_{max.} = 0.577 * \sigma_{max.} = 0.577 * 392.5 = 226.5\ MPa$$

$$d_F = \sqrt{\frac{16 * 629}{3 * \pi * 226.5}} = 2.17 mm$$

So this value is very small and the minimum inside diameter of roller bearings in text book =10mm

So say d_F=10mm

4- Point A

At this point there are axial load and vertical shearing force

$$\sigma_{max.} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_x = \frac{F_a}{A} = \frac{4 * 550}{\pi d_A^2} = \frac{700}{d_A^2}$$

$$\tau_{max.} = \frac{4 * V}{3 * A} = \frac{16\sqrt{(449)^2 + (550)^2}}{3 * \pi * d_A^2} = \frac{1206}{d_A^2}$$

$$\therefore 392.5 = \sqrt{\left(\frac{700}{d_A^2}\right)^2 + 3\left(\frac{1206}{d_A^2}\right)^2} = \frac{1}{d_A^2}\sqrt{(700)^2 + 3(1206)^2}$$

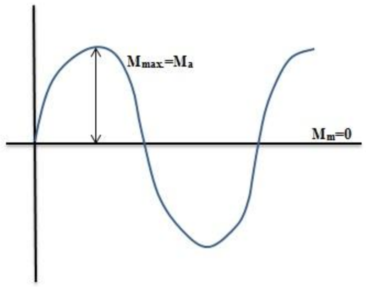
$\therefore$ d_A=2.4mm so say d_A=10mm (as before).

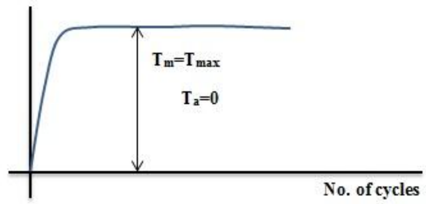
5- Point B

At this point there are torsion, bending moment, vertical shearing force and axial force.

Now neglect vertical shearing force and axial force.

∴ Use the following equations.





$$\sigma_{max.} = \frac{32}{\pi d^3}\sqrt{(K_t M)^2 + \frac{3}{4}T^2} \quad and \quad \frac{1}{N} = \frac{\sigma_a}{\acute{S}_n} + \frac{\sigma_m}{S_y}$$

$$\sigma_m = \frac{32}{\pi d_B^3} * \frac{\sqrt{3}}{2} * T_2 \qquad (M_m = Mean\ B.M. = 0)$$

$$\sigma_m = \frac{349500}{d_B^3}$$

$$\sigma_a = \frac{32}{\pi d_B^3}\left(1.6 * \sqrt{(62)^2 + (27.2)^2}\right) * 1000 = \frac{1104000}{d_B^3}$$

$$\therefore \frac{1}{2} = \frac{1104000}{203.5 d_B^3} + \frac{349500}{785 d_B^3}$$

∴d_B=22.7mm   say d_B=25mm (APP.2) P. A-3.

Also you can check as on point (P).

6- *Point C*

$$\sigma_m = \frac{349500}{d_C^3}$$

$$\sigma_a = \frac{32}{\pi d_C^3}\left(1.6 * \sqrt{(67.2)^2 + (55)^2}\right) * 1000 = \frac{1416000}{d_C^3}$$

$$\therefore \frac{1}{2} = \frac{1416000}{203.5 d_C^3} + \frac{349500}{785 d_C^3}$$

∴d_C=24.56mm   say d_C=30mm

Also you can check as on point (P).

Finally: you can solve other diameters for D, G, Q, S and T as before.

Now the complete construction of the gearbox may be similar to construction below:

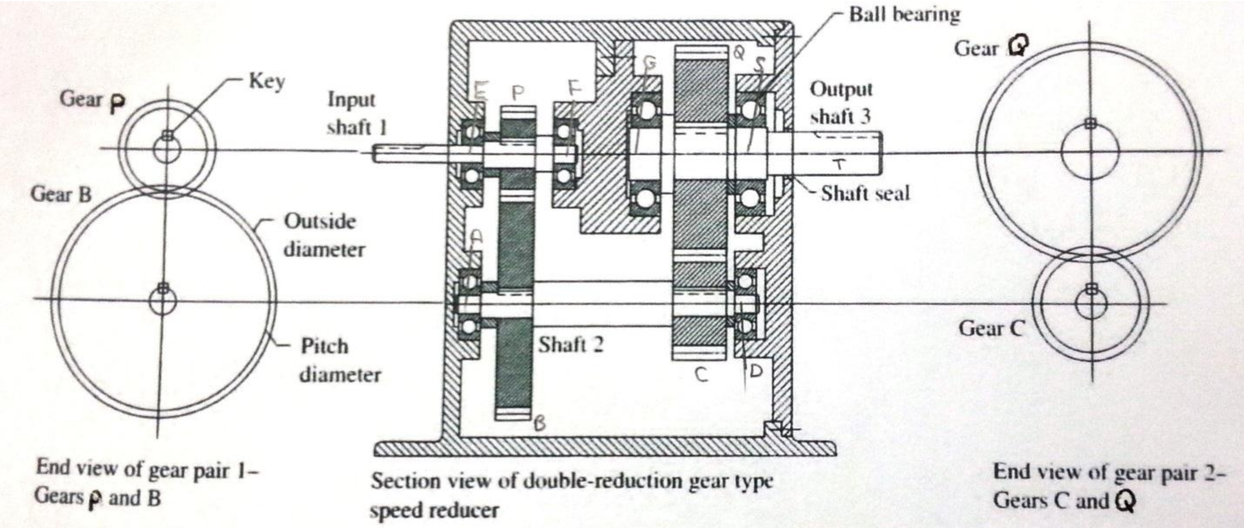


FIGURE 8–3 Conceptual design for a speed reducer