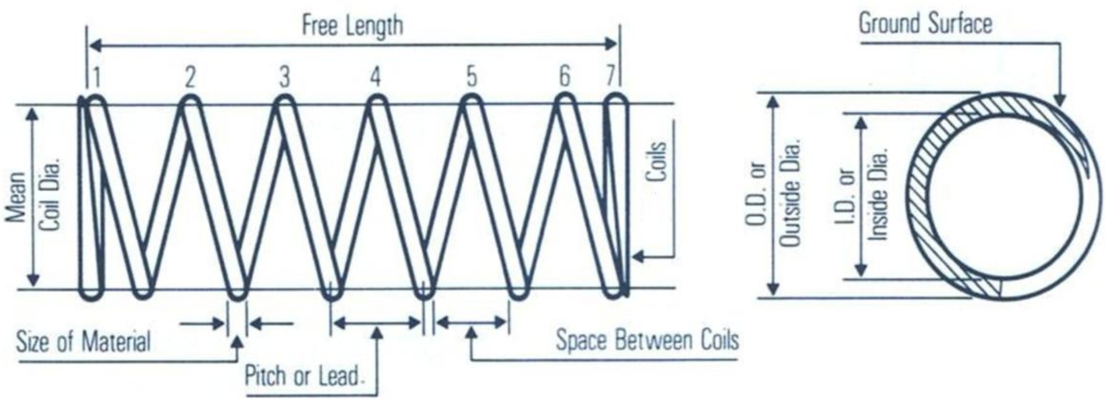
University of Technology
Mechanical Engineering Department

Machine Design I

Third Class for All Branches

LECTURES EIGHTTEEN & NINETEEN

SPRINGS



Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 19.

Introduction:

- A spring is a flexible element used to exert a force or torque and at the same time to store energy.
- Objective of this chapter (see section 19-1, page 732)

Kinds of springs (section 19-2, page 732):

See table (19-1) and Figure (19-2)

TABLE 19–1 Types of springs

Uses	Types of springs
Push	Helical compression spring
	Belleville spring
	Torsion spring: force acting at the end of the torque arm
	Flat spring, such as a cantilever or leaf spring
Pull	Helical extension spring
	Torsion spring: force acting at the end of the torque arm
	Flat spring, such as a cantilever or leaf spring
	Drawbar spring (special case of the compression spring)
	Constant-force spring
Radial	Garter spring, elastomeric band, spring clamp
Torque	Torsion spring, power spring

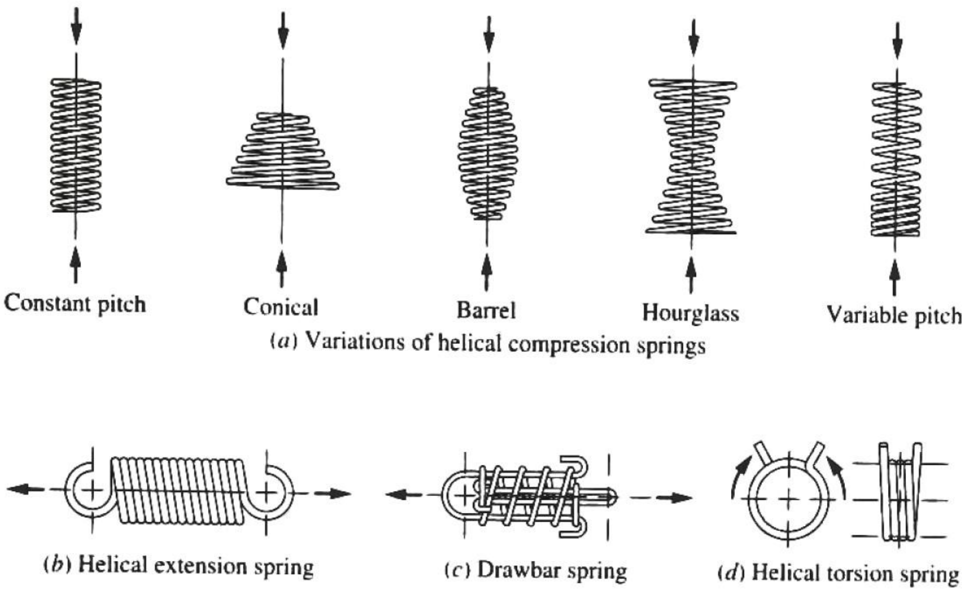
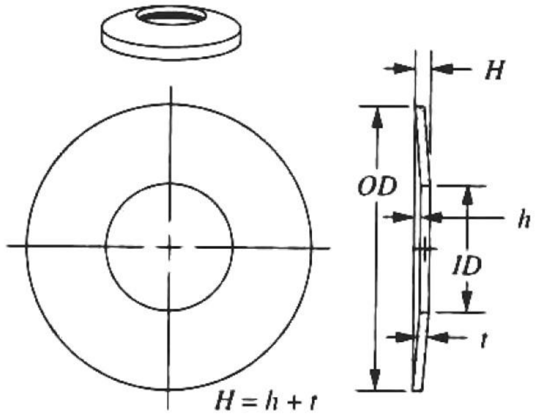
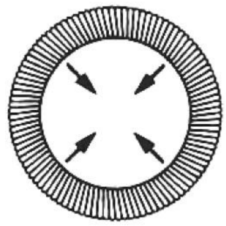


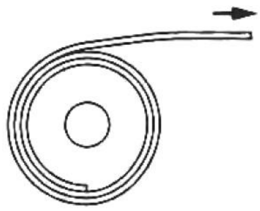
FIGURE 19–2
Several types of springs



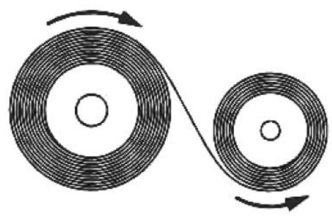
(e) Belleville spring



(f) Garter spring



(g) Constant force spring

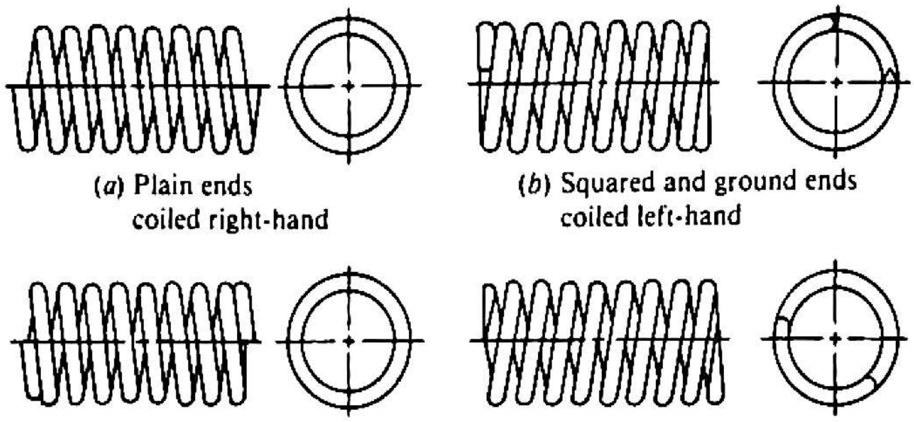


(h) Constant force spring motor

Helical compression springs (section 19-3, page 735):

In the most common form of helical compression spring, round wire is wrapped into a cylindrical form with a constant pitch between adjacent coils. This basic form is completed by a variety of end treatments as shown in fig. 19-3 (b, c, and d), page 734.

FIGURE 19–3
Appearance of helical compression springs showing end treatments



(a) Plain ends coiled right-hand

(b) Squared and ground ends coiled left-hand

(c) Squared or closed ends not ground, coiled right-hand

(d) Plain ends ground coiled left-hand

Note:

Figure 19-3 (b) is using for medium to large-size springs. Figure 19-3 (c) is using for springs with smaller wire. Figure 19-3 (d) is using for springs with unusual cases.

Diameters:

OD: outer diameter = $D_m + D_w$
ID: inside diameter = $D_m - D_w$
Where:
D_m: mean diameter of coil
D_w: wire diameter

Tables (19-2), page 736 shows Standard wire diameters

FIGURE 19–5
Notation for diameters

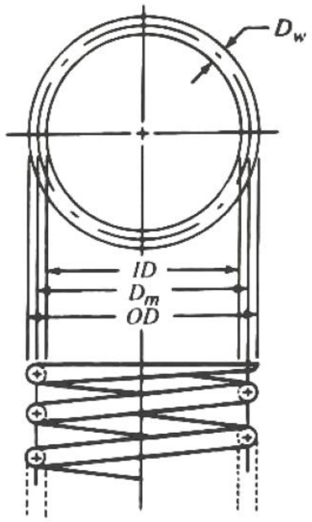


TABLE 19–2 Wire gages and diameters for springs

Gage no.	U.S. Steel Wire Gage (in)[a]	Music Wire Gage (in)[b]	Brown & Sharpe Gage (in)[c]	Preferred Metric Diameters (mm)[d]
7/0	0.4900			13.0
6/0	0.4615	0.004	0.5800	12.0
5/0	0.4305	0.005	0.5165	11.0
4/0	0.3938	0.006	0.4600	10.0
3/0	0.3625	0.007	0.4096	9.0
2/0	0.3310	0.008	0.3648	8.5
0	0.3065	0.009	0.3249	8.0
1	0.2830	0.010	0.2893	7.0
2	0.2625	0.011	0.2576	6.5
3	0.2437	0.012	0.2294	6.0
4	0.2253	0.013	0.2043	5.5
5	0.2070	0.014	0.1819	5.0
6	0.1920	0.016	0.1620	4.8
7	0.1770	0.018	0.1443	4.5
8	0.1620	0.020	0.1285	4.0
9	0.1483	0.022	0.1144	3.8
10	0.1350	0.024	0.1019	3.5
11	0.1205	0.026	0.0907	3.0
12	0.1055	0.029	0.0808	2.8
13	0.0915	0.031	0.0720	2.5
14	0.0800	0.033	0.0641	2.0
15	0.0720	0.035	0.0571	1.8
16	0.0625	0.037	0.0508	1.6
17	0.0540	0.039	0.0453	1.4
18	0.0475	0.041	0.0403	1.2
19	0.0410	0.043	0.0359	1.0
20	0.0348	0.045	0.0320	0.90
21	0.0317	0.047	0.0285	0.80
22	0.0286	0.049	0.0253	0.70

TABLE 19–2 (*continued*)

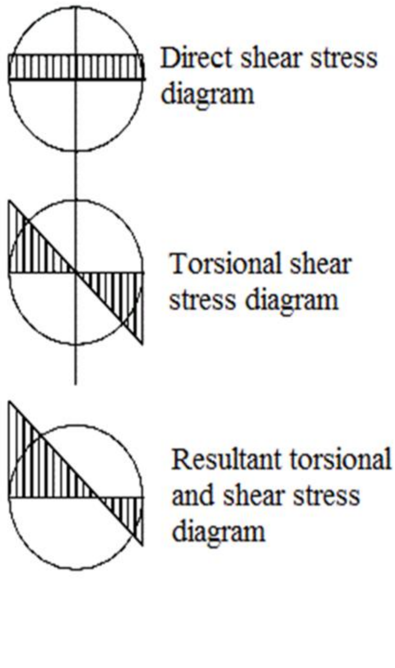
Gage no.	U.S. Steel Wire Gage (in)[a]	Music Wire Gage (in)[b]	Brown & Sharpe Gage (in)[c]	Preferred metric diameters (mm)[d]
23	0.0258	0.051	0.0226	0.65
24	0.0230	0.055	0.0201	0.60 or 0.55
25	0.0204	0.059	0.0179	0.50 or 0.55
26	0.0181	0.063	0.0159	0.45
27	0.0173	0.067	0.0142	0.45
28	0.0162	0.071	0.0126	0.40
29	0.0150	0.075	0.0113	0.40
30	0.0140	0.080	0.0100	0.35
31	0.0132	0.085	0.00893	0.35
32	0.0128	0.090	0.00795	0.30 or 0.35
33	0.0118	0.095	0.00708	0.30
34	0.0104	0.100	0.00630	0.28
35	0.0095	0.106	0.00501	0.25
36	0.0090	0.112	0.00500	0.22
37	0.0085	0.118	0.00445	0.22
38	0.0080	0.124	0.00396	0.20
39	0.0075	0.130	0.00353	0.20
40	0.0070	0.138	0.00314	0.18

Stresses on wire diameter of spring:

$\zeta_1 = \frac{F}{A}$ ……… (a)

$\zeta_2 = \frac{T.c}{J}$ ……….. (b)

$\zeta_3 = \zeta_1 + \zeta_2$



Lengths:

FIGURE 19–6
Notation for lengths and forces

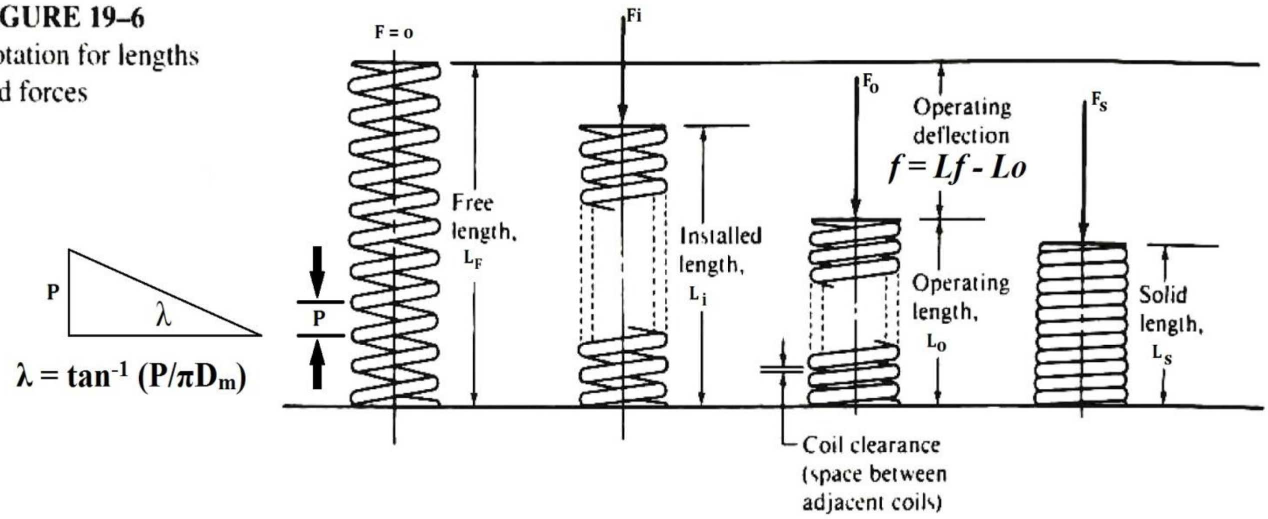
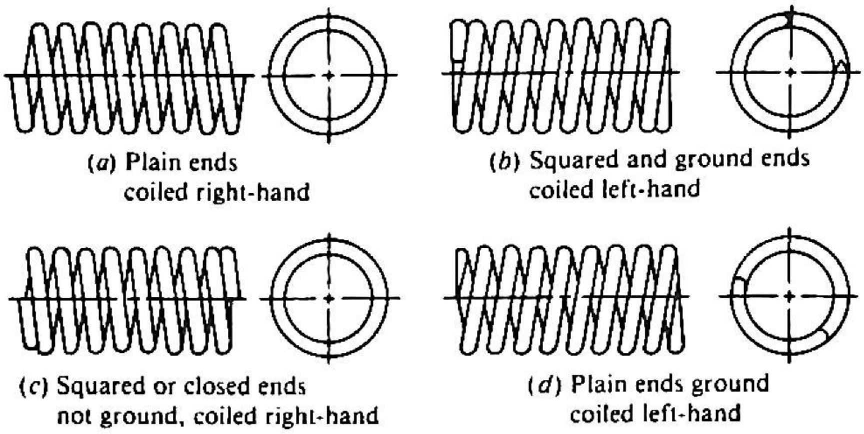


FIGURE 19–3
Appearance of helical compression springs showing end treatments



For above fig. (a) ⟶ $N_a = N$; $L_F = PN_a + D_w$

Fig. (b) ⟶ $N_a = N - 2$; $L_F = PN_a + 2D_w$

Fig. (c) ⟶ $N_a = N - 2$; $L_F = PN_a + 3D_w$

Fig. (d) ⟶ $N_a = N - 1$; $L_F = P\,(N_a + 1)$

Notes:

- **Forces:** see figure 19-6, page 737
- **Spring rate:** The relationship between force and its deflection (K)

$$K = \frac{\Delta F}{\Delta L} = \frac{F_O - F_i}{L_i - L_O} = \frac{F_O}{L_F - L_i} = \frac{Fi}{LF - Li} \quad \text{........} \quad (19\text{-}1)$$

- **Spring index:** $C = \frac{D_m}{D_w}$
- **Number of coils :** N
- **Number of active coil:** N_a
- **Pitch (P):** Axial distance from one coil to adjacent coil.
- **Pitch angle :** $\lambda = \tan^{-1} \left(\frac{P}{\pi D_m}\right)$
- **Materials used for springs:** see page 740 and table (19-3), page 741.
- **Types of loading and allowable stresses:**
  - ❖ Light service: for static load or up to 1000 cycles
  - ❖ Average service: for moderate load or up to 10^6 cycles
  - ❖ Severe service: for impact load or over 10^6 cycles

TABLE 19–3 Spring materials

Material type	ASTM no.	Relative cost	Temperature limits, °F
A. *High-carbon steels*			
Hard-drawn	A227	1.0	0–250
General-purpose steel with 0.60%–0.70% carbon; low cost			
Music wire	A228	2.6	0–250
High-quality steel with 0.80%–0.95% carbon; very high strength; excellent surface finish; hard-drawn; good fatigue performance; used mostly in smaller sizes up to 0.125 in			
Oil-tempered	A229	1.3	0–350
General-purpose steel with 0.60%–0.70% carbon; used mostly in larger sizes above 0.125 in; not good for shock or impact			
B. *Alloy steels*			
Chromium-vanadium	A231	3.1	0–425
Good strength, fatigue resistance, impact strength, high-temperature performance; valve-spring quality			
Chromium-silicon	A401	4.0	0–475
Very high strength and good fatigue and shock resistance			
C. *Stainless steels*			
Type 302	A313(302)	7.6	<0–550
Very good corrosion resistance and high-temperature performance; nearly nonmagnetic; cold-drawn; types 304 and 316 also fall under this ASTM class and have improved workability but lower strength			
Type 17-7 PH	A313(631)	11.0	0–600
Good high-temperature performance			
D. *Copper alloys:* All have good corrosion resistance and electrical conductivity.			
Spring brass	B134	High	0–150
Phosphor bronze	B159	8.0	<0–212
Beryllium copper	B197	27.0	0–300
E. *Nickel-base alloys:* All are corrosion-resistant, have good high- and low-temperature properties, and are nonmagnetic or nearly nonmagnetic (trade names of the International Nickel Company).			
Monel™			−100–425
K-Monel™			−100–450
Inconel™			Up to 700
Inconel-X™		44.0	Up to 850

Source: Associated Spring, Barnes Group, Inc. *Engineering Guide to Spring Design*. Bristol, CT, 1987. Carlson, Harold. *Spring Designer's Handbook*. New York: Marcel Dekker, 1978. Oberg, E., et al. *Machinery's Handbook*. 26th ed. New York: Industrial Press, 2000.

FIGURE 19–8
Design shear stresses for ASTM A227 steel wire, hard-drawn (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 144, by courtesy of Marcel Dekker, Inc.)

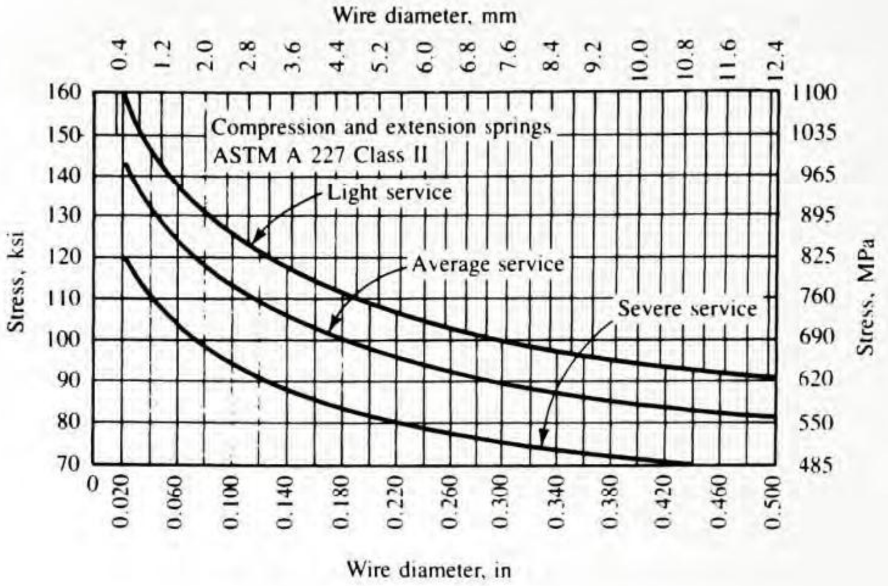


FIGURE 19–9
Design shear stresses for ASTM A228 steel wire (music wire) (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 143, by courtesy of Marcel Dekker, Inc.)

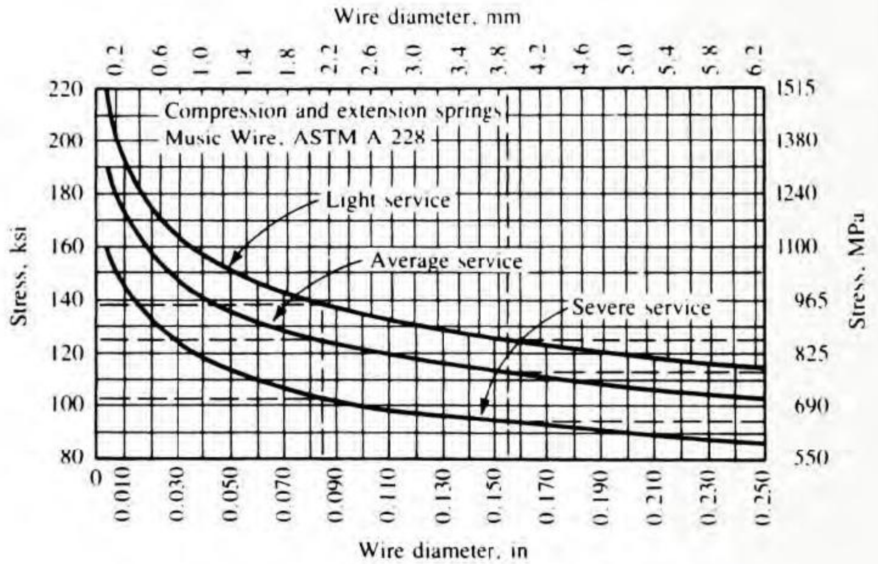


FIGURE 19–10
Design shear stresses for ASTM A229 steel wire, oil-tempered (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 146, by courtesy of Marcel Dekker, Inc.)

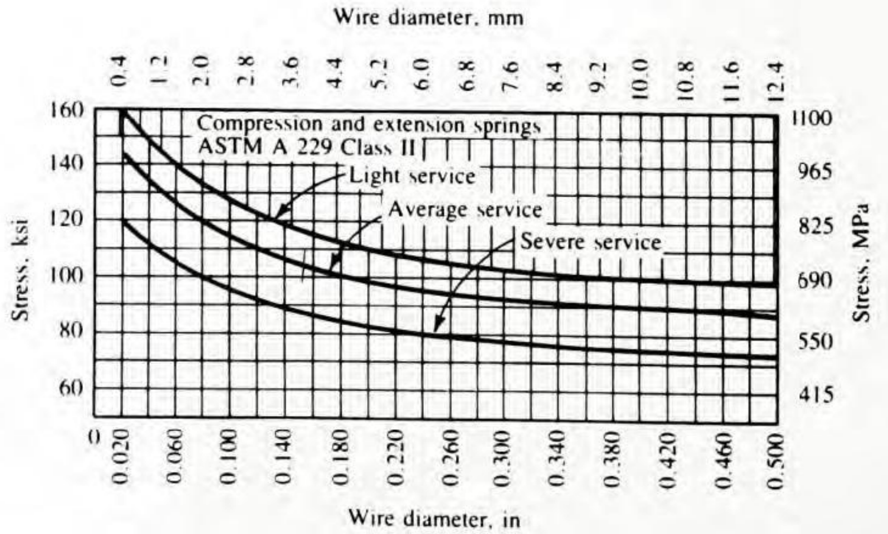


FIGURE 19–11
Design shear stresses for ASTM A231 steel wire, chromium-vanadium alloy, valve-spring quality (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 147, by courtesy of Marcel Dekker, Inc.)

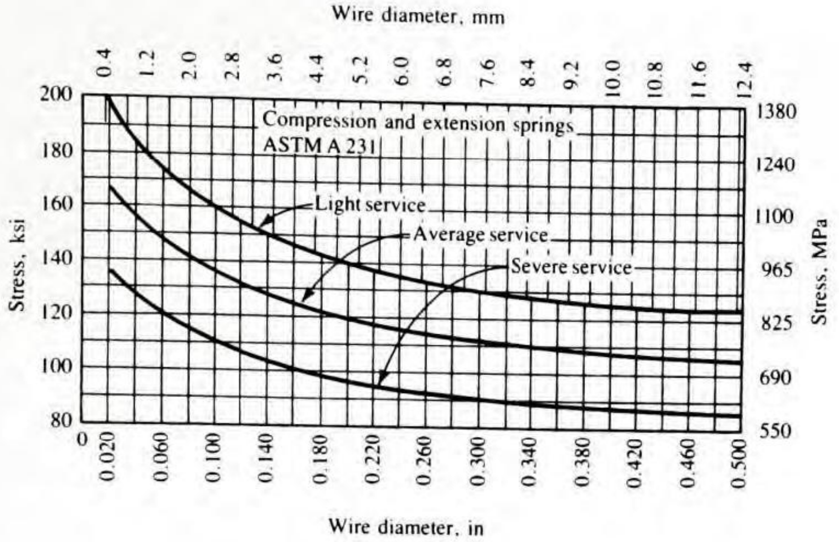


FIGURE 19–12
Design shear stresses for ASTM A401 steel wire, chromium-silicon alloy, oil-tempered (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 148, by courtesy of Marcel Dekker, Inc.)

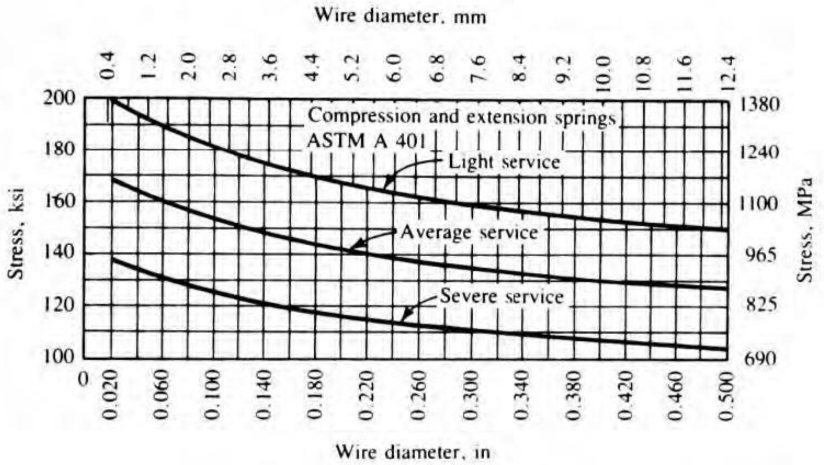
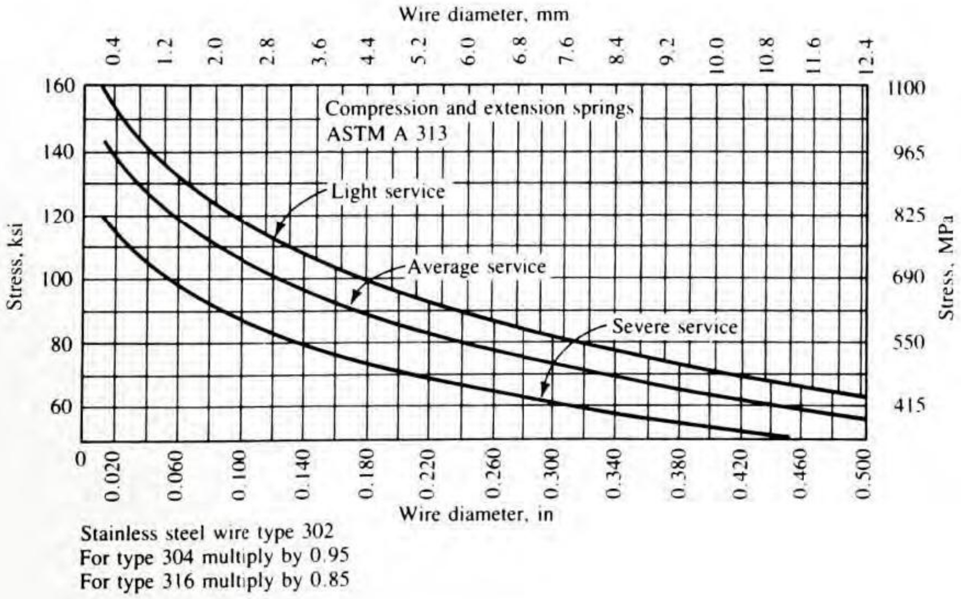


FIGURE 19–13
Design shear stresses for ASTM A313 corrosion-resistant stainless steel wire (Reprinted from Harold Carlson, *Spring Designer's Handbook*, p. 150, by courtesy of Marcel Dekker, Inc.)



Figures (19-8, 19-9, 19-10, 19-11, 19-12, and 19-13) are used for different materials.

Stresses and deflection for helical springs (section 19-4, page 744):

As a compression spring is compressed under an axial load the wire is twisted. Therefore the stress developed is Torsional shear stress $\left\{\zeta 1 = \frac{T*c}{J}\right\}$ combined with direct shear stress $\left(\zeta 2 = \frac{F}{A}\right)$then ($\zeta = \zeta_1 + \zeta_2$).

$$\zeta = \left\{\frac{\left(F*\frac{D_m}{2}*\frac{D_W}{2}\right)}{\frac{\pi}{32}D_W^4}\right\} + \frac{F}{\left(\frac{\pi D_W^2}{4}\right)}$$

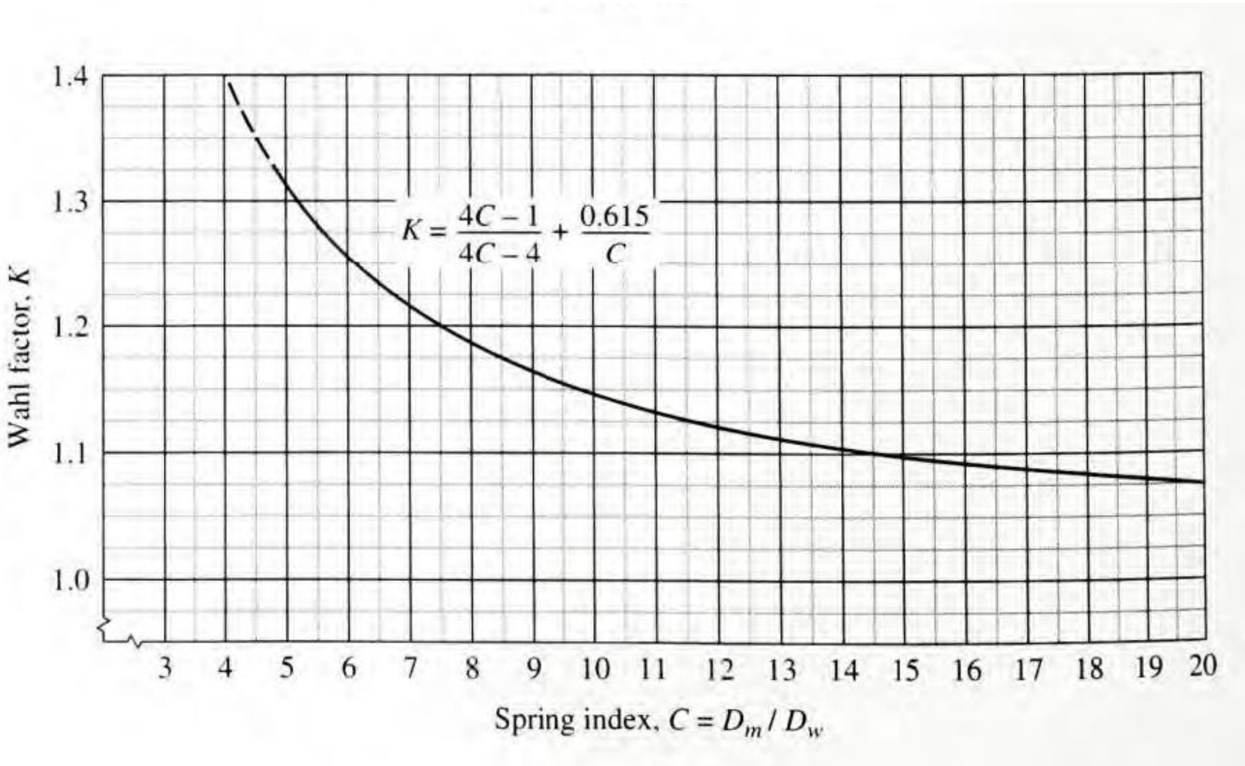
$$= \left(\frac{8FD_m}{\pi D_W^3}\right)\left(1+\frac{D_W}{2D_m}\right) = \left(\frac{8FD_m}{\pi D_W^3}\right)\left(1+\frac{1}{2C}\right)$$

$$\zeta = \left(\frac{8FD_m}{\pi D_W^3}\right)(K) = \left(\frac{8KFC}{\pi D_W^2}\right) \quad \text{.........} \ (19\text{-}4)$$

Where: C = spring index ; K = Wahl factor = $\frac{4C-1}{4C-4} + \frac{0.615}{C}$

Note:

1. Wahl factor is the term that account for the curvature of the wire and stress concentration.
2. See fig. (19-14), page 744 to find Wahl factor.
3. Recommended C is 12 > C > 5



Deflection:

$\theta = \frac{T*L}{G*J}$ & f = Deflection = $\theta * \frac{Dm}{2}$

$$f = \frac{T*L}{G*J} * \frac{D_m}{2} = \frac{\left\{\left(F*\frac{Dm}{2}\right)*(\pi D_m * N_a)\right\}}{\frac{G*\pi D_W^4}{32}} * \frac{D_m}{2}$$

$$\boldsymbol{f = \frac{(8F * {D_m}^3 * Na)}{G*{D_w}^4} = \frac{(8F * C^3 * Na)}{G*D_w}} \quad \text{………. (19-6)}$$

Where:

θ: Angle of twist in radians.

T: Applied torque = F * $\frac{D_m}{2}$

L: Length of wire = $\pi D_m * N_a$

G: Modulus of elasticity in shear (see table 19-4, page 745)

J: polar moment of inertia for wire =$\frac{\pi D_W^4}{32}$.

TABLE 19–4 Spring wire modulus of elasticity in shear (G) and tension (E)

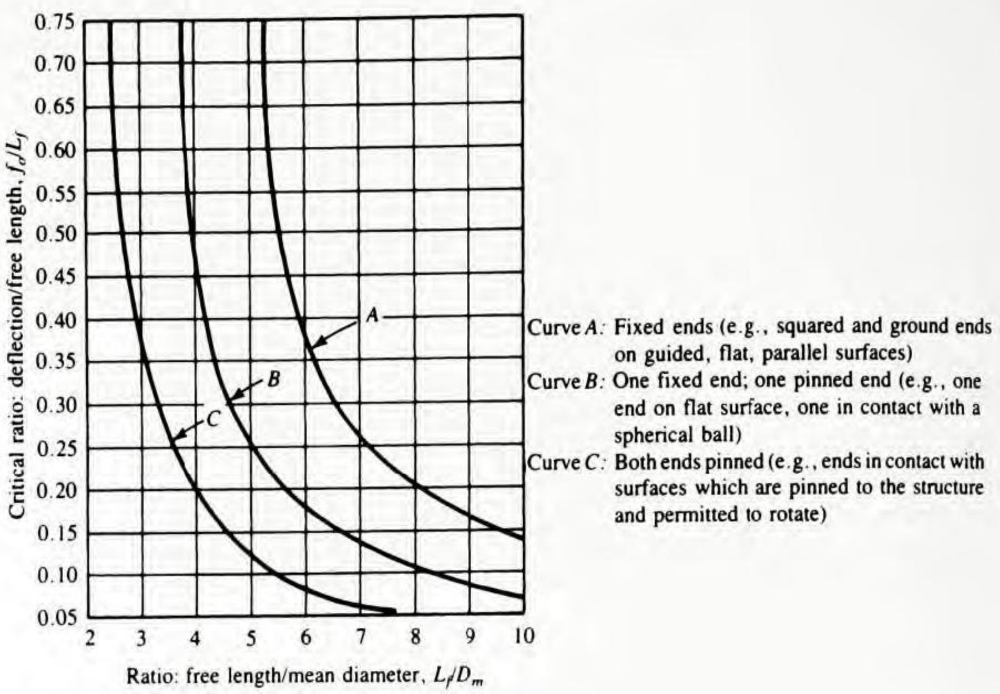
Material and ASTM no.	Shear modulus, G (psi)	Shear modulus, G (GPa)	Tension modulus, E (psi)	Tension modulus, E (GPa)
Hard-drawn steel: A227	11.5×10^6	79.3	28.6×10^6	197
Music wire: A228	11.85×10^6	81.7	29.0×10^6	200
Oil-tempered: A229	11.2×10^6	77.2	28.5×10^6	196
Chromium-vanadium: A231	11.2×10^6	77.2	28.5×10^6	196
Chromium-silicon: A401	11.2×10^6	77.2	29.5×10^6	203
Stainless steels: A313				
Types 302, 304, 316	10.0×10^6	69.0	28.0×10^6	193
Type 17-7 PH	10.5×10^6	72.4	29.5×10^6	203
Spring brass: B134	5.0×10^6	34.5	15.0×10^6	103
Phosphor bronze: B159	6.0×10^6	41.4	15.0×10^6	103
Beryllium copper: B197	7.0×10^6	48.3	17.0×10^6	117
Monel and K-Monel	9.5×10^6	65.5	26.0×10^6	179
Inconel and Inconel-X	10.5×10^6	72.4	31.0×10^6	214

Note: Data are average values. Slight variations with wire size and treatment may occur.

Buckling:

Figure (19-15) shows buckling criteria (page 746). First compute $\frac{L_f}{D_m}$ then find $\frac{f_O}{L_f}$ after that check the buckling from figure (19-15).

FIGURE 19–15
Spring buckling criteria. If the actual ratio of f_o/L_f is greater than the critical ratio, the spring will buckle at operating deflection.



Analysis of spring characteristics and design:

See example problem (19-1) to example (19-3).

Note:

- The following formula can be used for OD at the solid length condition:

$$OD_S = \sqrt{D_m^2 + \frac{P^2 - D_w^2}{\pi^2}} + D_w \quad \text{……….} \ (19\text{-}3)$$

- An initial diametric clearance of one- tenth of the wire diameter is recommended for springs having a diameter of 12 mm or greater.
- Coil clearance $C_C = (L_O - L_S)/N_a$
- Check that $(L_O - L_S) > 0.15(L_F - L_S)$

Example Problem 19-1, (Page 796): A spring is known to be made from music wire, ASTM A228 steel, but no other data are known. You are able to measure the following features using simple measurement tools:
Free length = L_f = 44.45 mm
Outside diameter = OD = 14.25 mm
Wire diameter = D_w= 1.4 mm
The ends are squared and ground.

The total number of coils (N) = 10.0.

Load=62.27 N at ≈300000 cycles.

Sol:

1. From table 19-2 the wire is 17-gage music wire,
   $D_m = OD - D_w = 14.25-1.4 = 12.85$ mm ;
   $ID = D_m - D_w = 12.85-1.4 = 11.45$ mm
   $C = D_m/D_w = 12.85/1.4 = 9.2$;
   $K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.158$

2. $F = F_0 = 62.27$ N
   $\zeta = \left(\frac{8KFC}{\pi D_w^2}\right) = \frac{8\,(1.158)\,(62.27)(9.2)}{\pi\,(1.4)^2} = 865.1$ MPa

3. $f = \frac{(8F * C^3 * Na)}{G * Dw} = \frac{8\,(62.27)(9.2^3)\,(8)}{81.7 * 10^9 * 1.4} = 27.2$ mm= f_o

   Na = N – 2 = 10 – 2 = 8 (for squared & ground) ; G= 81.7 GPa from table (19-4)

4. $L_0 = L_F - f_0 = 44.45 - 27.2 = 17.25$ mm ; $L_S = D_w * N = 1.4*10 = 14$ mm ;
   $K = \Delta F/\Delta L = F_o/(L_F - L_o) = F_0/f_0 = 62.27/27.2 = 2.29$ N/mm
5. $F_S = K\,(L_F - L_S) = 2.29(44.45-14) = 69.79$ N ;
   $\zeta = \frac{8\,F\,F_S\,C}{\pi\,D_w^2} = 970$ MPa
6. From fig. 19-9 for ASTM A228 steel for average service
   $\zeta_d = 930.8$ MPa at $D_w = 14$ mm > $\zeta_o = 865.7$ MPa **satisfactory**

7. $\zeta_S > \zeta_d$ so use light service ζ_d= 1034 MPa at $D_w = 1.4$ mm (in this case $\zeta_d > \zeta_S$ **satisfactory**)
8. $\frac{L_F}{D_m}$ = 49.5/12.85 = 3.46, so from fig. 19-15 use curve A (No Buckling)
9. $D_{hole} > OD + \frac{D_w}{10} = 14.4$ mm

Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-1 (Page 746)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	17.25	mm
Trial mean diameter	Dm =	12.85	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	= Music wire: A228		
Shear modulus of elasticity of spring wire	G =	81702.906	N/mm²
Maximum operating force	Fo =	62.27	N
Installed force	Fi =	0.0001	N
Load type	Average service		
Initial design stress	τid =	970	N/mm²

Results

Spring rate	k	=	2.289N/mm
Free length	Lf	=	44.450mm
Actual design stress	τd	=	926.366N/mm²
Maximum allowable stress	τmax	=	1032.697N/mm²
Computed wire diameter	Dw	=	1.372mm
Spring index	C	=	9.369
Actual expected stress due to operating force	τo	=	3.361e+006mm
Number of active coils	Na	=	7.441
Solid length	Ls	=	12.949mm
Force at solid length	Fs	=	72.116N
Stress at solid length	τs	=	1056.510N/mm²

Note

The Stress at solid length τs should be greater than the Maximum allowable stress τmax

Outside diameter	OD	=	14.222mm
Inside diameter	ID	=	11.478mm
Buckling ratio	Lf/Dm	=	3.459
Coil clearance	cc	=	0.578mm
If installed in hole, minimum hole diameter			14.359mm

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Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-2 (Page 749)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	31.75	mm
Trial mean diameter	Dm =	8.71	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	=	Chromium-vanadium: A231	
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	Fo =	53.38	N
Installed force	Fi =	35.6	N
Load type	Light service		
Initial design stress	τid =	992.9	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1231.283N/mm²
Maximum allowable stress	τmax	=	1231.283N/mm²
Computed wire diameter	Dw	=	1.206mm
Spring index	C	=	7.219
Actual expected stress due to operating force	τo	=	2.995e+006mm
Number of active coils	Na	=	22.109
Solid length	Ls	=	29.088mm
Force at solid length	Fs	=	57.107N
Stress at solid length	τs	=	869.639N/mm²
Outside diameter	OD	=	9.916mm
Inside diameter	ID	=	7.503mm
Buckling ratio	Lf/Dm	=	8.023
Coil clearance	cc	=	0.120mm
If installed in hole, minimum hole diameter			10.037mm

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Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-2 (Page 749)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	31.75	mm
Trial mean diameter	Dm =	8.71	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	=	Chromium-vanadium: A231	
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	Fo =	53.38	N
Installed force	Fi =	35.6	N
Load type	Light service		
Initial design stress	τid =	992.9	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1231.283N/mm²
Maximum allowable stress	τmax	=	1231.283N/mm²
Computed wire diameter	Dw	=	1.206mm
Spring index	C	=	7.219
Actual expected stress due to operating force	τo	=	2.995e+006mm
Number of active coils	Na	=	22.109
Solid length	Ls	=	29.088mm
Force at solid length	Fs	=	57.107N
Stress at solid length	τs	=	869.639N/mm²
Outside diameter	OD	=	9.916mm
Inside diameter	ID	=	7.503mm
Buckling ratio	Lf/Dm	=	8.023
Coil clearance	cc	=	0.120mm
If installed in hole, minimum hole diameter			10.037mm

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Tedata

Program : MDESIGN	User :	Customer :
Version : 1.1.2	Date : 22.12.2013	Proj. Nr :

Helical Compression Springs

Example: 19-3 (Page 754)

Input data:

Helical Compression Springs

Spring Geometry			
Installed length	Li =	44.45	mm
Operating length	Lo =	31.75	mm
Trial mean diameter	Dm =	15.24	mm
Ends type	Squared and ground		
Forces and Stresses			
Type of spring wire	= Chromium-vanadium: A231		
Shear modulus of elasticity of spring wire	G =	77221.312	N/mm²
Maximum operating force	Fo =	53.38	N
Installed force	Fi =	35.6	N
Load type	Light service		
Initial design stress	τid =	896.35	N/mm²

Results

Spring rate	k	=	1.400N/mm
Free length	Lf	=	69.879mm
Actual design stress	τd	=	1183.293N/mm²
Maximum allowable stress	τmax	=	1183.293N/mm²
Computed wire diameter	Dw	=	1.587mm
Spring index	C	=	9.600
Actual expected stress due to operating force	τo	=	2.196e+006mm
Number of active coils	Na	=	12.371
Solid length	Ls	=	22.815mm
Force at solid length	Fs	=	65.890N
Stress at solid length	τs	=	735.837N/mm²
Outside diameter	OD	=	16.827mm
Inside diameter	ID	=	13.652mm
Buckling ratio	Lf/Dm	=	4.585
Coil clearance	cc	=	0.722mm
If installed in hole, minimum hole diameter			16.986mm

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