LECTURE THREE. FOUR &FIVE

STRESS AND DEFORMATION ANALYSIS

1- Direct stresses, tension and compression:

Tensile and compressive stresses, called *normal stresses,* are shown acting perpendicular to opposite faces of the stress element. Tensile stresses tend to pull on the element, whereas compressive stresses tend to crush it.

$$\sigma_d = \frac{P}{A}$$

$$\delta = \frac{FL}{AE}$$

Where: σ_d= Design tensile or compression stress
F= direct axial load
A= cross sectional area
δ= total deformation
L=original total length
E= Modulus of elasticity

$$\sigma_d = \frac{Strength\ of\ material\ from\ which\ the\ component\ made}{Design\ factor\ (or\ safety\ factor)(N)}$$

For ductile material:

N = 1.25 – 2 (for static loads)
N = 2 – 2.5 (for dynamic loads with average confidence)
N = 2.5 – 4 (for dynamic loads with uncertainly about load)
N = 4 or higher (for shock load or to desire extra safety)

For brittle material:

N = 3 – 4 (for static load with high level of confidence)
N = 4 – 8 (for dynamic loading with uncertainly about load)

$\sigma_d = \frac{S_{ut}}{N}$ (For tensile stress)

$\sigma_d = \frac{S_{uc}}{N}$ (For compression stress)

Based on Ultimate tensile strength

OR

$\sigma_d = \frac{S_{yt}}{N}$ (For tensile stress)

$\sigma_d = \frac{S_{yc}}{N}$ (For compression stress)

Based on Ultimate yield strength

Example:

A large electrical transformer is to be suspended from a roof truss of a building. The total weight of transformer is 142.33 KN. Design the means of support.

Selection:

Choose N=3
Choose material AISI 1040 cold-drawn steel
Choose two straight cylindrical rod to support the transformer

Sol:

From APPENDIX 3, S_Y = 489.54 MPa

$$\sigma_d = \frac{S_y}{N} = \frac{489.54}{3} = 163.18\ MPa$$

$$\sigma_d = \frac{F}{A} \rightarrow A = \frac{142330\ N}{163.18\ \frac{N}{mm^2}} = 871\ mm^2 = \frac{\pi d^2}{4} \rightarrow d = 33.3\ mm$$

Key:

A *key* is a machinery component placed at the interface between a shaft and the hub of a power-transmitting element for the purpose of transmitting torque (see figure 11-1). It is installed in an axial groove machined into the shaft, called a *keyseat*. Keyseat in shafts are usually machined with either an end mill or circular milling cutter, producing the profile or sled runner key seat.

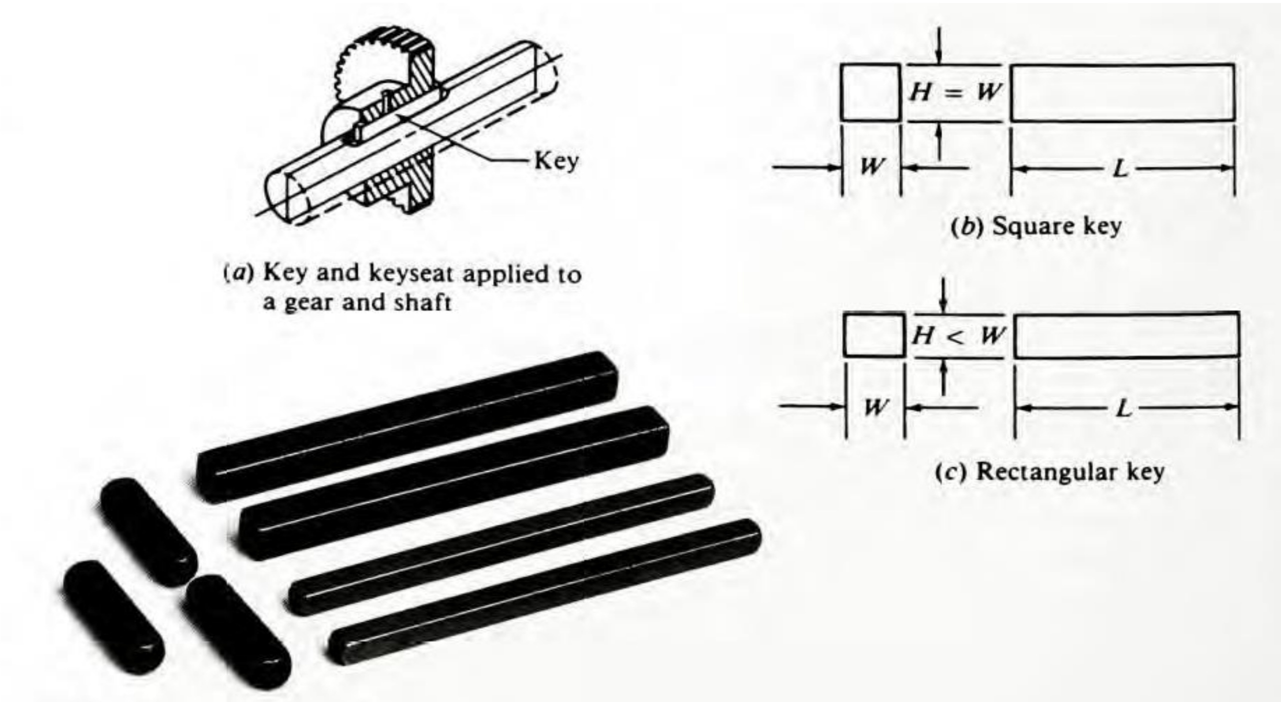
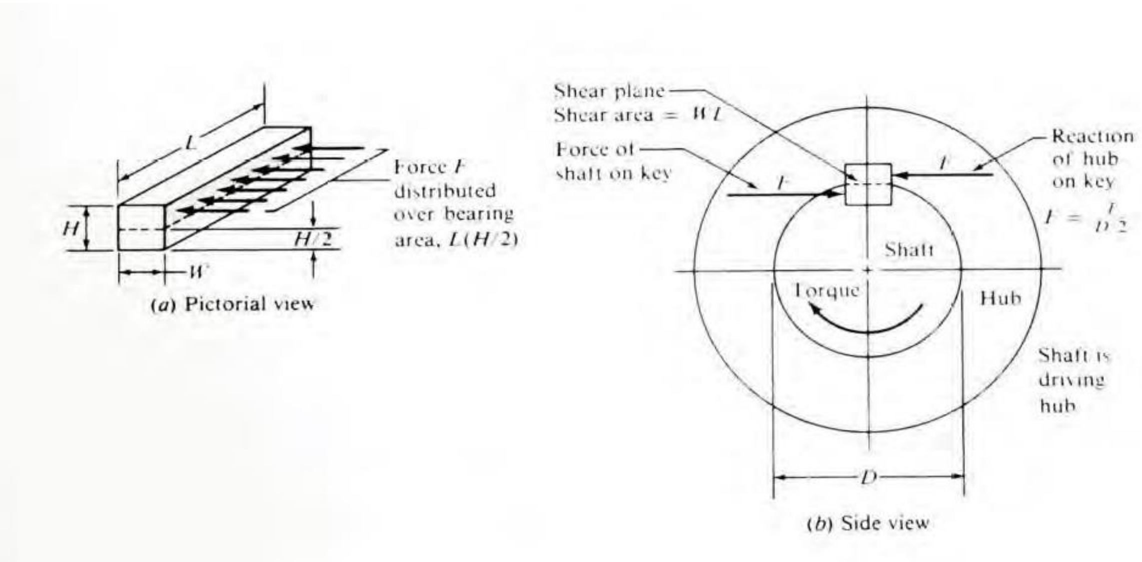


Figure (11-1) parallel keys



2- Direct shear stress:

Direct shear stress occurs when the applied force tends to cut through the member as scissors or shears do or when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine design is the tendency for a key to be sheared off at the section between the shaft and the hub of a machine element when transmitting torque.
Figure (3-7) shows the action.

$$\tau_d = \frac{F}{A}$$ Where: τ_d : Design shear stress

Example Problem (3-3), (page 93), [Ref. 1]:

Figure 3-7 shows a shaft carrying two sheaves that are keyed to the shaft. Part (b) shows that a force F is transmitted from the shaft to the hub of the sheave through a square key. The shaft is 57.15 mm in diameter and transmits a torque of 1589.119 N. The key has a square cross section, 12.7 mm on a side, and a length of 44.45 mm. compute the force on the key and the shear stress caused by this force.

Sol:

Torque = F * R $\rightarrow$ $F = \frac{T}{R} = \frac{1589.12 * 10^3 \ N.mm}{28.575 \ mm} = 55600 \ N$

Area in shear = b*L = 12.7*44.45 = 564.55 mm^2

$$\tau = \frac{F}{A_s} = \frac{55600 \ N}{564.55 \ mm^2} = 98598 \ kPa \quad (Ans.)$$

Note: see table 11-1 (Page 495) to find key size vs. shaft diameter.

FIGURE 3–7 Direct shear on a key

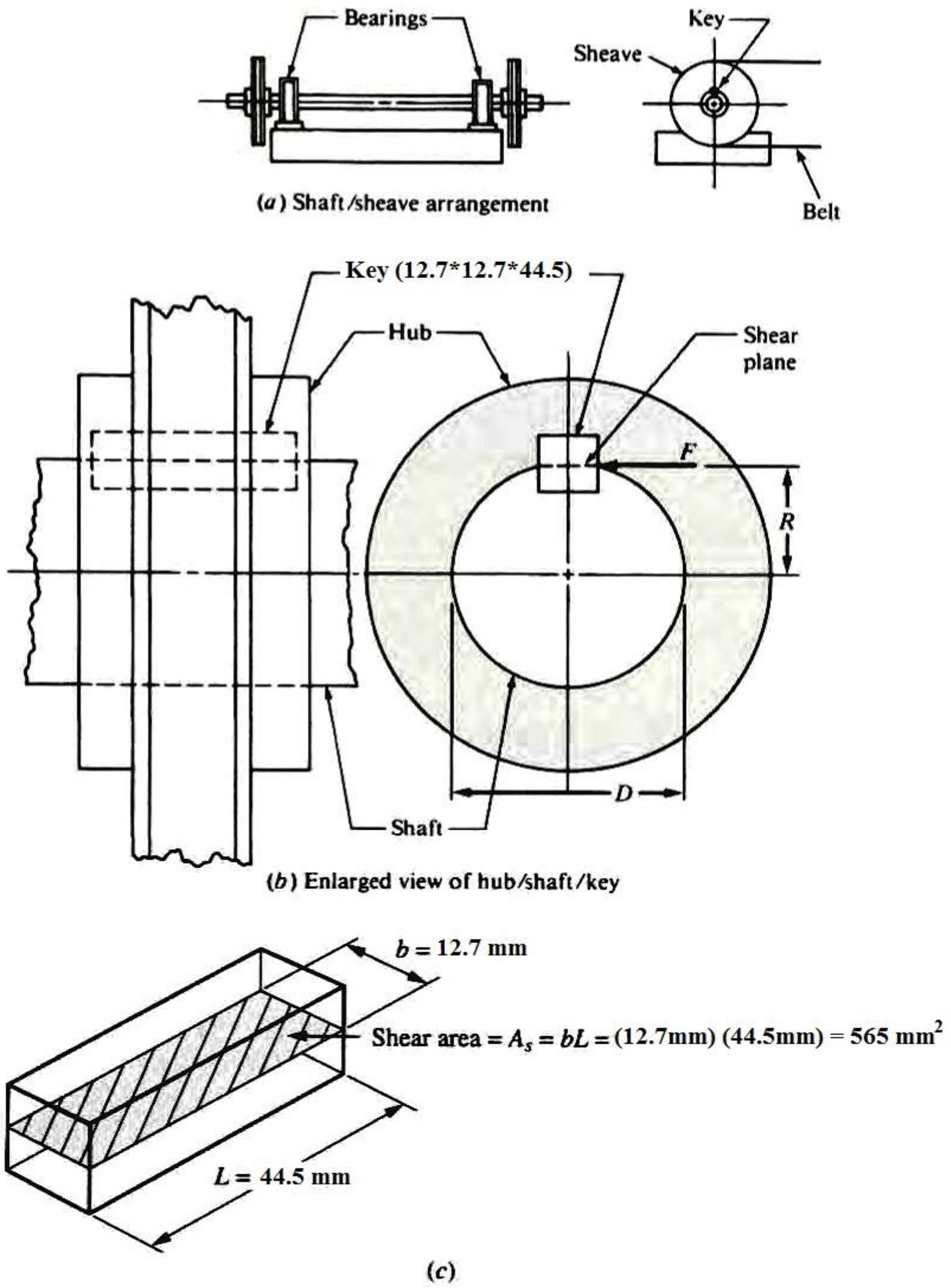


TABLE 11–1 Key size vs. shaft diameter

Nominal shaft diameter		Nominal key size		
			Height, *H*	
Over	To (incl.)	Width, *W*	Square	Rectangular
5/16	7/16	3/32	3/32	
7/16	9/16	1/8	1/8	3/32
9/16	7/8	3/16	3/16	1/8
7/8	$1\frac{1}{4}$	1/4	1/4	3/16
$1\frac{1}{4}$	$1\frac{3}{8}$	5/16	5/16	1/4
$1\frac{3}{8}$	$1\frac{3}{4}$	3/8	3/8	1/4
$1\frac{3}{4}$	$2\frac{1}{4}$	1/2	1/2	3/8
$2\frac{1}{4}$	$2\frac{3}{4}$	5/8	5/8	7/16
$2\frac{3}{4}$	$3\frac{1}{4}$	3/4	3/4	1/2
$3\frac{1}{4}$	$3\frac{3}{4}$	7/8	7/8	5/8
$3\frac{3}{4}$	$4\frac{1}{2}$	1	1	3/4
$4\frac{1}{2}$	$5\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	7/8
$5\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	1
$6\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{2}$
$7\frac{1}{2}$	9	2	2	$1\frac{1}{2}$
9	11	$2\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$
11	13	3	3	2
13	15	$3\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{2}$
15	18	4		3
18	22	5		$3\frac{1}{2}$
22	26	6		4
26	30	7		5

3- Bearing stress:

A localized compressive stress at the surface of contact between two members of a machine part that is relatively at rest is known as bearing stress or crushing stress or bearing pressure.

Example:

Find the bearing stress caused by force in last example for the key.

Sol:

$$\sigma_{\text{bearing}} = \frac{F}{A} = \frac{55600\ \text{N}}{\frac{12.7}{2} * 44.45} = 197.2\ \text{MPa}$$

(Give your comments for this result)

Note:

$$\left.\begin{array}{l} \sigma_{\text{bearing}} \text{ For CI Hub} \leq 50\ \text{MPa} \\ \sigma_{\text{bearing}} \text{ For steel Hub} \leq 90\ \text{MPa} \end{array}\right\} \boxed{\textbf{From Reference 2}}$$

Splines: A *spline* can be described as a series of axial keys machined into a shaft, with corresponding grooves machined into the bore of the mating part (gear, sheave, sprocket, and so on: see Figure 11-6). The splines perform the same function as a key in transmitting torque from the shaft to the mating element.

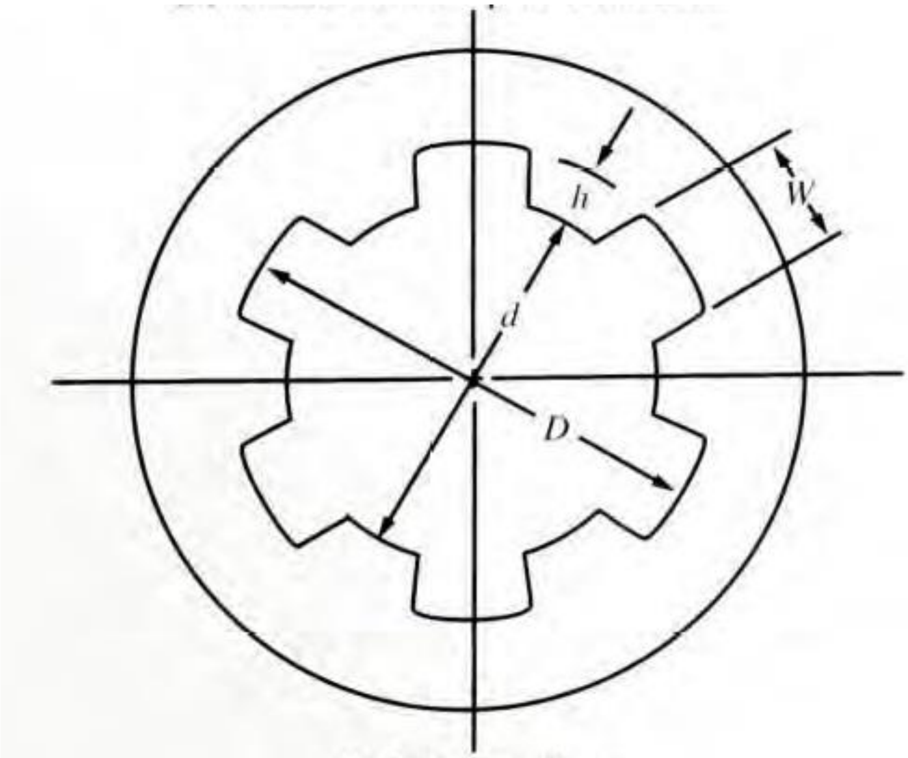


Figure (11-6) internal spline

TABLE 11–4 Formulas for SAE straight splines

No. of splines	W, for all fits	A: Permanent fit		B: To slide without load		C: To slide under load	
		h	d	h	d	h	d
Four	0.241D	0.075D	0.850D	0.125D	0.750D		
Six	0.250D	0.050D	0.900D	0.075D	0.850D	0.100D	0.800D
Ten	0.156D	0.045D	0.910D	0.070D	0.860D	0.095D	0.810D
Sixteen	0.098D	0.045D	0.910D	0.070D	0.860D	0.095D	0.810D

Note: These formulas give the maximum dimensions for *W*, *h*, and *d*.

Coupling:

The term *coupling* refers to a device used to connect two shafts together at their ends for the purpose of transmitting power. There are two general types of couplings: rigid and flexible.

Rigid Couplings:

Rigid couplings are designed to draw two shafts together tightly so that no relative motion can occur between them. This design is desirable for certain kinds of equipment in which precise alignment of two shafts is required and can be provided. In such cases, the coupling must be designed to be capable of transmitting the torque in the shafts.

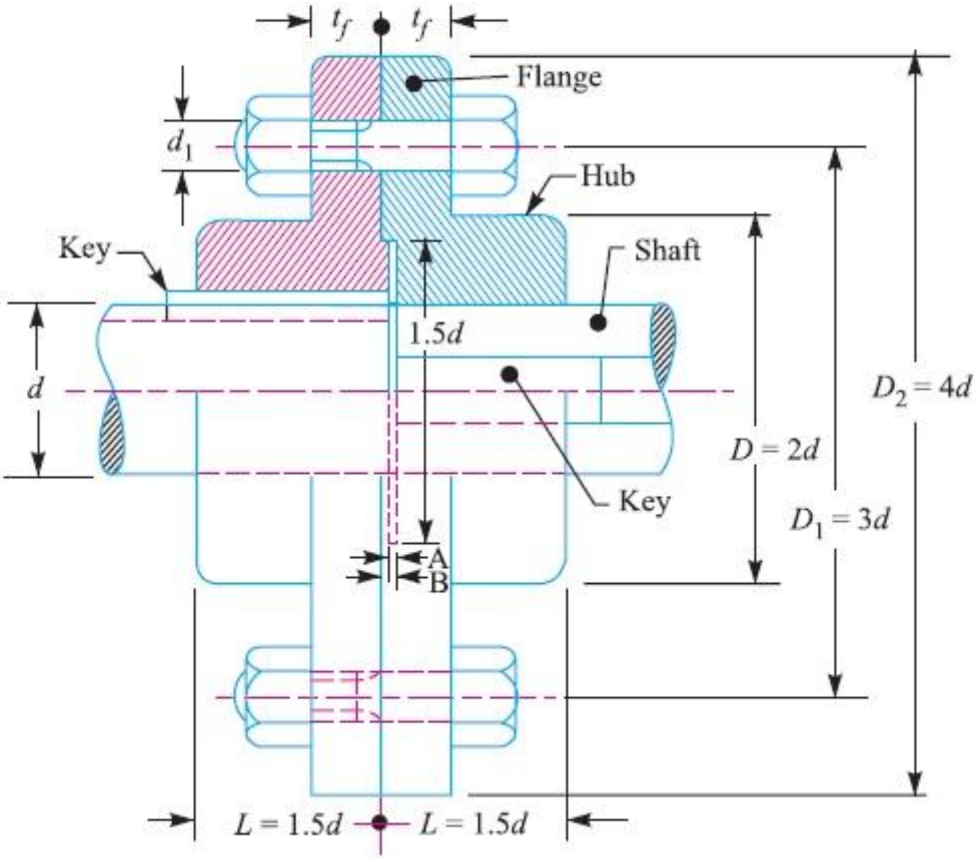


Figure (11-15) Unprotected type flange coupling

The usual proportions for an unprotected type cast iron flange couplings, as shown in figure (11-15) are as follows:

If **(d)** is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub, **D=2 d**

Length of hub, **L=1.5 d**

Pitch circle diameter of bolts, $\mathbf{D_1=3\ d}$

Outside diameter of flange, $\mathbf{D_2=D_1+(D_1-D)=2\ D_1-D=4\ d}$

Thickness of flange, $\mathbf{t_f=0.5\ d}$

Number of bolts = 3, for **(d)** up to 40mm

= 4, for **(d)** up to 100mm

= 6, for **(d)** up to 180mm

4- <u>Torsional shear stress:</u>

When a torque, or twisting moment, is applied to a member, it tends to deform by twisting, causing a rotation of one part of the member relative to another. Such twisting causes a shear stress in the member. For a small element of the member, the nature of the stress is the same as that experienced under direct shear stress.

$$\boldsymbol{\tau}_{\mathbf{max.}} = \frac{T*C}{J} = \frac{T}{Z_P}$$

Where: C = radius of the shaft to outside surface

J = Polar moment of inertia

Zp = Section modulus (J/C)

Note: see APPENDIX 1 [Ref. 1] for formulas for J

$$\boldsymbol{\theta} = \frac{\mathbf{T\ L}}{\mathbf{G\ J}}$$

Where : θ = angle of twist (in radian)

L = Length of shaft
G = Modulus of elasticity in shear (Modulus of Rigidity)

Example problem 3-6 (Page 96), [Ref. 1]:

Compute the maximum torsional shear stress in a shaft having a diameter of 10 mm when it carries a torque of 4.10 N.m.

Sol:

$$J = \frac{\pi D^4}{32} = \frac{\pi * 10^4}{32} = 982 \text{ mm}^4$$

$$\tau_{max} = \frac{(4.10 \text{ N.m})(5 \text{ mm})}{982 \text{ mm}^4} * \frac{10^3 \text{ mm}}{1 \text{ m}} = 20.9 \frac{\text{N}}{\text{mm}^2} = 20.9 \text{ MPa}$$

Example problem (3-7), (Page 97), [Ref.1]:

Compute the angle of twist of a 10 mm-diameter shaft carrying 4.10 N.m of torque if it is 250 mm long and made of steel with G = 80 GPa. Express the result in both radians and degrees.

Solution Objective Compute the angle of twist in the shaft.

Given Torque $= T = 4.10$ N·m; length $= L = 250$ mm.
Shaft diameter $= D = 10$ mm; $G = 80$ GPa.

Analysis Use Equation (3–11). For consistency, let $T = 4.10 \times 10^3$ N·mm and $G = 80 \times 10^3$ N/mm². From Example Problem 3–6, $J = 982$ mm⁴.

Results
$$\theta = \frac{TL}{GJ} = \frac{(4.10 \times 10^3 \text{ N}\cdot\text{mm})(250 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(982 \text{ mm}^4)} = 0.013 \text{ rad}$$

Using π rad $= 180°$,

$$\theta = (0.013 \text{ rad})(180 \text{ deg}/\pi \text{ rad}) = 0.75 \text{ deg}$$

Comment Over the length of 250 mm, the shaft twists 0.75 deg.

5- Vertical shearing stress:

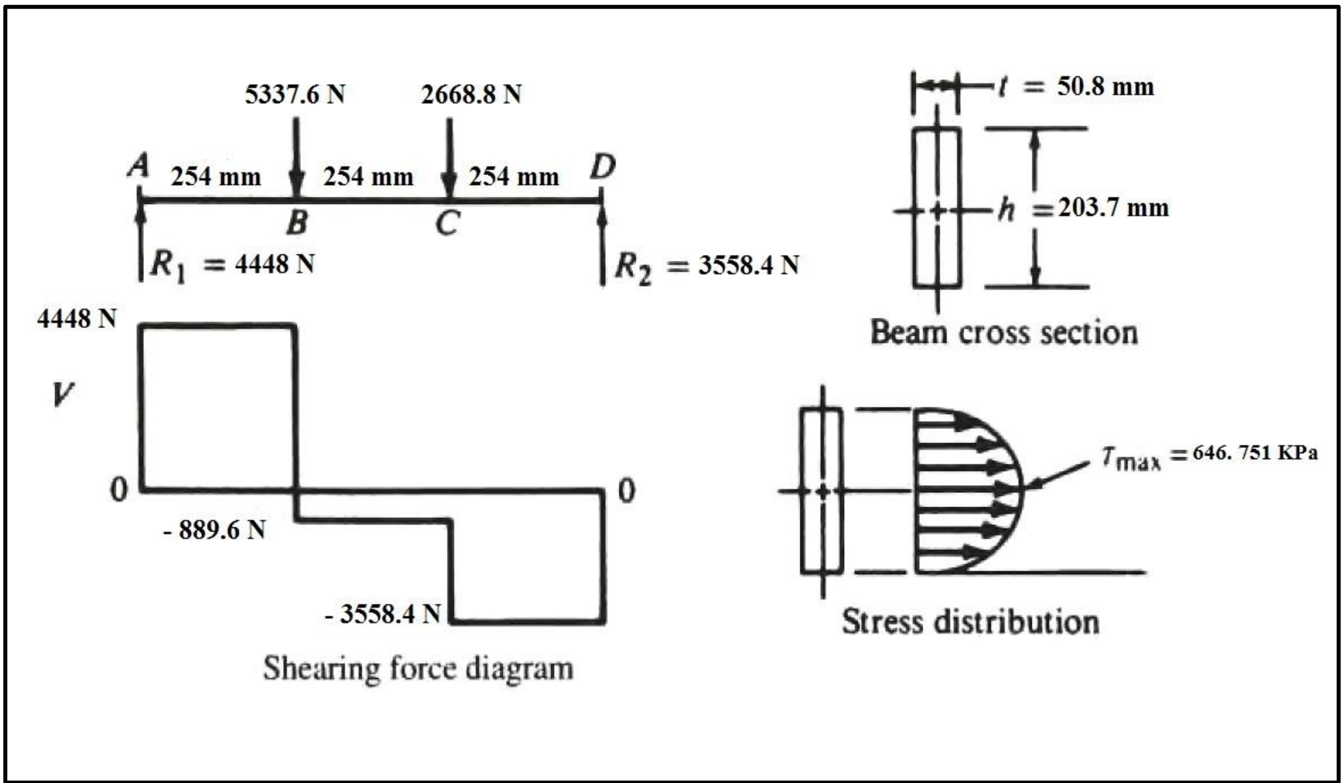
A beam carrying loads transverse to its axis will experience shearing forces denoted by V. In the analysis of beams, it is usual to draw the shearing force diagram. Then the resulting vertical shearing stress can be completed from:

$$\boldsymbol{\tau_{max} = \frac{3\,V}{2\,A}} \quad \textbf{(For rectangular section)}$$

$$\boldsymbol{\tau_{max} = \frac{4\,V}{3\,A}} \quad \textbf{(For circular section)}$$

Example problem (3-11), (Page 104), [Ref. 1]:

Compute the maximum shearing stress in the beam described below:



$$\tau_{max} = \frac{3\,V}{2\,A} = \frac{3 * 4448\,N}{2 * (50.8\,mm * 203.7\,mm)} = 646.7\,kPa$$

6- Stress due to Bending:

$$\boldsymbol{\sigma} = \frac{\mathbf{M\,C}}{\mathbf{I}} = \frac{\mathbf{M}}{\mathbf{S}}$$

Where: M= magnitude of the bending moment at the section

I = moment of inertia of the C.S. with respect to its neutral axis

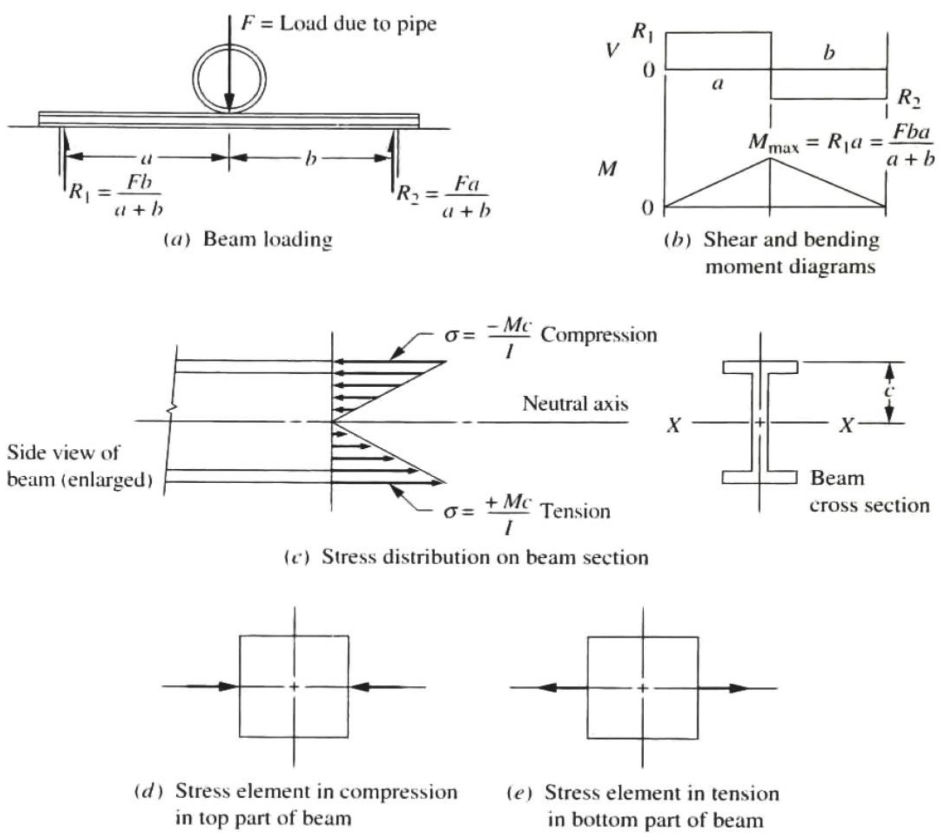
C= distance from the neutral axis to the outer most fiber of the beam C.S.

S = section modulus = I/C

Example Problem (3-12), (Page 107) [Ref.1]

For the beam shown in Figure 3-16, the load F due to the pipe is 53376 N. The distances are a = 1.2192 m and b = 1.8288 m. Determine the required section modulus for the beam to limit the stress due to bending to 206850 kPa, the recommended design stress for a typical structural steel in static bending.

FIGURE 3–16
Typical bending stress distribution in a beam cross section



Sol:

$$\sigma = \frac{M}{S} \quad \rightarrow \quad S = \frac{M}{\sigma} = \frac{R_1 * a}{\sigma} = \frac{F * b}{a + b} * \frac{a}{\sigma} = \frac{53376\,(1.83)}{1.22 + 1.83} * \frac{1.22}{206850}$$

$$= 18.85 * 10^4 \ \text{mm}^3$$

Now from table A16-3 & A16-4 choose (W203*22.1) wide-flange shape with

S = 19.3 * 10^4 mm^3

7-Stresses due to combined bending moment with axial load:

$$\sigma = \pm \frac{MC}{I} \pm \frac{F}{A}$$

Example problem (3-17), (Page 117), [Ref.1]:

The cantilever beam in Figure 3-24 is a steel American Standard beam. S6x12.5. The force F is 44480 N. and it acts at an angle of 30° below the horizontal, as shown. Use a = 609.6 mm and e = 152.4 mm. Draw the free-body diagram and the shearing force and bending moment diagrams for the beam. Then compute the maximum tensile and maximum compressive stresses in the beam and show where they occur.

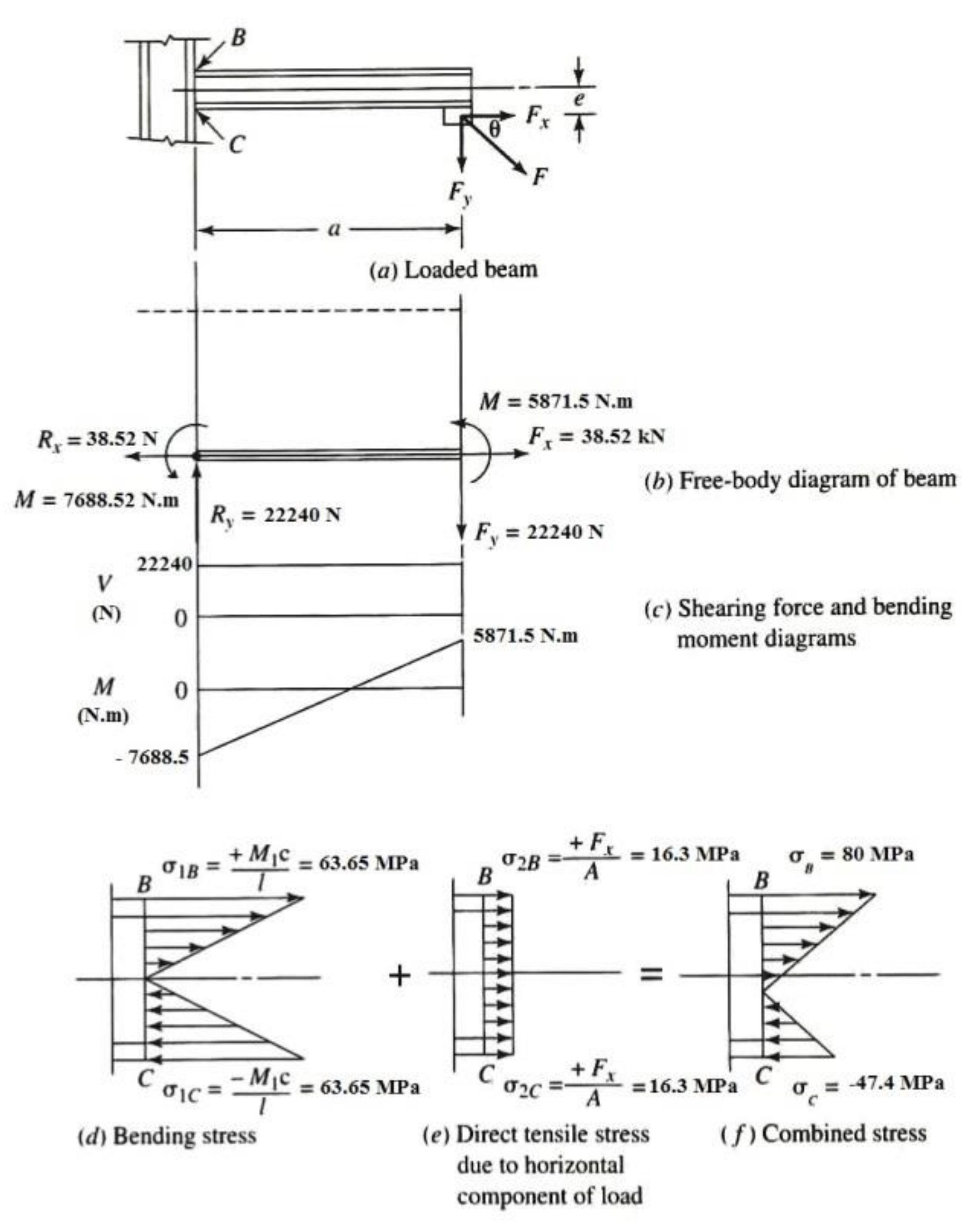


FIGURE 3–24 Beam subjected to combined stresses

Sol:

From table A16-4 ($S = 12.079*10^4\ mm^4$ & $A=2367.884\ mm^2$)

$F_X = F \cos 30 = 44480 \cos 30 = 38520$ N

$F_Y = F \sin 30 = 44480 \sin 30 = 22240$ N

$M_1 = F_X\ (0.152\ m) = 5871.5$ N.m

$M_{max} = 7688.5$ N.m occurs at left end of beam

$$\sigma_1 = \pm \frac{M}{S} = \frac{7688.5}{12 * 10^4} = \pm 63654.6\ \text{kPa}$$

$$\sigma_2 = \frac{F_x}{A} = \frac{38520}{2307.9} = 16.27\ \text{MPa}$$

$\sigma_B = +\sigma_1 + \sigma_2 = 63654.6 \text{ kPa} + 16.27\text{MPa} = 79.93 \text{ MPa}$

$\sigma_C = -\sigma_1 + \sigma_2 = -63654.6 \text{ kPa} + 16.27\text{MPa} = -47.4 \text{ MPa}$

8- Stress Concentrations:

The above simple stresses are applicable for the geometry of a member is uniform throughout the section of interest. But if there is a fillet, holes, key seals, grooves, etc, will cause the actual max. stress, so defining stress concentration factors by which the actual max. stress exceeds the nominal stress.

$$\boldsymbol{\sigma}_{\mathbf{max}} = \mathbf{K}\, \boldsymbol{\sigma}_{\mathbf{nom.}} \quad \& \quad \boldsymbol{\tau}_{\mathbf{max}} = \mathbf{K}\, \boldsymbol{\tau}_{\mathbf{nom.}}$$

Where K can be found from **APPENDIX 15** [Ref.1]

Example Problem (3-18), (Page 121), [Ref.1]:

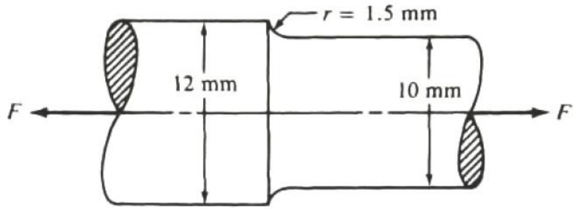
Compute the maximum stress in a round bar subjected to an axial tensile force of 9800 N. The geometry is shown in Figure 3-26.

Solution Objective Compute the maximum stress in the stepped bar shown in Figure 3–26.

Given The layout from Figure 3–26. Force = F = 9800 N.
The shaft has two diameters joined by a fillet with a radius of 1.5 mm.
Larger diameter = D = 12 mm; smaller diameter = d = 10 mm.

Analysis The presence of the change in diameter at the step causes a stress concentration to occur. The general situation is a round bar subjected to an axial tensile load. We will use the top graph of Figure A15–1 to determine the stress concentration factor. That value is used in Equation (3–27) to determine the maximum stress.

FIGURE 3–26
Stepped round bar subjected to axial tensile force



Results Figure A15–1 indicates that the nominal stress is computed for the smaller of the two diameters of the bar. The stress concentration factor depends on the ratio of the two diameters and the ratio of the fillet radius to the smaller diameter.

$$D/d = 12\text{ mm}/10\text{ mm} = 1.20$$
$$r/d = 1.5\text{ mm}/10\text{ mm} = 0.15$$

From these values, we can find that K_t = 1.60. The stress is

$$\sigma_{nom} = F/A = (9800\text{ N})/[\pi(10\text{ mm})^2/4] = 124.8\text{ MPa}$$
$$\sigma_{max} = K_t\sigma_{nom} = (1.60)(124.8\text{ MPa}) = 199.6\text{ MPa}$$

Comments The maximum tensile stress of 199.6 MPa occurs in the fillet near the smaller diameter. This value is 1.60 times higher than the nominal stress that occurs in the 10-mm-diameter shaft. To the left of the shoulder, the stress reduces dramatically as the effect of the stress concentration diminishes and because the area is larger.