LECTURE SIX & SEVEN

COMBINE STRESSES, MOHR'S CIRCLE & DESIGN FOR DIFFERENT TYPE OF LOADING

1. Maximum normal stresses: Principle stresses, [Ref.1]:

The combination of the applied normal and shear stresses that produce the maximum normal stresses is called the max. & min. principle stresses σ_1 & σ_2 are:

$$\boldsymbol{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

The angle of inclination of planes is:

$$\boldsymbol{\emptyset_\sigma = \frac{1}{2}\, tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

FIGURE 4–3
General two-dimensional stress element

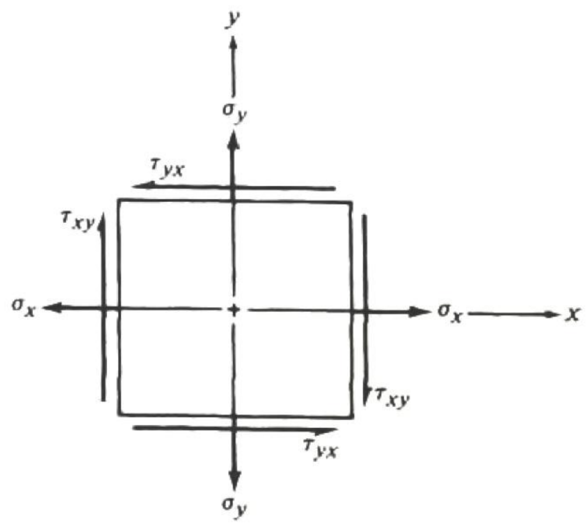
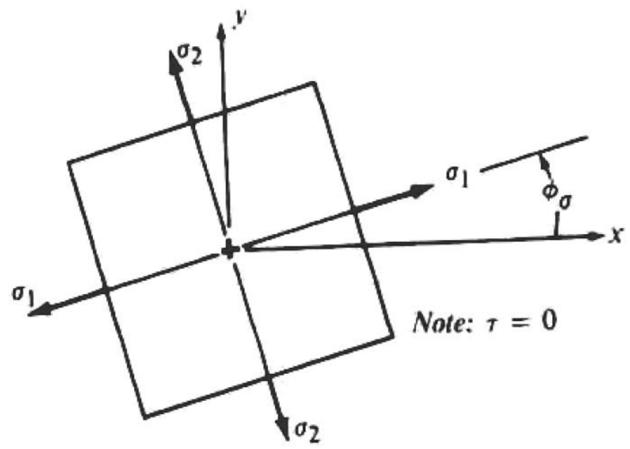


FIGURE 4–4
Principal stress element



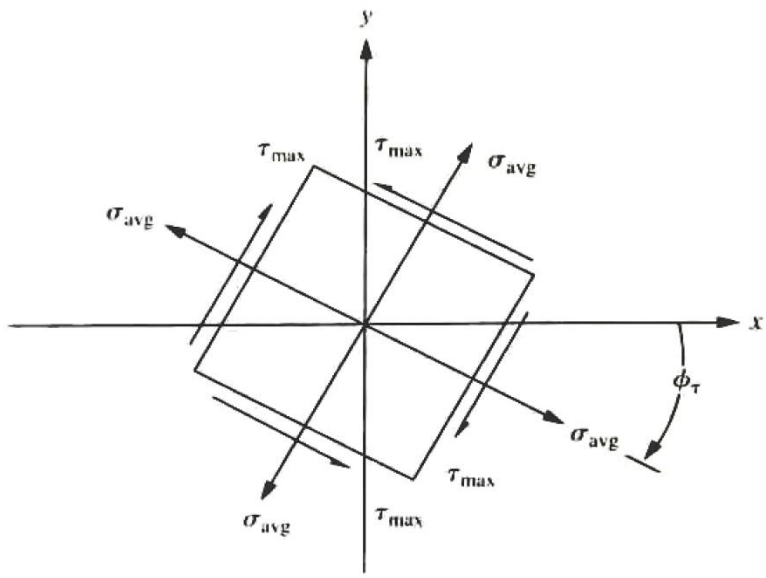
2. Maximum shear stress, [Ref.1]:

On different orientation of stress element, the maximum shear stress will occur. It magnitude can be computed from:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad \phi_\tau = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

FIGURE 4–5
Maximum shear stress element



Note: the angle between principle stress element and the max. shear stress element is always 45 degree.

Example Problem 4-1, (page 141), [Ref.1]:

The shaft shown in Figure 4-7 is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the X-Z plane. Sheave B exerts a clockwise torque on the shaft when viewed toward the origin of the coordinate system along the X-axis. Sheave C exerts an equal but opposite torque on the shaft. For the loading condition shown, determine the principal stresses and the maximum shear stress on element K on the front surface of the shaft (on the positive Z-side) just to the right of sheave B, Follow the general procedure for analyzing combined stresses given in this section.

Sol:

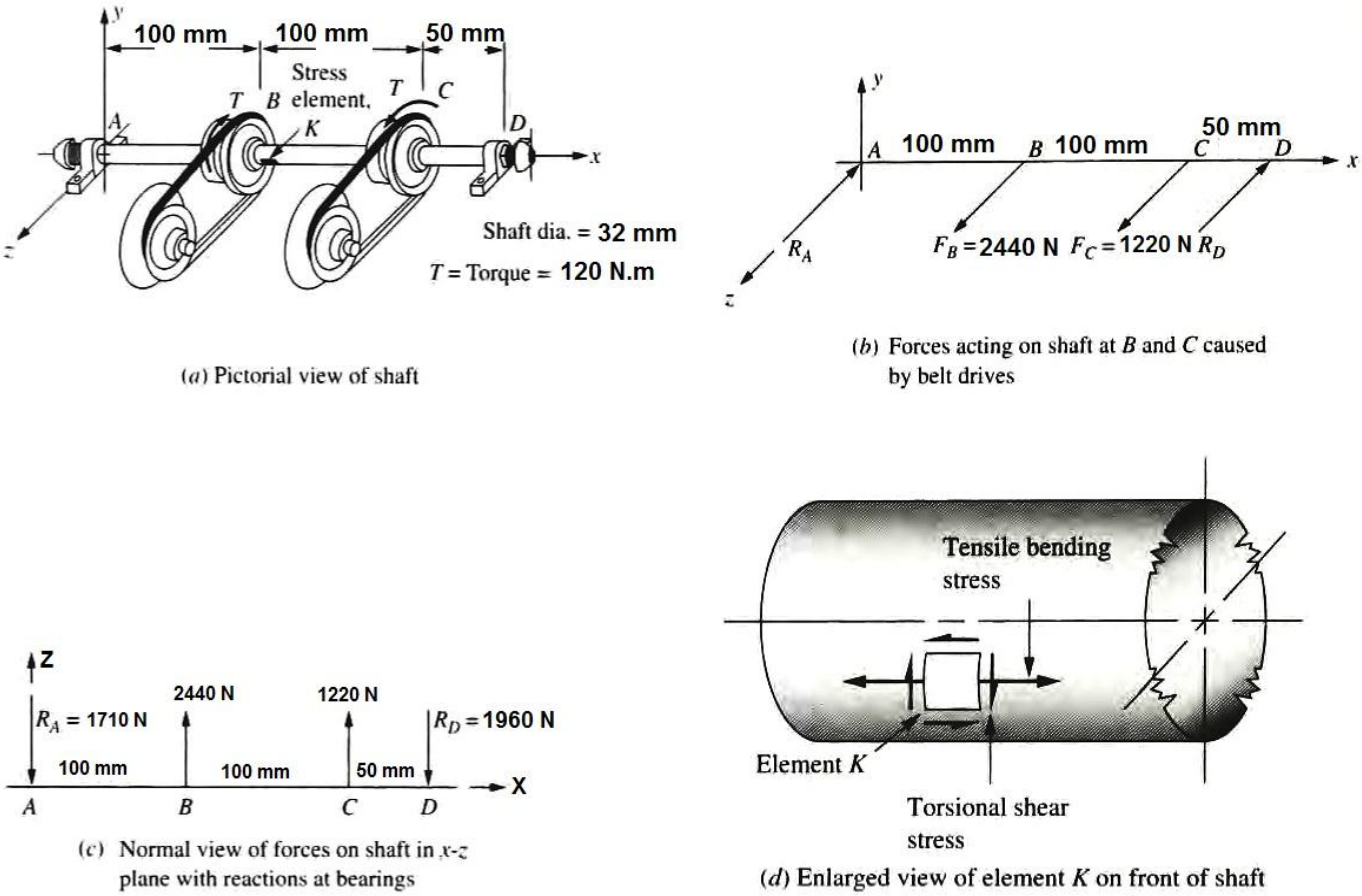


FIGURE 4–7 Shaft supported by two bearings and carrying two V-belt sheaves

FIGURE 4–8
Shearing force and bending moment diagrams for the shaft

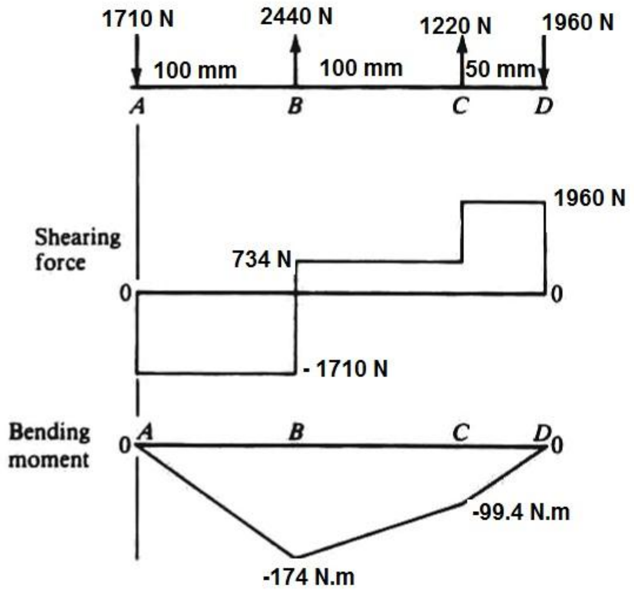
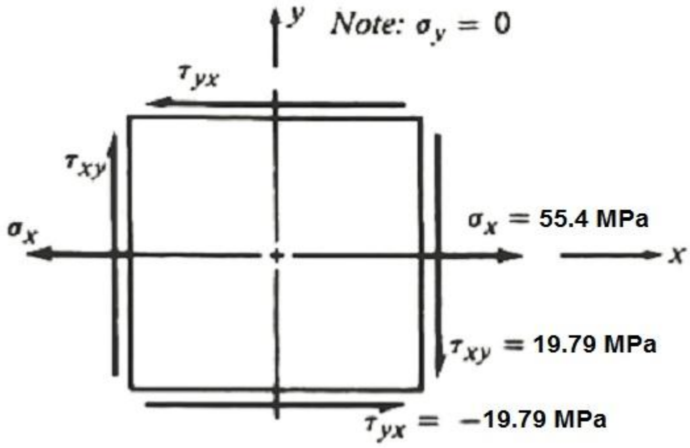


FIGURE 4–9
Stresses on element K



$$\sigma_x = \frac{M}{S} \quad ; \quad S = \frac{\pi D^3}{32} = \frac{\pi\,(31.75)^3}{32} = 3146.3\,mm^3$$

$$\sigma_x = \frac{174\,N.m}{3146.3\,mm^3} = 55.4\,MPa$$

$$\tau_{xy} = \frac{T}{Z_p} \quad ; \quad Z_p = \frac{\pi D^3}{16} = \frac{\pi\,(31.75)^3}{16} = 6276.2\,mm^3$$

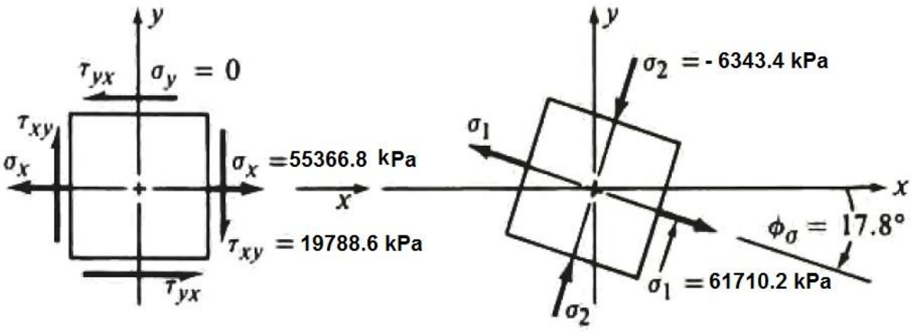
$$\tau_{xy} = \frac{120\,N.m}{6276.2\,mm^3} = 19.79\,MPa$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{55.4}{2}\right) + \sqrt{\left(\frac{55.4}{2}\right)^2 + (19.79)^2} = 61.7\,MPa$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{55.4}{2}\right) - \sqrt{\left(\frac{55.4}{2}\right)^2 + (19.79)^2} = -\,6.3\,MPa$$

$$\emptyset_\sigma = \frac{1}{2}\,tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\,tan^{-1}\frac{2 * 19.79}{55.4} = 17.8°$$

FIGURE 4–10
Principal stress element



(a) Original stress element (b) Principal stress element

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{55366.8}{2}\right)^2 + (19788.6)^2} = \pm 34026.8\ kPa$$

$$\emptyset_\tau = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(-\frac{55366.8}{2 * 19788.6}\right) = -27.2°$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{55366.8}{2} = 27683.4\ kPa$$

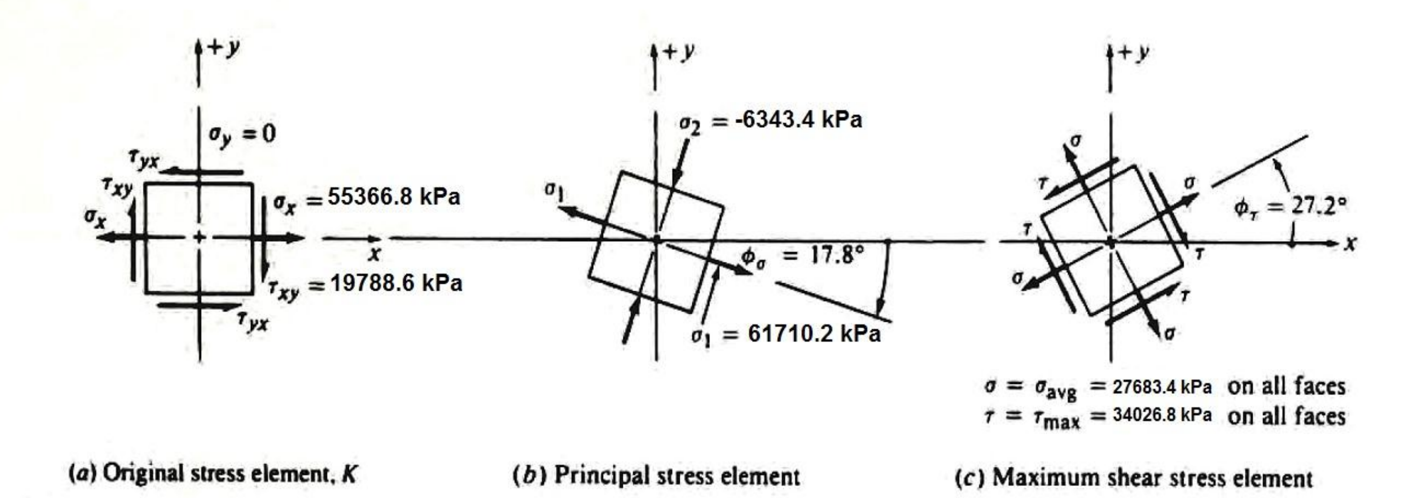


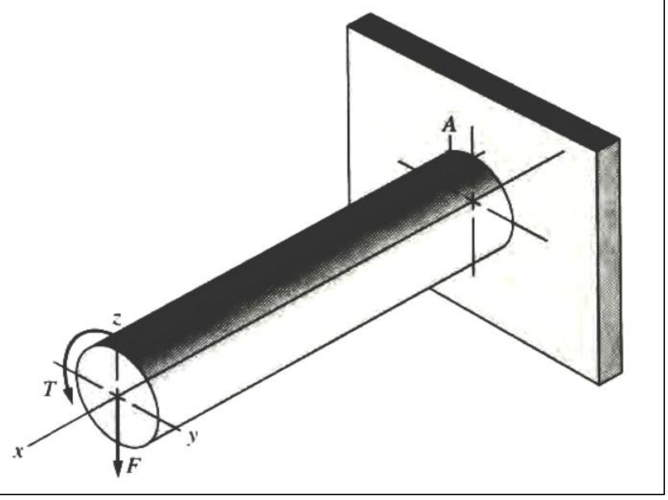
FIGURE 4–11 Relation of maximum shear stress element to the original stress element and the principal stress element

3. Mohr's circle for different stress conditions:

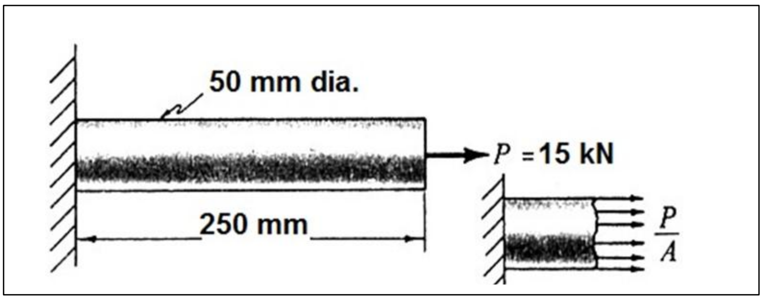
Use the Mohr's circle module from M-design to complete the following cases:

Example: A hypothetical machine member 50mm diameter by 250mm long and supported at one end as a cantilever will be used to demonstrate how numerical tensile, compressive, and shear stresses are determined for various types of uniaxial loading. In this example not that $(\boldsymbol{\sigma})_{\boldsymbol{y}}$ ***= 0*** for all arrangements, at the critical points. Compute the following cases:

FIGURE 4–33
Circular bar in bending and torsion



a) Axial load only

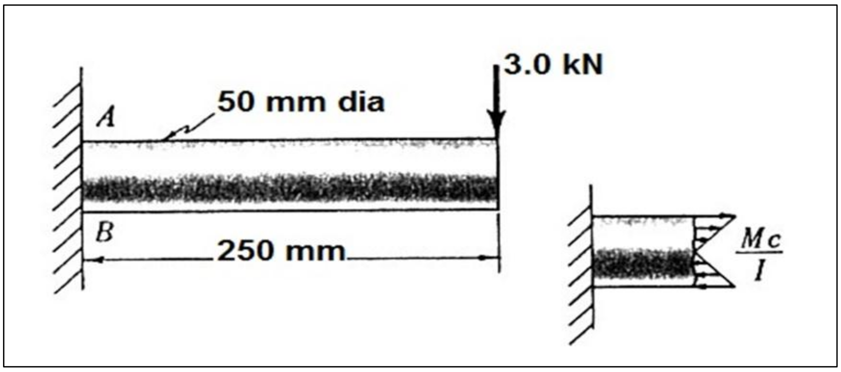


$$\sigma_x = \frac{P}{A} = \frac{15000\ N}{\frac{\pi}{4}(50)^2} = 7.65\ MPa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{7.65 + 0}{2} \pm \sqrt{\left(\frac{7.65 - 0}{2}\right)^2 + 0}$$

$$\sigma_1 = 7.65\ MPa \ ; \quad \sigma_2 = 0 \ ; \quad \tau_{xy} = 0 \ ; \quad \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3.83\ MPa$$

b) Bending only



$\sigma_x = +\frac{MC}{I}$ For point A

$\sigma_x = -\frac{MC}{I}$ For point B

$\tau_{xy} = 0\ at\ points\ A\ \&\ B$

$$\sigma_x = +\frac{3*10^3*250*10^{-3}*25*10^{-3}*64}{\pi(50*10^{-3})^4} = 61.1\ MPa \quad \text{(For point A)}$$

$$\sigma_x = -61.1\ MPa \quad \text{(For point B)}$$

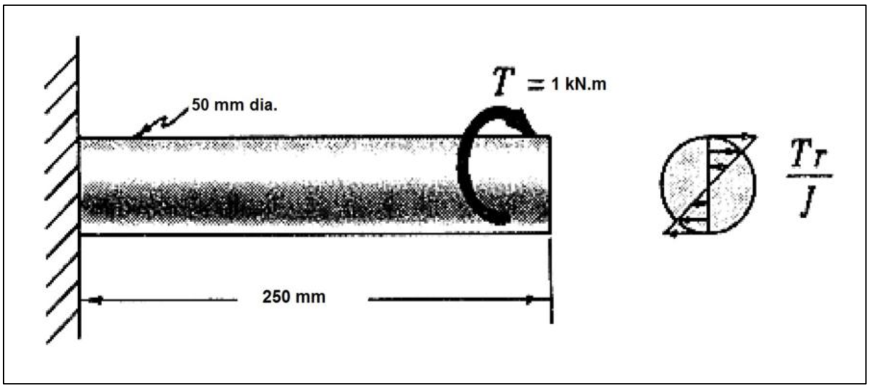
$$\boldsymbol{\sigma_{1,2}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} \pm \sqrt{\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}}$$

$\therefore\ \sigma_1 = 61.1\ MPa \quad \& \quad \sigma_2 = 0 \qquad at\ point\ A$

$\sigma_1 = 0 \quad \& \quad \sigma_2 = -61.1\ MPa \qquad at\ point\ B$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{61.1}{2} = 30.6\ MPa \qquad at\ point\ A\ \&\ B$$

c) Torsion only



$\sigma_x = 0$

$$\tau_{xy} = \frac{T * r}{J}$$

$$\tau_{xy} = \frac{1 * 10^3 * 25 * 10^{-3} * 32}{\pi(50 * 10^{-3})^4}$$

$$\tau_{xy} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

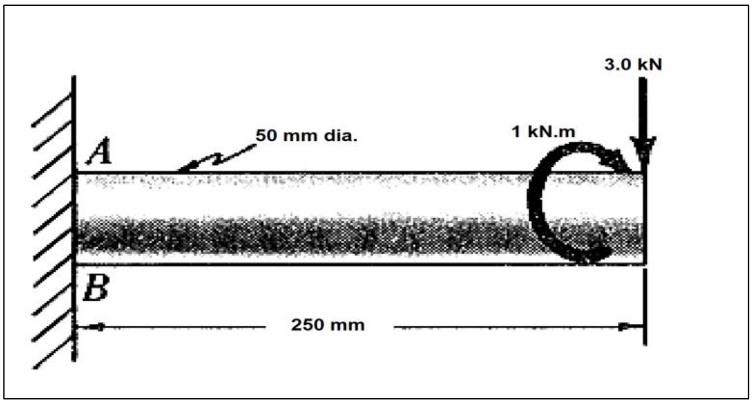
$$\therefore \sigma_1 = 40.7\ MPa\ \ (Tension)\ \&\ \sigma_2 = -40.7\ MPa\ (Compression)\ \&\ \tau_{max} = 40.7\ MPa$$

d) <u>Bending and torsion</u>

$$\sigma_x = \frac{M * C}{I} = +61.1\ MPa\ \ (at\ A)$$

$$\sigma_x = -61.1\ MPa\quad (at\ B)$$

$$\tau_{xy} = \frac{T * r}{J} = 40.7\ MPa\ \ (at\ A\ \&\ B)$$



$$\boldsymbol{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\therefore \sigma_1 = 81.4\ MPa \quad \& \quad \sigma_2 = -20.3\ MPa \quad \& \quad \tau_{max} = +50.9\ MPa \qquad (at\ point\ A)$$

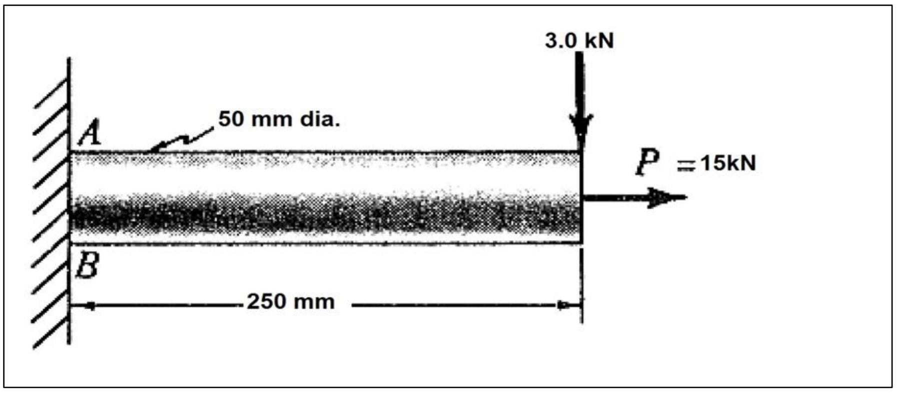
$$\sigma_1 = 20.3\ MPa \quad \& \quad \sigma_2 = -81.4\ MPa \quad \& \quad \tau_{max} = -50.9\ MPa \qquad (at\ point\ B)$$

e) <u>Bending and axial load</u>

At point A

$$\sigma_x = +\frac{P}{A} + \frac{M * c}{I}$$

$$= 68.8\ MPa \quad (Tension)$$



$$\boldsymbol{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \quad \& \quad \boldsymbol{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

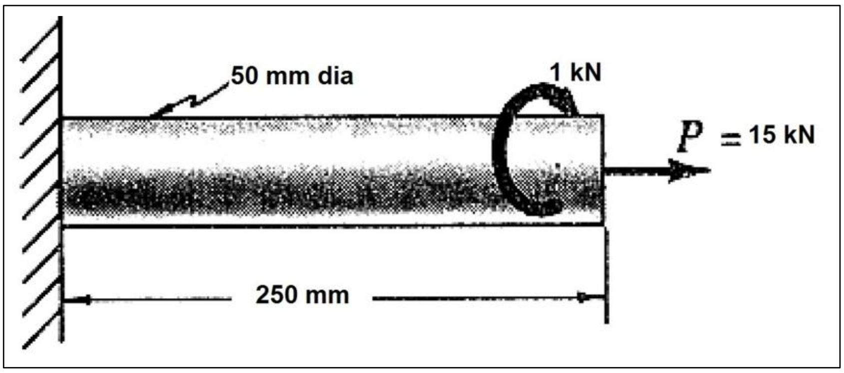
$$\therefore \sigma_1 = \sigma_x = 68.8\ MPa \quad \& \quad \sigma_2 = 0 \quad \& \quad \tau_{max} = \frac{\sigma_x}{2} = 34.4\ MPa \qquad (\boldsymbol{at\ point\ A})$$

At point B

$$\sigma_x = +\frac{P}{A} - \frac{M * c}{I} = -53.5\ MPa \quad (Compression)$$

$$\therefore \sigma_1 = 0 \quad \& \quad \sigma_2 = -53.5\ MPa \quad \& \quad \tau_{max} = \frac{\sigma_x}{2} = -26.7\ MPa \qquad (\boldsymbol{at\ point\ B})$$

f) Torsion and axial load



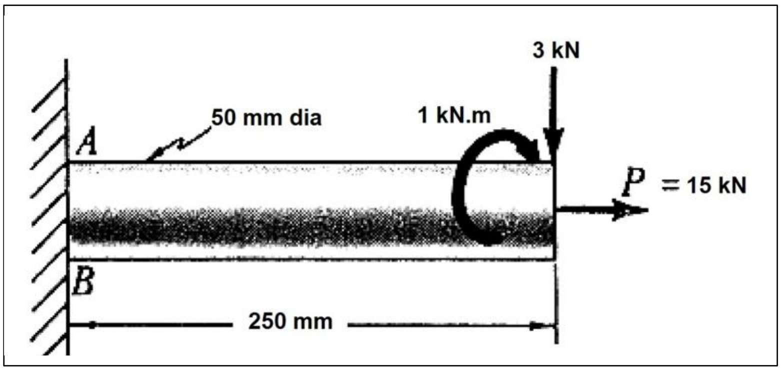
$$\sigma_x = \frac{P}{A} = 7.65\ MPa$$

$$\tau_{xy} = \frac{T * r}{J} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 44.7\ MPa\ (tension)$ & $\sigma_2 = -37.1\ MPa\ (comp.)$ & $\tau_{max} = 40.9\ MPa$

g) Bending, axial load and torsion



At point A

$$\sigma_x = \frac{M * c}{I} + \frac{P}{A}$$

$$= 61.1 + 7.65 = 68.8\ MPa$$

$$\tau_{xy} = \frac{T * r}{J} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 87.7\ MPa\ (tension)$ & $\sigma_2 = -19\ MPa\ (comp.)$ & $\tau_{max} = 53.3\ MPa$

At point B

$$\sigma_x = -\frac{M * c}{I} + \frac{P}{A} = -61.1 + 7.65 = -53.5\ MPa$$

$$\tau_{xy} = \frac{T * r}{J} = 40.7\ MPa$$

$$\boldsymbol{\sigma_{1,2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \& \quad \boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore \sigma_1 = 21.9\ MPa\ (tension)$ & $\sigma_2 = -75.5\ MPa\ (comp.)$ & $\tau_{max} = 48.7\ MPa$