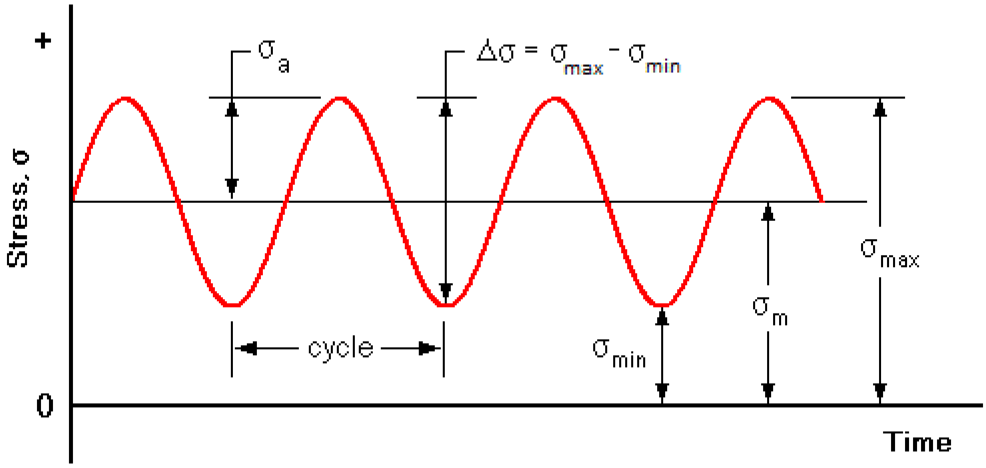
LECTURES EIGHT, NINE & TEN

DESIGN FOR DIFFERENT TYPE OF LOADING

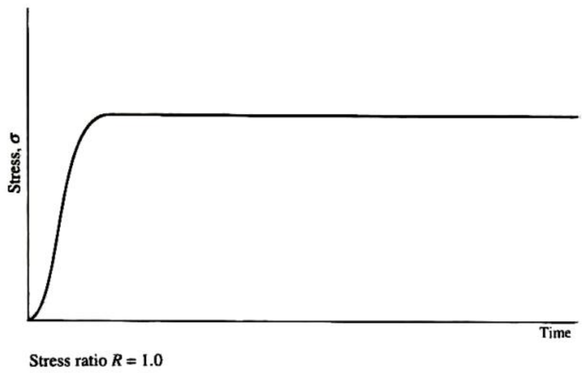
Reference: "Machine Elements in Mechanical Design" 4th Edition in SI units, By: Robert L. Mott, Chapter 5.



Loading Types

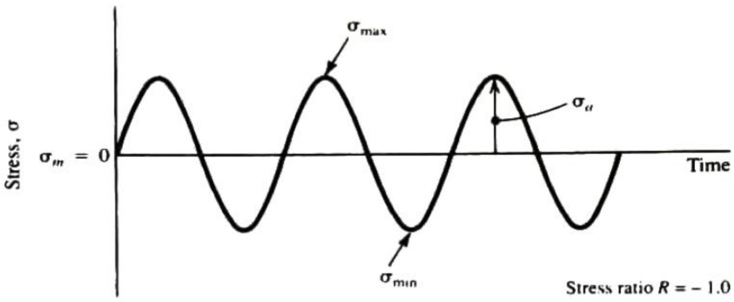
1. Static

- Load applied slowly
- $\sigma_{max.} = \sigma_{min.} = \sigma$
- Stress ratio (R) = 1



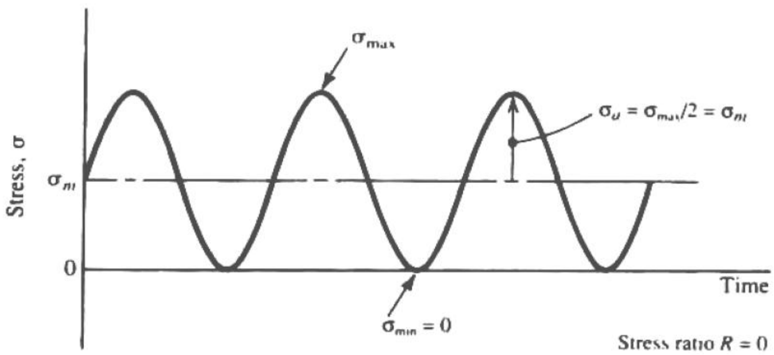
2. Reversed

- $\sigma_m = 0$
- $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$
- $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$
- R = -1



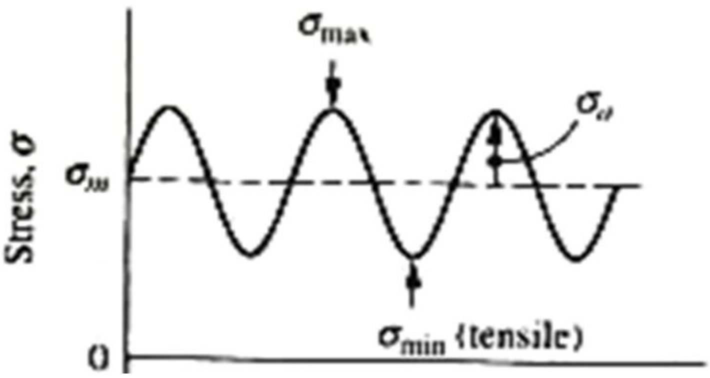
3. Repeated

- $\sigma_a = \sigma_m = \dfrac{\sigma_{max}}{2}$
- $\sigma_{min} = 0$
- R = 0



4. Fluctuating

- $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
- $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
- Stress ratio ($0 < R < 1$)



5. Shock or Impact

- Load applied suddenly & rapidly

6. Random

- When load not regular in their amplitude

Where:

σ_{max} = Maximum Stress & σ_{min} = Minimum Stress

σ_m = Mean (average) Stress

σ_a = Amplitude Stress (Alternating stress)

R = Stress ratio = $\frac{\text{min. stress } (\sigma_{min})}{\text{max. stress } (\sigma_{max})}$

A = Stress ratio = $\frac{\sigma_a}{\sigma_m}$

Repeated and Reversed Stress:

An important example in machine design is a rotating circular shaft loaded in bending such as that shown in Figure (5-3). All parts of the shaft that are in bending see repeated, reversed stress. This is a description of the classical loading case of reversed bending. This machine is called a standard R.R. Moore fatigue test device.

FIGURE 5–2
Repeated, reversed stress

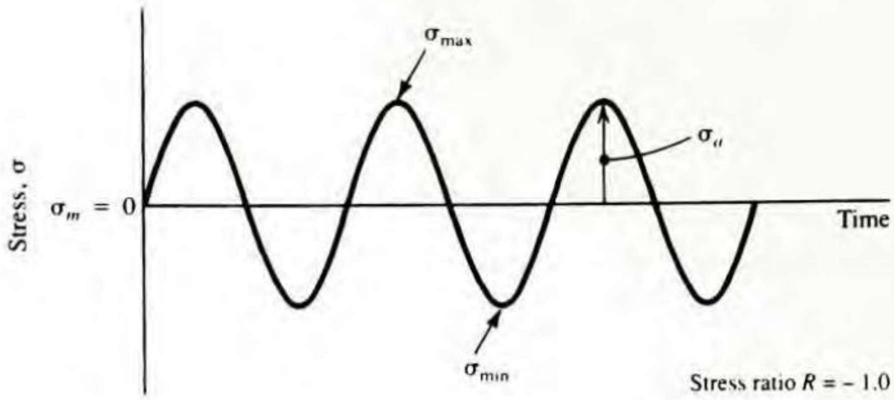
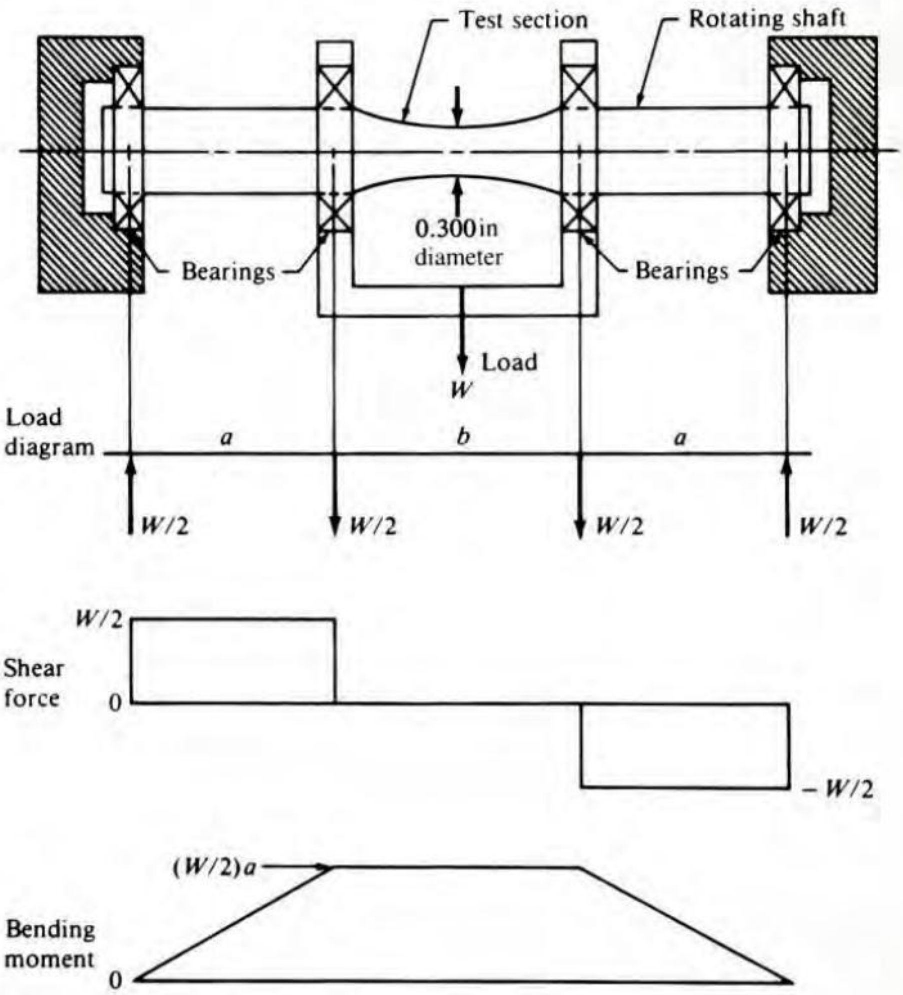


FIGURE 5–3
R. R. Moore fatigue test device



Endurance Strength:

Its ability to withstand fatigue loads. In general, it is the stress level that Material can survive for given number of cycles of loading. If the number of cycles is infinite, the stress level is called the endurance strength. Figure (5-7) (page 173), (Ref. 1), is called the S-N diagram.

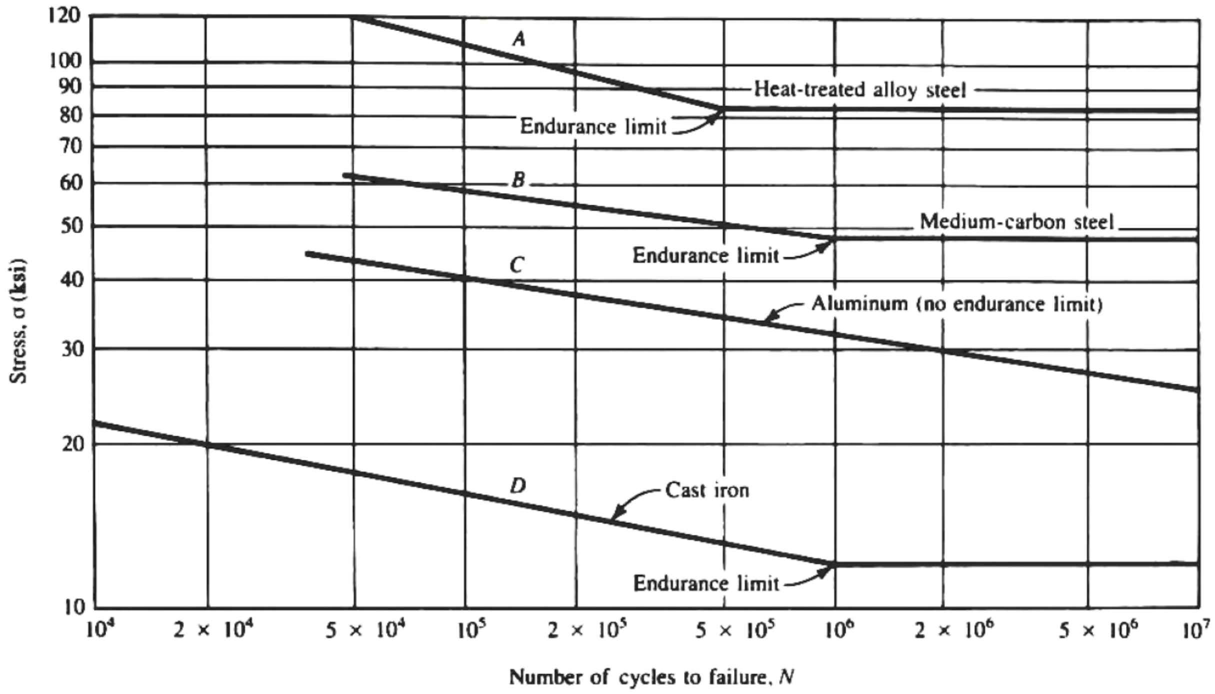


Figure (5-7) Representative endurance strengths.

The approximate value for endurance strength for wrought steel:

Endurance Strength = 0.5 (Ultimate tensile strength)

$$S_n = 0.5\ S_u$$

OR

See figure (5-8) page 175, (Ref. 1), for various surface conditions.

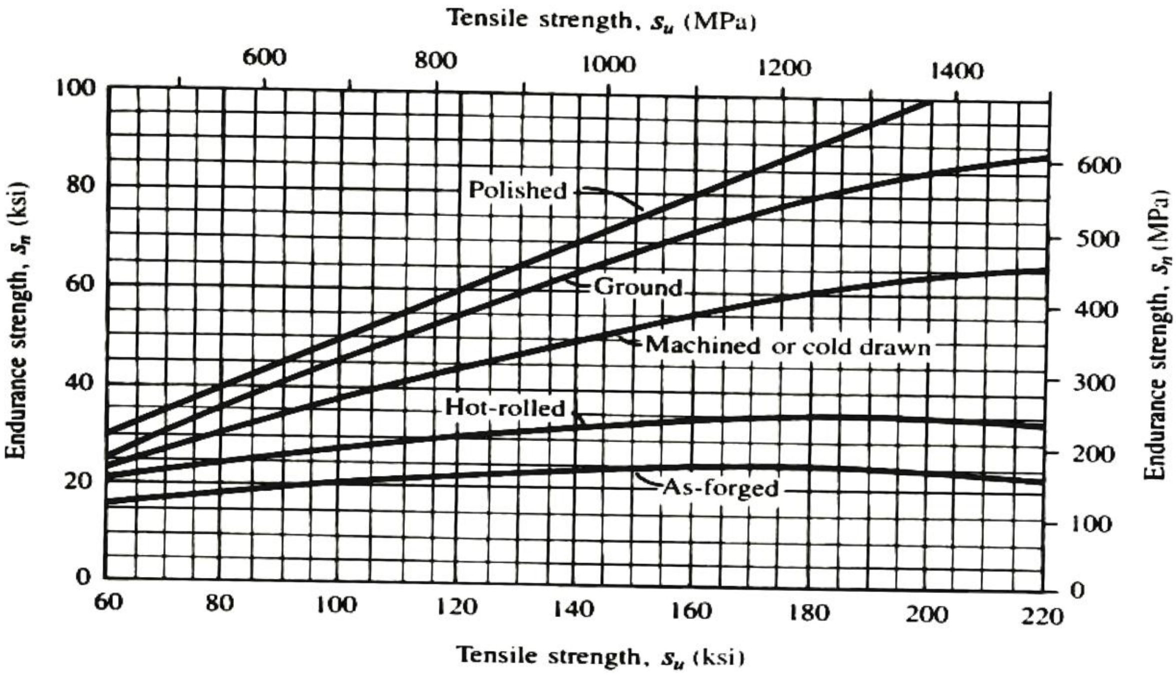


Figure (5-8) Endurance strength S_n versus tensile strength S_u for wrought steel for various surface conditions.

Estimated Actual Endurance Strength (S'_n):

1. Specify the material for the part and determine its ultimate tensile strength, $\mathbf{S_u}$, considering its condition, as it will be used in service.
2. Specify the manufacturing process used to produce the part with special attention to the condition of the surface in the most highly stressed area.
3. Use Figure 5-8 to estimate the endurance strength, S_n
4. Apply a material factor, $\mathbf{C_m}$, from the following list.

Wrought steel: C_m = 1.00 ; Malleable cast iron: C_m = 0.80

Cast steel: C_m = 0.80 ; Gray cast iron: C_m = 0.70

Powdered steel: C_m = 0.76 ; Ductile cast iron: C_m = 0.66

5. Apply a type-of-stress factor: C_{st} = 1.0 for bending stress; C_{st} = 0.80 for axial tension.
6. Apply a reliability factor, C_R , from Table 5-1.
7. Apply a size factor, C_S using Figure 5-9 and Table 5-2 as guides.
8. Compute the estimated actual endurance strength, S'_n from

$$S'_n = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S) \quad \ldots\ldots\ (5\text{-}4)\ Page\ 174$$

Figure (5-9) Size Factor

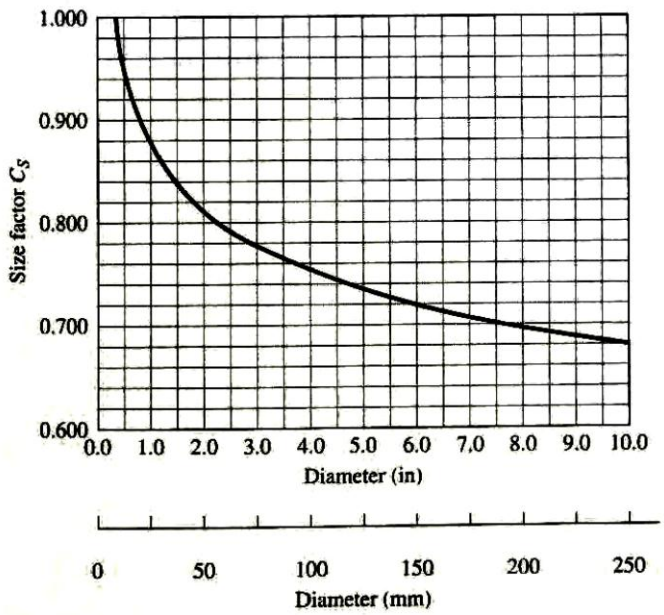


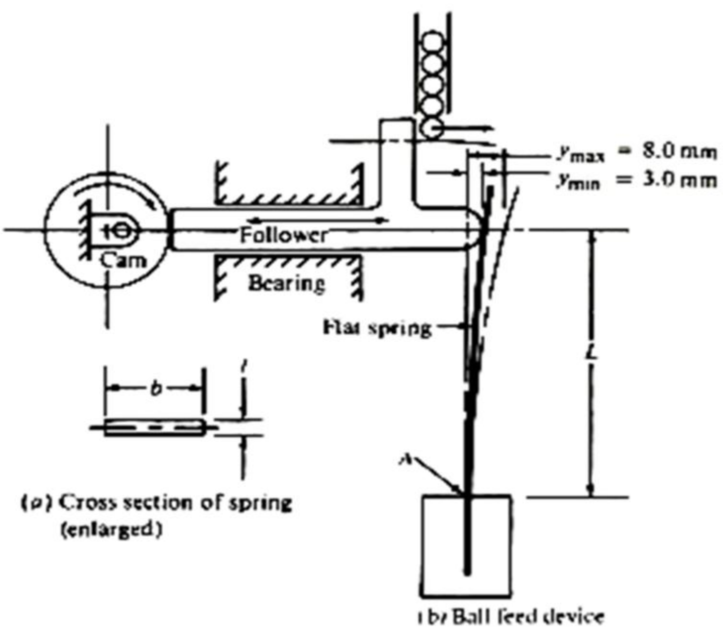
TABLE 5–1 Approximate reliability factors, C_R

Desired reliability	C_R
0.50	1.0
0.90	0.90
0.99	0.81
0.999	0.75

TABLE 5–2 Size factors

U.S. customary units	
Size Range	**For D in inches**
$D \le 0.30$	$C_S = 1.0$
$0.30 < D \le 2.0$	$C_S = (D/0.3)^{-0.11}$
$2.0 < D < 10.0$	$C_S = 0.859\text{-}0.02125D$
SI units	
Size Range	**For D in mm**
$D \le 7.62$	$C_S = 1.0$
$7.62 < D \le 50$	$C_S = (D/7.62)^{-0.11}$
$50 < D < 250$	$C_S = 0.859\text{-}0.000837D$

Example 5-1 (Page 170). (Ref. 1): For the flat steel spring shown below, compute the maximum stress, the minimum stress, the mean stress, the alternating stress and the stress ratio R. The length L is 65 mm. The dimensions of the spring cross section are t = 0.80 mm and b=6.0 mm. If E= 207 GPa.



Deflection (Y) = $\frac{PL^3}{3EI}$ $\rightarrow$ $P = \frac{3YEI}{L^3}$; $I = \frac{bt^3}{12} = \frac{6*0.8^3}{12}$ *0.256 mm*^{4}

$P_{min} = \frac{3\,(207*10^9)\,(0.256)\,(Y=3)}{65^3} * \frac{1}{10^6} = 1.74\ N$

$P_{max} = \frac{3\,(207*10^9)\,(0.256)\,(Y=8)}{65^3} * \frac{1}{10^6} = 4.63\ N$

Bending Moment at point (A) = P*L

$M_{min} = P_{min} * L = 1.74*65 = 113$ *N.mm* ; $M_{max} = P_{max} * L = 4.63*65 = 301$ *N.mm*

$\sigma = \frac{M\,C}{I}$

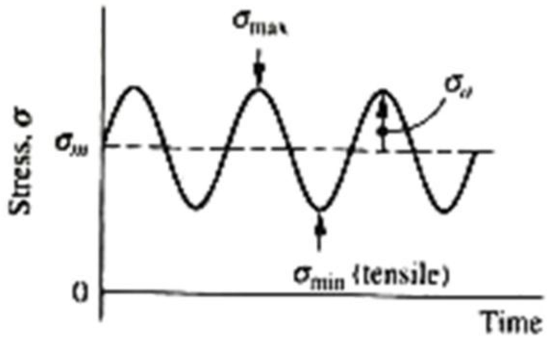
$\sigma_{max} = \frac{301*0.4}{0.256} = 470\ \frac{N}{mm^2}$; $\sigma_{min} = \frac{113*0.4}{0.256} = 176\ N/mm^2$

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$ = *(470+176)/2 = 323 MPa*

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$ = *(470-176)/2 = 147 MPa*

Stress ratio R = $\frac{\sigma_{min}}{\sigma_{max}}$ *176/470 = 0.37*

Type of loading is Fluctuating.



<u>Example 5-2 (Page 181), Ref.1:</u>

Estimate the actual endurance strength of AISI 1050 cold-drawn steel when used in a circular shaft subjected to rotating bending only. The .shaft will be machined to a diameter of approximately 44.45 mm.

<u>Solution:</u>

S_u *from Appendix 3 = 689.5 MPa* $\longrightarrow$ S_n *= 262 MPa*

From figure 5-8 (curve machine or cold drawn).

C_m $\longrightarrow$ *for wrought steel = 1*

C_R $\longrightarrow$ *(Design decision) for (Reliability = 0.99) = 0.81*

C_{st} $\longrightarrow$ *reversed Bending = 1*

C_S $\longrightarrow$ *from figure 5-9 at D= 44.45 mm,* C_s*= 0.83*

$S'_n = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S) = 262\,(1)\,(1)\,(0.81)\,(0.83) = 176$ *MPa*

Predictions of Failure:

See section 5-8 (page 186) (For the application of some of the followings theories, see Design Example 5-1 to 5-4 (Page 201 to Page 213)), Ref.1.

1. Maximum normal stress (uniaxial static stress on Brittle Materials):

For tensile stress $Kt\,\sigma < \sigma_d = S_{ut}/N$ .... (5-9) or

For compressive stress $Kt\,\sigma < \sigma_d = S_{uc}/N$ ...... (5-10)

2. Modified Mohr (Biaxial Static Stress on Brittle materials).

3. Yield strength (uniaxial static stress on Ductile materials). Equations (5-11) & (5-12), this theory is called "**Rankine**".

For tensile stress: $\sigma < \sigma_d = S_{yt}/N$ .... (5-11)

For compressive stress: $\sigma < \sigma_d = S_{uc}/N$ .... (5-12)

For most wrought ductile metals, $S_{ut} = S_{uc}$

4. Maximum shear stress (Biaxial static stress on ductile material) (Mod. Cons.), this theory is called "**Tresca**".

$$\zeta_{max} < \zeta_d = S_{sy}/N = 0.5S_y/N \quad \text{......... (5-13)}$$

5. Distortion energy (Biaxial or Triaxial stress on ductile material) (Good Predictor). (**Von-Mises theory**)

$$\sigma' = \text{Von Mises stress} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \quad \text{......... (5-14)}$$

$$\sigma' < \sigma_d = S_y/N \quad \text{......... (5-15)} \quad \text{or}$$

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \quad \text{.........(5-16)}$$

6. Goodman (Fluctuating stress on ductile material) (Slightly cons.)

$$\frac{\sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = 1 \quad \text{.......... (5-19)}$$

Or (Design equation) $\frac{K_t\,\sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = \frac{1}{N}$ .... (5-20)

7. Gerber (Fluctuating stress on ductile material) (Good Predictor)

$$\frac{\sigma_a}{S_n'} + [\frac{\sigma_m}{S_u}]^2 = 1 \quad \text{........... (5-23)}$$

Or

$$\frac{K_t\,\sigma_a}{S_n'} + [\frac{\sigma_m}{S_u}]^2 = \frac{1}{N}$$

8. Soderberg (Fluctuating stress on Ductile material) (Mod. Cons.)

$$\frac{K_t\,\sigma_a}{S'_n} + \frac{\sigma_m}{S_y} = 1 \quad \text{.......... (5-24)}$$

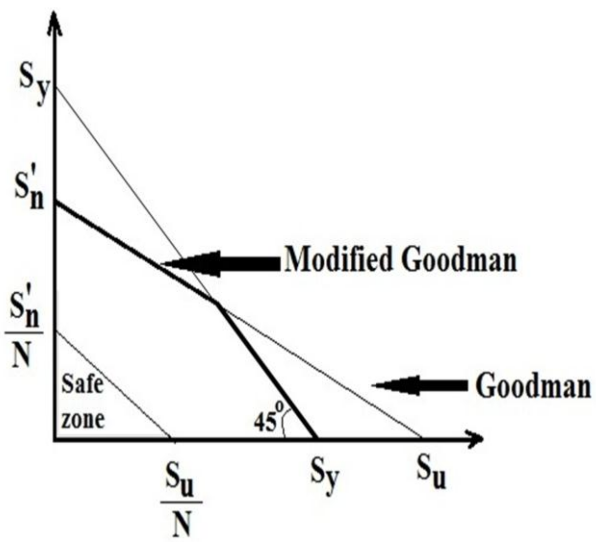
Or

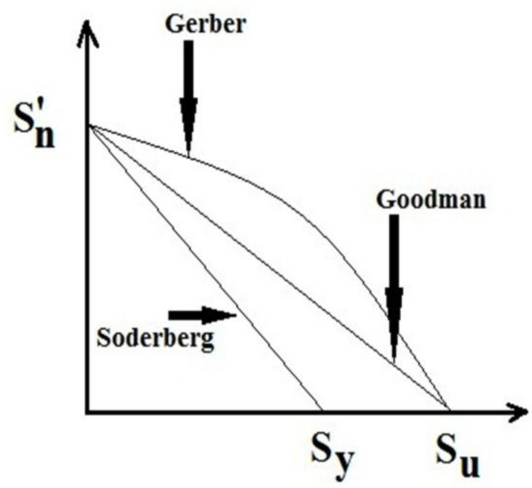
$$\frac{K_t\,\sigma_a}{S'_n} + \frac{\sigma_m}{S_y} = \frac{1}{N}$$

Note: There are many recommended methods for design analysis based on:

1. Material (Brittle or Ductile)
2. Nature of load (Static or Cyclic)
3. Type of stress (Uniaxial or Biaxial).

So there are 16 cases should be discussed (see figure 5-17 to end of page 197)



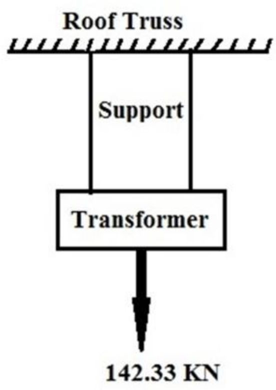


	Type of loading	Tension or Bending	Compression	Torsion
Static ultimate strength	Static	S_u or S_{ut}	S_{uc}	S_{su}
Yield strength	Static	S_y	S_{yc}	S_{sy}
Endurance strength	General or Fluctuating	S_n	S_n	S_{sn}
Endurance strength under actual condition	General or Fluctuating	S'_n	S'_n	S'_{sn}
Mean stresses Amplitude stresses	General or Fluctuating	σ_{xm} σ_{xa}	σ_{-xm} σ_{-xa}	τ_{xym} τ_{xya}

	S_n	S_{sn}	S_{su}	S_{sy}
Wrought steel	**0.5 Su**	**0.577 Sn**	**0.75 Su**	**0.577 Sy**

Design Example 5-1 (Page 201). Ref.1:

A large electrical transformer is to be suspended from a roof truss of a building. The total weight of the transformer is. 32 000 lb. Design the means of support.



- The load is static
- Two rod was assumed
- The end of rod will be threaded
- Only one rod assumed to carry the load during instillation.
- From Appendix 3, AISI 1040 cold-drawn steel assumed as a material of rod.

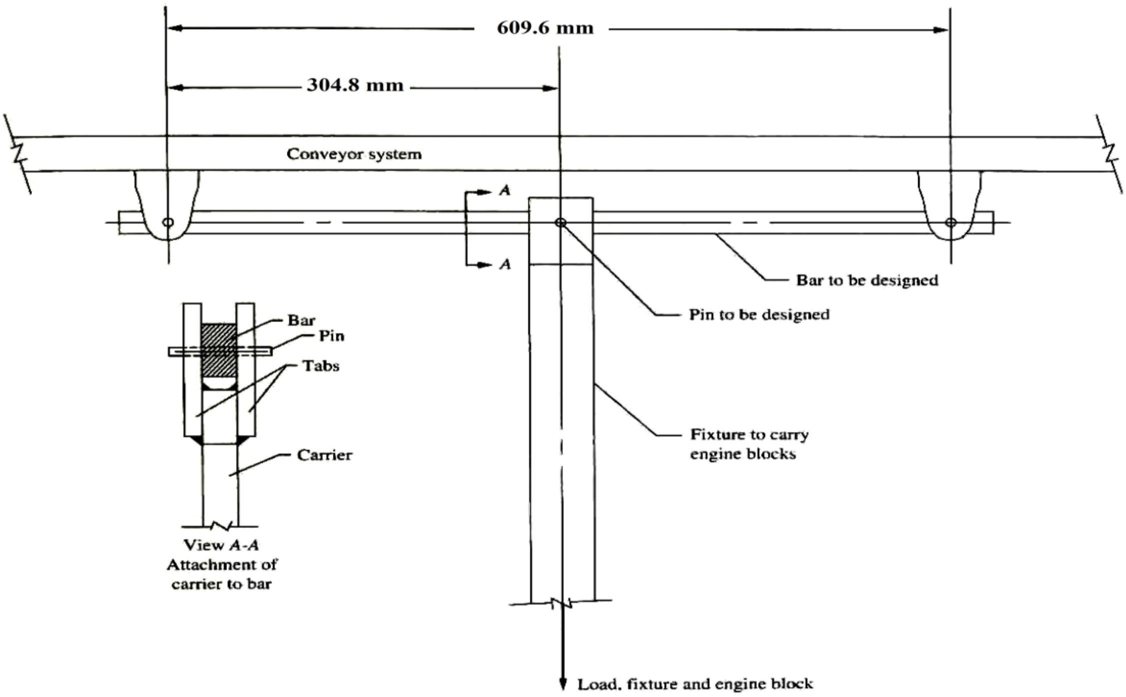
$$[\,S_y = 489.54\ MPa]$$

- Critical place of stress in the threaded part of rod.
- $\sigma_{design} = \frac{S_{yt}}{N}$ & σ *should be* $\le \sigma_d$ …….. (5-11)

$$\sigma_d = \frac{489.54}{3} = 163.18\ MPa\ , and\ \ \sigma_d = \frac{F}{A} \rightarrow A = \frac{142000}{163.18} = 871\ mm^2$$

- Find standard diameter of thread with higher tensile stress area.
- Stress concentration (K_t) is neglected in this case (give your comments).

Design Example 5-2 (Page 202). Ref.1:



A part of a conveyor system for a production operation is shown in Figure 5-18. Design the pin that connects the horizontal bar to the fixture. The empty fixture weighs 378 N. A cast iron engine block weighing 1000.8 N is hung on the fixture to carry it from one process to another where it is then removed. It is expected that the system will experience many thousands of cycles of loading and unloading of the engine blocks

- Weight of empty fixture = 378 N
- Weight of C.I. engine block = 1000.8 N
- Many thousands of Cycles of loading & unloading.
- To design a pin then:

$\tau = \frac{F}{2A}$ & $F_{min} = 378\, N$, $F_{max} = 1379\, N$

$F_{mean} = \frac{F_{max} + F_{min}}{2}$ = *879 N &* $F_a = \frac{F_{max} - F_{min}}{2} = 501\, N$

$$\tau_m = \frac{F_m}{2A} \quad \& \quad \tau_a = \frac{F_a}{2A} \quad \& \quad \frac{K_t\, \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$

- **Note:** comment for using equation $\frac{K_t\, \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$
- From figure if Goodman equation are used.
- If there is shear so *use* $\frac{K_t\, \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$
- Choose material for pin AISI 1020 cold-drawn steel. From **APPENDIX 3** Sy = 351.64 & Su = 420.59 MPa
- In the absence of shear strength data use estimates:

$S'_{sn} = 0.577\, S'_n$ & $S_{su} = 0.75\, S_u$

$S_{su} = 0.75\, S_u = 0.75 * 420.5 = 315.4\, MPa$

From figure (5-8), S_n*=144.9 MPa and find data for (*C_m, C_{st}, C_R *and* C_s*)*

$S'_{sn} = 0.577\, S'_n = (0.577) * [\, S_n * C_m * C_{st} * C_R * C_s]$

*= 0.577*144.8*1*1*0.75*1 = 62.6 MPa*

$$\frac{K_t\, \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$

$\frac{1}{(N=4)} = \frac{(K_t=1) * \tau_a}{62.6} + \frac{\tau_m}{315.4}$ $(N = 4\, for\, mild\, shock)$, the pin will be uniform diameter, $K_t = 1$.

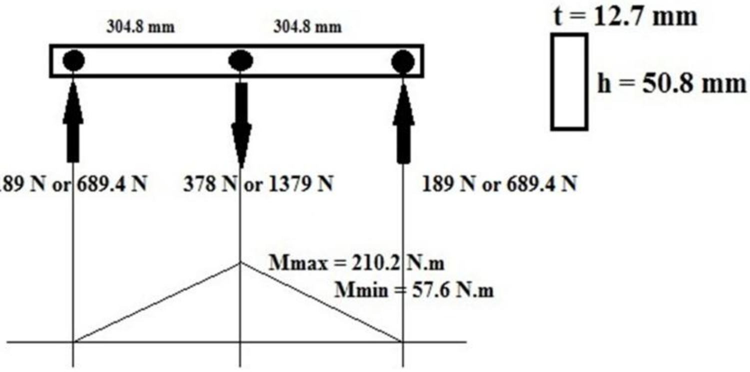
A = 21.579 mm² & D = 5.24 mm (we should choose higher diameter D=12.7 mm)

Design example 5-3, (Page 204)Ref.1: Find the safety factor (N) of the horizontal arm in Ex.5-2 it is propose to make the bar from steel in the form of rectangular bar. Assume pin in the middle = 12.7 mm and at end are 9.52 mm, use material, from Appendix 3, AISI1020 hot-rolled with S_y = 206.85 MPa & S_u = 379.2 MPa.

$\sigma = \frac{M}{S} = \frac{M\,C}{I}$,

$M_{mean} = \frac{M_{max} + M_{min}}{2} = 133.9\ N.m$

$M_a = \frac{M_{max} - M_{min}}{2} = 76.3\ N.m$



Use Goodman line.

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\,\sigma_a}{S_n'}\ ,\quad \sigma_m = \frac{M_m}{S}\quad \&\quad \sigma_a = \frac{M_a}{S}\quad ,\quad S = Section\ Modulus$$

$S_n' = S_n\,(C_m)\,(C_{St})\,(C_R)\,(C_S)$

C_m = 1 for wrought-hot rolled steel

C_{st} = 1 for repeated bending stress

C_R = let = 0.75 to achieve a reliability of 0.999 (table 5-1)

C_s = size factor (fig. (5-9) page 175 [curve between CS & diameter]

If it is not round then diameter = 0.808 $\sqrt{ht}$

C_S = 0.9 from figure (5-8) at D = 0.808 $\sqrt{50.8 * 12.7}$ = 20.5 mm

S_n = 137.9 MPa from figure 5-8 for hot rolled steel & Su = 379 MPa

S_n' = 137.9*1*1*0.75*0.9 = 93.08 MPa

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t\,\sigma_a}{S_n'}$$

$$\frac{1}{N} = \frac{M_m}{(S)(S_u)} + \frac{K_t\,M_a}{S\,(S_n')} = \frac{1}{S}\left[\frac{M_m}{S_u} + \frac{K_t\,M_a}{S_n'}\right] \rightarrow \quad N \cong 4\ \ at\ K_t = 1$$

Because the ratio of d/h < 0.5 from Appendix (15),

$S = \frac{t\,(h^3 - d^3)}{6h} = 0.46 * 10^4\ \ mm^4$ *And t = 12.7 mm, then h =46.99 mm*

<u>EX.1</u>: A bar of steel has S_{ut} = 700 MPa, Sy= 500 MPa & S_n'= 200 MPa. Find safety factor (N) against static and fatigue failures for:

a. $\tau_m = 140\ MPa$ ***b***. $\tau_m = 140\ MPa\ \ ,\ \ \tau_a = 70\ MPa$

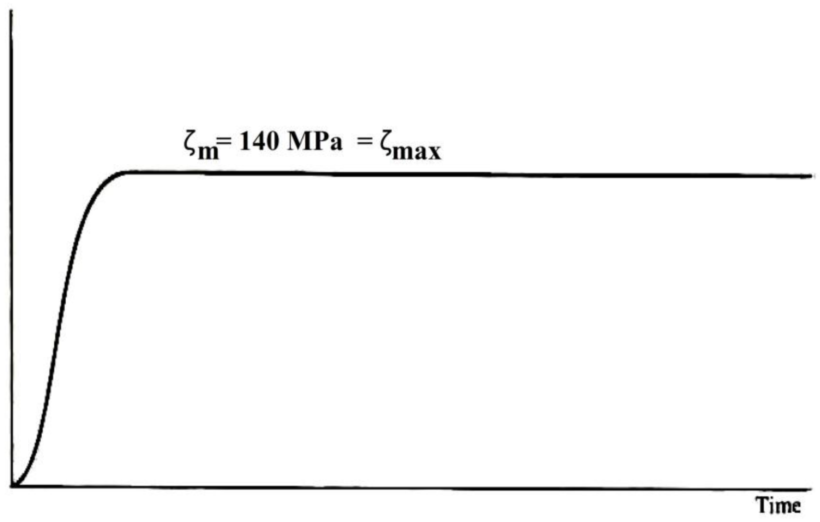
c. $\tau_{xym} = 100\ MPa\ \ ,\ \sigma_{xa} = 80\ MPa$

d. $\sigma_{xm} = 60\ MPa\ \ ,\ \sigma_{xa} = 80\ MPa\ \ ,\ \tau_{xym} = 70\ MPa\ \ ,\ \tau_{xya} = 35\ MPa$

<u>Solution:</u>

a) $S_{Sy} = 0.577\ S_y = 0.577*500 = 288\ MPa$

$$N = \frac{S_{Sy}}{\tau_m} = \frac{288}{140} = 2.06$$



ζm= 140 MPa = ζmax = ζmin , ζa =0

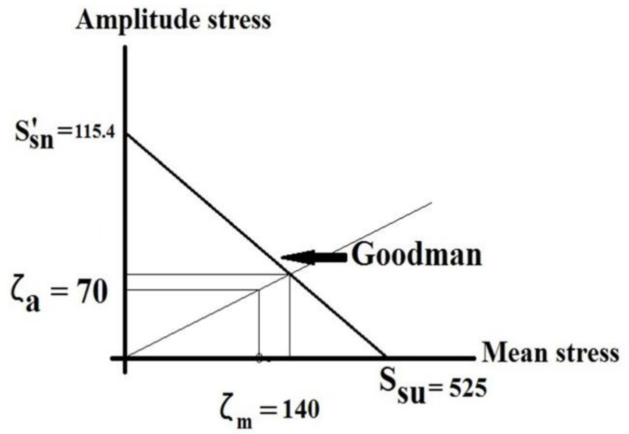
b) $\tau_{max} = \tau_m + \tau_a = 140 + 70 = 210\ MPa$

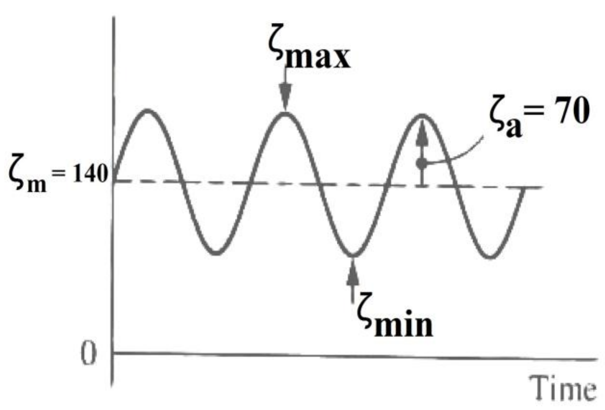
$$N\ (static) = \frac{S_{sy}}{\tau_{max}} = \frac{288}{210} = 1.37$$

$S_n' = 200\ MPa$ $\quad S_{sn}' = 0.577 * S_n' = 0.577*200 = 115.4\ MPa$

$S_{su} = 0.75 S_u = 0.75*700 = 525\ MPa$

$$\frac{\tau_a}{S_{sn}'} + \frac{\tau_m}{S_{su}} = \frac{1}{N} \quad \rightarrow \quad \frac{70}{115.4} + \frac{140}{525} = \frac{1}{N} \quad \rightarrow \quad N = 1.145$$





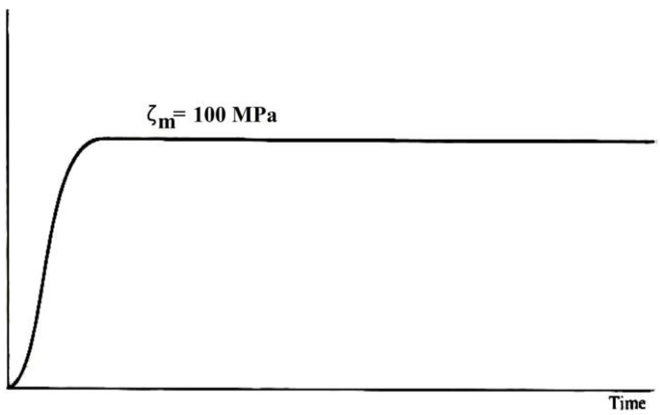
c) Use Distortion energy theory (equation 5-16):

$$\sigma' < \sigma_d = \frac{S_y}{N} \quad \& \quad \sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma' = \sqrt{(80)^2 + 3(100)^2} = 191\ MPa$$

$$N\ (static) = \frac{S_y}{\sigma'} = \frac{500}{191} = 2.62$$

$$N\ (fatigue) = \frac{S_m}{\sigma'_m} \quad or \quad = \frac{S_a}{\sigma'_a}$$



$$\sigma'_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \sqrt{0 + 3(100)^2} = 173\ MPa$$

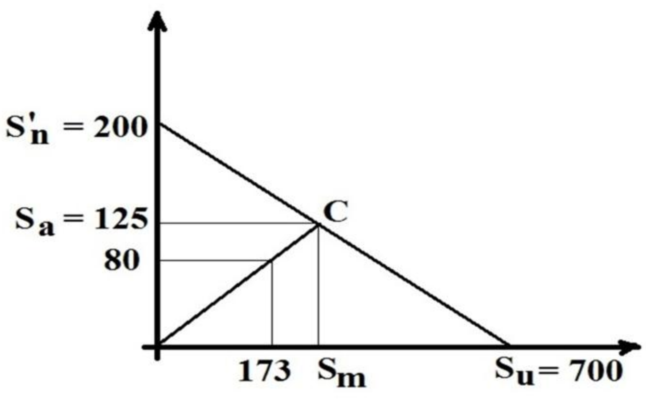
$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \sqrt{80^2 + 0} = 80\ MPa$$

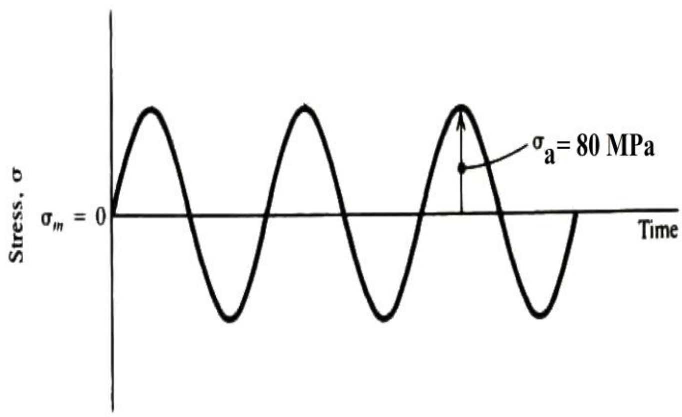
Now, from figure Sm = 270 MPa & Sa = 125 MPa

N (fatigue) = 270/173 = 125/80 = 1.56

OR

$$\frac{1}{N} = \frac{\sigma'_m}{S_u} + \frac{\sigma'_a}{S'_n} \quad \rightarrow \quad \frac{1}{N} = \frac{173}{700} + \frac{80}{200} \quad \rightarrow \quad N = 1.55$$



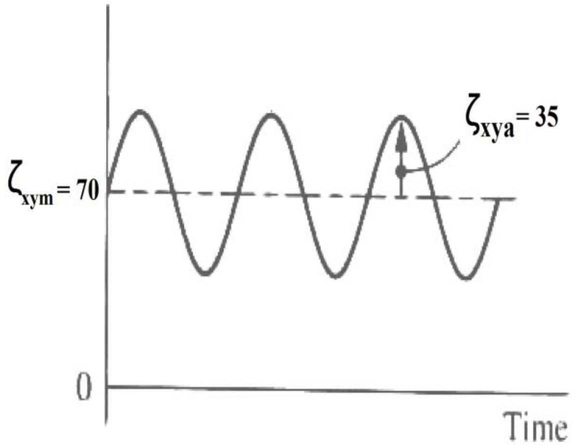


d) From figure $\sigma_{x(max)} = \ 60 + 80 = 140 \ MPa$

$\tau_{xy(max)} \ = 70 + 35 = 105 \ MPa$

$\sigma'_{max} = \sqrt{140^2 \ + 3(105)^2} \ = \ 229 \ MPa$

N (fatigue) = $\frac{S_y}{\sigma'_{max}} \ = \ \frac{500}{229} \ = \ 2.18$



<u>For fatigue (as before):</u>

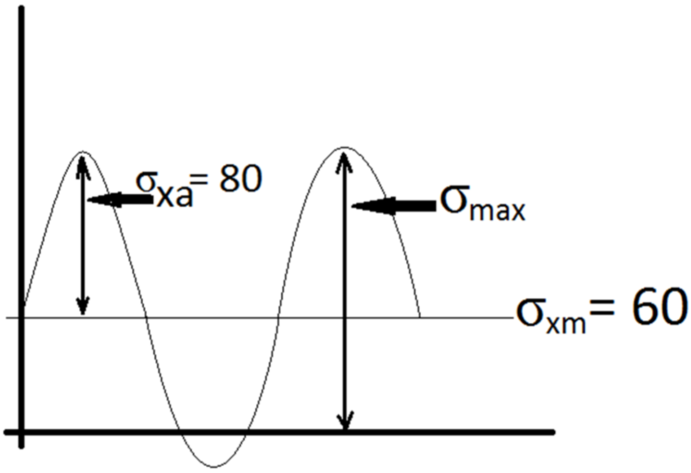
$\sigma'_m \ = \ \sqrt{60^2 \ + \ 3(70)^2} \ = \ 135 \ \ MPa$

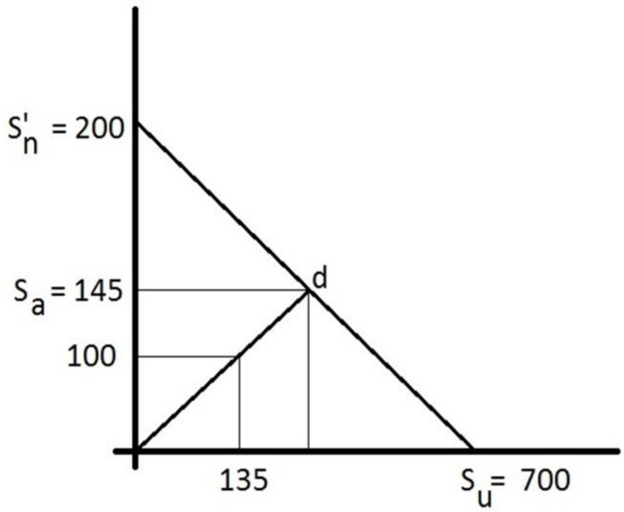
$\sigma'_a \ = \ \sqrt{80^2 \ + \ 3(35)^2} \ = \ 100 \ MPa$

N (fatigue) = $\frac{S_m}{\sigma'_m} \ = \ \frac{S_a}{\sigma'_a} \ = \ \frac{145}{100} \ = \ 1.45$

Or

$\frac{1}{N} \ = \ \frac{\sigma'_m}{S_u} + \frac{\sigma'_a}{S'_n} \quad \rightarrow \quad \frac{1}{N} \ = \ \frac{135}{700} + \frac{100}{200} \quad \rightarrow \quad N = 1.45$





1-Solve problems in chapter 5 (Page 219), [Q1 – Q8 – Q9 –Q11 – Q19 – Q28 – Q30 – Q42 – Q67 – Q77].