The “Apron” means a foundation of regulating hydraulic structures. This Apron generally founded on alluvial soils which are highly pervious. Moreover, these soils are easily scoured when the high velocity water passes over the structures. The “Hydraulic” failure of apron constructed on the permeable foundation may occur due to various causes, which may be broadly classified into the following two categories:

1. Failure due to subsurface flow  
2. Failure due to surface flow

The hydraulic failure can also be categorized as follows:

- Formation of deep scour downstream of apron (failure due to surface flow),
- Formation of a hydraulic jump outside the floor (failure due to surface flow),
- Excessive uplift pressure over the weight of apron (failure due to subsurface flow),
- Under mining or piping due to excessive exit gradient (failure due to subsurface flow).

This lecture will be focusing just on “Theories” that can be used for analysis and design of hydraulic structures floors (Aprons) to overcome the failure causes of sub-surface flow (seepage), under structure.

1. Failure due to subsurface flow

The failure due to subsurface flow may occur by piping or by rupture of floor due to uplift.
(a) **Failure by piping (or undermining);** occurs below the floor if the water percolating through the foundation has a large seepage force when it emerges at the downstream end of the impervious floor. When the seepage force exceeds a certain value, the soil particles are lifted up at the exit point of the seepage. With the removal of the surface soil particles, there is further concentration of flow in the remaining portion and more soil particles are removed. This process of backward erosion progressively extends towards the upstream side, and a pipe-like hollow formation occurs beneath the floor. The floor ultimately subsides in the hollows so formed and fails. This type of failure is known as piping failure. (see Figure Below)
(b) Failure by rupture of floor; the water percolating through the foundation exerts an upward pressure on the impervious floor, called the uplift pressure. If the weight of the floor is not adequate to counterbalance the uplift pressure, it may fail by rupture. (see Figure below)

Design aspects for Subsurface Flow:

The basic principles for the design of all irrigation structures on pervious foundations are as follows:
1. The structure should be designed such that the piping failure does not occur due to subsurface flow.

2. The downstream pile must be provided to reduce the exit gradient and to prevent piping.

3. An impervious floor of adequate length is provided to increase the path of percolation and to reduce the hydraulic gradient and the seepage force.

4. The seepage path is increased by providing piles and impervious floor to reduce the uplift pressure.

5. The thickness of the floor should be sufficient to resist the uplift pressure due to subsurface flow. The critical section is D/S of the location of a control section (e.g. gate).

6. A suitably graded inverted filter should be provided at the downstream end of the impervious floor to check the migration of soil particles along with water. The filter layer is loaded with concrete blocks. Concrete blocks are also provided at the upstream end.

**Floor Thickness**

The floor should have appropriate thickness to counteract the uplift pressure acting on it. At selected point let the residual head is “h” which is the subsoil H.G.L. measured from the top surface of the floor. If h' is the head measured above the bottom surface of the floor, then:-
Where “\( t \)” is the thickness of floor. Fig. shows the uplift pressure diagram on the bottom surface. It is more convenient to measure the intercept \( h \) than the intercept \( h' \). The intercept \( h' \) above the bottom surface of the floor can be determined only after the thickness “\( t \)” has been determined or has been assumed. For the determination of the floor thickness let us consider the force acting on the unit area of the floor (shown hatched) so as:-

\[
\gamma_w h' = \gamma_c t
\]

\[
\Rightarrow h + t = \left( \frac{\gamma_c}{\gamma_w} \right) t
\]

\[
\Rightarrow t = h (G_s - 1)
\]
where “Gs” is a specific gravity of the floor material. For plain concrete floor, the value of Gs usually varies from 2.0 to 2.3 depending upon the type of aggregates used. A value of 2.24 is usually adopted. Generally, a factor of safety of 4/3 is adopted. Thus

\[ t = \frac{4h}{3(Gs-1)} = \frac{4P_{uH}}{3(Gs-1)} \]  

where \( P_{uH} = h \) is uplift pressure head at that point above the top surface of the floor.

**Seepage Theories:**

1-Bligh's Theory

In 1910, W.G. Bligh gave creep theory. According to this theory, the percolating water creeps along the contact surface of the base profile of the structure with the subsoil. The length of the path thus traversed by the percolating water is called the length of creep or the creep length. As the water creeps from the upstream end to the downstream end, the head loss occurs. The head loss is proportional to the creep distance travelled. Bligh made no distinction between the creep in the horizontal direction below the floor and the creep in the vertical direction along the faces of the piles. The theory assumes that the head loss variation is linear, while the actual head loss variation is non-linear.

If \( H_L \) is the total head loss between upstream and downstream and \( L \) is the length of the creep, then the loss of head per unit of creep
length (i.e. $\frac{H_1}{L}$) is called the hydraulic gradient. Bligh’s theory makes no discrimination between horizontal and vertical creeps.

Consider a section as shown in the figure below. Let $H$ be the difference of water levels between upstream and downstream ends (no water is shown in the downstream end). Water starts percolating at A and emerges at B.

Total creep length is:

$$L = 2d_1 + L_1 + 2d_2 + L_2 + 2d_3$$

$$= (L_1 + L_2) + 2 (d_1 + d_2 + d_3)$$

$$L = b + 2 (d_1 + d_2 + d_3)$$

Head loss per unit length (hydraulic gradient) is:
Then the head loss at any point of apron can be calculated by “Multiply the hydraulic gradient by the distant location of this point from the beginning of creep”, such that:

Head loss occurs on upstream cutoff = \( \frac{H}{L} \times 2d1 \)

Head loss occurs on intermediate cutoff = \( \frac{H}{L} \times 2d2 \)

Head loss occurs on downstream cutoff = \( \frac{H}{L} \times 2d3 \)

Head at Point C = Total Head – Head loss occurs on U/S cutoff;

\[ H_c = H - \frac{H}{L} \times 2d1 = \frac{H}{L} (L - 2d1) \]

So the hydraulic gradient drop at upstream cutoff is \( H-H_c \), then;

\[ H - H_c = H - \left( H - \frac{H}{L} \times 2d1 \right) = \frac{H}{L} \times 2d1 \]

The Figure below illustrate the head for any point of Apron;
Safety against Piping or Undermining

Safety against piping can be ensured by providing sufficient creep length given by:

\[ L = C H \]  \hspace{2cm} (3)

C is Bligh’s coefficient for the soil

And hence: \[ \frac{H}{L} = \frac{1}{C} \]

Bligh’s creep coefficient “C” depends upon the bed material as shown in Table below. Therefore for known seepage head and creep coefficient the uplift pressure at any point can be determined by Bligh theory and then required thickness to counteract it.
Creep Coefficient by Bligh

<table>
<thead>
<tr>
<th>No.</th>
<th>Type of soil</th>
<th>Value of C</th>
<th>Safe exit gradient less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fine sand</td>
<td>15</td>
<td>1/15</td>
</tr>
<tr>
<td>2</td>
<td>Coarse grained sand</td>
<td>12</td>
<td>1/12</td>
</tr>
<tr>
<td>3</td>
<td>Sand mixed with boulders and gravel</td>
<td>5 to 9</td>
<td>1/5 to 1/9</td>
</tr>
<tr>
<td>4</td>
<td>Light sand and mud</td>
<td>8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Hydraulic gradient \( H \times L < 1 \) \( C \) for safety against piping.

**Safety against Uplift Pressure**

If the uplift head at any point is known and denoted \( P_{uh} \) or \( h \) (meter of water) then uplift head has to be counterbalanced by the weight of floor thickness. This suitable thickness can be determined by using Eq.1, where;

\[
t = \frac{4h}{3(Gs-1)} = \frac{4P_{uh}}{3(Gs-1)} \quad \ldots \ldots (1)
\]

**Lane’s Weighted Creep Theory**

From the analysis of 200 dams all over the world, Lane’s concluded that horizontal creep is less effective in reducing uplift than vertical creep. Therefore, he suggested a factor of 1/3 for horizontal creep against 1 for the vertical creep. For the structure in figure the creep length is:

\[
L = 2d1 + (1/3)L1 + 2d2 + (1/3) L2 + 2d3 \quad \ldots \ldots (4)
\]
As know, L= b, then:

\[ L = \frac{1}{3}b + 2(d_1 + d_2 + d_3) \]

Note that; the slope steeper than 45° taken as vertical.

In Bligh Method, the creep coefficient is equal to L/H, where;

\[ L = CH \quad \text{leads to} \quad C = \frac{L}{H} \]

In Lane’s Method the “Actual” length is greater than the “creep” length, so;

\[ L > CH \quad \text{leads to} \quad C < \frac{L}{H} \]

The following Table gives the Lane coefficient for different types of soil
Lane’s Creep Coefficient

<table>
<thead>
<tr>
<th>Material</th>
<th>Ratio of C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very fine sand or silt</td>
<td>8.5</td>
</tr>
<tr>
<td>Fine sand</td>
<td>7</td>
</tr>
<tr>
<td>Medium sand</td>
<td>6</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>5</td>
</tr>
<tr>
<td>Fine gravel</td>
<td>4</td>
</tr>
<tr>
<td>Medium gravel</td>
<td>3.5</td>
</tr>
<tr>
<td>Coarse gravel</td>
<td>3</td>
</tr>
<tr>
<td>Boulders gravel</td>
<td>2.5</td>
</tr>
<tr>
<td>Soft clay</td>
<td>3</td>
</tr>
<tr>
<td>Medium clay</td>
<td>2</td>
</tr>
<tr>
<td>Hard clay</td>
<td>1.8</td>
</tr>
<tr>
<td>Very hard clay</td>
<td>1.6</td>
</tr>
</tbody>
</table>

**Khosla’s Theory**

*Khosla* provided a complete rational solution of the problem based on potential flow theory and *Schwartz-Christoffel* transformation. Various cases were analyzed and studied by them. The resultant Khosla’s theory gives uplift pressure at various points of the structure, depending upon its profile. It also gives the exit gradient. To ensure that the piping failure does not occur, there must be a downstream pile and the exit gradient should be safe. Moreover, the thickness of the floor should be adequate to resist uplift pressure. As can be shown from Figure below, the distribution of U.P under the floor is non-linear as adopted by Khosla, thus a pressure head “P” at any point beneath the Apron is a “Fraction, ϕ” of a total head “Hs”, where:-
\( \phi = \frac{P}{Hs} \quad \text{or} \quad P = \phi Hs \) \hspace{1cm} (5)

And Khosla given the following form of \( \phi \):

\[ \phi = \frac{1}{\pi} \cos^{-1} \left( \frac{2x}{b} \right) \] \hspace{1cm} (6)

So, the U.P at any point of Apron will be:

\[ P = \frac{Hs}{\pi} \cos^{-1} \left( \frac{2x}{b} \right) \] \hspace{1cm} (7)

\( b \) = the horizontal length of Apron (the distance between entry “Point A” and exit “Point B” as shown in Fig.),

\( x \) = the distance from floor center (point “o” in Figure) to the point where U.P needs.

Also Khosla proposed the following formula for determination of Exit Gradient;

\[ G_E = \frac{Hs}{d} \frac{1}{\pi \sqrt{\lambda}} \] \hspace{1cm} (8)
\[ \lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2} \] .......................... (9)

\[ \alpha = \frac{b}{d} \]

d= the depth of D/S cutoff (pile).

Therefore for any given profile of the Apron of a weir/barrage/hydraulic structure on pervious foundation the following three steps may be adopted to compute uplift pressure at any point:

1. Decompose the general profile into elementary profiles
2. Assemble/superposition of uplift pressures at key points
3. Correction and interpolation

In this method a composite barrage or weir floor section is split up into a number of simple elementary profiles at a standard forms of known analytical solution, and the uplift pressures have been determined at “key-points”, by finding the solution of the expression (2x/b) of Eq.7 as follows:-

\( (i) \) Pile at Downstream End

Referring to Eq.7 the U.P that affected on any point of Apron take the form; \[ P = \frac{H_s}{\pi} \cos^{-1} \left( \frac{2x}{b} \right), \] Khosla found the analytical solution of (2x/b) for the specified Key-Points as illustrated in figure, where;
At point E: \[
\frac{2x}{b} = \left(\frac{\lambda - 2}{\lambda}\right)
\]

At point D: \[
\frac{2x}{b} = \left(\frac{\lambda - 1}{\lambda}\right)
\]

At Point C: \(P_c = 0\)

Where:

\[
\lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2} \quad \text{and} \quad \alpha = \frac{b}{d}
\]

(ii) Pile at Upstream End

As done in the above, the finding of U.P of key points shown in the Figure below, is as follows:-
(i) At point C1: \[ \frac{2x}{b} = \left( \frac{2-\lambda}{\lambda} \right) \]

(ii) At point D1: \[ \frac{2x}{b} = \left( \frac{1-\lambda}{\lambda} \right) \]

(iii) At Point E1: \[ P_{E1} = Hs \]

Also: \[ \lambda = \frac{1+\sqrt{1+\alpha^2}}{2} \] and \[ \alpha = \frac{b}{d} \]

(iii) Intermediate Pile

If there is a cutoff located between U/S and D/S, the U.P at its key-points will be calculated as follow (see Fig. below)

- At point E: \[ \frac{2x}{b} = \left( \frac{\lambda 2 - 1}{\lambda 1} \right) \]
- At point D: \[ \frac{2x}{b} = \left( \frac{\lambda 2}{\lambda 1} \right) \]
- At Point C: \[ \frac{2x}{b} = \left( \frac{\lambda 2 + 1}{\lambda 1} \right) \]
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\[
\lambda_1 = \frac{\sqrt{1+\alpha_1^2} + \sqrt{1+\alpha_2^2}}{2} \quad \text{and} \quad \lambda_2 = \frac{\sqrt{1+\alpha_1^2} - \sqrt{1+\alpha_2^2}}{2}
\]

\[
\alpha_1 = \frac{b_1}{d}, \quad \alpha_2 = \frac{b_2}{d}
\]

(iv) Depressed floor

The uplift pressure head at key points D’1 and D’ as shown in Fig is given by:

\[
P_{D'} = P_D - \frac{2}{3}(P_E - P_D) + \frac{3H_S}{\alpha^2} \quad \text{and} \quad P_{D'_1} = H_S - P_{D'}
\]

Where the \(E, D\) are the key-points of D/S pile and the calculations of \(P_E\) and \(P_D\) from the procedure illustrated for key points of D/S pile are used to finding the U.P of key-points of depressed floor.
Assemble/superposition of uplift pressures at key points

Correction and interpolation

In actual structure the floor has some thickness and it may not be horizontal and also there may be more than one line of piles, so the following corrections have to be applied to the superposed values of the uplift pressures:

1. Correction for thickness of floor.
2. Correction for mutual interference of piles.
3. Correction for slope of the floor.

Correction for thickness of floor

The thickness of floor is assumed to be negligible and the pressure is found at point E, C from the procedure of Khosla. The pressure at point E', C' (as shown in Figure below), are interpolated by assuming straight line variation as follows.

\[ P_{E'} = P_E - C_E \quad \text{and} \quad P_{C'} = P_C + C_C \]
HYDRAULIC CONSIDERATIONS FOR APRON DESIGN

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Where;

\[ C_E = \frac{t}{d} (\Phi_E - \Phi_D) \]

\[ C_C = \frac{t}{d} (\Phi_D - \Phi_C) \]

**Correction for interference of piles**

\[ C_p = \pm 19 \sqrt{\frac{D}{b'}} \left( \frac{d + D}{b} \right) \]

\( C_p \) = correction in a percentage.

\( b' \) = the distance between two piles.

\( b \) = total floor length.

\( d \) = depth of pile on which the effect is to be taken.

\( D \) = depth of the intermediate pile its influence on U.P of neighboring pile of depth “d”.

The “\( C_p \)” is a % of \( H_s \), this percentage value was “Added” or “subtracted” from the U.P of key-point according to its location relative to “intermediate” pile where:

- Added (+\( C_p \)) to the value of calculated U.P of U/S pile.
- Subtracted (-\( C_p \)) to the value of calculated U.P of D/S pile.

**Note that: The correction “C” was neglected (have not effect) if:-**

\[ D < d \quad \text{and} \quad b' = 2d \]
Correction due to slope

In the derivation of the expressions for the uplift pressure at the key points, the floor has been assumed to be horizontal. If the floor is sloping, the correction is applicable to key points at beginning and end of sloping floor as given by;

\[ \text{Cs} = \text{or} - \frac{bs}{b'} \cdot C \]

<table>
<thead>
<tr>
<th>Slope V: H</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>11.2</td>
</tr>
<tr>
<td>1:2</td>
<td>6.5</td>
</tr>
<tr>
<td>1:3</td>
<td>4.5</td>
</tr>
<tr>
<td>1:4</td>
<td>3.3</td>
</tr>
<tr>
<td>1:5</td>
<td>2.8</td>
</tr>
<tr>
<td>1:6</td>
<td>2.5</td>
</tr>
<tr>
<td>1:7</td>
<td>2.3</td>
</tr>
<tr>
<td>1:8</td>
<td>1</td>
</tr>
</tbody>
</table>

The correction being **plus** for the **downslope** and **minus** for the **upslope** following the direction of water.

It may be noted that the correction is applicable only to the **key points** of the pile lines which lie either at the **beginning** or at the **end** of the sloping floor.
Permissible exit-gradient (GE)

For alluvial soils the critical (safe) exit gradient is (1/1), and the permissible exit gradient are as follows:-

- For fine sand, \( G_E = 1/6 – 1/7 \)
- For coarse sand, \( G_E = 1/5 – 1/6 \)
- For shingle, \( G_E = 1/4 – 1/5 \)

Regime Scour Depth

The channel or river bed is scoured during high velocities or during excessive flood, then a large scour hole may developed progressively adjacent to concrete Apron which may cause undermining of floor.

The scour depth measured below High Flood Level (HFL) corresponding to the regime width is called “Regime scour depth”, this depth estimated by “Lacey’s formula”:-

\[
Rs = 0.475 \left( \frac{Q}{\sqrt{f}} \right)^{3/2} \quad \ldots (10)
\]

Eq.10 used if the actual width of channel “B” is greater or equal to regime width.

\[
Rs = 1.35 \left( \frac{q^2}{\sqrt{f}} \right)^{3/2} \quad \ldots (11)
\]

Eq.11 used if actual width “B” is less than the regime width.
\[ f = 1.75 \sqrt{d_{50}} \quad \ldots \quad (12) \]

\[ B_r = 4.75 \sqrt{Q} \quad \ldots \quad (13) \]

Where:-

\( B_r \) = regime width,

\( R_s \) = regime scour depth,

\( Q \) = the total flow discharge,

\( q \) = the intensity of discharge,

\( f \) = Lacey’s silt factor, and

\( d_{50} \) = mean diameter of bed material in “mm”.

See Figure below for regime scour depth demonstrations.