Torsional Behavior of High-Strength Reinforced Concrete Beams

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ABSTRACT:
Recent methods for torsional design of reinforced concrete beams tend to the use of space truss analogy, instead of the earlier skew bending theory. A total of \((4)\) rectangular beams made of high strength concrete (HSC) that failed under pure torsion are considered in this work. These have been taken from the literature.

Regression analysis was performed on the results to obtain two representative equations to predict: cracking torsional moment \(T_{cr}\) and torsional resistance moment \(T_r\). The first equation is based on \((4)\) major parameters that include concrete compressive strength \(f'_c\) and sectional dimensions, while the second one is based on \((7)\) major parameters which include the quantification of the influence of both transverse and longitudinal reinforcement.

When the ACI 413-50 Code design equation was applied, it gave a coefficient of variation (COV) of \((3.23)\) percent for the ratio of tested / calculated torsional strength \((T_u-test / T_r-calc)\), however, the proposed equation has led to a COV of \((112.1)\) percent.

Keywords: beams; cracking torsional moment; high strength concrete; longitudinal reinforcement; torsional resistance moment; transverse reinforcement.

 السلوك اللي في العتبات الخرسانية المسلحة المصنوعة من خرسانة عالية المقاومة

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الخلاصة:
تعتمد الطرق الحديثة لتصميم اللي في العتبات الخرسانية المسلحة على نظرية المسنم الفضائي (Space Truss Bending). تم دراسة (34) عتبة خرسانية مسلحة مستطيلة المقطع مصنوعة من خرسانة عالية المقاومة فشلت تحت تأثير اللي الخلاصي مأخوذة من بحوث سابقة.
INTRODUCTION:

Use of HSC leads to more economical building structures resulting from small sections of structural members and large usable floor areas. It also leads to a reduction in overall building height and dead loads resulting from the use of thinner slabs and shallower beams [1,7,11].

Pure torsion only occurs infrequently in practice. Normally, it arises as a combined action with bending and/or shear. It can become a predominant action in structures such as eccentrically loaded box beams, curved girders, spandrel beams, structures of irregular shapes, and spiral staircases [5,13]. However, in bridges, torsion constitutes a significant design action because of eccentric forces. Since large bridge construction is an obvious application of HSC, an investigation of reinforced HSC beams subjected to pure torsion is of interest. In 1964, the then ACI Committee 934 published its report recommending torsional design based on the skew bending theory [4]. The ACI 318 Committee used this theory starting from the 1981 Code [14] which continued up to the 1984 Code [15]. BS-85 [11] and BS-57 [15] Code versions also used the same approach.

The most recognized theoretical model of pure torsion in reinforced concrete is the space truss model. Based on post-doctoral research published by MacGregor and Ghoneim [11], the ACI Code in 1995 [17] accepted this model. This is now included in the latest ACI 318-2011 Code [14]. The Canadian [18], AASHTO-LRFD [16], and European [19] Codes also use space truss analogy for torsional design.

There are a number of more accurate but more complex design procedures in the literature, [10,14] but they are not considered in this work.

RESEARCH SIGNIFICANCE:

This paper provides an evaluation of the design provisions for pure torsion based on (Y) different code approaches: (Y) using skew bending theory (ACI 318M-85[5] and BS-85[11]); plus (Y) using space truss analogy (ACI 318M-94[11], ACI 318M-2011, Canadian [18], AASHTO-LRFD [16], and EURO [19]). In addition, a number of equations adopted by some researchers to predict $T_{cr}$ value are included. A total of (Y) tests of torsional failure of tested beams is used to evaluate the previous (Y) methods. Two proposed equations which are based on regression analysis are also introduced. The first one estimates the cracking torsional
moment \( T_{cr} \) of HSC beams, while the second one predicts the torsional resistance moment of such beams.

**EXPERIMENTAL RESULTS:**
All available experimental results from test series on pure torsion are obtained from the literature. The ranges of the variables of these (43) rectangular solid section beams are listed in Table (1). The main significant parameters are concrete compressive strength \( f'_{c} \), aspect ratio \( \frac{y}{x} \), sectional area \( A_{cp} \), nominal stirrup strength \( \rho_{v} f_{yt} \), and nominal longitudinal steel strength \( \rho_{f} f_{yt} \). These beams include \( \circ, \dagger, \ddagger, \wedge, \) and \( \vee \) specimens from the references 22, 23, 28, and 22 respectively.

<table>
<thead>
<tr>
<th>Detail</th>
<th>( f'_{c} ) (MPa)</th>
<th>( \frac{y}{x} )</th>
<th>( A_{cp} ) (mm(^2))</th>
<th>( \rho_{v} f_{yt} ) (MPa)</th>
<th>( \rho_{f} f_{yt} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>41,730</td>
<td>1,420</td>
<td>4,500</td>
<td>1,700</td>
<td>7,800</td>
</tr>
<tr>
<td>High</td>
<td>109,810</td>
<td>4,000</td>
<td>17,500</td>
<td>7,200</td>
<td>24,700</td>
</tr>
<tr>
<td>High/Low</td>
<td>27,733</td>
<td>2,797</td>
<td>4,375</td>
<td>4,265</td>
<td>8,576</td>
</tr>
</tbody>
</table>

Where:

- \( f'_{c} \) = cylinder compressive strength of concrete, MPa
- \( x \) = the shorter side of the cross section, mm
- \( y \) = the longer side of the cross section, mm
- \( A_{cp} \) = area enclosed by outside perimeter of concrete cross section, mm\(^2\)
- \( \rho_{v} = \) stirrups ratio = \( \frac{2A_{v}}{b.s} \)
- \( f_{yt} \) = specified yield strength of transverse reinforcement, MPa
- \( \rho_{f} \) = longitudinal steel ratio = \( \frac{A_{f}}{b.h} \)
- \( f_{yt} \) = specified yield strength of longitudinal reinforcement, MPa

**EVALUATION OF EXPERIMENTAL RESULTS:**

**Cracking Torsional Moment Equations:**
Following are the methods considered in this work to estimate the cracking torsional moment of the beams:

1. **ACI 318M-18 Code** [1] method:
\begin{equation}
T_{cr} = \frac{1}{6} \sqrt{f'_c} \sum x^2 \cdot y \tag{1}
\end{equation}

Where:

\( T_{cr} \) = cracking torsional moment, N.mm

\( \sum \)

\begin{enumerate}
\item ACI 318-M-05 Code \cite{1} method:

\begin{equation}
T_{cr} = 0.33 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \tag{2}
\end{equation}

Where:

\( P_{cp} \) = outside perimeter of concrete cross section, mm.

\item Canadian-84 Code \cite{2} method:

\begin{equation}
T_{cr} = 0.4 \phi_c \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \tag{3}
\end{equation}

Where \( \phi_c = \cdot \cdot \cdot \)

\item Hsu and Mo's \cite{3} method:

\begin{equation}
T_{cr} = 0.5 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \tag{4}
\end{equation}

\item Koutchoukali and Belarbi's\cite{4} method:

\begin{equation}
T_{cr} = 0.46 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \tag{5}
\end{equation}

\item Fang and Shiau's\cite{5} method:

\begin{equation}
T_{cr} = 0.095 \sqrt{f'_c} \cdot x^2 \cdot y \tag{6}
\end{equation}

\end{enumerate}

\textbf{Torsion Design Equations:}

\( \forall \) Methods of existing design codes are included in this study to predict the torsional resistance moment of the beams. To make comparison between design methods, torsional
resistance $T_{r\text{-calc.}}$ is used instead of nominal $T_{n\text{-calc.}}$ throughout (e.g. $T_{r\text{-calc.}} = \cdot, \cdot, \cdot \cdot T_{n\text{-calc.}}$ per ACI M-41 Code\textsuperscript{[1]} method).

The design code methods are based on two approaches:

**a. Skew Bending Theory:**

Torsional strength of beams is composed of two parts: the concrete contribution $T_c$ and the reinforcement contribution $T_s$.

1. **ACI M-41 Code\textsuperscript{[1]} method:**

$$T_{r\text{-calc.}} \leq T_{r\text{,ACI-M-89}} = 0.85 \left[ \frac{\sqrt{f'_c}}{15} \sum x'^2 \cdot y + \alpha_t \cdot \frac{A_t \cdot x_1 \cdot y_1 \cdot f_y}{S} \right] (Y, Y)$$

$$T_{r\text{-calc.}} \leq T_{r\text{,ACI-M-89}} = 0.85 \left[ \frac{\sqrt{f'_c}}{15} \sum x'^2 \cdot y + \alpha_t \cdot \frac{A_t \cdot x_1 \cdot y_1 \cdot f_y}{P_h} \right] (Y, Y)$$

Where:

- $T_{r\text{,ACI-M-89}}$ = torsional resistance moment provided by concrete and stirrups, calculated by ACI-M-89 method, N.mm.
- $\alpha_t = \cdot, \cdot, \cdot, \cdot (y/f_y) \leq 1, \cdot$
- $A_t$ = area of one leg of closed stirrup resisting torsion within spacing $S$, mm$^2$.
- $x'$ = shorter centre-to-centre dimension of closed rectangular stirrup, mm.
- $y'$ = longer centre-to-centre dimension of closed rectangular stirrup, mm.
- $S$ = spacing of transverse torsional reinforcement in direction parallel to longitudinal reinforcement, mm.
- $T_{r\text{,ACI-M-89}}$ = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by ACI-M-89 method, N.mm.
- $A_l$ = area of longitudinal reinforcement required for torsion, mm$^2$.
- $P_h$ = perimeter of centerline of outermost closed transverse torsional reinforcement, mm.

2. **BS M-41 Code\textsuperscript{[1]} method:**

$$T_{r\text{-calc.}} \leq T_{r\text{,BS-M-84}} = 0.0375 x^2 \left( y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6 A_t \cdot x_1 \cdot y_1 (0.95 f_y)}{S} (\cdot, 1)$$

$$T_{r\text{-calc.}} \leq T_{r\text{,BS-M-84}} = 0.0375 x^2 \left( y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6 A_t \cdot x_1 \cdot y_1 (0.95 f_y)}{P_h} (\cdot, Y)$$
Where it is assumed that $f'_c = \cdot \cdot \cdot f_{cu}$,

$T_{r,BS-t} = \text{torsional resistance moment provided by concrete and stirrups, calculated by BS-3V method, N.mm.}$

$T_{r,BS-\ell} = \text{torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by BS-3V method, N.mm.}$

b. Space Truss Analogy:

This new method is considerably simpler to understand and apply than the previous one. It can also be used for prestressed concrete loaded in torsion, a case not covered by the ACI 314M-91 Code. It assumes that the concrete contribution $T_c = \cdot \cdot \cdot$. In this method, the beam cross section is idealized as a tube. After cracking, the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners, and concrete compression diagonals approximately centered on the stirrups. The diagonals are at an angle $\theta$ to the member longitudinal axis.

The most significant difference between the torsion provisions of the ACI Codes and the AASHTO-LRFD specifications is the specified value of $\theta$. For non-prestressed sections, the ACI Code recommends ($\cdot \cdot \cdot$) degrees, while the AASTHO provisions permit a value of about ($\cdot \cdot \cdot$) degrees (based on the longitudinal strain at mid-span of the section). The methods adopted this analogy are:

1. **ACI 314M-94 Code** method:

$$T_{r-calc} \leq T_{r,ACI-t-99} = 0.85 \left[ \frac{1.7A_{oh}\cdot f_{yt}}{S} \right]$$

$$T_{r-calc} \leq T_{r,ACI-\ell-99} = 0.85 \left[ \frac{1.7A_{oh}\cdot f_{yt}}{P_h} \right]$$

Where:

$T_{r,ACI-t-99} = \text{torsional resistance moment provided by stirrups, calculated by ACI-94 method, N.mm.}$

$T_{r,ACI-\ell-99} = \text{torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-94 method, N.mm.}$

$A_{oh} = \text{area enclosed by centerline of outermost closed transverse torsional reinforcement, mm}^2$.

2. **ACI 314M-94 Code** method:
\[ T_{r\text{-calc}} \leq T_{r\text{-ACI-}05} = 0.75 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{S} \right] \] (1.1)

\[ T_{r\text{-calc}} \leq T_{r\text{-ACI-}11} = 0.75 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{P_h} \right] \] (1.1, 1)

Where:
\[ T_{r\text{-ACI-}05} = \text{torsional resistance moment provided by stirrups, calculated by ACI-}05\text{ method, N.mm.} \]
\[ T_{r\text{-ACI-}11} = \text{torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-}11\text{ method, N.mm.} \]

\section{Canadian-\&\& Code method:}

\[ T_{r\text{-calc}} \leq T_{r\text{-Can-}t} = 0.85 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{S} \right] \] (1.1, 1)

\[ T_{r\text{-calc}} \leq T_{r\text{-Can-}ℓ} = 0.85 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{P_h} \right] \] (1.1, 1)

Where:
\[ T_{r\text{-Can-}t} = \text{torsional resistance moment provided by stirrups, calculated by Canadian Code method, N.mm.} \]
\[ T_{r\text{-Can-}ℓ} = \text{torsional resistance moment provided by longitudinal torsion reinforcement, calculated by Canadian Code method, N.mm.} \]

It can be seen that the Canadian Code \(^{14}\) method is symmetric with the ACI \(^{1}\) method.

\section{AASHTO-LRFD-\& Bridge Design Specifications method:}

\[ T_{r\text{-calc}} \leq T_{r\text{-AASHTO-}t} = 0.85 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{S} \cdot \cot \theta \right] \] (1.1, 1)

\[ T_{r\text{-calc}} \leq T_{r\text{-AASHTO-}ℓ} = 0.85 \left[ \frac{1.7A_{oh}\cdot A_f\cdot f_{yf}}{0.9P_h} \cdot \tan \theta \right] \] (1.1, 1)

Where:
\[ T_{r\text{-AASHTO-}t} = \text{torsional resistance moment provided by stirrups, calculated by AASHTO method, N.mm.} \]

\[ \text{\&} \]
\[ T_{r \ AASHTO-t} = \text{torsional resistance moment provided by longitudinal torsion reinforcement, calculated by AASHTO method, N.mm.} \]

\[ \theta = \text{angle of inclination of compression diagonals to the member longitudinal axis, equal to } 39 \text{ degrees}. \]

## 4. EURO-\& Code \[71\] method:

\[ T_{r \ EU} = 1.7 A_{th} \sqrt{\frac{A_t}{S} \cdot f_{st} \cdot A_t} \cdot f_{\text{st}} \]

(13)

Where:

\[ T_{r \ EU} = \text{torsional resistance moment calculated by EURO method, N.mm.} \]

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**Statistical Evaluation of Existing Methods:**

Table (14) shows the results of the cracking torsional moment of (14) specimens (out of 36 tested beams- not all the values of \( T_{cr} \) are included in the references). The comparison between these results and predicted values \( \left( T_{cr-test} / T_{cr-calc.} \right) \) leads to a range of \( (\cdot, 84, 11, 98) \) for the mean of this ratio. It can be seen that the ACI 718M-\& Code \[71\] method is the one with the greatest amount (all the 36 specimens) of unacceptable predictions-based on the value of \( \left( T_{cr-test} / T_{cr-calc.} \right) < 1 \). The lowest ratio for this code is \( (\cdot, 86, 1, 91) \).

In contrast, the ACI 718M-\& Code \[71\], Canadian [10], and Fang and Shiaw [11] methods lead to good predictions with no results of the previous ratio < 1, but the ACI 718M-\& Code \[71\] method seems to be the best due to the lowest values of low and high of the ratio \( \left( T_{cr-test} / T_{cr-calc.} \right) \) among the other two methods. The coefficient of variation (COV) gives a good indication as a measure of the relevance of the prediction method for the ratio \( \left( T_{cr-test} / T_{cr-calc.} \right) \). It can be seen that the difference in COV values of all methods is very small (ranging between 87848 - 87981 percent), therefore this coefficient does not indicate which method is the best.

### Table 14 - Statistical analysis of the ratio \( \left( T_{cr-test} / T_{cr-calc.} \right) \) for 36 tests.

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<td>S.D.</td>
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<tr>
<td>COV, %</td>
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<td>High</td>
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<tr>
<td>High/Low</td>
<td>( 8,948 )</td>
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<td>( 8,948 )</td>
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<td>Number &lt;</td>
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</tr>
</tbody>
</table>
Table (\(\gamma\)) shows the values of the results of the (\(\xi\)) tested beams, compared with the predicted strength (\(T_{u-test} / T_{r-calc}\)). The range of the mean of this ratio is (\(1,1\xi-1,8\xi\)). Based on the value of (\(T_{u-test} / T_{r-calc}\) < 1, the EURO \(^{[17]}\) method leads to unsafe predictions (\(1\xi\) specimens). The lowest ratio for this code is (\(1,9/\xi\)). On the other hand, the ACI \(\xi\Lambda\text{M-}91\) Code \(^{[11]}\) method is the most conservative of the existing methods with only (\(8\)) results with the previous ratio < 1. From table (\(\gamma\)) it can be seen that the ACI \(\xi\Lambda\text{M-}91\), Canadian \(^{[18]}\), and ACI \(\xi\Lambda\text{M-}91\) Code methods lead to the least relevant prediction with a high COV of (\(4,2,\Lambda\gamma\)) percent for each one of them. From this point of view, the best COV is (\(7,1,\Lambda\gamma\)) percent for the ACI \(\xi\Lambda\text{M-}91\) Code \(^{[5]}\) method. The COV values are (\(7,2,\Lambda\gamma, 3,7,1,\gamma\), and \(4,1,\Lambda\gamma\)) percent for BS \(^{[1]}\), AASHTO \(^{[1]}\), and EURO \(^{[1]}\) methods, respectively.

Table \(\gamma\)- Statistical analysis of the ratio (\(T_{u-test} / T_{r-calc}\)) for \(\xi\) tests.

| Detail | ACI- \(\Lambda\text{M-}9\) \(^{[5]}\) | BS \(^{[1]}\) | ACI- \(\Lambda\text{M-}9\) \(^{[1]}\) and Canadian \(^{[18]}\) | ACI- \(\Lambda\text{M-}9\) \(^{[1]}\) | AASHTO \(^{[1]}\) | EURO \(^{[1]}\) | Proposed Eq. (\(\xi\)) |
|--------|--------------------------------|-------|--------------------------------|--------------------------------|--------------|--------------|----------------|------------------|
| \(\bar{x}\) | 1,788 | 1,784 | 1,741 | 1,809 | 1,477 | 1,141 | 1,250 |
| S.D. | 0.776 | 0.718 | 0.772 | 0.766 | 0.532 | 0.473 | 0.140 |
| COV, % | 4,2,\Lambda\gamma | 3,7,\Lambda\gamma | 4,2,\Lambda\gamma | 3,7,\Lambda\gamma | 3,7,1,\gamma | 3,7,1,\gamma | 1,1,\gamma |
| Low | 0.801 | 0.773 | 0.777 | 0.787 | 0.801 | 0.799 | 0.799 |
| High | 2,1,11 | 2,1,17 | 2,4,2 | 2,4,33 | 2,4,34 | 2,4,33 | 2,4,33 |
| High/Low | 2,2,135 | 2,4,36 | 2,4,37 | 2,4,38 | 2,4,38 | 2,4,38 | 2,4,38 |
| Number< 1 | 7 | 12 | 8 | 0 | 10 | 14 | 1 |

Regression Analysis of Test Results:

Using regression analysis, the (\(\gamma\)) and (\(\xi\)) test results of cracking and resistance moment, respectively were analyzed by computer. The aim is to obtain simple and conservative equations to predict cracking torsional moment and torsional resistance moment.
of HSC rectangular section beams under pure torsion, that give the lowest possible COV values of the ratios \( \frac{T_{cr-test}}{T_{cr-calc}} \) and \( \frac{T_{u-test}}{T_{r-calc}} \). This has led to the following prediction equations:

\[
T_{cr-proposed} = 0.115 \left( f' \right)^{0.6} \cdot x^{1.92} \cdot y
\]  

(1) 

Where:

\( T_{cr-proposed} \) = cracking torsional moment calculated by proposed method, N.mm

\[
T_{r-proposed} = 6.2 \frac{A_{0h}^{1.2}}{S^{0.55}} \left( \frac{A_t \cdot A_r}{P_h} \right)^{0.26} \left( f_{yf} \cdot f_{yt} \right)^{0.026}
\]  

(1) 

Where:

\( T_{r-proposed} \) = torsional resistance moment provided by stirrups and longitudinal torsion reinforcement, calculated by proposed method, N.mm.

Equation (1) is based on the \( \gamma \) main parameters \( f' \), \( x \), and \( y \), while equation (1) is based on the \( \gamma \) main parameters \( A_{0h}, S, A_t, A_r, P_h, f_{yf}, \) and \( f_{yt} \). Tables \( \gamma \) and \( \gamma \) had show the summary of statistical evaluation of the proposed methods. The proposed equation (1) which estimates \( T_{cr} \) gives the best COV value of \( \{9,4\} \) percent among all other methods with no result having the ratio of \( \frac{T_{cr-test}}{T_{cr-calc.}} < 1 \) (Table \( \gamma \)).

As shown in Table (1), when the proposed equation (1) [that predicts \( T_r \)] was applied, it led to much safer prediction with only one specimen (out of \( \xi \)) having the ratio of \( \frac{T_{u-test}}{T_{r-calc.}} < 1 \) essentially \( \xi, 9,4 \) \( \cong 1 \). It can be seen that there is a great reduction in the COV value that was obtained by applying the proposed equation (1) [COV = \( \{9,4\} \) percent]. In addition, the value of high/low of the previous ratio was \( \{9,4\} \) for this equation, while the range of this ratio was \( \{9,4,9,4,9,4\} \) for all other methods.

To illustrate the relevance of the proposed method – equation (1), the ratio of \( \frac{T_{u-test}}{T_{r-calc}} \) had been compared by this method with that of the latest available ACI \( T_{\lambda - \lambda} \) Code \( [11] \) procedure (which is the same as the procedure of the ACI \( T_{\lambda - \lambda} \) Code \( [11] \)). These are shown in Figs. \( \lambda, \gamma, \gamma, \gamma, \gamma \), and \( \delta \).

The comparison in Fig.1 between the ACI \( T_{\lambda - \lambda} \) Code \( [11] \) method and the proposed equation (1) shows clearly that for the range of \( f' \) \( \{9,4,9,4,9,4\} \) MPa, the proposed method shows much less scatter in the results. In addition, the number of unsafe results \( \frac{T_{u-test}}{T_{r-calc.}} < 1 \) is greater for the ACI \( T_{\lambda - \lambda} \) Code \( [11] \) method, despite the fact that this ratio is high in several cases (up to \( \gamma, 4,4,4 \)). It is to be noted that there is a tendency toward greater safety with rising \( f' \) values for both methods which is an important advantage since much fewer tests are made on HSC beams in torsion.

Similar conclusions regarding the much greater scatter and the number of unsafe results by the ACI \( T_{\lambda - \lambda} \) Code \( [11] \) method can be seen in Fig.\( \gamma \) (influence of the aspect
ratio $x$, $\gamma$ (influence of sectional area $A_{cp}$), $\delta$ (influence of the nominal stirrup strength $\rho_{v}f_{st}$), and $\sigma$ (influence of the nominal longitudinal steel strength $\rho_{\ell}f_{y\ell}$). For the ACI 394.1-88 code method, there is a significant rise in the factor of safety with rising value of $x$, while the safety factor of the proposed method – equation (98) is not influenced by variation of $x$ value- Fig.7. The influence of $A_{cp}$ is indicated in Fig.8 which shows that for ACI 394.1-88 method, the factor of safety decreases with increasing $A_{cp}$ value. In contrast, the safety factor of the proposed method is approximately constant with variation of $A_{cp}$ value.

Figs. 1 and 8 show clear trends for the overestimation of the influence of the nominal steel strength ($\rho_{v}f_{st}$ and $\rho_{\ell}f_{y\ell}$) by the ACI 394.1-88 method. On the other hand, the proposed method shows no variation in the safety factor with rising value of $\rho_{v}f_{st}$ (ranging between 979-9728 MPa), and $\rho_{\ell}f_{y\ell}$ (ranging between 2744-2879 MPa).

**Fig.1** - Influence of compressive strength of concrete $f'_{c}$ on test results
Fig. 3 - Influence of aspect ratio $y/x$ on test results

Fig. 4 - Influence of sectional area $A_{cp}$ on test results
CONCLUSIONS:

Based on this work, the following conclusions are made:
A simple equation (14) is presented to estimate cracking torsional moment \( T_{cr} \) in HSC rectangular section beams.

Another equation (15) is suggested for predicting torsional resistance moment \( T_r \) of such beams. This method agrees with the recent trend of space truss analogy of shear flow that bases strength only on the contribution of reinforcement as in ACI \( 394^{M-88} \) and later ACI Code versions, Canadian, AASHTO-LRFD, and EURO methods.

The existing methods, give COV values between (1734-1784) percent for the ratio \( T_{cr-test}/T_{cr-calc} \), while the proposed equation (14) leads to a COV value of (4713) percent for this ratio.

The COV value of the existing code design methods ranges between (28791-82742) percent for the ratio \( T_u-test/T_r-calc \). On the other hand, a significant reduction in COV value has been obtained when the proposed equation (14) was applied, which led to the best value of COV - 99799 percent for this ratio.

The proposed method - equation (15) is similar to the EURO one - equation (14), with one major difference. Proposed equation (15) uses powers of values less than 8 for parameters: \( A_t, A_f, P_h, f_{yt} \), and \( f_{yr} \). Therefore, for the ratio \( T_{u-test}/T_{r-calc} \), equation (15) gives low value of essentially 99\% (Table 3). In contrast, EURO method has respective values of 99\% and 11\%.

For a range of \( f'_c \) between (9791-98174) MPa, the proposed equation (15) gives safe prediction (Fig. 1), as well as a rising factor of safety with increasing \( f'_c \). This is considered useful, since the number of available HSC tests in torsion is limited, compared to normal strength concrete \( (f'_c < 97318) \) MPa.

Figs. 2, 3, 8, and 1 show that the factor of safety of the proposed equation (15) is not influenced by rising values of \( \frac{A}{A}, A_{cp}, \rho_y f_{yt} \), and \( \rho_t f_{yt} \). On the other hand, the safety factor of the ACI \( 394^{M-88} \) Code method increases with rising value of \( \rho_y f_{yt} \) and decreases with rising values of \( A_{cp}, \rho_y f_{yt} \), and \( \rho_t f_{yt} \). This may be that the proposed equation (15) relates to these factors more closely than ACI \( 394^{M-88} \) method with practical tests.

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