FLUID MACHINERY

Third Year – Power Engineering
Electromechanical Engineering Department

Lecturer

Dr. Hussein Majeed Salih

Chapter One
Dynamic action of fluid

1. Turbo machines

Are devices in which energy is transferred either to, or from, a continuously flowing fluid by the dynamic action of moving blades on the runner.

Dynamic action of fluid

A stream of fluid entering in a machine such as a hydraulic or steam turbine, a pump or fan has more or less a defined direction. A force is always required to act upon the fluid to change its velocity either in direction or in magnitude. Newton's Third law of motion states that to every action there is an equal and opposite reaction. According to this law an equal and opposite force is exerted by the fluid upon the body that cause the change. This force exerted by virtue of fluid motion is called a "Dynamic force".

The major problem in turbo-machinery is to find the power developed (or consumed) by (or in) a particular machine. The power is determined from the dynamic force or forces which are being exerted by the following fluid on the boundaries of flow passage and which are due to change of momentum. These are determined by applying "Newton's Second Law of Motion".

Newton's Second Law of Motion, linear momentum equation and its application

The fundamental principle of dynamics is Newton's Second Law of Motion which states that " The rate of change of momentum is proportional to the applied force and take place in the direction of the force". More precisely this statement may be written as "The resultant of an external force F_x acting on the particle of mass m along any arbitrarily chosen direction x is equal to the time rate change of linear momentum of the particle in the same direction i.e., x-direction.

Momentum of the body is the product of its mass and velocity.

Let m be the mass of fluid moving with velocity v and let the change of velocity be dv in time dt.

$\therefore$ change of momentum = $m.dv$

And rate of change of momentum = $m.\dfrac{dv}{dt}$

According to the above law;
Dynamic force applied in x-direction = Rate of change of momentum in x-direction.
i.e., $F_x = m.\dfrac{dv_x}{dt}$
For a control volume with fluid entering with uniform velocity v_{x1}, and leaving after time t with uniform velocity v_{x2}, thus:

$$\sum F_x = \frac{m}{t}(v_{x2} - v_{x1})$$

i.e., $\sum F_x = \rho Q(v_{x2} - v_{x1})$

Where Q is the rate of flow and ρ the density.

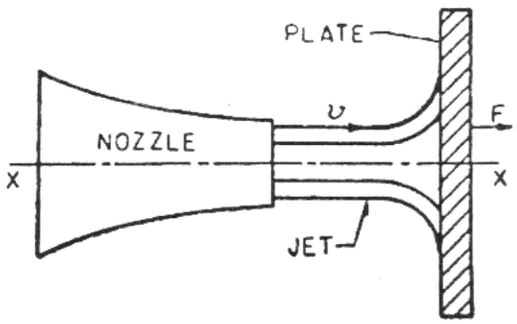
External force F_x may be three kinds:
1. Pressure force and those acting between the fluid and boundary surfaces, or between any two adjacent fluid layer.
2. Inertia force : are those caused by the action of gravity and or centrifugal effects. These are also known as " body forces".
3. Drag forces: are those existing between boundary surfaces and flow. These are also known as " viscous forces".

There are two kinds of applications of linear-momentum equation:
1. To determine the forces exerted by the flowing fluid on the boundaries of flow passage due to change of momentum.
2. To determine the flow characteristics when there is some loss of known quantity of energy in the flow system such as sudden enlargement of a pipe cross-section and hydraulic jump in an open channel flow.

In this cores we are concerned with the applications under (1) above.

1.2 Dynamic force exerted by fluid on fixed and moving flat plates:
1.2.1 Plate normal to jet :

A fluid jet issues from a nozzle and strikes a flat plate with a velocity *v*. The plate is held stationary and perpendicular to the centre line of the jet.



Applying the following equation :

$$\sum F_x = \rho Q(v_{x2} - v_{x1})$$

$$-F_x = \rho Q(0 - v) = -\rho Q v$$

The minus sign on right hand side of the equation indicates that the velocity is decreasing, while this sign used with F_x indicates that the force is acting in the negative direction of x-axis.
Now the force exerted by the fluid on the plate is given by " Newton's Law of Action and Reaction" which will be equal and opposite,

$$F_x = \rho Q v$$

1.2.2 Inclined plate

The dynamic force acting normal to the plate is given by:

$$F = \rho Q v \sin\theta$$

Component of this force F in the direction of jet

$$F_x = F.\sin\theta = \rho Q v \sin^2\theta$$

Let F_S be the force along the inclined surface of plate, and Q_1 and Q_2 the quantities of flow along the surface as shown. As there is no change in pressure elevation before and after the impact and neglecting losses due to impact, no force is exerted on the fluid by the plate in s-direction,

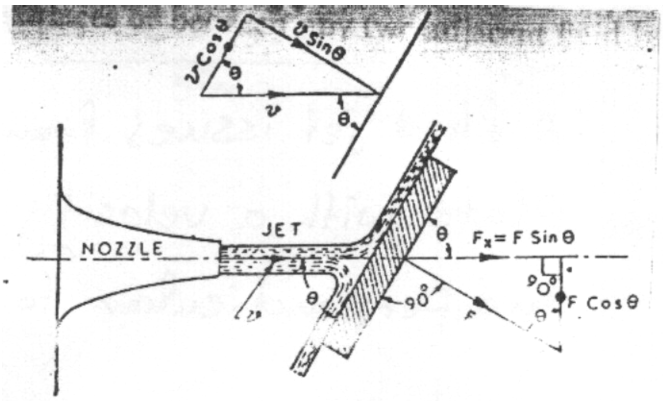
$$F_S = 0 = \rho Q v \cos\theta = \rho Q_1 v - \rho Q_2 v$$

But

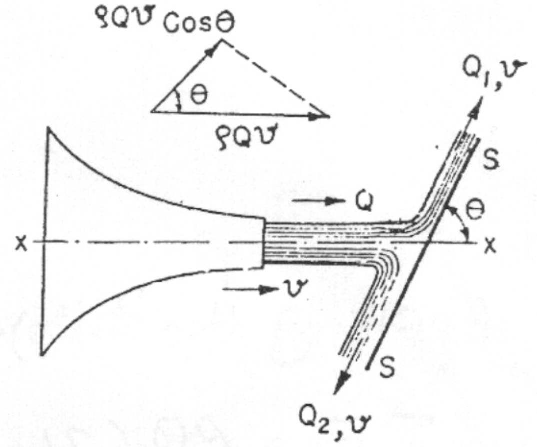
$$Q\cos\theta = Q_1 - Q_2$$

From equation of continuity $Q = Q_1 - Q_2$
From the above two equations:

$$Q_1 = \frac{1}{2}Q(1+\cos\theta) \text{ and } Q_2 = \frac{1}{2}Q(1-\cos\theta)$$



Fluid Jet on Stationary Inclined Plate



Division of Flow Along Surface of Inclined Fixed Plate

1.2.3 Force on moving flat plate

Let the plate in Fig.1 move with a velocity u in the same direction as the jet, then the jet with velocity v has struck the plate. The change in velocity is (u-v).

Thus $Q = a.w = a(v-u)$

Where

a: cross-sectional area.

w: velocity of jet relative to the motion of plate.

v: absolute velocity of jet.

$\therefore$ Force exerted on the fluid by the vane F_x is equal:

$$F_x = \rho Q(u - v)$$

And force exerted by the fluid on the vane is:

$$F_x = \rho Q(v - u) = \rho a(v - u)^2$$

Here the distance between plate and nozzle is constantly increasing by u m/s. A single moving plate is, therefore, not a practical case. If, however, a series of plates as shown in figure, were so arranged that each plate appeared successively before the jet in the same position always moving with a velocity u in the direction of jet, then whole flow from the nozzle is utilized by the plates.

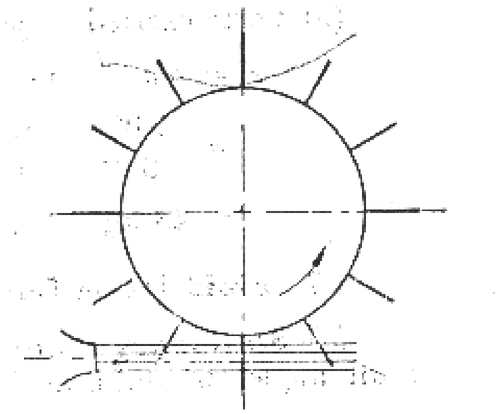


Fig 1.6 Fluid Jet on a Series of Moving Plates

$$\therefore \quad F = \rho av(v - u)$$

Work done on the plates $= F.u$

$$= \rho Q(v - u).u$$

$\because$ kinetic energy of jet $= \frac{1}{2}.m.v^2 = \frac{1}{2}.\rho.Q.v^2$

Where m is the mass of fluid

$\therefore$ Efficiency of system ,

$$\eta = \frac{work\ obtaine}{energy\ input}$$

$$= \frac{\rho.Q(v-u).u}{\frac{1}{2}\rho.Q.v^2} = \frac{2(v-u).u}{v^2}$$

For $\eta_{\max} \Rightarrow \dfrac{d\eta}{du} = 0$

$$\therefore \frac{d}{du}\left(vu - u^2\right) = 0 \quad \Rightarrow v - 2u = 0$$

$$\therefore u = \frac{v}{2}$$

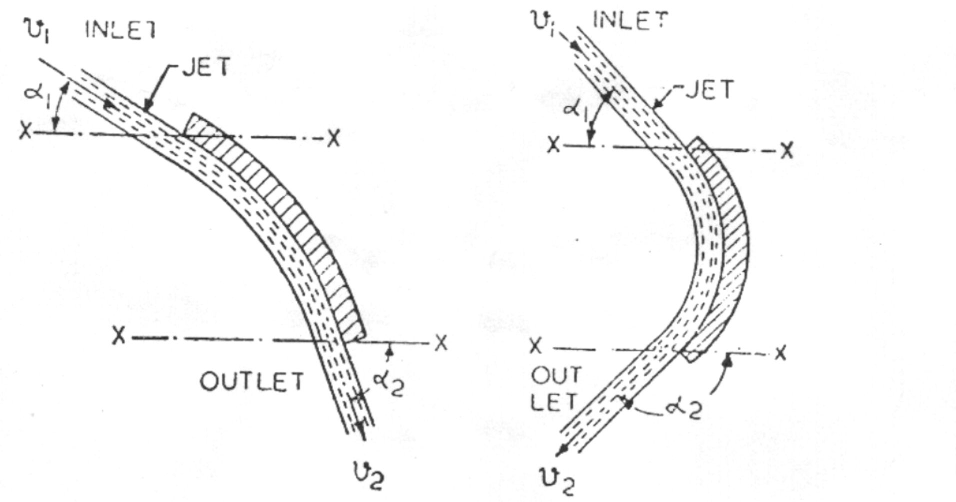
Substituting the value of u in equation of η

$$\eta_{\max} = \frac{2(v-\frac{v}{2}).\frac{v}{2}}{v^2} = 0.5 \quad or \quad 50\%$$

1.3 dynamic force exerted by fluid on stationary and moving plates

1.3.1 on stationary curved plates

The jet impinges on a curved plate at an angle α_1 *and* α_2 at inlet and exit respectively both angles measured with respect to x-direction, as shown in figure:



Jet Falling on Stationary Curved Plate with Acute Discharge Angle Fig Jet Falling on Stationary Curved Plate with Obtuse Discharge Angle

Let v_1 and v_2 be the velocities of jet at inlet and outlet respectively. The velocities v_1 and v_2 will be same as long as there is no friction on the plate.

Velocity of jet at inlet in x-direction $= v_1 \cos\alpha_1$

Velocity of jet at outlet in x-direction $= v_2 \cos\alpha_2$

$\therefore$ Force exerted on the jet by the plate in x-direction can be determine by applying linear momentum equation.

$$F_x = \frac{m}{t} * \text{change of velocity in x-direction.}$$

$$\therefore F_x = \rho Q(v_2 \cos\alpha_2 - v_1 \cos\alpha_1)$$

And force exerted on the plate by the jet in x-direction.

$$\therefore F_x = \rho Q(v_1 \cos\alpha_1 - v_2 \cos\alpha_2)$$

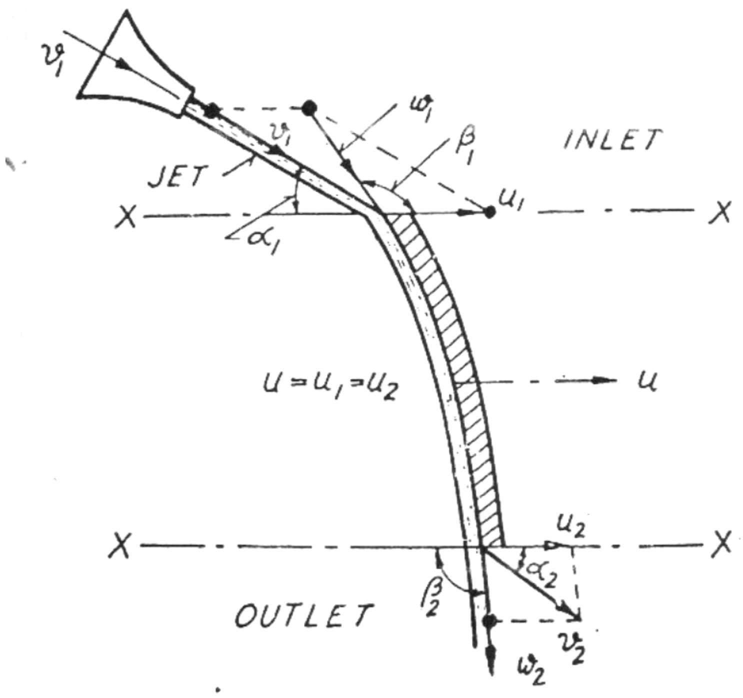
Where $Q = a.v_1$

$$\therefore F_x = \rho.a.v_1.(v_1 \cos\alpha_1 - v_2 \cos\alpha_2)$$

If the curvature of the plate at outlet is such that outlet angle α_2 is more than 90°, then the second term in the bracket i.e., $v_2 \cos\alpha_2$ will be negative. Hence in order to get more force, the curvature of the plate at outlet should be with an obtuse angle α_2.

1.3.2 Single moving plate

Let the angle of curvature of the plate of inlet and outlet with the reversed direction of motion of plate i.e., $-u_1$ be β_1 *and* β_2, see figure. The plate is moving with a velocity u in x-direction. Thus, the velocity of jet relative to the motion of the plate is denoted by w_1. Its direction will tangential to the point of inlet. Its magnitude is determined by the vector sum of u and v_1.



When the jet leaves the plate, its relative velocity will remain equal to w_1 provided there is no decrease in velocity due to friction on the surface of flow. i.e., w_1=w_2. Now the absolute velocity of water at outlet v_2 will be vector sum of w_2 and u.

$\therefore$ Force exerted by the jet on the plate in x-direction or in the direction of motion is determined by applying linear momentum equation:

$$F_x = \frac{m}{t} * \text{change of velocity in x-direction.}$$

$$F_x = \rho Q\left(v_1 \cos\alpha_1 - v_2 \cos\alpha_2\right)$$

Where $Q = a\left(v_1 - u\right)$

$$F_x = \rho a\left(v_1 - u\right)\left(v_1 \cos\alpha_1 - v_2 \cos\alpha_2\right)$$

For $\alpha_2 \rangle \frac{\pi}{2}$, $\cos\alpha_2 \langle 0$

Then the second term in the bracket ($v_2 \cos\alpha_2$) will be negative. Hence in order to get more force, the curvature of the plate should be such that α_2 is obtuse.

1.4 Absolute path of fluid through the machine.

When the jet strikes the moving plates, its position is given by full lines as shown in figure below. As the plate moves with velocity u, it reaches the position shown by dotted lines when the jet leaves it. Now there are two paths traced by water jet, one over the plate surface which is relative to the motion of plate and therefore appears to be moving with the plate; and the other is known as absolute path which appears to be stationary with respect to earth. To determine the absolute path of water particle, take any six points (0 to 5) from inlet to outlet of the plate as shown in figure below. Take the distances $S_{w0-1}, S_{w0-2}, S_{w0-3}$, etc., along the curved path of the plate from the point of entrance 0 to points 1,2,3,etc. These are the distance traversed by the water particle with *w*, the velocity of water relative to the motion of the plate in times t_1, t_2, t_3, etc., respectively. Now take the distances $S_{u0-1}, S_{u0-2}, S_{u0-3}$,etc., in the horizontal direction from points 1,2,3,etc., respectively. These are the distances traveled by the plate moving with u, its peripheral velocity, in time t_1, t_2, t_3, etc., respectively. Join the points $S_{u0-1}, S_{u0-2}, S_{u0-3}$, etc., taken in horizontal direction with a curve which indicates the absolute

path of water particles. The direction of absolute velocity of water at any point will be tangential to the absolute path of water. Similarly the direction of relative velocity of water at any point will be tangential to the relative path of water. The direction of the peripheral velocity of plate is always horizontal. The direction of all the three velocities *u,v,w* being known. The velocity triangle can be drawn at any point of the path. The velocity triangles have been shown at points 0, 3 and 5 in the figure.

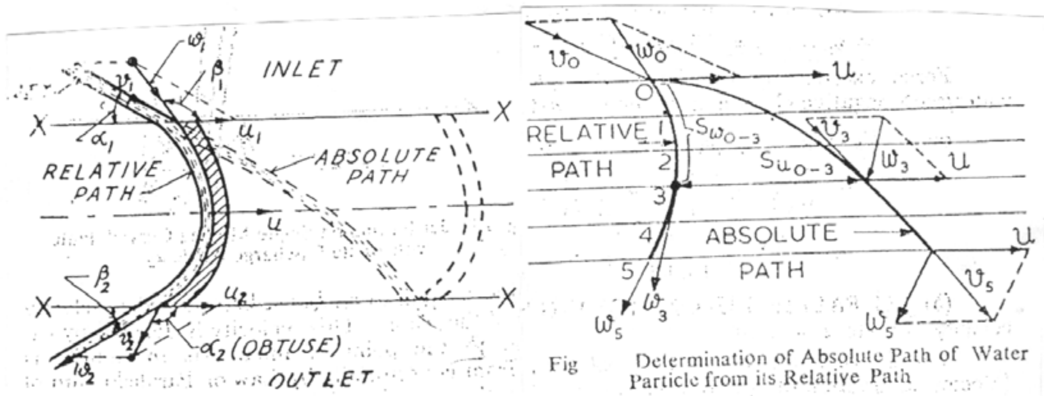


Fig Determination of Absolute Path of Water Particle from its Relative Path

1.5 Velocity diagrams for pump and turbine blades

The velocity is a vector quantity, therefore the velocity triangle is a vector diagram.

Inlet

With refer to figure below, draw $\overline{AC} = v_1$ the absolute velocity of water at inlet at an angle of α_1 to the wheel tangent. Draw $\overline{AB} = u_1$, the peripheral velocity of wheel in the horizontal direction. Join $\overline{BC}$ which gives w_1, the velocity of water relative to wheel motion at inlet, making angle β_1 with wheel tangent. Resolve the absolute velocity of water at inlet into two components v_{u1}, the velocity of whirle at inlet which is the tangent component, and v_{m1}, the velocity of flow which is the normal

and radial component. Mark the directions of the velocities with arrows as shown in the figure.

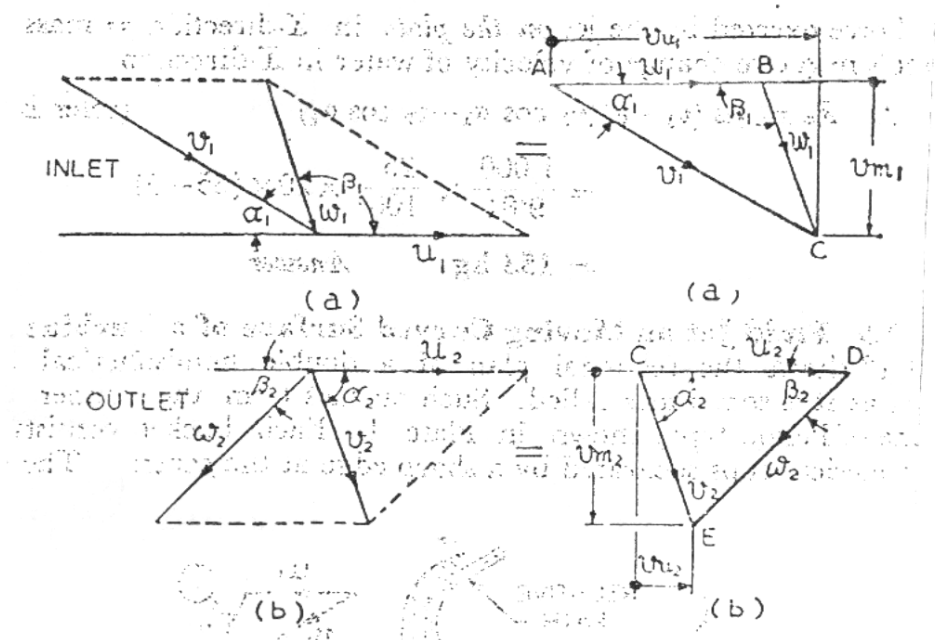


Fig Typical Velocity Triangles for the Flow over Turbine Blade
(a) Inlet (b) Outlet

Outlet

Refer to the previous figure. Draw $\overline{CD} = u_2$, the peripheral velocity of wheel at outlet in the horizontal direction. Draw $\overline{DE} = w_2$, the relative velocity of water at outlet at an angle β_2 to u_2. Join $\overline{CE}$ which gives v_2, the absolute velocity of water at outlet making an angle α_2 to the wheel motion.

$$\overline{v_2} = \overline{w_2} + \overline{u_2}$$

Resolve the absolute velocity of water at outlet into two components v_{u2} *and* v_{m2}, as discussed in inlet. The velocity of whirle at outlet v_{u2} may be positive or negative, depending upon the angle α_2 being acute or obtuse respectively.

Chapter Two
Unit and specific quantities

2. Unit and specific quantities

The rat of flow, speed, power, etc., of hydraulic machines are all function of the working head which is one of the most fundamental of all quantities that go to determine the flow phenomena associated with machines such as turbines and pumps. To facilitate correlation, comparison and use of experimental data, these quantities are usually reduced to unit heads and known as unit quantities e.g. unit flow, unit speed, unit force, unit power and unit torque, etc. Thus two similar turbines having different data can be compared by reducing the data of both turbines under unit head.

For similar reasons it is also convenient to use some specific quantities. A specific quantity is obtained by reducing any quantity to a value corresponding to unit head and some unit size. The later dimension is the inlet diameter of runner in case of reaction turbines and least jet diameter in Pelton turbines. When two different turbines are to be compared, it can be done by reducing their data to specific quantities.

2.1 Unit quantities

2.1.1 Unit rate of flow

Rate of flow = cross-sectional area * velocity of flow

$$Q \propto v_{mo}$$

But

$$v_{mo} = k_{v_{mo}} \cdot \sqrt{2g.H}$$

Where H is the head and $k_{v_{mo}}$ some velocity coefficient.

$$\therefore Q \propto \sqrt{H}$$

or $\therefore Q = k_1 \sqrt{H}$

Now when H=1

$$\therefore Q = k_1 \sqrt{1} = k_1 \quad \Rightarrow \quad k_1 = Q_1$$

Where Q_1 is the unit rate of flow.

$\therefore$ The unit rate of flow = $Q_1 = \dfrac{Q}{\sqrt{H}}$

2.1.2 Unit speed

Let N rpm be the speed of the turbine, then linear or peripheral velocity of runner at inlet.

$$u_1 = \frac{\pi . D_1 . N}{60}$$

Also

$$u_1 = k_{u1} . \sqrt{2g.H}$$

$$\therefore N \propto u_1 \propto \sqrt{H}$$

or $N = k_2 . \sqrt{H}$

Where k_2 is some coefficient.

Now, by definition, unit speed

$$N_1 = k_2 \sqrt{1} = k_2 \quad \Rightarrow \quad k_2 = N_1$$

$$\therefore N_1 = k_2 = \frac{N}{\sqrt{H}}$$

2.1.3 Unit power

The available power of a turbine:

$$P_a = \gamma . Q . H$$

And the developed power is :

$$P_t = \eta_t . \gamma . Q . H$$

Where η_t : turbine overall efficiency

In general turbine power is:

$$P \propto . Q . H$$

But $Q \propto \sqrt{H}$

$\therefore P \propto H.\sqrt{H}$

or $P = k_3 .H^{3/2}$

Where k_3 is some coefficient.

Now, by definition, unit power.

$$P_1 = k_3 (1)^{3/2} = k_3 \quad \Rightarrow \quad k_3 = P_1$$

$$\therefore P_1 = k_3 = \frac{P}{H^{3/2}}$$

2.1.4. Unit force

The force exerted by jet on Pelton runner at its periphery is given:

$$F = \rho Q(v_{u1} - v_{u2})$$

i.e., $F \propto .Q.v_u$

But $Q \propto \sqrt{H}$

And $v_u \propto \sqrt{H}$

$\therefore F \propto H$

or $F = k_4 .H$

Where k_4 is some coefficient.

Now, by definition, unit force.

$$F_1 = k_4 (1) = k_4 \quad \Rightarrow \quad k_4 = F_1$$

$$\therefore F_1 = k_4 = \frac{F}{H}$$

2.1.5 Unit torque:

Torque or turning moment on runner = force at periphery * radius.

$$T = F.R$$

or $T \propto F$

But $F \propto H$

$$\therefore T \propto H$$

or $T = k_5 .H$

Where k_5 is some coefficient.

Now, by definition, unit torque.

$$T_1 = k_5\,(1) = k_5 \quad \Rightarrow \quad k_5 = T_1$$

$$\therefore T_1 = k_5 = \frac{T}{H}$$

2.2 Specific quantities:

2.2.1 Specific rate of flow, or specific flow for a reaction turbine:

For a reaction turbine

$$Q = (\pi.D_o.B_o).v_{mo}$$

The dimension B_o and D_o generally have linear relations with D_1, the runner diameter at inlet, and therefore, since.

$$v_{mo} \propto \sqrt{H}$$

$$Q \propto D_1^2.\sqrt{H}$$

or $Q = k_6.D_1^2.\sqrt{H}$

Now, by definition, specific rate of flow.

$$Q_{11} = k_6.1^2.\sqrt{1} = k_6 \quad \Rightarrow \quad k_6 = Q_{11}$$

$$Q_{11} = k_6 = \frac{Q}{D_1^2.\sqrt{H}}$$

For a Pelton turbine

$$Q = \frac{\pi}{4}.d_1^2.v_1$$

i.e., $Q \propto d_1^2.\sqrt{H}$

where d_1 the least diameter of water jet falling on turbine runner.

$$Q_{11} = \frac{Q}{d_1^2.\sqrt{H}}$$

2.2.2 ***Specific power***

Power, $P \propto Q.H$

Since $Q \propto D_1^2.\sqrt{H}$ for a reaction turbine

$$\therefore P \propto D_1^2.H^{3/2}$$

or $P = k_7 .D_1^2.H^{3/2}$

Now, by definition, the specific power.

$$P_{11} = k_7 .1^2.(1)^{3/2} = k_7 \quad \Rightarrow \quad k_7 = P_{11}$$

$$\therefore P_{11} = k_7 = \frac{P}{D_1^2.H^{3/2}}$$

Similarly for a Pelton turbine.

$$\therefore P_{11} = \frac{P}{d_1^2.H^{3/2}}$$

2.2.3 Specific force of jet on periphery of runner

$F = \rho Q(v_{u1} - v_{u2})$

or $F \propto .Q.v_u$

But $Q \propto \sqrt{H}$

And $v_u \propto d_1^2 . \sqrt{H}$ and $v_u \propto \sqrt{H}$

$\therefore F \propto d_1^2 . H$

or $F = k_8 .d_1^2 .H$

By definition, the specific force.

$F_{11} = k_8 .1^2 .(1) = k_8 \quad \Rightarrow \quad k_8 = F_{11}$

$$\therefore F_{11} = k_8 = \frac{F}{d_1^2 .H}$$

2.2.4 Specific torque

Torque = peripheral force * radius of runner.

$T \propto F$

or $T \propto d_1^2 . H$

or $T = k_9 .d_1^2 . H$

by definition, the specific torque,

$T_{11} = k_9 .1^2 .(1) = k_9 \quad \Rightarrow \quad k_9 = T_{11}$

$$\therefore T_{11} = k_9 = \frac{T}{d_1^2 . H}$$

Alternatively ,

$$T = \frac{P}{\omega}$$

Where ω is the angular velocity

$$\omega \propto \sqrt{H}$$

$$\therefore P \propto D_1^2 . H^{3/2} \quad \Rightarrow \quad T \propto \frac{D_1^2 . H^{3/2}}{H^{1/2}}$$

or $T \propto D_1^2 . H$

$$\therefore \text{ specific torque } \quad T_{11} = \frac{T}{d_1^2 . H}$$

2.2.5 Specific speed of a turbine

$$u_1 = \pi . D_1 . N$$

and $u_1 \propto \sqrt{H}$

$$\therefore D_1 \propto \frac{\sqrt{H}}{N}$$

$$\because P_t \propto Q.H$$

Where $Q \propto D_1^2 . \sqrt{H}$

$$\therefore P_t \propto D_1^2 . H^{3/2}$$

Substituting for D_1,

$$P_t \propto \frac{H}{N^2} . H^{3/2} \quad \Rightarrow \quad P_t \propto \frac{H^{5/2}}{N^2}$$

or $N \propto \sqrt{\dfrac{H^{5/2}}{P_t}}$

or $N = N_s.\dfrac{H^{5/4}}{\sqrt{P_t}}$

where $N_s = \dfrac{N.\sqrt{P_t}}{H^{5/4}}$

If P_t=1 and H=1 $\Rightarrow N_s$=N

N_s is, therefore, by definition, the specific speed of a turbine.

Chapter Three
Hydroelectric power plants

3.1 Introduction

The purpose of a hydroelectric power plant is to harness power from water flowing under pressure. As such it incorporates a number of water driven prime-movers known as water turbines.

Water flowing under pressure has two forms of energy kinetic and potential. The kinetic energy depends on the mass of water flowing and its velocity while the potential energy exists as result of the difference in water level between two points which is known as "head". The water or hydraulic turbine, as it is sometimes named, converts the kinetic and potential energies possessed by water into mechanical power.

3.2 Head and flow rate or discharge

Head is the difference in elevation between two levels of water. The head of a hydroelectric power plant is entirely dependent on the topographical conditions. Head can be characterized as: gross head, and net or effective head.

3.2.1 Gross head

Is defined as the difference in elevation between the head race level at the intake and the tail race level at the discharge side, naturally, both the elevations have to be measured simultaneously. The gross head may vary as both the elevations of water do not remain the same at all times. It is essential to known the maximum and minimum as well as the normal values of the gross head. The normal value would be that for which the plant works most of the time. In rainy season the flood may raise the elevation of tail race, thus, reducing the gross head. On the other hand at the time of draught the same may be increased.

3.2.2 Net or effective head

Is the head obtained by subtracting from gross head all losses in carrying water from the head race to the entrance of the turbine. The losses are due to friction occurring in tunnels, canals and penstocks which lead the water into the turbine. Net or effective head is, therefore, the true pressure difference between the entrance to the turbine casing and the tail race water elevation.

3.2.3 ***Flow rate or discharge of water***

It is the quantities of water used by the water turbine in unit time and is generally measured in (m^3/s) or (l/s).

3.3 ***Essential components of hydroelectric power plant.***

3.3.1 ***Storage reservoir***

The water available from a catchment area is stored in a reservoir, so that it can be utilized to run the turbines for producing electric power according to the requirement through out the year. The storage reservoir may be natural or artificial.

3.3.2 ***Dam with its control works***

Dam is a structure erected on suitable site to provide for the storage of water and to create head. Dam may be built to make an artificial reservoir from a valley or it may be erected in a river to control the flowing water.
Structures and appliances to control the supply of water from the storage reservoir through the dam, are known as control works or head works. The principal elements of control works are:

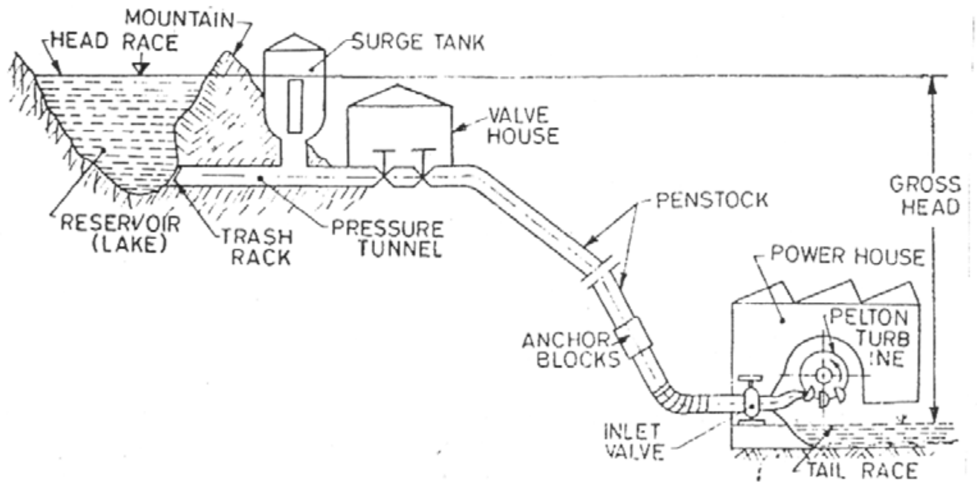
a. Gates and valves.
b. Structures necessary for their operation.
c. Devices for the protection of gates and hydraulic machines, which consist of:

   i. Trashracks: They are made up of a row of rectangular cross sectional structural steel bars placed across the intake opening is an inclined position. They are used to obstruct debris from going into the intake.
   ii. Debris cleaning device fitted on the trashrack.
   iii. Heating element against ice troubles.

3.3.3 ***Waterways with their control works.***

Is a passage through which the water is carried from the storage reservoir to the power house. It may consist of tunnels, canals, forebays and pipes (i.e., penstocks) as shown in figure below. The control works for the tunnels, canals, forebays and pipes may be different types of gates in additional to these, surge tank which is reservoir fitted at some opening made on a long pipe line to receive the rejected flow when the pipeline is suddenly closed by a valve at its steep end. The surge tank, therefore,

controls the pressure variations resulting from the rapid changes in pipeline flow thus eliminating water hammer effects.



High Head Water Power Plant Layout

3.3.4 Power house

Is a building to house the turbines, generators and other accessories for operating the machines.

3.3.5 Tail race

Is a waterway to conduct the water discharged from the turbines to a suitable point where it can be safely disposed of or stored to be pumped back into the original reservoir.

3.3.6 Generation and transmission of electric power

It consists of electrical generating machines, transformers, switching equipments and transmission lines.

3.4 Classification of water power plants

3.4.1 High head water plants

Such plants works under heads ranging from (25 to 2000) m. Water is usually stored up in lakes on high mountains during the rainy season or during the season when the snow melts. The rate of flow should be such

that water can last through out the year. From one end of the lake, tunnels are constructed which lead the water into smaller reservoirs known as forebays. The forebays distribute the water to penstocks through which it is lead to the turbines. These forebays help to regulate the demand of water according to the load on the turbines.

3.4.2 Low head water power plants

Work within the range of (25-80) m of head. These plants usually consist of a dam across a river shown in figure below:

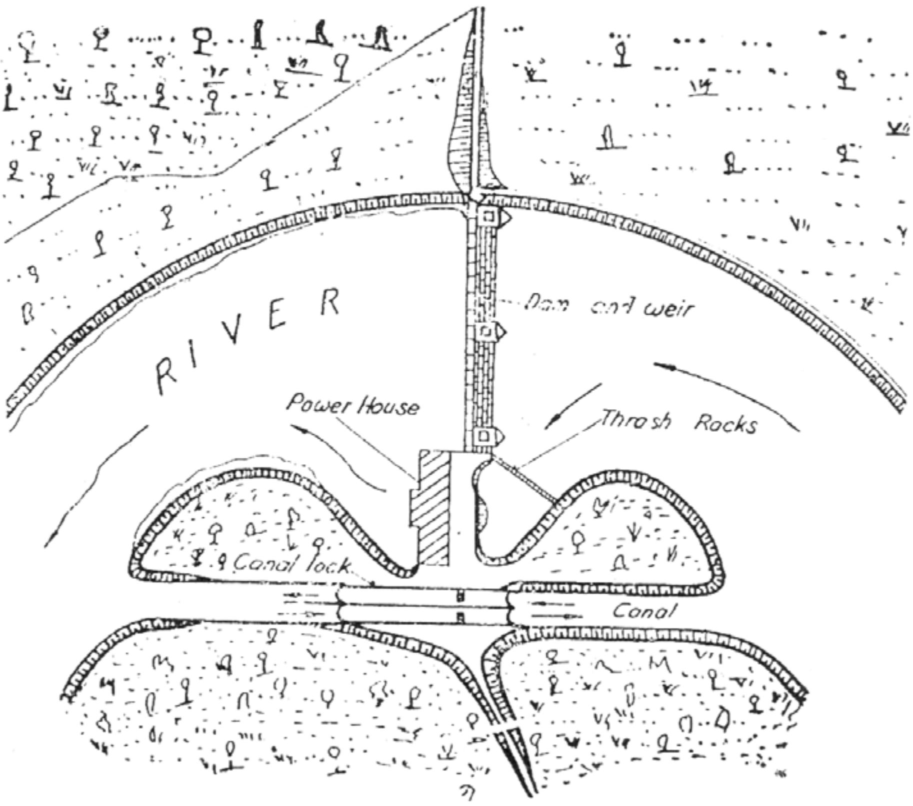
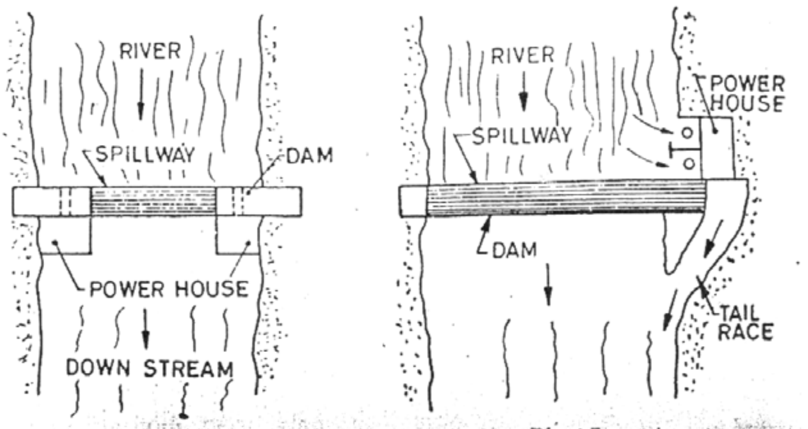


Fig 4.3 Low Head Water Power lant Layout



Low Head Water Power Plant Layouts

3.4.3 Medium head water power plants

Work within (30-500) m.

It is to be noted that the above plants overlap each other. Therefore, it is difficult to classify the plants directly on the basis of head alone. The basis, therefore, technical adopted is the specific speed of the turbine used

for a particular plant, as explained in the previous chapter from the above one can classify the hydraulic turbine according to:

a. Head at the inlet of turbine

- i. High head turbine (250-1800) m. Example: Pelton wheel.
- ii. Medium head turbine (50-250) m. Example: Francis turbine.
- iii. Low head turbine (less than 50) m. Example: Kaplan turbine.

b. Specific speed of the turbine.

- i. Low specific speed turbine ($\langle$ 50) m.
  Example: Pelton wheel.
- ii. Medium specific speed turbine ($50 \langle N_S \langle 250$) m.
  Example: Francis turbine.
- iii. Low head turbine ($\rangle$ 250) m.
  Example: Kaplan turbine.

Chapter Four
Pelton turbine or Impulse turbine

4.1 Introduction
The Pelton wheel turbine is a pure impulse turbine in which a jet of fluid leaving the nozzle strikes the buckets fixed to the periphery of a rotating wheel. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. The turbine is used for high heads ranging from (150-2000) m. The turbine is named after L. A. Pelton, an American engineer.

4.2 Parts of the Pelton turbine
4.2.1 Nozzle and flow control arrangement
The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle converts the total head at the inlet of the nozzle into kinetic energy. The amount of water striking the curved buckets of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit.

4.2.2 Runner and buckets
The rotating wheel or circular disc is called the runner. On the periphery of the runner a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as the splitter. The jet of water strikes on the splitter. The splitter then divides the jet into two equal parts and the water comes out at the outer edge of the bucket. The buckets deflect the jet through an angle between (160^o-165^o) in the same plane as the jet. Due to this deflection of the jet, the momentum of the fluid is changed reacting on the buckets. A bucket is therefore, pushed away by the jet.

4.2.3 Casing
The casing prevents the plashing of the water and discharges the water to tail race. The spent water falls vertically into the lower reservoir or tail race and the whole energy transfer from the nozzle outlet to tail race take place at a constant pressure. The casing is made of cast iron or fabricated steel plates.

4.2.4 ***Breaking jet***

To stop the runner within a short time, a small nozzle is provided which directs the jet of water on to the back of the vanes. The jet of water is called the "breaking jet ". If there is no breaking jet, the runner due to inertia goes on revolving for a long time.

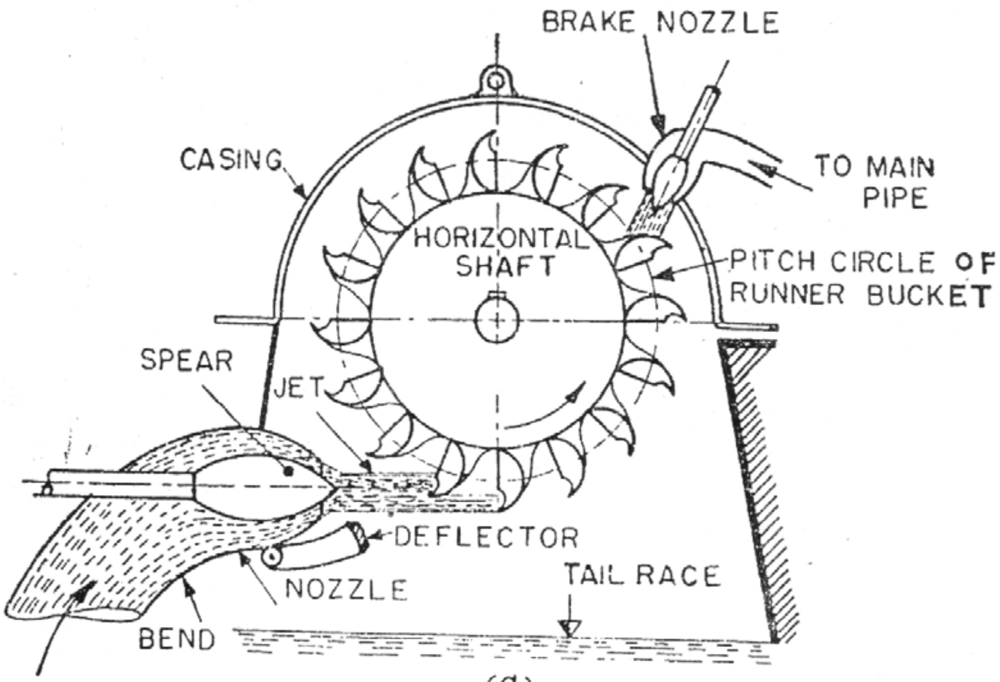


Fig 5.4 (*a*) Single Jet, Horizontal Shaft Pelton Turbine

4.3 ***Force, Power and efficiency***

4.3.1 ***Velocity triangles***

In a Pelton wheel the jet strikes a number of buckets simultaneously. It commences to strike the bucket before it has reached a position directly under the centre of the wheel. The angle which the striking jet makes with the direction of motion of the bucket is denoted by symbol α_1 and in practice it varies from (8^o-20^o). As discussed in chapter one the force exerted by the jet can be calculated with the help of velocity triangles at inlet and outlet.

In drawing the typical velocity triangles for a Pelton runner, the following points should be kept in mind:

$u_1 = u_2$ since $r_1=r_2$

$w_1 = w_2$ assuming there is no friction at the blades

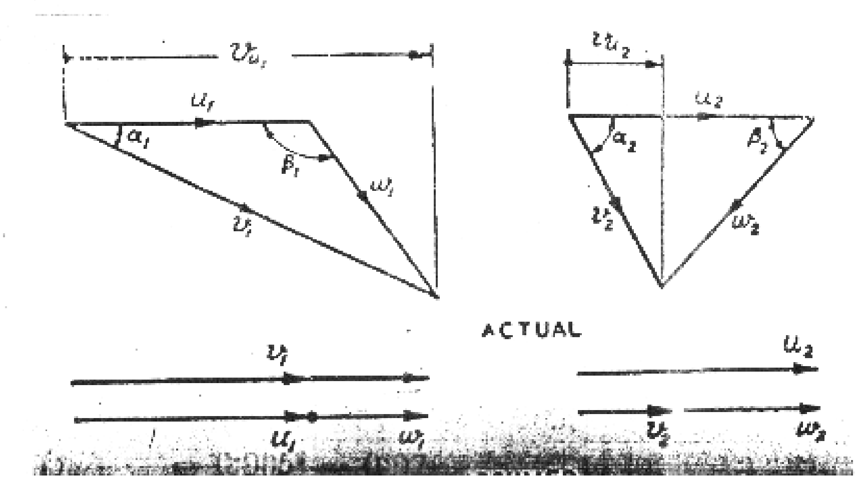
$\alpha_1 \langle \beta_1$

$v_1 \rangle\rangle\rangle v_2$

$\alpha_2 \rangle \alpha_1$

$u_1 \langle v_1$

$v_2 \langle w_2$



4.3.2 ***Force exerted by the jet***

For the calculation of the force exerted by the jet, it is assumed that $\alpha_1 = 0$ i.e., the bucket face is perpendicular to the jet.

If $\alpha_1 = 0$, $\beta_1 = 180^o$

Then $v_{u1} = v_1 \cos\alpha_1 = v_1 = u_1 + w_1$

or $w_1 = v_1 - u_1$

From velocity triangle at outlet

$$v_{u2} = v_2 \cos\alpha_2 = u_2 - w_2 \cos\beta_2$$

For ideal case

$\beta_2 = 0$ i.e., water is deflected back by 180^o

$$\therefore v_{u2} = u_2 - w_2 \quad (\cos 0 = 1)$$

But $u_1 = u_2$ and $w_1 = w_2$

$$\therefore v_{u2} = u_1 - w_1$$

$$= u_1 - (v_1 - u_1) = 2u_1 - v_1$$

Force exerted by the jet in the direction of u_1

$$F_u = \rho Q (v_{u1} - v_{u2})$$

Assuming that the total quantity of Q strikes the bucket.

or

$$F_u = \rho Q [v_1 - (2u_1 - v_1)]$$

$$= 2\rho Q (v_1 - u_1)$$

Also

$$Q = \frac{\pi}{4} . d_1^2 . k_{v1} . \sqrt{2gH}$$

$$v_1 = k_{v1} . \sqrt{2gH} \quad , \; u_1 = k_{u1} . \sqrt{2gH}$$

Substituting these values in F_u

$$F_u = 2\rho\left(\frac{\pi}{4}.d_1^2.k_{v1}\sqrt{2qH}\right)(k_{v1}-k_{u1})\sqrt{2gH}$$

$$= \rho\pi.k_{v1}(k_{v1}-k_{u1}).d_1^2.g.H$$

$$= \gamma\pi.k_{v1}(k_{v1}-k_{u1}).d_1^2.H$$

Hence, force for unit head and unit diameter

$$F_{u11} = \pi.k_{v1}(k_{v1}-k_{u1}).\gamma$$

Per unit head and unit jet diameter.

Force will be maximum when $k_{u1} = 0$, i.e., wheel is at rest.

$$(F_{u11})_{\max} = \pi.k_{v1}{}^2.\gamma$$

Substituting average values $k_{u1} = 0.985$ and $\gamma = 9800\,N/m^3$

$$(F_{u11})_{\max} = \pi.(0.985)^2 * 9800 = 29.87\,kN$$

Per unit head and unit jet diameter.
Under normal working conditions $k_{u1\cong 0.45}$

$$\therefore F_{u11} = \pi * 0.985(0.985-0.45) * 9800 = 16.22\,kN$$

For running speed (i.e., speed at no load or in other words, the vanes or wheel running away from the jet with the same velocity as that of the jet.

$$\therefore k_{u1} = k_{v1}$$

Then $F_{u11} = 0$

Theoretically for maximum efficiency $\dfrac{k_{v1}}{k_{u1}} = 2$

This can be proved as follows:

Power $P = F_u.u_1 = 2\rho Q(v_1 - u_1).u_1$

For given v_1, the power attains maximum value when:

$$\frac{dP}{du_1} = 2\rho Q(v_1 - 2u_1) = 0$$

or $v_1 = 2u_1$ or $\frac{v_1}{u_1} = 2$

but in practice, on account of losses $\frac{k_{v1}}{k_{u1}} = 1.8$

4.3.3 Work done and power developed by the jet

$P_H = F_u.u$ (kW)

$$= \gamma.\pi.k_{v1}.(k_{v1} - k_{u1}).d_1^{\,2}.H.k_{u1}\sqrt{2gH}$$

Power developed per unit head and unit jet diameter:

$$P_{H11} = \frac{P_H}{d_1^{\,2}.H^{3/2}} = \gamma.\pi.k_{v1}.k_{u1}.(k_{v1} - k_{u1}).\sqrt{2g}$$

Substituting average values $k_{v1} = 0.985$ and $k_{u1} = 0.45$

$P_{H11} = 32.32$ kW per unit head and unit jet diameter.

4.3.4 Turbine efficiency

4.3.4.1 Jet efficiency or head efficiency

Head efficiency

$$\eta_H = \frac{P_H}{P_a} = \frac{\gamma.\pi.k_{v1}.(k_{v1} - k_{u1}).d_1^2.H.k_{u1}\sqrt{2gH}}{\gamma.\frac{\pi}{4}.d_1^2.k_{v1}\sqrt{2gH}.H} = 4.k_{u1}.(k_{v1} - k_{u1})$$

For maximum efficiency, assuming k_{v1} as constant,

$$\frac{d\eta_H}{dk_{u1}} = 0 \quad \text{or} \quad 4.(k_{v1} - 2k_{u1}) = 0$$

$$\therefore k_{u1} = \frac{k_{v1}}{2} \quad \Rightarrow \quad u_1 = \frac{v_1}{2}$$

$$\therefore (\eta_H)_{\max} = 4\frac{.k_{v1}}{2}.\left(k_{v1} - \frac{k_{v1}}{2}\right) = k_{v1}^2$$

Taking the average value of $k_{v1} = 0.985$

$$\therefore (\eta_H)_{\max} = 0.985^2 = 0.97$$

In the ideal case $(\eta_H)_{\max} = 1$ but actually it is within 0.96 to 0.98.

4.3.4.2 Volumetric efficiency

The total quantity of water contained in the jet does not strike the bucket and always there is some amount of water slips and falls in the tail race without doing any useful work. Thus, a new factor called volumetric efficiency is introduced.

If ΔQ be the quantity of water lost on account of slip.

$$\eta_Q = \frac{Q - \Delta Q}{Q}$$

Actual value of η_Q is between 0.97 and 0.99.

4.3.4.3 ***Hydraulic efficiency***

Considering the hydraulic losses of the turbine, the hydraulic efficiency can be written as:

$$\eta_h = \left(\frac{H - \Delta H}{H}\right)\left(\frac{Q - \Delta Q}{Q}\right) = \eta_H . \eta_Q$$

4.3.4.4 ***Mechanical efficiency***

There are always some mechanical losses in the transmission of power by the turbine, $\eta_m = 0.97 - 0.99$.

4.3.4.5 ***Final power output from turbines.***

If P_a be the natural available power produced by jet,

$$P_H = P_a . \eta_H$$

Hydraulic power generated by turbines:

$$P_h = P_H . \eta_Q = P_a . \eta_H . \eta_Q$$

Net brake power developed by the turbine shaft,

$$P_t = P_h . \eta_{mech} = P_a . \eta_H . \eta_Q . \eta_{mech}$$

Hence, final or overall efficiency of the turbine:

$$\eta_t = \frac{P_t}{P_a} = \frac{P_a . \eta_H . \eta_Q . \eta_{mech}}{P_a} = \eta_H . \eta_Q . \eta_{mech}$$

Now, it must be remember that in calculating the above values of force, power and efficiencies it was presumed that:

$$\beta_1 = 180^o \quad , \quad \beta_2 = 0^o \quad and \quad w_1 = w_2$$

In practice,

$$\beta_1 = (95 - 110)^o \quad , \quad \beta_2 = (10 - 20)^o \quad and \quad w_2 = (0.96 - 0.98)\ of\ w_1$$

More accurate calculations for force, power and efficiencies can be made by taking into account these facts and making the necessary corrections.

Chapter Five
Reaction turbine (Francis and Kaplan)

5.1 Francis turbine

5.1.1 Main components

- Penstock

Penstock is a waterway to carry water from the reservoir to the turbine casing. Trashracks are provided at the inlet of penstock in order to obstruct the debris entering in it.

- Casing

The water from penstocks enter the casing which is of spiral shape. In order to distribute the water around the guide ring evenly, the area of cross section of the casing goes on decreasing gradually. The casing is usually made of concrete, cast steel or plate steel.

- Guide vanes

The stationary guide vanes are fixed on stationary circular wheel which surrounds the runner. The guide vanes allow the water to strike the vanes fixed on the runner without shock at the inlet. This fixed guide vanes are followed by adjustable guide vanes. The cross sectional area between the adjustable vanes can be varied for flow control at part load.

- Runner

It is circular wheel on which a series of radial curved vanes are fixed. The water passes into the rotor where it moves radially through the rotor vanes and leaves the rotor blades at a smaller diameter. Later, the water turns through 90^o into the draft tube.

- Draft tube
- The pressure at the exit of the rotor of a reaction turbine is generally less than the atmospheric pressure. The water at exit can not be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging the water from the turbine exit to the tail race. In other words, the draft tube is a tube of increasing cross sectional area which converts the kinetic energy of water at the turbine exit into pressure energy.

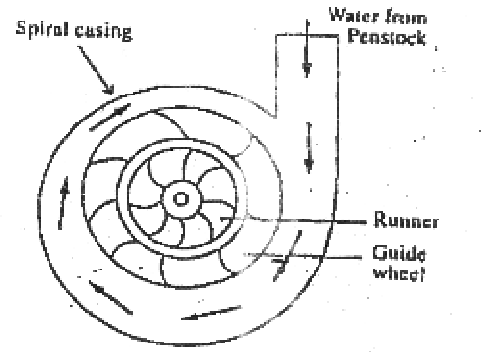


Figure 9.5 Radial flow turbine

5.1.2. Force, power and efficiencies

5.1.2.1 Force and torque

The resultant dynamic force exerted by the water on the runner vanes in the direction of rotation.

$$F_u = \rho.Q(v_{u1} - v_{u2})$$

Force equivalent of motion at inlet $\equiv F_{u1} = \rho.Q.v_{u1}$
Force equivalent of motion at outlet $\equiv F_{u2} = \rho.Q.v_{u2}$

The action of the stream on the vanes of a radial flow runner can be determined by finding the total torque produced by all elementary forces over the vanes. The runner is considered to be divided into a number of parts of equal area, each constituting what may be called a functional turbine.
Let dM_H be the turbine moment of a functional turbine and dQ the quantity of water flowing through it.
Equivalent turning moment of fluid motion at inlet :

$$dM_{H1} = dF_{u1}.R_1 = \rho.dQ.v_{u1}.R_1$$

Similarly equivalent turning moment at outlet:

$$dM_{H2} = dF_{u2}.R_2 = \rho.dQ.v_{u2}.R_2$$

Resultant torque:

$$dM_H = dM_{H1} - dM_{H2} = \rho.dQ(v_{u1}.R_1 - v_{u2}.R_2)$$

$$\therefore M_H = \int dM_H = \rho \int_0^Q (v_{u1}.R_1 - v_{u2}.R_2).dQ$$

$$\therefore M_H = \rho.Q.(v_{u1}.R_1 - v_{u2}.R_2)$$

5.1.2.2 Power

Let P_H be the power developed by the turbine.
Then the power of a functional turbine:

$$dP_H = dM_H.\omega$$

$$= \rho.dQ.(v_{u1}.R_1.\omega - v_{u2}.R_2.\omega)$$

$$= \rho.dQ.(v_{u1}.u_1 - v_{u2}.u_2)$$

$$\therefore P_H = \int dP_H = \rho \int_0^Q (v_{u1}.u_1 - v_{u2}.u_2).dQ$$

$$\therefore P_H = \rho.Q.(v_{u1}.u_1 - v_{u2}.u_2)$$

5.1.3 Efficiency

- ***Head efficiency***

  Let the total head loss in turbine be ΔH and the net operating head H.

Then efficiency, $\eta_H = \dfrac{H - \Delta H}{H} = 1 - \dfrac{\Delta H}{H}$

This is known as the "Head efficiency"

But $P_H = P_a.\eta_H = \gamma.Q.H.\eta_H$

$\therefore \rho.Q.(v_{u1}.u_1 - v_{u2}.u_2) = \gamma.Q.H.\eta_H$

or $\quad \eta_H = \dfrac{(v_{u1}.u_1 - v_{u2}.u_2)}{g.H}$

$$= \frac{2(v_{u1}.u_1 - v_{u2}.u_2)}{2g.H}$$

Substituting $\quad v_{u1} = k_{v_{u1}}.\sqrt{2gH}$

$$v_{u2} = k_{v_{u2}}.\sqrt{2gH}$$

$$u_1 = k_{u1}.\sqrt{2gH}$$

$$u_2 = k_{u2}.\sqrt{2gH}$$

$$\therefore \eta_H = 2(k_{v_{u1}}.k_{u1} - k_{v_{u2}}.k_{u2})$$

but $u_2 = \dfrac{R_2}{R_1}u_1$ $\qquad$ (since $\omega = \dfrac{u_1}{R_1} = \dfrac{u_2}{R_2}$)

$$k_{u_2} = \frac{R_2}{R_1}.k_{u_1}$$

$$\therefore \eta_H = 2.k_{u_1}\left(k_{v_{u1}} - k_{v_{u2}}.\frac{k_{u_2}}{k_{u_1}}\right)$$

$$= 2.k_{u_1}\left(k_{v_{u1}} - k_{v_{u2}}.\frac{R_2}{R_1}\right)$$

If the discharge in radial, i.e., $\alpha_2 = \dfrac{\pi}{2}$

Then, $\cos\alpha_2 = 0$, and $k_{v_{u2}} = 0$

$$\therefore \eta_H = 2k_{u1}.k_{v_{u1}}$$

- ***Volumetric efficiency***

Let ΔQ be the amount of water that passes over to the tail race through some passage.

$$\Delta Q = \Delta Q_u + \Delta Q_l$$

where ΔQ_u : upper clearance loss
ΔQ_l : lower clearance loss

Volumetric efficiency $\eta_Q = \dfrac{Q - \Delta Q}{Q} = 1 - \dfrac{\Delta Q}{Q}$

- ***Hydraulic efficiency***

Total hydraulic loss in turbine is made up of total head loss and volumetric loss. The actual hydraulic power of the turbine is obtained by considering the total loss.

Thus, $P_h = \gamma(Q - \Delta Q)(H - \Delta H)$

and $P_a = \gamma.Q.H$

Hydraulic efficiency $\eta_h = \dfrac{P_h}{P_a} = \dfrac{\gamma(Q - \Delta Q)(H - \Delta H)}{\gamma.Q.H}$

$$= \left(1 - \frac{\Delta Q}{Q}\right)\left(1 - \frac{\Delta H}{H}\right)$$

$$\eta_h = \eta_Q . \eta_H$$

- ***Mechanical efficiency***

Brake power of a turbine is the hydraulic power minus the mechanical losses.

$$P_t = P_h - \Delta P_{mech.}$$

Mechanical loss may be due to bearing friction and winding.

Now, mechanical efficiency:

$$\eta_{mech.} = \frac{P_h - \Delta P_{mech.}}{P_h} = 1 - \frac{\Delta P_{mech.}}{P_h}$$

$\therefore$ Brake power $P_t = P_h . \eta_{mech.}$

- ***Overall efficiency***

Let η_t be the overall efficiency of the turbine, then:

$$\eta_t = \frac{P_t}{P_a} = \frac{P_h . \eta_{mech.}}{P_a}$$

$$= \frac{P_a . \eta_Q . \eta_H . \eta_{mech.}}{P_a} = \eta_Q . \eta_H . \eta_{mech.}$$

5.1.4 Discharge through Francis turbine

Q=area across flow * velocity of flow

Area across radial flow at inlet $= (\pi D_1 - z_2 t).B_o$

where D_1: the inlet diameter of runner.
z_2 : the number of blades in runner.
t : the thickness of blades.
B_o : the height of runner $\cong$ height of guide vanes.

Radial velocity of flow at inlet:

$$v_{m1} = v_1 . \sin \alpha_1$$

$$\therefore Q = (\pi D_1 - z_2 t) . B_o . v_{m1}$$

Now the area occupied by blade edges is usually 5 to 10 % of πD_1.

In general, $(\pi D_1 - z_2 t) = k.\pi.D_1$

where k = percentage of net flow area (0.9 – 0.95)

also B_o is proportional to D_1

let $\dfrac{B_o}{D_1} = k_B \quad \Rightarrow \quad B_o = k_B . D_1$

and $v_{m1} = k_{v_{m1}} . \sqrt{2gH}$

$$\therefore Q = k.\pi.k_B.k_{v_{mi}} . D_1^2 . \sqrt{2gH}$$

The factor $\left(k.\pi.k_B.k_{v_{mi}} . \sqrt{2g}\right)$ which is constant for geometrically similar turbines is called specific flow and is denoted by Q_{11}.

Then, $Q = Q_{11} . D_1^2 . \sqrt{H}$

Q_{11} is therefore, the quantity of water required by the turbine when working under unit head and with unit runner inlet diameter (D_1).

5.2 Axial flow reaction turbine

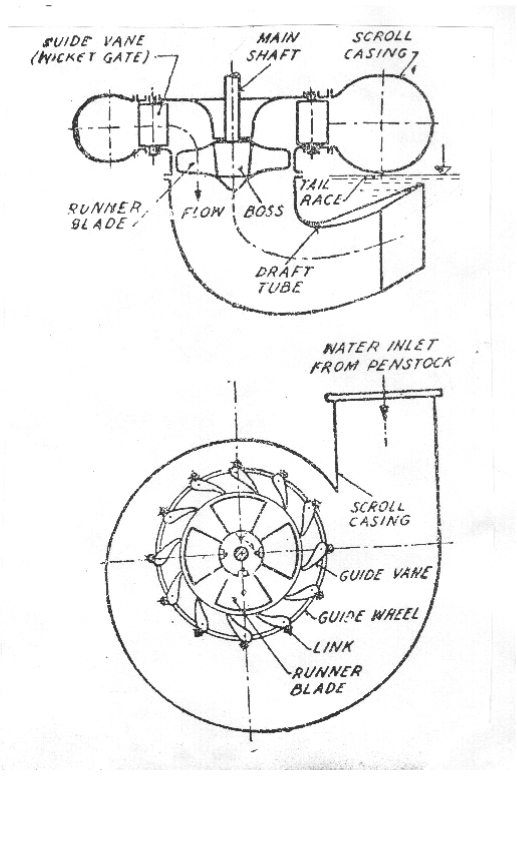
In axial flow reaction turbine, water flows parallel to the axis of rotation of the shaft. In a reaction turbine, the head at the inlet of the turbine is the sum of pressure energy and kinetic energy and a part of the pressure energy is converted into kinetic energy as the water flows the runner. For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft which is made larger is known as "hub" or "boss". The vanes are fixed on the hub and hence the hub acts as a runner for the axial flow reaction turbine. The two important axial flow turbine are:

- Propeller turbine.
- Kaplan turbine.

The vanes are fixed to the hub and are not adjustable, then the turbine is known as propeller turbine, on the other hand if the vanes on the hubare adjustable, the turbine is known as Kaplan turbine. It is named after V. Kaplan, an Austrian engineer. Kaplan turbine is suitable where a large quantity of water at low heads (up to 400 m) is available.

The main parts of Kaplan turbine are:

- Scroll casing.
- Guide vanes.
- Hub with vanes.
- Draft tube.



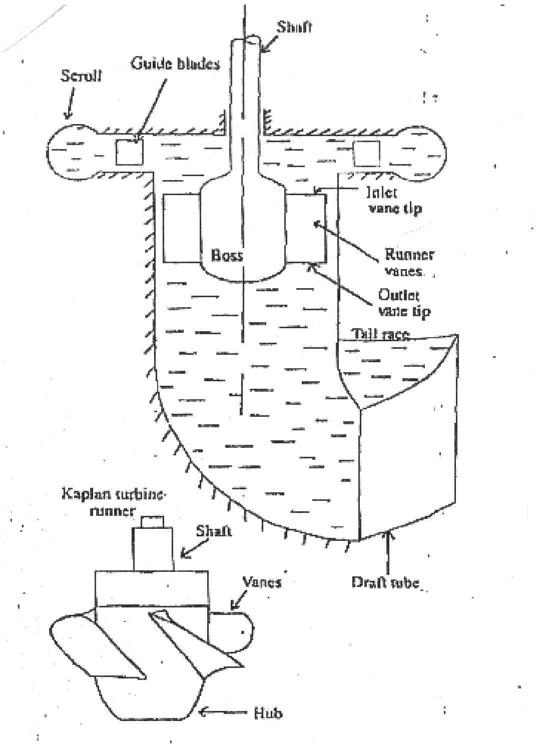
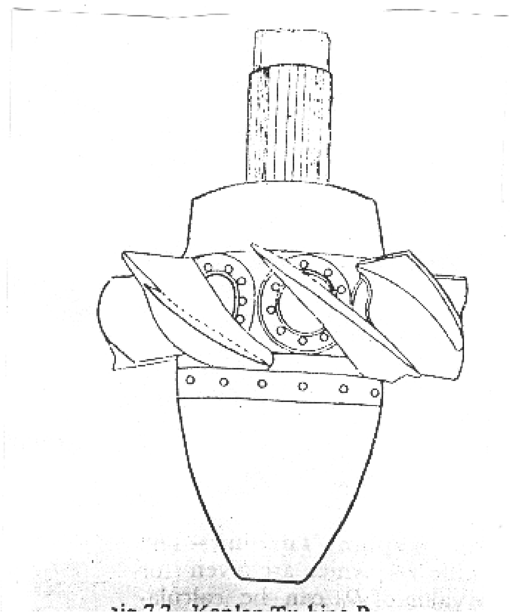


Figure 9.9 Kaplan turbine



5.2.1 Force, torque, power and efficiency

For the calculations of force, torque, power and various efficiencies, the formulae derived for Francis turbine, will hold good for propeller or Kaplan turbine also.

5.2.2 Rate of flow through propeller or Kaplan runner

Q= area across flow * velocity of flow.

Area across flow= $k\,\frac{\pi}{4}\left(D_1^2 - d_1^2\right)$

Where k : percentage of net flow area obtained after deducting area occupied by the blades.

$D_1 = D_2$: the external diameter of the runner.
d_1 : the diameter of the runner boss or hub.

Velocity of flow $v_{m1} = k_{vm1}\sqrt{2gH}$

$$\therefore Q = \frac{\pi}{4}.k.k_{vm1}.\sqrt{2g}.\left(D_1^2 - d_1^2\right).\sqrt{H}$$

or $Q = Q_{11}.\left(D_1^2 - d_1^2\right).\sqrt{H}$

where $Q_{11} = \frac{\pi}{4}.k.k_{vm1}.\sqrt{2g}$

Q_{11} is a factor, constant for geometrically similar turbines (specific flow). Its value from 0.6 to 2.175 depending upon N_s .

Chapter Six
Pumps

6.1 Reciprocating pumps, definition and working principle

The reciprocating pump is a positive acting type which means it is a displacement pump which creates lift and pressure by displacing liquid with a moving member or piston. A reciprocating pump consists primarily of a piston or plunger reciprocating inside a close fitting cylinder, thus performing the suction and delivery strokes. The chamber or cylinder is alternately filled and emptied by forcing and drawing the liquid by mechanical motion. Suction and delivery pipes are connected to the cylinder. Each of the two pipes is provided with a non-return valve. The function of the non-return or one way valve is to ensure unidirectional flow of liquid. Thus the suction pipe valve allows the liquid only to enter the cylinder while the delivery pipe valve permits only its discharge from the cylinder. Volume or capacity delivered is constant regardless of pressure, and is varied only by speed changes. If H_s and H_d be the suction and delivery heads respectively of the pump, then $\left(H = H_s + H_d\right)$ is known as its "static head".

6.1.1 Applications

Reciprocating pump generally operates at low speeds and is therefore to be coupled to an electric motor with V-belt. It is best suited for relatively small capacities and high heads.

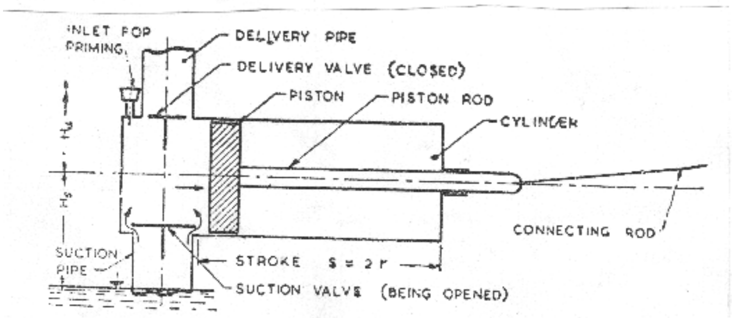


Fig 11.1 Reciprocating Single Acting Piston Pump. (Piston may be direct connected to the connecting rod)

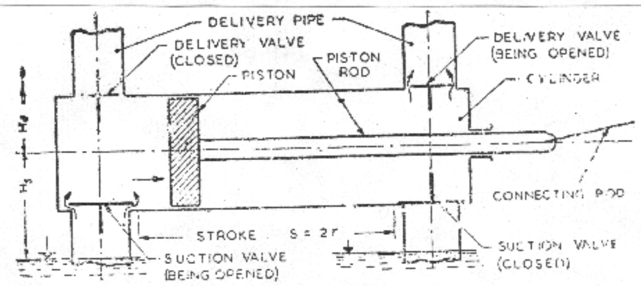


Fig 11.2 Double Acting Single Cylinder Piston Pump

6.1.2 Piston pump

a) Single acting

It consists of one suction and one delivery pipe simply connected to one cylinder as shown in the previous figure, let:

A: the cross sectional area of the piston in m^2.
a: the cross sectional area of the piston rod in m^2.
S: the stroke of the piston in m.
N: the speed of crank in rpm.

Then, average rate of flow= $\dfrac{A.S.N}{60}$ $\left(m^3/s\right)$.

Force on piston forward stroke= $\gamma.H_s.A$ (kN).

Force on piston backward stroke= $\gamma.H_d.A$ (kN).

Neglecting head losses in transmission and at values, power of the pump.

$$= \gamma.Q.H = \frac{\gamma.(A.S.N).(H_s + H_d)}{60} \qquad (kW)$$

b) Double acting single cylinder pump.

It has two suction and two delivery pipes connected to one cylinder.

$$Q = A.S.\frac{N}{60} + (A - a).S.\frac{N}{60}$$

$$= \frac{S.N.(2A - a)}{60} \cong \frac{2A.S.N}{60} \qquad (\text{ m}^3/\text{s})$$

Force acting on piston in forward stroke:

$$= \gamma.H_s.A + \gamma.H_d.(A - a) \qquad (\text{kN})$$

Force acting on piston during backward stroke:

$$= \gamma.H_s.(A - a) + \gamma.H_d.A \qquad (\text{kN})$$

c) Two-throw pump.

It has two cylinders each equipped with one suction and one delivery pipe. The pistons reciprocating in the cylinders are moved with the help of connecting rods fitted with a crank at 180^o.

$$Q = \frac{2A.S.N}{60} \qquad (\text{m}^3/\text{s})$$

d) Three-throw pump.

It has three cylinders and three pistons working with three connecting rods fitted with a crank at 120^o.

$$Q = \frac{3A.S.N}{60} \qquad (m^3/s)$$

6.1.3 Rate of delivery

The reciprocating pumps are run by crank and connecting rod mechanism which gives the motion of piston as simple harmonic. In simple harmonic motion (SHM) the velocity of piston is equal to $\omega r \sin\theta$.

The rate of delivery= cross sectional area of the pipe * velocity of water.

The velocity of water in the pipe $= \omega . r . \sin\theta . \frac{A}{a}$

Where A: cross sectional area of the piston.
a : cross sectional area of the pipe.

Thus, the rate of delivery into or out of the pump varies as $\sin\theta$ and it is therefore not uniform.

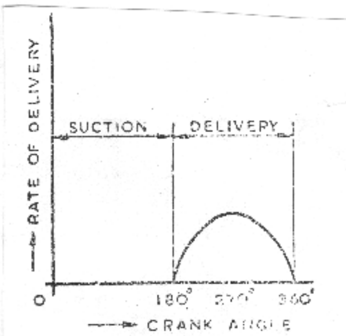


Fig 11.7 Rate of Delivery *vs* Crank Angle for Single Acting Piston or Plunger Pump

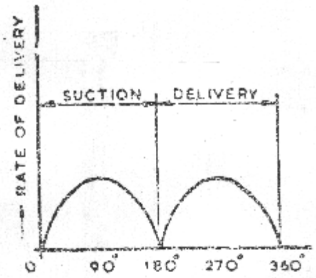


Fig 11.8 Rate of Delivery *vs* Crank Angle for Double Acting Single Cylinder Piston or Plunger Pump

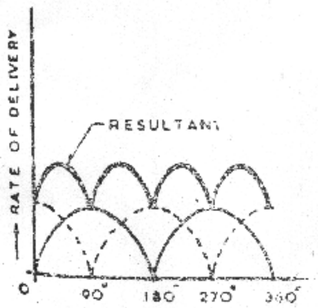


Fig 11.9 Rate of Delivery *vs* Crank Angle for Double Acting Driven by Two Cranks at Right Angles Piston or Plunger Pump

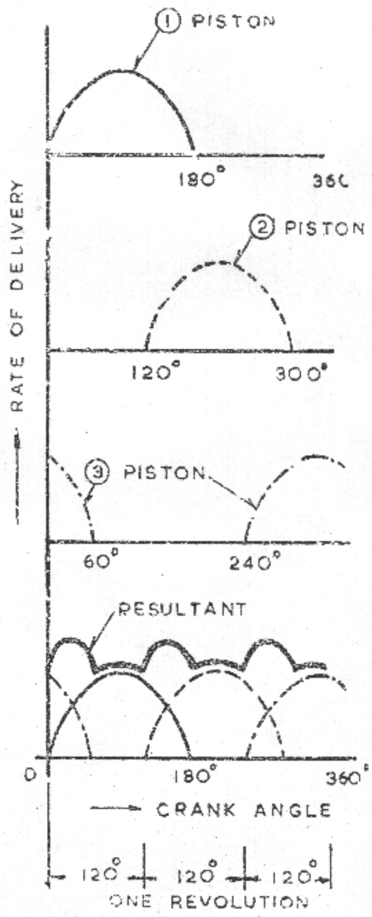


Fig 11.10 Rate of Delivery *vs* Crank Angle for Three-Throw Pump

6.1.4 Velocity and acceleration of water in reciprocating pumps

If at any instant separation takes place (discontinuity of flow), it will result in a sudden change of momentum of the moving water. This causes an impulsive force which is responsible for phenomenon of "water hummer" in reciprocating pump causes heavy shocks. As a result of this, pump may be fracture. To eliminate this, driving force must be sufficient

to accelerate the mass of water following the piston at the same rate as the piston itself. Assuming that the pressure inside the cylinder is zero when the piston moves forward, total suction pressure is equal to atmospheric pressure and it has to work against the following forces:

- Work against gravity equivalent to suction height H_s.
- Work against inertial force equivalent to head H_{as}.
- Work against frictional forces equivalent to head H_f.
- Work against force required to open the non-return valve equivalent to head H_{vs}.
- Work against friction in the valve equivalent to head H_{vfs}.
- Work against kinetic head due to velocity of water in the suction pipe equivalent to head $\frac{v_s^2}{2g}$.
- Work against vapor pressure equivalent to head H_{vap}.

$$\therefore \quad H_{atm} = H_s + H_{as} + H_{fs} + H_{vs} + H_{vfs} + \frac{v_s^2}{2g} + H_{vap}$$

Now, let:

f_p : the acceleration of piston.
A : the cross-sectional area of piston.
a_s : the cross-sectional area of suction pipe.

Then acceleration of water in suction pipe.

$$f_s = f_p.\frac{A}{a_s}$$

Acceleration force= mass * acceleration

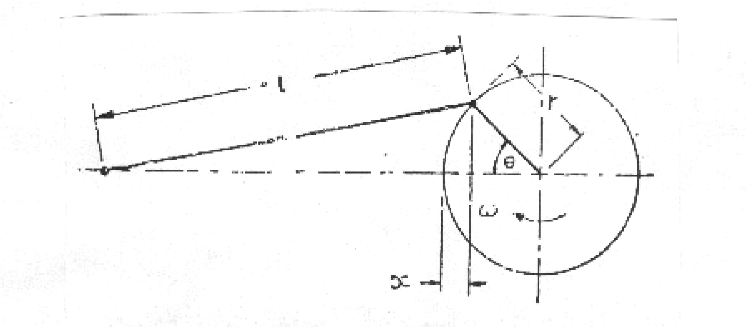
$$F_{as} = \rho.a_s.L_s.f_s \qquad \text{(kN)}$$

Where L_s: the length of suction pipe.

Force per unit cross-sectional area $f_{as} = \rho.L_s.f_s$ (kN/m^2)

Head due to this force $H_{as} = L_s.\frac{f_s}{g}$ (m of water).

Now, consider the following figure:



$x = r - r\cos\theta$
where r is the radius of the crank.

or $x = r - r\cos\omega t$

$\therefore$ velocity of piston $\frac{dx}{dt} = \omega.r.\sin\theta$

Acceleration of piston $\frac{d^2x}{dt^2} = \omega^2.r.\cos\theta$

Now, $f_s = f_p.\frac{A}{a_s} = \omega^2.r.\cos\theta.\frac{A}{a_s}$

and $H_{as} = L_s.\frac{f_s}{g} = \frac{L_s}{g}.\omega^2.r.\cos\theta.\frac{A}{a_s}$

This is maximum when $\cos\theta = 1 \quad or \quad \theta = 0^o$, i.e., when piston is at its dead centre.

$\therefore \quad (H_{as})_{\max} = \frac{L_s}{g}.\frac{A}{a_s}.\omega^2.r$

6.2 Centrifugal pumps

6.2.1 Definition

The hydraulic machines that convert mechanical energy into pressure energy, by means of centrifugal force acting on the fluid are called as "centrifugal pumps". The centrifugal pump is similar in construction to Francis turbine. But the difference is that the fluid flow is in a direction opposite to that in the turbine.

6.2.2 Principle of operation

The first step in the operation of a pump is priming that is, the suction pipe and casing are filled with water so that no air pocket is left. Now the revolution of the pump impeller inside a casing full of water produces a forced vortex which is responsible for imparting a centrifugal head to the water. Rotation of impeller effects a reduction of pressure of the centre. This causes the water in the suction pipe to rush into the eye. The speed of the pump should be high enough to produce centrifugal head sufficient to initiate discharge against the delivery head.

6.2.3Classificationof centrifugal pumps

Centrifugal pumps can be classified according to :

a) Working head
- Low lift centrifugal pumps (up to 15 m).
- Medium lift centrifugal pumps (15-40 m).
- High lift centrifugal pumps (above 40 m).

b) Type of casing
- Volute pump.
- Turbine pump or diffusion pump.

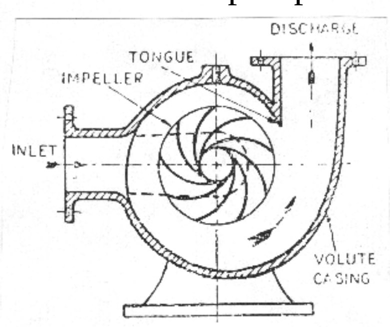
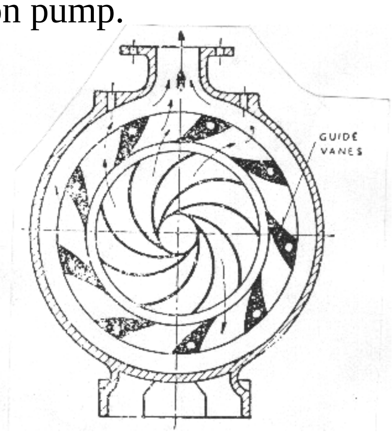


Fig 12.2 Volute Pump



c) Number of impeller
- Single stage centrifugal pump.
- Multi stage centrifugal pump.

d) Number of entrances to the impeller
- Single entry.
- Double entry.

e) Disposition of shaft
- Horizontal.
- Vertical.

f) Liquid handled.

g) Specific speed.

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \;, \qquad H = \frac{v^2}{2g}$$

h) Non-dimensional factor k_s.

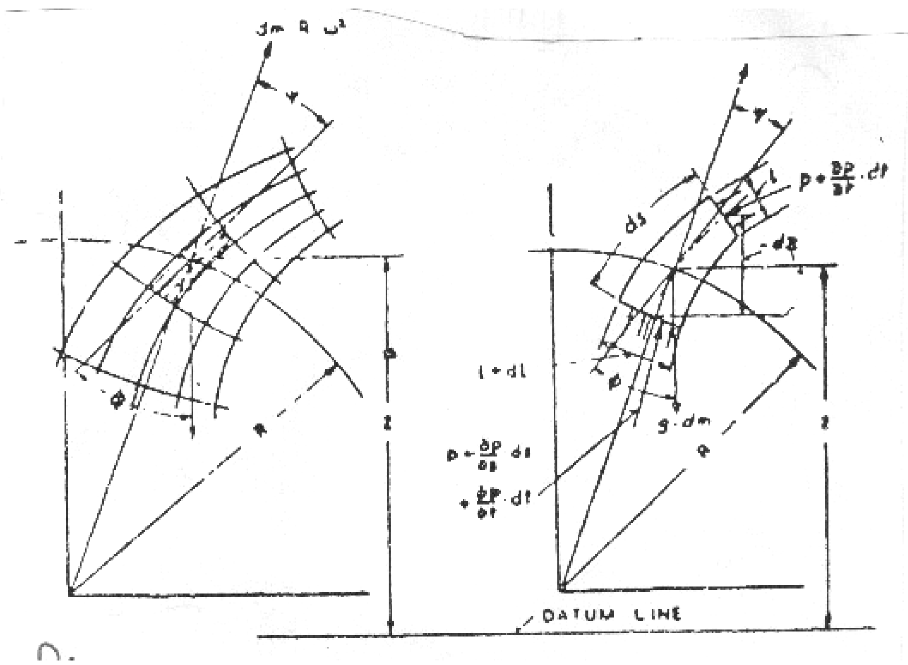
The specific speed N_s is a dimensional quantity, but a k_s is a non-dimensional quantity.

$$k_s = \frac{Q.N^2}{v^3}$$

Q: flow rate. (m^3/s)
N: speed. (rpm)
v: velocity of water $= \sqrt{2gH}$ (m/s) , H: total head.

6.2.4 Bernoulli's equation for relative motion

Consider the motion of fluid inside a turbine runner or an impeller of a centrifugal pump.



For steady flow:

$$\frac{\partial P}{\partial t} = 0, \qquad \frac{\partial w}{\partial t} = 0$$

Acceleration in the direction of relative velocity w,

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial t} + w\frac{\partial w}{\partial s_w}$$

$$= 0 + w\frac{dw}{ds_w}$$

Mass of fluid accelerated $= \rho.(b.l.ds_w)$

Force acting on the element are:

- Weight $=\gamma.b.l.ds_w$.
- Centrifugal force $=\rho.b.l.ds_w.R.\Omega^2$.
- Pressure difference $=b.l.P-(P+dP).b.(l+dl)$
  $=-b.l.dP$ (neglecting dl)

Now resultant force in the direction of stream line:

= mass * acceleration

$$\gamma.b.l.ds_w\cos\phi-b.l.dP+\rho.b.l.ds_w.R.\Omega^2.\cos\psi=\rho.b.l.ds_w.w.\frac{dw}{ds_w}$$

Simplifying

$$w.\frac{dw}{g}=ds_w\cos\phi-\frac{dP}{\gamma}+\frac{R.\Omega^2}{g}.ds_w.\cos\psi$$

Sub. $ds_w.\cos\phi=-dz$ and $ds_w.\cos\psi=dR$

Then $\dfrac{w.dw}{g}=-dz-\dfrac{dP}{\gamma}+\dfrac{R.\Omega^2}{g}.dR$

or $\dfrac{w.dw}{g}+dz+\dfrac{dP}{\gamma}-\dfrac{R.\Omega^2}{g}.dR=0$

integrating:

$$\frac{w^2}{2g}+z+\frac{P}{\gamma}-\frac{R^2.\Omega^2}{2g}=cons.$$

$R.\Omega=$ centrifugal velocity (u)

$$\boxed{\therefore \quad \frac{w^2}{2g} - \frac{u^2}{2g} + \frac{P}{\gamma} + z = cons.}$$

(Bernoulli's equation for relative motion).

6.2.5 Fundamental equation of centrifugal pump

Consider the figure below. The equation of flow between any two consecutive points can be obtained by applying the Bernoulli's theorem.

a) For flow from (i) to (1) i.e., through the stationary suction pipe, since v_1 (absolute velocity of water).

$$\frac{v_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{v_i^2}{2g} + \frac{P_i}{\gamma} + z_i - H_{L(i-1)}$$

b) From (1) to (2) i.e., through the movable impeller.

$$\frac{w_2^2}{2g} - \frac{u_2^2}{2g} + \frac{P_2}{\gamma} + z_2 = \frac{w_1^2}{2g} - \frac{u_1^2}{2g} + \frac{P_1}{\gamma} + z_1 - H_{L(1-2)}$$

c) From (2) to (d) i.e., through the stationary casing inside which the motion of water is absolute:

$$\frac{v_d^2}{2g} + \frac{P_d}{\gamma} + z_d = \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + z_2 - H_{L(2-d)}$$

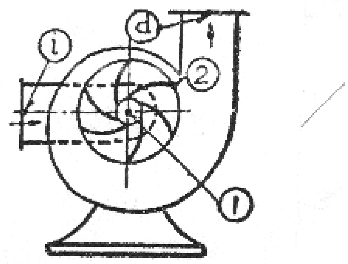


Fig 12.19 Liquid Flow Though a Centifugal Pump

Now, adding the above three equations and re-arranging:

$$\frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} = \left[\left(\frac{v_d^2}{2g} + \frac{P_d}{\gamma} + z_d\right) - \left(\frac{v_i^2}{2g} + \frac{P_i}{\gamma} + z_i\right)\right]$$
$$+ \left(H_{L(i-1)} + H_{L(1-2)} + H_{L(2-d)}\right)$$

The first term on the right hand side is the gross manometric head of the pump, while the second term stands for the total pump losses due to the fluid resistance inside the pump only.

$$\therefore \quad \frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} = H_{mano} + \Delta H_{mano}$$

This is known as the fundamental equation of centrifugal pump.

Consider the manometric efficiency of the pump:

$$\eta_{mano} = \frac{H_{mano}}{H_{mano} + \Delta H_{mano}}$$

$$\therefore \quad \frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} = \frac{H_{mano}}{\eta_{mano}}$$

With aid of velocity triangles at inlet and outlet:

$$w_1^2 = u_1^2 + v_1^2 - 2u_1 v_1 \cos\alpha_1$$

$$w_2^2 = u_2^2 + v_2^2 - 2u_2 v_2 \cos\alpha_2$$

$$\therefore \quad w_1^2 - w_2^2 = u_1^2 - u_2^2 + v_1^2 - v_2^2 - 2u_1 v_1 \cos\alpha_1 + 2u_2 v_2 \cos\alpha_2$$

Substituting this in fundamental equation:

$$\frac{H_{mano}}{\eta_{mano}} = \frac{v_2^2 - v_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{u_1^2 - u_2^2 + v_1^2 - v_2^2 - 2u_1 v_1 \cos\alpha_1 + 2u_2 v_2 \cos\alpha_2}{2g}$$

$$\frac{H_{mano}}{\eta_{mano}} = \frac{u_2 v_2 \cos\alpha_2 - u_1 v_1 \cos\alpha_1}{g}$$

Generally $\alpha_1 = 90^o \quad \Rightarrow \quad \cos\alpha_1 = 0$, neglecting prerotation.

$$\frac{H_{mano}}{\eta_{mano}} = \frac{u_2 v_2 \cos\alpha_2}{g} = \frac{u_2 v_{u2}}{g}$$

$$\therefore \quad H_{mano} = \eta_{mano} * \frac{\textit{Peripheral speed at outlet} * \textit{velocity of whirl at outlet}}{g}$$

This is the form of the fundamental equation, which is used in practice.

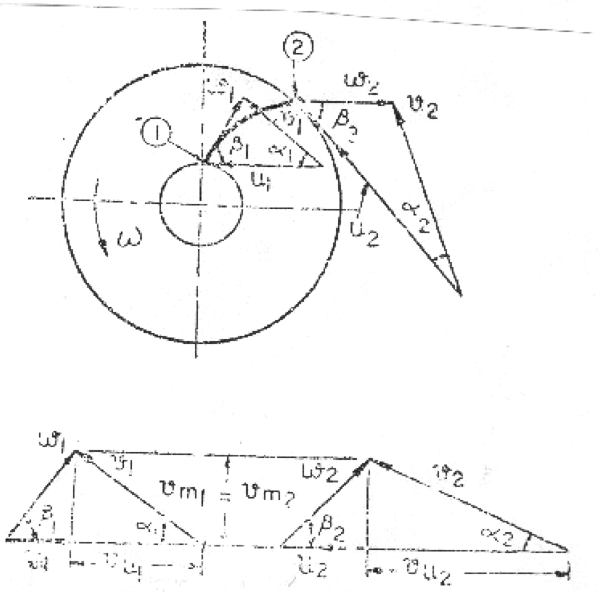


Fig 12.20 Velocity Triangles of a Centrifugal Pump

6.2.6 Work done and manometric efficiency

Work done/sec by the pump impeller is:

$$P = \rho.Q.(u_1 v_{u1} - u_2 v_{u2}) \qquad \text{(kW)}$$

The suffices (1) and (2) used in this equation will hold true if point (1) denotes the pressure side and point (2) denotes suction side of the pump. However if point (1) denotes inlet and point (2) the outlet of the pump impeller, then the above equation will be written as:

$$P = \rho.Q.(u_2 v_{u2} - u_1 v_{u1})$$

Since $\alpha_1 = 90^o$ (radial entrance) $\Rightarrow \cos\alpha_1 = 0$

$\therefore$ The energy supplied to the fluid by the impeller per kN per sec is:

$$= \frac{u_2 v_{u2}}{g}$$

and this is equal to the head generated, provided there is no losses inside the pump.
i.e., Head generated by the pump=Difference between the total energy of fluid at inlet and outlet of the pump $\cong$ M|anometric head.

$\dfrac{u_2 v_{u2}}{g} = H_{mano}$ if there is no internal loss of the pump.

In practice there are always some head losses inside the pump.

$$\therefore \quad \frac{u_2 v_{u2}}{g} = H_{mano} + \Delta H_{mano}$$

$$\eta_{mano} = \frac{H_{mano}}{H_{mano} + \Delta H_{mano}}$$

but $H_{mano} = H_{static} + H_f + \dfrac{v_d^2}{2g}$

$$\therefore \quad \eta_{mano} = \frac{H_{static} + H_f + \dfrac{v_d^2}{2g}}{\dfrac{u_2 v_{u2}}{g}}$$

6.2.7 Pressure rise by pump impeller

Applying Bernoulli's theorem between inlet and outlet edge of impeller.

Energy at inlet = Energy at outlet – useful work done by impeller.

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} - \frac{u_2 v_{u2}}{g}$$

$\therefore$ Pressure rise between outlet and inlet edges of impeller:

$$\frac{P_2 - P_1}{\gamma} = \frac{v_1^2 - v_2^2}{2g} + \frac{u_2 v_{u2}}{g}$$

6.2.8 Efficiency of centrifugal pump

a) Overall efficiency

$$\eta_{overall} = \frac{ouput\ water\ power}{input\ shaft\ power}$$

This is known also "gross efficiency" or "actual efficiency"

$P_{shaft} = P_{input\ to\ impeller} + P_{leakage} + P_{mech.\ loss}$

P_{shaft} = B.P of driving unit – $P_{lost\ in\ coupling}$

$$P_{input\ to\ impeller} = \frac{u_2 v_{u2}}{g} \qquad \text{per kN per sec}$$

$= P_{water} + P_{hydraulic}$

$$P_{water} = \gamma . Q . H_{mano} \qquad \text{(kW)}$$

$$P_{hydraulic} = \gamma . Q . \Delta H_{mano} \qquad \text{(kW)}$$

= Power required to overcome head losses due to:

i. Circulatory or secondary flow.
ii. Frictions of volute and impeller.
iii. Turbulence.

$P_{leakage}$ = Power required to overcome leakage.
$P_{mech.\ loss}$ = Power required to overcome mechanical losses

b) Mechanical efficiency

$$\eta_{mech} = \frac{\text{power delivered by the impeller to the water}}{\text{power input to the pump shaft}}$$

$$= \frac{\gamma (Q + \Delta Q)\left(\frac{u_2 v_{u2}}{g}\right)}{shaft\ power} = \frac{sh.P - P_{mec.\ loss}}{sh.P}$$

c) Volumetric efficiency

$$\eta_Q = \frac{Q}{Q + \Delta Q}$$

where Q: discharge delivered by the pump
Δ Q : amount of leakage.

d) Manometric efficiency

$$\eta_{mano} = \frac{actual\ measured\ head\ or\ gross\ lift}{head\ imparted\ to\ fluid\ by\ impeller}$$

$$\therefore \quad \eta_{mano} = \frac{H_{static} + H_f + \frac{v_d^2}{2g}}{\frac{u_2 v_{u2}}{g}} = \frac{H_{mano}}{\frac{u_2 v_{u2}}{g}}$$

$$= \frac{\gamma.(Q + \Delta Q).H_{mano}}{sh.P - P_{mec.\,loss}}$$

This is also known as hydraulic efficiency. From above:

$$\eta_{overall} = \eta_{mech.} . \eta_Q . \eta_{mano}$$

*6.2.9 **Cavitation in pump***

When the liquid is flowing in the pump, it is possible that the pressure at any part of the pump may fall below the vapor pressure, then the liquid will vaporize and the flow will no longer remain continuous. The vaporization of the liquid will appear in the form of bubbles released in the low pressure region of the pump. These bubbles are carried along with water stream and when these pass through a region of high pressure, these collapse suddenly. When the bubbles collapse on a metallic surface such as tips of impeller blades, the cavities are formed. Successive bubble collapsing at the same metallic surface produces pitting since penetration in the grain boundaries take place. Once the pitting takes place, the liquid rushes to fill the pits causing mechanical destruction and the liquid hits the blades with such a great force that it damages the impeller. This phenomena is known as "Cavitation". A great noise is experienced due to cavitation leading to vibration of the pumping set.

Since the cavitation occurs when the pressure falls below atmospheric, the trouble is experienced mainly at the impeller vane inlet due to high suction lift which must be brought within limits.

6.2.10 Net Positive Suction Head (NPSH)

(NPSH) is the head required at the pump inlet to keep the liquid from cavitation or boiling. The pump inlet or suction side is the low pressure point where cavitation will first occur.

$$NPSH = \frac{P_i}{\gamma} + \frac{v_i^2}{2g} - \frac{P_{vap}}{\gamma}$$

Where P_{vap} is the vapor pressure of the liquid. NPSH is also defined as a measure of the energy available on the suction side of the pump. NPSH is a commercial term used by the pump manufactures and indicates the suction head which the pump impeller can produce. In other words, it is the height of the pump axis from the water reservoir which can be permitted for installation.

Chapter Seven
Compressors

7.1 Centrifugal compressors and fans

Compressors as well as pumps and fans are the devices used to increased the pressure of a fluid, but they differ in the tasks they perform. A fan increase the pressure of a gas slightly and it is mainly used to move a gas around. A compressor is capable of compressing the gas to very high pressure. Pumps work very much like compressors except that they handle liquids inside of gases.

Centrifugal compressors and fans are turbo-machines employing centrifugal effects to increase the pressure of the fluid. The centrifugal compressor is mainly found in turbo chargers.

7.1.1 components

- Impeller.
- Diffuser.
- Spiral casing (scroll or volute).

As shown in figure.

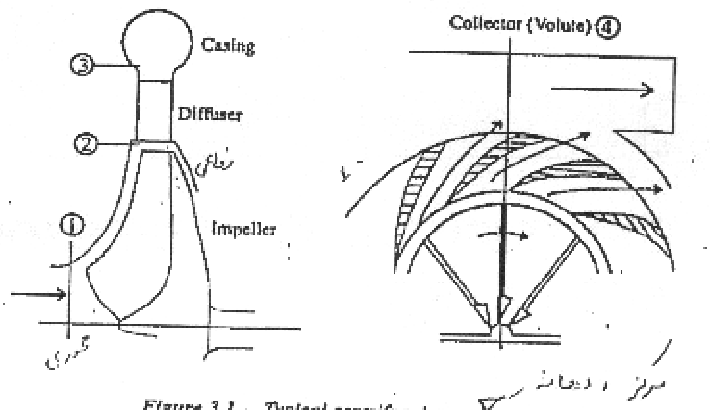
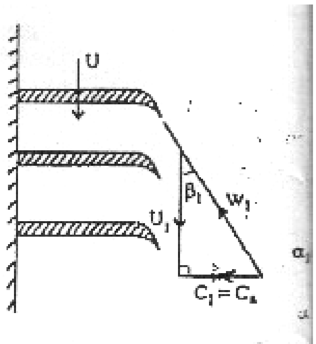


Figure 3.1 Typical centrifugal compressor

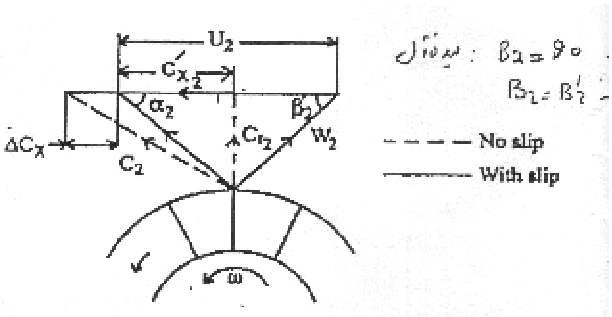
7.1.2 Velocity diagrams for a centrifugal compressor

The gas enters the compressor at the eye, in an axial direction ($\alpha_1 = 90^o$), as shown in figure below.



(a) Inlet velocity triangle

And for radial blade ($\beta_2 = 90^o$) as a result of slip, the relative velocity vector (W_2) is at angle ($\beta_2' \langle \beta_2$), for zero slip ($\beta_2 = 90^o$)and so $\left(C_{x2} = U_2\right)$ and $\left(C_{r2} = W_2\right)$, as shown in figure.



(b) Outlet velocity triangle

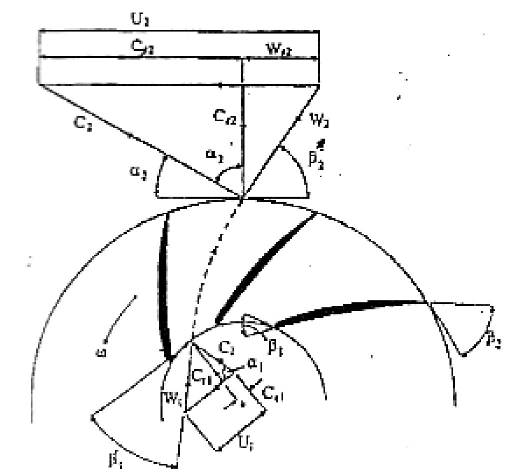


Figure 3.4 Velocity triangles for a backward curved impeller

7.1.3 ***Slip factor***

The fluid leaves the impeller at an angle β_2' other than the actual blade angle β_2. This is due to "fluid slip". Angle β_2' is less than angle β_2. In centrifugal compressors, the air trapped between the impeller vanes is reluctant to move around with the impeller, and this results in a higher static pressure on the leading face of the vane than on the trailing face of the vane. This problem is due to the inertia of the air. Then the air tends to flow around the edges of the vanes in the clearance space between impeller and casing.

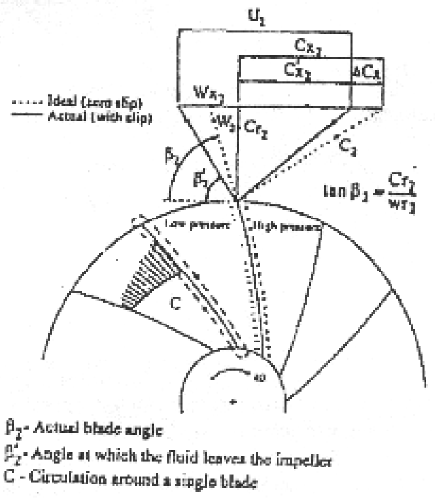


Figure 3.5 Slip and velocity distribution in centrifugal pump impeller blades

Slip factor is defined as:

$$\sigma_s = \frac{C'_{x2}}{C_{x2}} = \frac{(C_{x2} - \Delta C_x)}{C_{x2}}$$

Referring to the above figure , for the no slip condition:

$$C_{x2} = U_2 - W_{x2}$$

and

$$W_{x2} = C_{r2}.\cot\beta_2 \qquad , \qquad C_{x2} = U_2 - C_{r2}.\cot\beta_2$$

Stodola proposed the existence of a relative eddy within the blade passages as shown in figure below. By definition, a frictionless fluid which passes through the blade passages have no rotation. Therefore at the outer of the passage the rotation should be zero. Now, the impeller has an angular velocity "ω" , so that, relative to the impeller, the fluid must have an angular velocity "-ω" to match the zero-rotation condition.

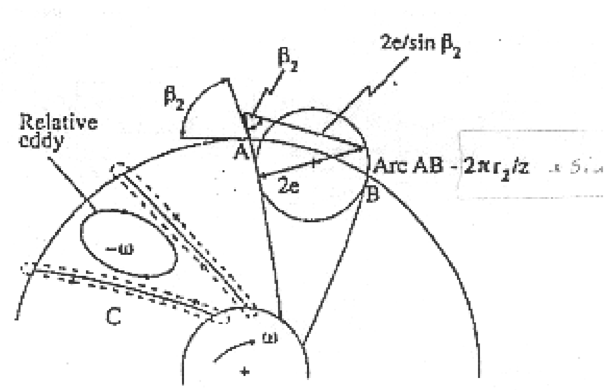


Figure 3.6 The relative eddy between impeller blades

$$\therefore \Delta C_x = \omega . e$$

Assume $\dfrac{2e}{\sin\beta_2} = \dfrac{2\pi . r_2}{z}$

$$\therefore \Delta C_{x2} = \frac{U_2 . \pi . \sin\beta_2}{z}$$

$$\therefore \sigma_s = 1 - \frac{U_2 . \pi . \sin\beta_2}{z(U_2 - C_{r2} . \cot\beta_2)}$$

The Stodola slip factor equation gives best results for the blade angle in the range $20^o \langle \beta_2 \langle 30^o$, for another angle range there are another equations.

7.1.4 ***Energy transfer***

By Euler's pump equation, without slip

$$E = \frac{U_2 . C_{x2} - U_1 . C_{x1}}{g}$$

From inlet velocity triangle $C_{x1} = 0$, for ideal condition $U_2 = C_{x2}$, from the outlet velocity triangle:

$$E = \frac{U_2 . C_{x2}}{g} = \frac{U_2^2}{g}$$

And with slip, the theoretical work is

$$E = \frac{\sigma_s . U_2^2}{g}$$

7.1.5 ***Power input factor***

Power input factor or work factor, or stage loading coefficient:

$$\psi = \frac{Actual\ work\ sup\ plied}{Theoretical\ work\ sup\ plied}$$

ψ typically takes values from 1.035-1.041

So, the actual energy transfer becomes:

$$E = \frac{\psi . \sigma_s . U_2^2}{g}$$

7.1.6 The energy equation along a streamline

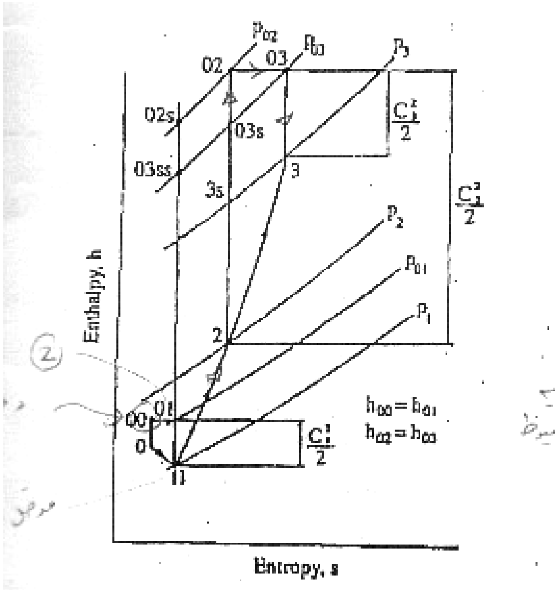


Figure 3.7 Mollier chart for a centrifugal compressor

1. inlet casing

total enthalpy, $h_o = h + \frac{C^2}{2} = constant$

therefore, for the fluid drawn from the atmosphere into the inducer section, the total enthalpy is:

$$h_{oo} = h_o + \frac{C_o^2}{2}$$

Total enthalpy at section -1, i.e., inlet of the impeller, is

$$h_{o1} = h_1 + \frac{C_1^2}{2}$$

And since no shaft work has been done and assuming that adiabatic steady flow occurs

$h_{oo} = h_{o1}$, thus $h_o + \frac{C_o^2}{2} = h_1 + \frac{C_1^2}{2}$

2. Impeller

Work is done on the fluid across the impeller and the static pressure is increased from P_1 to P_2 . Writing the work done per unit mass on the fluid in terms of enthalpy, we get:

$$W/m = h_{o2} - h_{o1}$$

From Euler's pump equation

$$W/m = U_2.C_{x2} - U_1.C_{x1}$$

Equating the two equation and after substituting for h_o

$$I = h_1 + \frac{C_1^2}{2} - U_1.C_{x1} = h_2 + \frac{C_2^2}{2} - U_2.C_{x2}$$

Where I is the impeller constant.

In general :

$$I = h + \frac{C^2}{2} - U.C_x$$

After substituting $C^2 = C_r^2 + C_x^2$ and $C_r^2 = W^2 - W_x^2$ and rearrange:

$$I = h + \frac{W^2}{2} - \frac{U^2}{2}$$

Thus :

$$h_2 - h_1 = \frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}$$

Usually C_{x2}=0 is assumed in preliminary design calculation.

$$\therefore \quad h_{o2} - h_{o1} = \psi.\sigma_s.U_2^2$$

Substituting $h_o = C_p.T_o$ and rearranging the equation , we get:

$$T_{o2} - T_{o1} = \frac{\psi.\sigma_s.U_2^2}{C_p}$$

Where C_p is the mean specific heat over temperature range.

Since , no work is done in the diffuser, $h_{o2} = h_{o3}$ and so

$$T_{o3} - T_{o1} = \frac{\psi.\sigma_s.U_2^2}{C_p}$$

Now define the isentropic efficiency.

$$\eta_c = \frac{Total\ isentropic\ enthalpy\ rise\ between\ inlet\ and\ outlet}{Actual\ enthalpy\ rise\ between\ same\ total\ \ pressure\ limits}$$

$$= \frac{h_{03_{ss}} - h_{01}}{h_{03} - h_{01}}$$

Where the subscript "ss" represents the end state on the total pressure line P_{03} when the process is isentropic:

$$\eta_c = \frac{T_{03_{ss}} - T_{01}}{T_{03} - T_{01}}$$

$$= T_{01} \frac{\left(\left(T_{03_{ss}} / T_{01}\right) - T_{01}\right)}{T_{03} - T_{01}}$$

But

$$\frac{P_{03}}{P_{01}} = \left(\frac{T_{03_{ss}}}{T_{01}}\right)^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{\eta_c \left(T_{03} - T_{01}\right)}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{\eta_c . \psi . \sigma_s . U_2^2}{C_p . T_{01}}\right]^{\gamma/(\gamma-1)}$$

7.1.7 ***Stage pressure rise and loading coefficient***

The static pressure rise in a centrifugal compressor occurs in the impeller, diffuser and the volute. No change in stagnation enthalpy occurs in the diffuser and volute.

$$Work\ sup\ plied = h_{02_s} - h_{01}$$

$$= C_p \left(T_{02_s} - T_{01}\right)$$

$$= C_p . T_{01} \left(\frac{T_{02_s}}{T_{01}} - 1 \right)$$

$$= C_p . T_{01} \left(\left(\frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\gamma} - 1 \right)$$

$$= C_p . T_{01} \left(R_0^{(\gamma-1)/\gamma} - 1 \right)$$

Where R_o is stagnation pressure ratio

From Euler's equation

$$Work\ sup\ plied = U_2 . C_{x2}$$

$$= U_2 \left(U_2 - C_{r2} \cot \beta_2 \right)$$

$$= U_2^2 \left(1 - \frac{C_{r2}}{U_2} \cot \beta_2 \right)$$

$$= U_2^2 \left(1 - \phi_2 . \cot \beta_2 \right)$$

Where ϕ_2 is the flow coefficient at the impeller exit $= \dfrac{C_{r2}}{U_2}$

Equating the two equations:

$$C_p . T_{01} \left(R_0^{(\gamma-1)/\gamma} - 1 \right) = U_2^2 \left(1 - \phi_2 . \cot \beta_2 \right)$$

Thus

$$R_0 = \left[1 + \frac{(1 - \phi_2 . cot\,\beta_2)U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

The loading or pressure coefficient is defined as:

$$\psi_p = \frac{Work\ done / kg}{U_2^2}$$

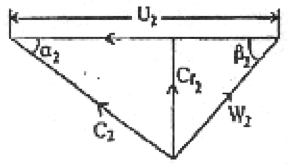


Figure 3.8 (a) Velocity triangle

From the outlet velocity triangle

$$C_{x2} = U_2 - C_{r2}.\cot\beta_2$$

From Euler's equation

$$E.g = U_2 . C_{x2}$$

Substitute $C_{x2} = U_2 - C_{r2}.cot\,\beta_2$ and $\phi_2 = \dfrac{C_{r2}}{U_2}$, yields :

$$\psi_p = (1 - \phi_2 . cot\,\beta_2)$$

Substitute this in stagnation pressure ratio equation R_0 :

$$R_0 = \frac{P_{02}}{P_{01}} = \left[1 + \frac{\psi_p . U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

7.1.8 ***Pressure coefficient***

The pressure or loading coefficient is also defined as the ratio of isentropic work to Euler's work:

$$\psi_p = \frac{Isentropic\ work}{Euler's\ work}$$

$$\psi_p = \frac{C_p\left(T_{o2_s} - T_{o1}\right)}{U_2 . C_{x2}}$$

For radial vane impeller $C_{x2} = U_2$

$$\therefore \quad \psi_p = \frac{C_p\left(T_{o2_s} - T_{o1}\right)}{U_2^2}$$

Now, isentropic work = actual work * isentropic efficiency

$$Isentropic\ work = \eta_c . C_p\left(T_{o2} - T_{o1}\right)$$

Then,

$$\psi_p = \eta_c . \frac{C_p\left(T_{o2} - T_{o1}\right)}{U_2^2}$$

But $C_p(T_{o2} - T_{o1}) = \psi.\sigma_s.U_2^2$

$\therefore \quad \psi_p = \eta_c.\psi.\sigma_s$

7.1.9 Degree of reaction

The degree of reaction of a centrifugal compressor stage is given by:

$$R = \frac{Change\ in\ static\ enthalpy\ in\ the\ impeller}{Change\ in\ stgnation\ enthalpy\ in\ the\ stage}$$

$$= \frac{h_2 - h_1}{h_{o2} - h_{o1}} \quad [h_{o3} - h_{o1} = h_{o2} - h_{o1}\ as\ h_{o2} = h_{o3}]$$

If the velocity of the gas approaching the compressor inlet is negligible $(C_1 \cong 0)$ *,then* $h_1 \cong h_{o1}$, and

$$h_2 = h_{o2} - \frac{C_2^2}{2}$$

$$\therefore \quad R = \frac{(h_{o2} - h_{o1}) - C_2^2/2}{(h_{o2} - h_{o1})}$$

$$\because \ (h_{o2} - h_{o1}) = U_2.C_{x2} = U_2(U_2 - W_{x2})$$

$$= U_2^2(1 - \phi_2 \cot\beta_2)$$

By substituting for $C_2^2 = C_{x2}^2 + C_{r2}^2$, $C_{x2} = U_2 - C_{r2}\cot\beta_2$, yields :

$$R = \frac{U_2^2\left(1-\phi_2 \cot\beta_2\right) - \frac{1}{2}U_2^2\left[\phi_2^2 + \left(1-\phi_2 \cot\beta_2\right)^2\right]}{U_2^2\left(1-\phi_2 \cot\beta_2\right)}$$

By some rearranging :

$$R = \frac{1-\phi_2^2 \operatorname{cosec}\beta_2}{2\left(1-\phi_2 \cot\beta_2\right)}$$

For radial vane $\beta_2 = 90^o$, then

$$R = \frac{1}{2}\left(1-\phi_2^2\right), \quad and \quad \psi_p = 1$$

7.1.10 Effect of impeller blade shape on performance

The different blade shapes utilized in impellers of a centrifugal compressors can be classified as :

a. Backward – curved blades.

$$\beta_2 \langle 90^o$$

From outlet velocity triangle,

$$C_{x2} = U_2 - C_{r2} \cot\beta_2$$

And the energy transfer $E = \dfrac{U_2 . C_{x2}}{g}$

Thus $E = \dfrac{U_2 .\left(U_2 - C_{r2} \cot\beta_2\right)}{g}$

or $E = \dfrac{U_2^2}{g} - \dfrac{mU_2 \cot\beta_2}{\rho g A}$

where $\frac{m}{\rho A} = C_{r2}$, the above equation is in the form of $E = a - bm$, where $a = \frac{U_2^2}{g}$ and $b = \frac{U_2 \cot \beta_2}{\rho g A}$. As m increases , E decreases. The characteristic is therefore falling.

b. Radial blades.

$\beta_2 = 90^o \Rightarrow \cot \beta_2 = 0$

$\therefore \; E = a$

The energy transferred is constant at all flow rate and hence characteristic is neutral.

c. Forward – curved blades.

$\beta_2 \rangle \; 90^o$

$\therefore \; E = a + bm$

When m increases , E is increased. The characteristic will then be raising .

The typical value of β_2 for a multi-blades centrifugal fan is 140°.

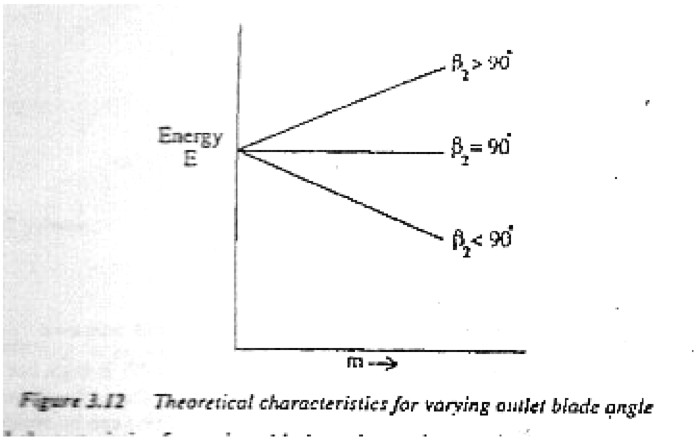


Figure 3.12 Theoretical characteristics for varying outlet blade angle

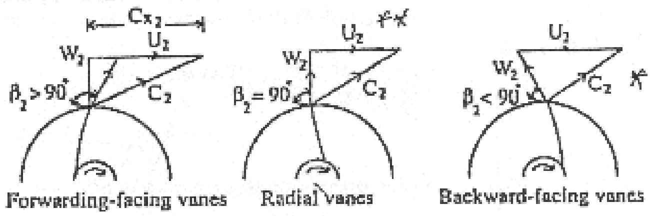


Figure 3.14 Centrifugal compressor outlet velocity triangles for varying blade outlet angle

7.2 Axial flow compressors and fans

Axial flow compressors and fans are turbo-machines that increase the pressure of the gas flowing continuously in the axial direction. The efficiency of the axial flow compressor is very sensitive to the mass flow rate. Thus the axial flow compressor is ideal for constant load applications such as in aircraft gas turbine engines. They are also used in fossil fuel power stations, where gas turbines are used to meet the load exceeding the normal peak load.

7.2.1 Advantages of an axial flow compressors

a. Axial flow compressor has higher efficiency than radial flow compressor.
b. Axial flow compressor gives higher pressure ratio on a single shaft with relatively high efficiencies.
c. Pressure ratio of 8:1 or even higher can be achieved using multi-stage axial flow compressors.
d. The greatest advantage of the axial flow compressor is its high thrust per unit frontal area.
e. It can handle large amount of air, inspite of small frontal area.

The main disadvantages are its complexity and coast.

7.2.2 Description of an axial flow compressors

An axial flow compressor consist of fixed and movable set of blades in alternating sequence. Moving blades are attached to the periphery of a rotor hub followed by fixed blades attached to the walls of the outer

casing as shown in the following figure. At the inlet of the compressor, an extra row of fixed vanes called inlet guide vanes are fitted. These do not form part of the compressor stage but are solely to guide the air at the correct angle on to the first row of moving blades.

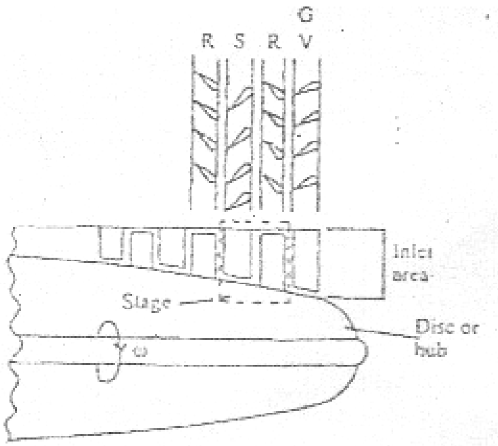


Figure 4.1 An axial compressor stage

7.2.3 Working principle

The kinetic energy is imparted to the air by the rotating blades. Which is then converted into a pressure rise. So, the basic principle of working is similar to that of the centrifugal compressor. Referring to the previous figure, the air enters axially from the right into the inlet guide vanes, where it is deflected by a certain angle to impinge on the first row of rotating blades with the proper angle of attack. The rotating vanes add kinetic energy to the air. There is a slight pressure rise to the air. The air is then discharged at the proper angle to the first row of stator blades, where the pressure is further increased by diffusion. The air is then directed into the second row of moving blades and the same process is repeated through the remaining compressor stages. This process is shown clearly by the following figure for the velocity triangles for an axial flow compressor stage.

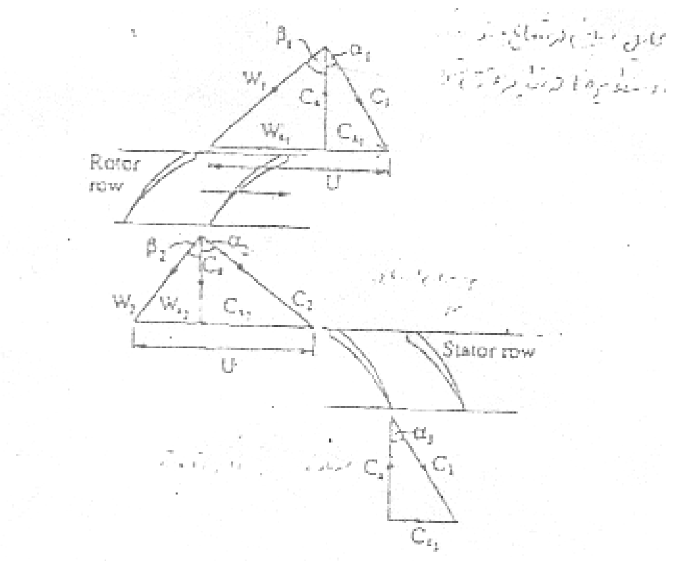


Figure 4.2 Velocity triangles for an axial flow compressor stage

7.2.4 Energy transfer

By Euler's equation:

$$E = \frac{U_2 . C_{x2} - U_1 . C_{x1}}{g}$$

From the velocity triangles, C_a is constant through the stage and $U_1 = U_2 = U$.

$$C_{x2} = U - C_a \tan\beta_2$$

and

$$C_{x1} = U - C_a \tan\beta_1$$

$$C_{x2} - C_{x1} = C_a(\tan\beta_1 - \tan\beta_2)$$

Therefore $\quad E = \dfrac{U . C_a(\tan\beta_1 - \tan\beta_2)}{g}$

The energy transfer may also be written in terms of the absolute velocity flow angles.

$$E = \frac{U.C_a(tan\alpha_2 - tan\alpha_1)}{g}$$

7.2.5 ***Mollier chart***

The flow through the axial flow compressor stage is shown thermodynamically on the Mollier chart as shown in the following figure.

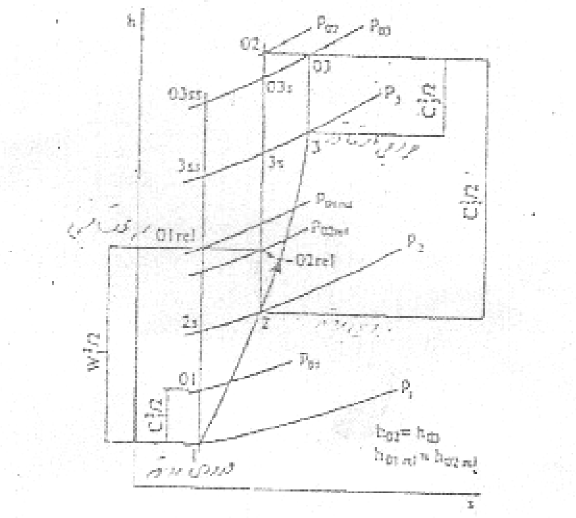


Figure 4.3 Mollier chart for an axial flow compressor stage

$$E.g = h_{o2} - h_{o1}$$

$$\because \; h_o = h + \frac{C^2}{2} = h + \frac{(C_a^2 + C_x^2)}{2}$$

Then,

$$h_{o2} - h_{o1} = (h_2 - h_1) + \frac{(C_{x2}^2 - C_{x1}^2)}{2} = U(C_{x2} - C_{x1})$$

or $\quad (h_2 - h_1) - (C_{x2} - C_{x1})\dfrac{[2U - (C_{x2} - C_{x1})]}{2} = 0$

substituting for $U - C_{x2} = W_{x2}$, $U - C_{x1} = W_{x1}$ and rearrange :

$$(h_2 - h_1) + \frac{\left(W_{x2}^2 - W_{x1}^2\right)}{2} = 0$$

Since C_a is constant , then $\left(W_{x2}^2 - W_{x1}^2\right) = \left(W_2^2 - W_1^2\right)$, therefore:

$$h_2 + \frac{W_2^2}{2} = h_1 + \frac{W_1^2}{2}$$

Rewrite the change in enthalpy for a centrifugal compressor :

$$h_2 - h_1 = \frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}$$

The comparison between the above two equations, indicates why the enthalpy change in a single stage axial flow compressor is so low compared to the centrifugal compressor. The relative velocities may be of the same order of magnitude, but the axial flow compressor receives no contribution from the change in tangential velocity (U).

Now, the isentropic or overall total-to-total efficiency is written as:

$$\eta_c = \frac{Ideal\ isentropic\ work\ input}{Actual\ work\ input}$$

$$= \frac{Toatal\ isentropic\ enthalpy\ rise\ in\ the\ stage}{Actual\ enthalpy\ rise\ between\ the\ same\ total\ \ pressure\ limits}$$

$$= \frac{h_{o3ss} - h_{o1}}{h_{o3} - h_{o1}}$$

Which reduces to:

$$\eta_c = T_{o1}\frac{((T_{o3ss}/T_{o1})-1)}{T_{o3}-T_{o1}}$$

Putting $\dfrac{P_{o3}}{P_{o1}} = \left(\dfrac{T_{o3ss}}{T_{o1}}\right)^{\frac{\gamma}{\gamma-1}}$

The pressure ratio becomes:

$$\frac{P_{o3}}{P_{o1}} = \left(1+\eta_c\frac{T_{o3}-T_{o1}}{T_{o1}}\right)^{\frac{\gamma}{\gamma-1}}$$

Where $T_{o3}-T_{o1} = U.C_a\dfrac{(tan\,\beta_1 - tan\,\beta_2)}{C_p}$

The energy input to the fluid will be absorbed in raising the pressure and velocity of the air and some will be wasted in overcoming various frictional losses.

7.2.6 Work done factor

In practice "C_a" is not constant along the length of the blade , thus the work done factor is introduced. It is defined as:

$$\lambda = \frac{Actual\ work\ absorbing\ capacity}{Ideal\ work\ absorbing\ capacity}$$

Hence,

$$T_{o3} - T_{o1} = \lambda . U . C_a \frac{(\tan\beta_1 - \tan\beta_2)}{C_p}$$

7.2.7 Stage loading or pressure coefficient

$$\psi_p = \frac{Work\ input}{mU^2}$$

$$\psi_p = \frac{(h_{o3} - h_{o1})}{U^2} = \lambda \frac{C_{x2} - C_{x1}}{U}$$

$$= \lambda \left(\frac{C_a}{U} \right) (\tan\alpha_2 - \tan\alpha_1)$$

$\psi_p = \lambda . \phi . (\tan\alpha_2 - \tan\alpha_1)$, where ϕ is the flow coefficient.

7.2.8 Reaction ratio

$$R = \frac{Static\ enthalpy\ rise\ in\ rotor}{Static\ enthalpy\ rise\ in\ stage}$$

$$= \frac{h_2 - h_1}{h_3 - h_1}$$

Since, $h_2 - h_1 = \dfrac{W_1^2 - W_2^2}{2}$

Also if C_1=C_3 then, $h_3 - h_1 = h_{o3} - h_{o1} = U\left(C_{x2} - C_{x1}\right)$

and substituting for (h_2-h_1) and (h_3-h_1), yields :

$$R = \frac{W_1^2 - W_2^2}{2U\left(C_{x2} - C_{x1}\right)}$$

$$= \frac{\left(C_a^2 + W_{x1}^2\right) - \left(C_a^2 + W_{x2}^2\right)}{2U\left(C_{x2} - C_{x1}\right)}$$

$$= \frac{\left(W_{x1} + W_{x2}\right)\left(W_{x1} - W_{x2}\right)}{2U\left(C_{x2} - C_{x1}\right)}$$

But $C_{x2} = U - W_{x2}$ and $C_{x1} = U - W_{x1}$, therefore:

$$C_{x2} - C_{x1} = W_{x1} - W_{x2}$$

Hence,

$$R = \frac{\left(W_{x1} + W_{x2}\right)}{2U}$$

$$= \frac{C_a\left(tan\,\beta_1 + tan\,\beta_2\right)}{2U} = \frac{C_a}{U} tan\,\beta_m$$

$$\therefore \; R = \phi\, tan\,\beta_m$$

Where , $tan\,\beta_m = \dfrac{(tan\,\beta_1 + tan\,\beta_2)}{2}$

and β_m is the mean relative velocity vector angle.

By some arrangement, one can verify :

$R = \dfrac{1+\phi(tan\,\beta_2 - tan\,\alpha_1)}{2}$ and $R = \dfrac{1+\phi(tan\,\beta_1 - tan\,\alpha_2)}{2}$

For the case of incompressible and reversible flow :

$$R = \frac{P_2 - P_1}{P_3 - P_1}$$

7.2.9 Effect of reaction ratio on the velocity triangles

Case – 1 when R=0.5

The reaction ratio R is

$$R = \frac{h_2 - h_1}{(h_3 - h_2) + (h_2 - h_1)}$$

When R=0.5 $(h_3 - h_1) = (h_3 - h_2)$

For a reaction ratio of 50% , the static enthalpy and temperature increase in the stator and rotor are equal.

$$\because \quad R = \frac{1+\phi(tan\,\beta_2 - tan\,\alpha_1)}{2}$$

When R=0.5 , thus $\beta_2 = \alpha_1$

So, when the outlet and inlet velocity triangles are superimposed, the resulting velocity diagram is symmetrical.

Case – 2 when $R \rangle 0.5$

From the previous equation for R , it is seen that $\beta_2 \rangle \alpha_1$, therefore, the static enthalpy rise in the rotor is greater than in the stator.

Case – 3 when $R \langle 0.5$

$\beta_2 \langle \alpha_1$, and the static enthalpy rise and pressure rise are greater in the stator than in the rotor.

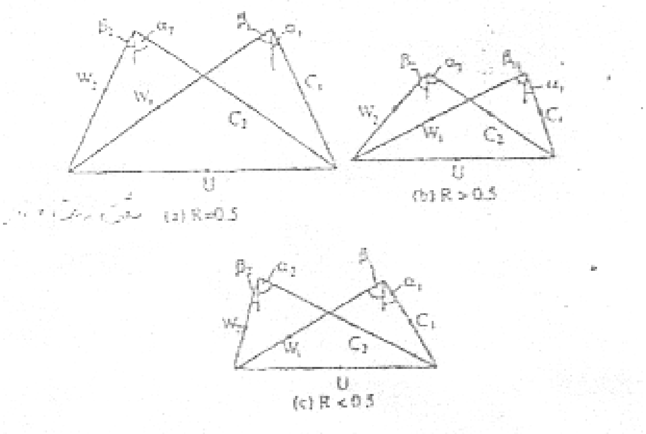


Figure 4.6 Effect of reaction ratio on the velocity triangles

7.2.10 **Static pressure rise**

The main function of a compressor is to raise the static pressure of the air:

$$P_2 - P_1 = \frac{1}{2}\rho\left(W_1^2 - W_2^2\right)$$

Across the stator row:

$$P_3 - P_2 = \frac{1}{2}\rho\left(C_2^2 - C_3^2\right)$$

Adding the above two equations, and considering a normal stage $\left(C_3 = C_1\right)$, gives:

$$\frac{2}{\rho}\left(P_3 - P_1\right) = \left(C_2^2 - W_2^2\right) + \left(W_1^2 - C_1^2\right)$$

$$\therefore \ (\Delta P)_{stage} = (\Delta P)_{rotor} + (\Delta P)_{stator}$$

From the velocity triangles, the cosine rule gives:

$$C^2 = U^2 + W^2 - 2UW\, cos\left(\frac{\pi}{2} - \beta\right)$$

And $W\, sin\,\beta = W_x$, then

$$C^2 - W^2 = U^2 - 2UW_x$$

Substituting this equation in the stage pressure difference equation, yields:

$$\frac{2}{\rho}\left(P_3 - P_1\right) = \left(U^2 - 2UW_{x2}\right) - \left(U^2 - 2UW_{x1}\right)$$

$$= 2U\left(W_{x1} - W_{x2}\right)$$

From the velocity diagram , we get:

$$U_1 = U_2 = U$$

$$\therefore\ W_{x1} + C_{x1} = W_{x2} + C_{x2} \qquad \text{or} \qquad W_{x1} - W_{x2} = C_{x2} - C_{x1}$$

$$\therefore\ \frac{(P_3 - P_1)}{\rho} = U(C_{x2} - C_{x1}) = h_3 - h_1$$

Since, for an isentropic process :

$$Tds = 0 = dh - \frac{dP}{\rho} \quad \text{and therefore} \quad (\Delta h)_s = \frac{\Delta P}{\rho}$$

The pressure rise in the real stage (involving irreversible process) can be determined if the isentropic (stage) efficiency is known.