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Power Electronics and Electrical Drives
EME 401
Energy- Systems branch

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Contents

1) Definition of power electronics.
2) Power Semiconductor devices.
3) Power electronics application s.
4) Classification of power electronics.
5) Characteristics of different power semiconductor devices.

Objectives

1) Identify the types of power semiconductor devices.
2) Identify the major applications of power electronics.
3) Classify the power electronics converters.
4) Understand the characteristics of power semiconductor devices

References

1) N.Mohan, et al , Power Electronics, Converters, Applications, and Design, 3rd Edition , John Wiley and Sons,2003.
2) P.C.Sen, Principles of Electric Machines and Power Electronics, 3rd Edition, John Wiley and Sons, 2014.
3) B.K.Bose, Modern Power Electronics and AC Drives, Prentice Hall Inc, 2002.
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1-1 Power Electronic: Solid state electronic used to control the flow of electrical power by supplying voltages, currents, and frequencies in a form that is optimally suited for user load. Fig.1-1 shows a simple power electronic system.

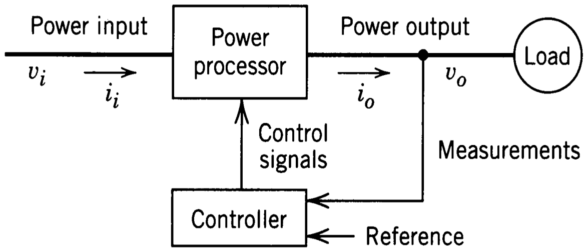

Figure 1-1 Block diagram of a power electronic system.

1-2 Power semiconductor devices: An Electronic switches (power processor) controlled by control signals that is generated by controller (integrated circuits and/ or digital signal processor).These power semiconductor devices can be broadly classified into three groups according to their degree of controllability:

1. Uncontrolled turn on and off (Power Diode).
2. Controlled turn on uncontrolled turn off (Thyristors).
3. Controlled turn on and off (GTO, BJT, MOSFET, IGBT).

1-3 POWER ELECTRONICS APPLICATIONS:

Power electronics has a wide range of applications including:-

1. Power supplies (TV, Radio, Receiver, PC and its Peripherals, UPS, mobile phone battery charger).
2. Air Conditioning and Refrigeration.
3. Elevators.
4. Electric drives.
5. Light control (dimmer).
6. High Voltage Direct Current (HVDC) systems.
7. Flexible AC Transmission (FACT) system.
8. Solar power.
9. Micro grid.
10. Wind generation.

1-4 CLASSIFICATION OF POWER ELECTRONIC CONVERTERS

Power electronics converters can be divided into the following categories:

1. **AC-DC converter (rectifier):** Convert input AC voltage into a DC output voltage.
2. **DC-DC converter (DC Chopper):** Convert fixed input DC voltage into variable DC output voltage or vice versa.
3. **AC-AC converter (AC voltage regulator also called cycloconverter):** Convert fixed input AC voltage and frequency into variable AC output voltage and variable (lower) frequency.
4. **DC-AC converter (Inverter):** Convert input DC voltage into a variable AC output voltage

Finally, Figure 1.1 presents a categorization of power electronic converters into families according to their type of electrical conversion.

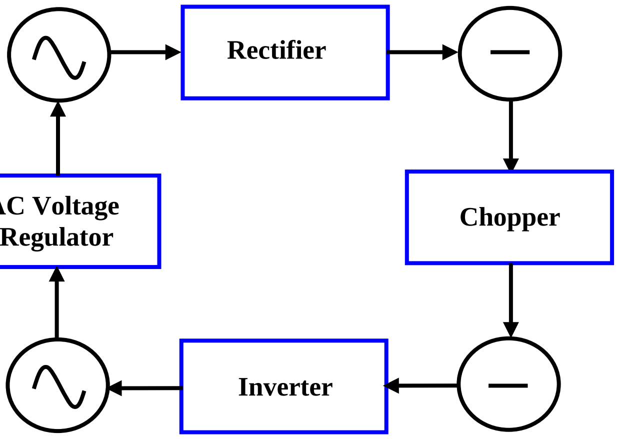

Fig.1-2 Families of solid state power converters categorized according to their conversion function..

1-5 CHARACTERISTICS OF POWER ELECTRONIC DEVICES

1-5-1 Power Diode

Power diode has two terminals anode (A) (positive) and cathode (K) (negative).When anode is positive with respect to the cathode, the diode is said to be forward biased and it begins to conduct with only small forward across it, which is on the order of 1v. Fig.1.3 (a) show the circuit symbol for the diode and (b) & (c) its steady state i-v and idealized characteristics respectively.

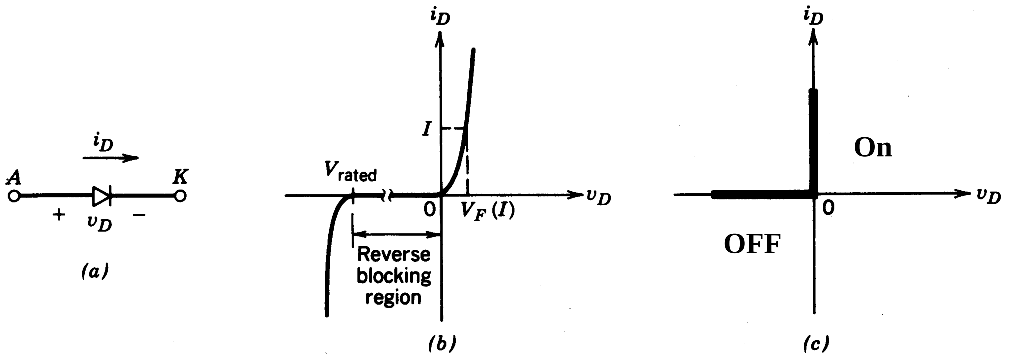

Fig.1-3 Diode (a) symbol,
(b) i-v characteristic, (c) idealized characteristic.

Power diode is used in power electronics circuits to perform one the following functions:

1) Switches in rectifiers.
2) Freewheeling in switching regulators.
3) Charge reversal of capacitor.
4) Voltage isolation.

Power diodes are similar to ordinary PN junction signal diode with slight difference. Power diode is capable to handle high power, voltage, and current. Frequency response or switching speed is low compared with that of signal diodes.

POWER DIODES TYPES

Power diodes can be classified as

- General purpose diodes: Used in rectifier circuits with voltage rating upto 5KV and current 3.5KA.
- Fast and ultra fast recovery diodes: Used in high frequency circuits with voltage rating up to 3KV and current 1KA
- Schottky diode: Used in low voltage, high current application such as switched mode power supplies with voltage rating up to 0.1KV and current 0.3KA.

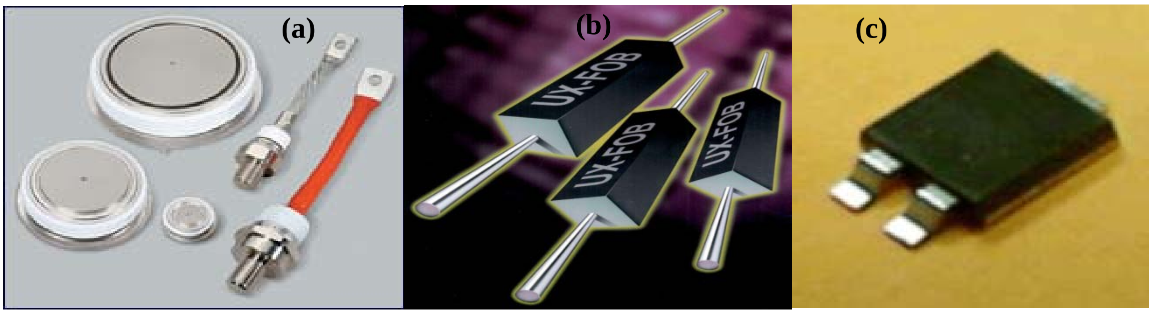

Fig.1-4 Photos for different types of diode (a) General purpose, (b) fast and ultra fast recovery , (c) schottky.

1-5-2 Thyristor or silicon controlled rectifier (SCRs)

Thyristors has three terminals anode, cathode, and gate. They are available with voltage rating up to (5KV) and current rating up to (3KA**).** The circuit symbol for the thyristor, and its *i-v* characteristics along with the idealized characteristics are shown in Figs.1.4a through Figs.1.4c respectively. Thyristor can be turned on by applying a pulse of positive gate current for a short duration, and then remain conducting as a diode until the gate current falls to zero. For successful to turn off thyristor, the anode current should be goes to negative by using an external circuit.

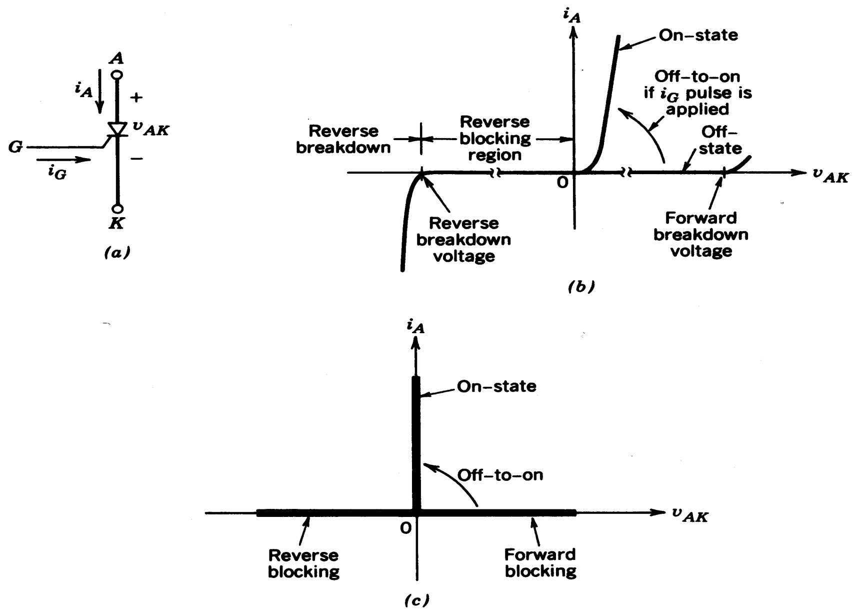

Fig.1.4 Thyristor : (a) symbol ,(b) i-v characteristics ,(c) idealized.

1-5-3 Gate Turn Off thyristor (GTO)

The circuit symbol for the GTO is shown in Fig. 1.5a and its steady state *i-v* as well as idealized characteristics is shown in Fig. 1.5b and Fig. 1.5c respectively. The GTO belong to a thyristor family. It can be turned on by a short pulse of gate current and turned off by a reverse gate pulse. This reverse gate current must be (20 %) of the anode current which is considered very large. The GTOs are available with voltage rating (4.5 kV), current rating (3kA), and switching frequency (10 KHz).Fig 1.6 show photo of GTOs.

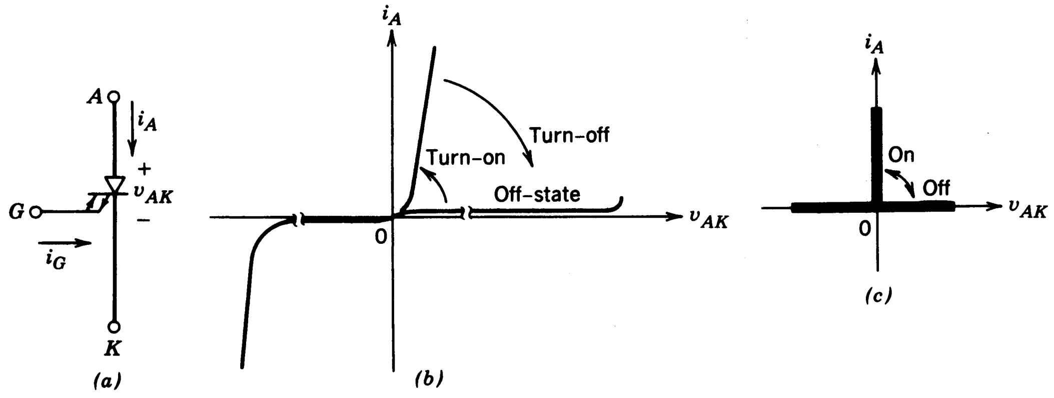

Fig.1.5 A GTO: (a) symbol, (b) i-v characteristics, (c) idealized.

Fig 1.6 photos for GTOs

CLASSIFICATION OF THYRISTORS

1) **Phase control thyrisors:** Suitable for use in AC and DC motor drives as well as in HVDC power transmission. Available with voltage rating (5-7 KV) and current (4KA)
2) **Inverter grade thyrisors**: Used in inverter and chopper with voltage rating (2.5kV) and current rating (1.5KA). It can be turned-on using force-commutation method.
3) **Light activated thyrisors:** Similar to phase controlled, but triggered by pulse of light guide by optical fibers . It can be used in very high power applications.
4) **TRIAC**: Dual polarity thyristors (4KV) (3KA).

1-5-3 Metal Oxide Semiconductor Field Effect Transistors (MOSFETs)

The circuit symbol of an *n*-channel MOSFET is shown in Fig.1.7 and its steady state *i*-*v* as well as idealized characteristics is shown in Fig. 1.7b and Fig. 1.7c respectively. Fig.1.8 show photos for different types of MOSFET. The MOSFET is on when the gate source voltage is below the threshold value, V_{GS}(th).The main features of these devices are fast switching typically (100-1000 KHz) and available in voltage rating up to 1KV and current rating up to 0.1 KA.

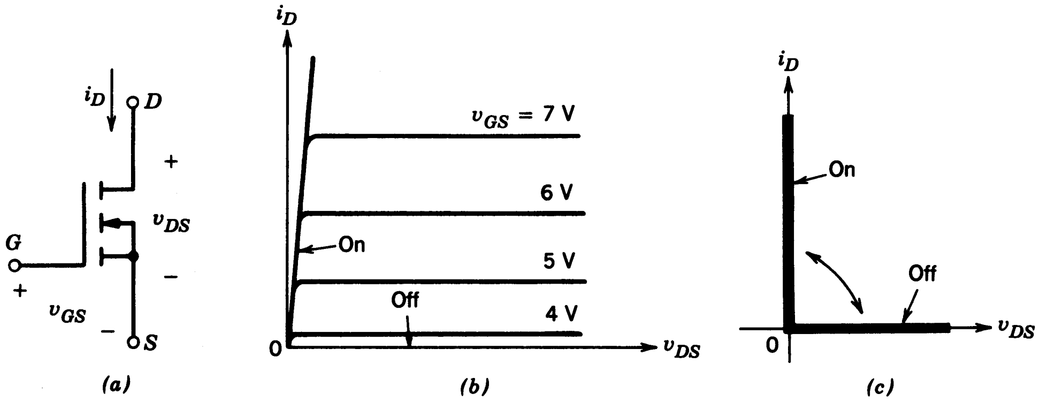

Fig.1.7 N-channel MOSFET: (a) symbol, (b) i-v characteristics, (c) idealized.

Fig.1.8 Photos for different types of MOSFET.

1-5-3 Insulated Gate Bipolar Transistors (IGBTs)

The circuit symbol for an IGBT is shown in Fig.1.9a and its steady state *i*-*v* as well as idealized characteristics is shown in Fig. 1.9b and Fig. 1.9c respectively.

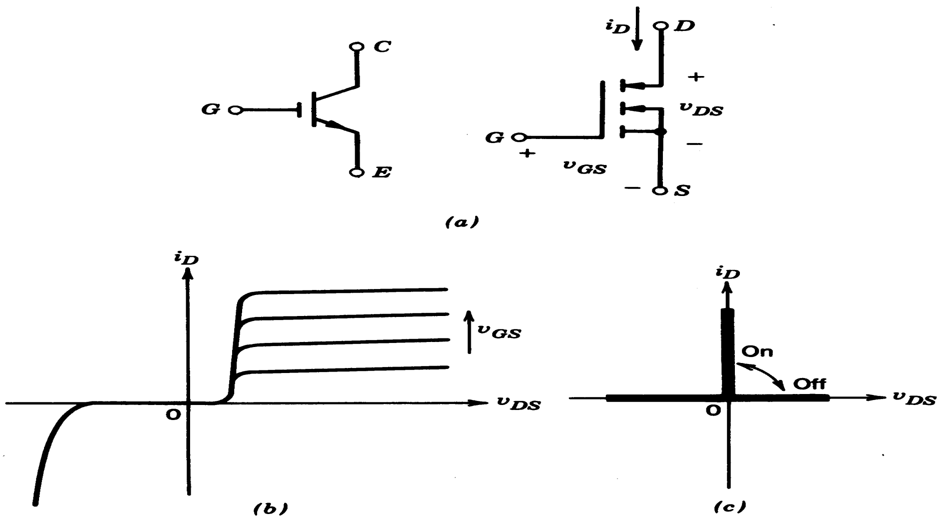

Fig.1.9 An IGBT: (a) symbol, (b) i-v characteristics, (c) idealized.

IGBTs are voltage controlled four-layer devices that combine the characteristics of BJT and MOSFET. They are currently available with voltage rating 4.5 KV, current rating 1.2 KA and switching frequency up to 100 KHz. Fig.1.10 show photo for IGBTs.

Fig.1.10 Photos for IGBTs.

1-5-4 MOS CONTROLLED THYRISTORS (MCTs)

The MCT is voltage controlled device like IGBT and the MOSFET. The current density of MCT is high compared to a power MOSFET, and IGBT. The circuit symbol for an MCT is shown in Fig.1.11a and its steady state *i-v* as well as idealized characteristics is shown in Fig. 1.11b and Fig. 1.11c respectively.

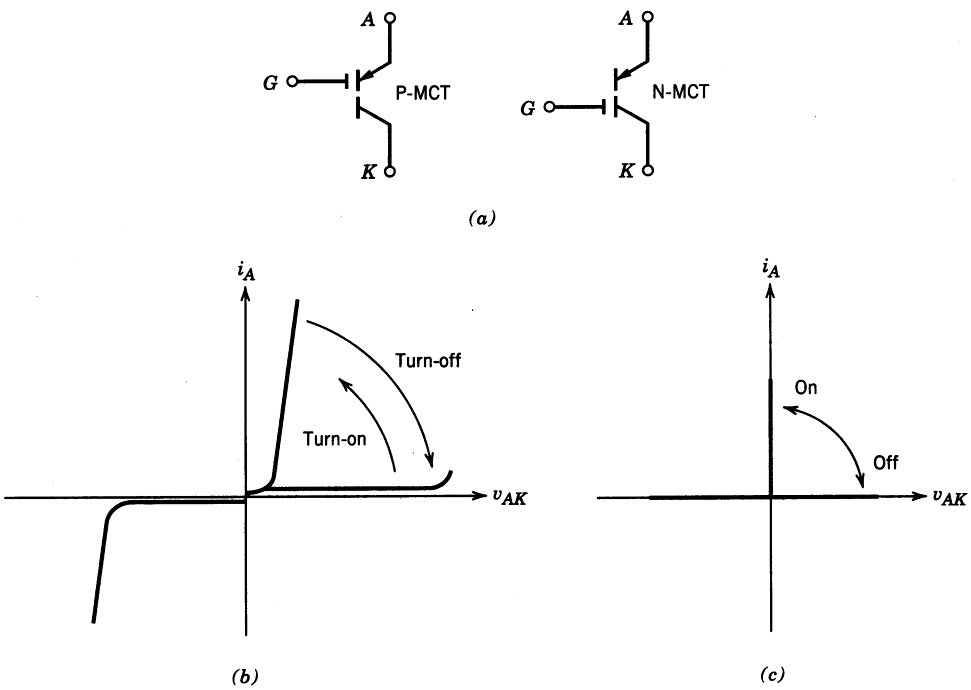

Fig.1.11 An MCT : (a) symbol, (b) i-v characteristics, (c) idealized.

1-5-5 TRIAC

TRIAC is similar to two back to back (anti parallel) connected thyristors but with only three terminals. Unlike thyristor the TRIAC can conducts in both directions from anode to cathode and vice versa. Ratings from (2-50 A) and (200-800 V). Used in lamp dimmers, home appliances, and hand tools. Not as rugged as many other device types, but very convenient for many ac applications. . The circuit symbol for the TRIAC is shown in Fig.1.12a and its steady state *i-v* is shown in Fig. 1.12b. Fig.1.13 show photo for TRIACs.

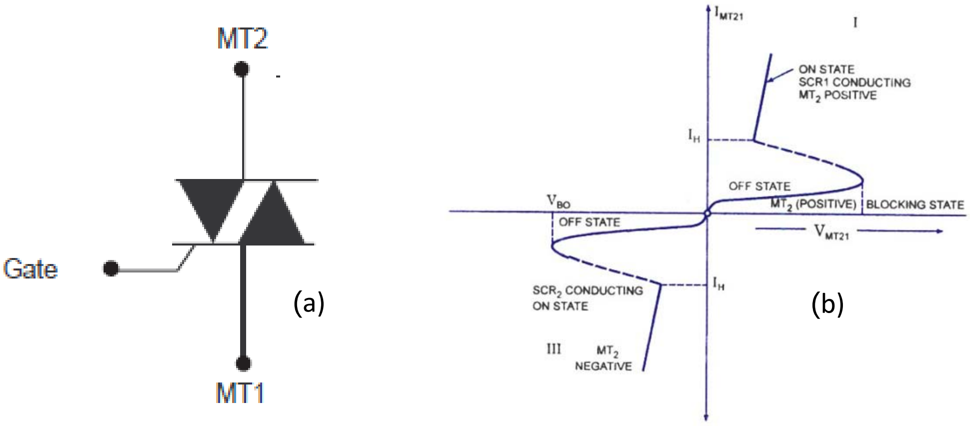

Fig.1.12 TRIAC: (a) symbol, (b) i-v characteristics.

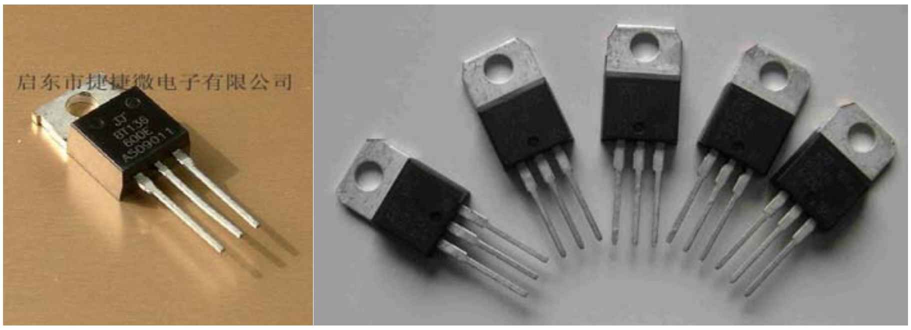

Fig.1.13 Photos for TRIACs.

Table 1.1 Summery of the Power Semiconductor Devices.

Current	**Uncontrollable**	**On Controllable**	**On and Off Controllable**
Uni-Direction	Diode	Thyristor	GTO MOSFET IGBT MCT
Bi-Direction		TRIAC	

AC/DC Converter (Rectifier)

Single and Three Phase AC/DC Converters

Fig. (1) shows the inputs and outputs of AC to DC converters. The input is single phase or three phase AC supply normally available from the mains. The output is the controlled DC voltage and current.

Fig. (1) AC to DC converters

The AC to DC converters include diode rectifiers as well as controlled rectifiers. The controlled rectifiers mainly use SCRs. Since the input is AC supply, the SCRs are turned off natural commutation. Hence external commutation circuits are not required. Hence AC to DC converters are also called as line (supply) commutated converters.

These converters are used for DC drives, uninterruptible power supplies (UPS), high voltage DC transmission (HVDC) systems.

1- Single phase diode rectifiers

Rectification is the process of conversion of alternating input voltage to direct output voltage. As stated before, a rectifier converts ac power to dc.

In diode –based rectifiers, the output voltage cannot be controlled.

In this section, uncontrolled single phase rectifiers are studied. The diode is assumed ideal has no forward voltage drop (V_d =0).

1-1 Single phase half wave uncontrolled rectifier

This is the simplest type of uncontrolled rectifier. It is never used in industrial applications because of its poor performance. Its study is, however, useful in understanding the principle of rectifier operation.

In a single phase half wave rectifier, for one cycle of supply voltage, there is one half cycle of output, or load, voltage.

The load on the output side of rectifier may be R, RL or RL with a freewheeling diode .These are now discussed briefly.

a- R load

The circuit diagram of a single half wave rectifier is shown in fig. (2-a). The waveforms of v_s, v_o, i_o and v_d are sketched in fig.(2-b). For a resistive load, output current i_o has the same waveform as that of the output voltage v_o. Diode voltage is zero when diode conducts.

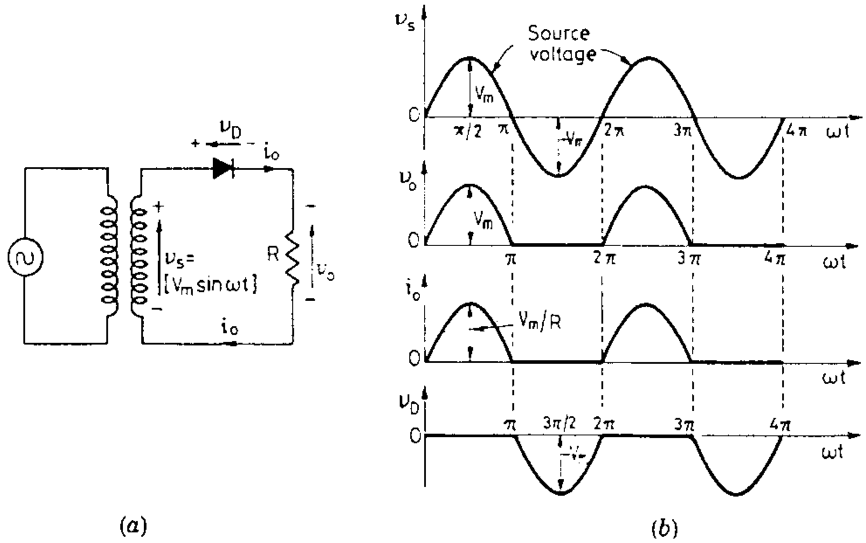

Fig. (2) Single phase half wave diode rectifier with R load
(a) circuit diagram and (b) waveforms

$v_s = v_D + v_0$

When diode is forward biased ,it is therefore turned on

$v_D = 0$ (ideal)

$v_s = v_o$

$$V_{mean} = V_o = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta d\theta\right]$$

$$= \frac{Vm}{2\pi} |-\cos\ \theta|_0^{\pi} = \frac{Vm}{\pi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$I_{mean} = I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \qquad \text{……………………..(2)}$$

$$P_{o\ mean} = V_o\ I_o \qquad \text{……………………..(3)}$$

$$V_{o\ rms} = \left[\frac{1}{2\pi}\int_0^{\pi}(V_m \ \sin\theta)^2\ d\ \theta\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos\ 2\theta}{2}\right)d\theta\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left|\theta - \frac{\sin\ 2\theta}{2}\right|_0^{\pi}\right]^{1/2} = \frac{V_m}{2} \qquad \text{………………………(4)}$$

$$I_{rms} = I_o = \frac{V_{o\ r}}{R} = \frac{V_m}{2R} \qquad \text{………………………(5)}$$

$$P_{o\ \ rms} = V_o\ I_o \qquad \text{………………………(6)}$$

The efficiency of a rectifier

$$\eta = \frac{P_{mean}}{P_{rms}} \qquad \text{………………………(7)}$$

The form factor (FF), which is a measure of the shape of output voltage

$$\text{FF} = \frac{V_{rms}}{V_{mean}} \qquad \text{………………………(8)}$$

The ripple factor (RF), which is a measure of the ripple content

$$\text{RF} = \frac{V_{rms}}{V_{mean}} = \frac{\sqrt[2]{V_{rms}^2 - V_{mean}^2}}{V_{mean}} = \sqrt[2]{\left(\frac{V_{rms}}{V_{mean}}\right)^2 - 1} = \sqrt[2]{FF^2 - 1} \qquad \text{…………….....(9)}$$

Ex:

The rectifier in fig.(1) has a purely resistive load of R.

Determine a) the efficiency, b) the form factor , c) the ripple factor.

Solution:

$$V_{mean} = \frac{\boldsymbol{V_m}}{\boldsymbol{\pi}} = 0.318\ \mathrm{V}_{\boldsymbol{m}}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{0.318V_m}{R}$$

$$P_{mean} = \frac{(0.318\, V_m)^2}{R}$$

$$V_{rms} = \frac{V_m}{2} = 0.5\, V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.5\, V_m}{R}$$

$$P_{rms} = \frac{(0.5\, V_m)^2}{R}$$

a) $\eta = \frac{(0.318\, V_m)^2}{(0.5\, V_m)^2} = 40.5\ \%$ (Low)

b) FF = $\frac{0.5\, V_m}{0.318\, V_m} = 1.57 = 157\ \%$ (high)

c) RF = $\sqrt[2]{(1.57)^2 - 1} = 1.21 = 121\%$ (high)

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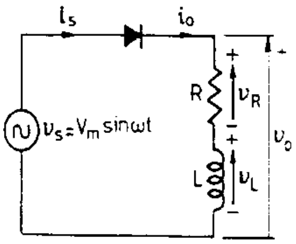
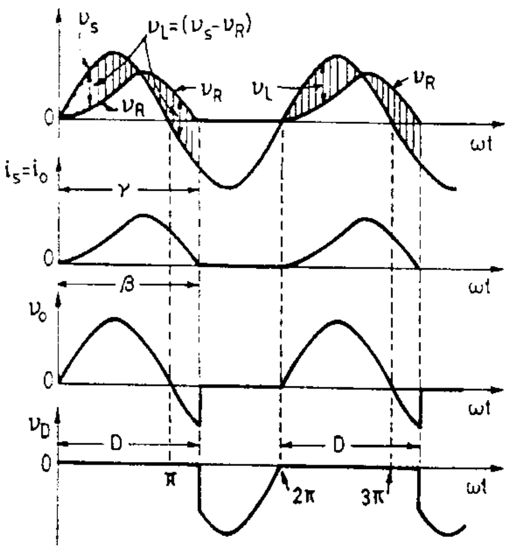
b- RL load

A single phase half wave rectifier feeding RL load is shown in fig. (3-a). Current continues to flow-even after source voltage v_s has become negative; this is because of the presence of inductance L in the load circuit.

Voltage $v_R = i_o R$ has the same wave shape as i_o. Inductor voltage $v_L = v_s - v_R$ is also shown. The current i_o flows till the two areas A and B are equal. Area A represents the energy stored by L and area B the energy released by L. It must be noted that mean value of voltage v_L across inductor L is zero.



(a)



(b)

Fig.(3) Single phase half wave diode rectifier with RL load

(a) Circuit diagram and (b) waveforms

When $\mathrm{i}_o = 0$ at $= \beta$, $\mathrm{v}_L = 0$, $\mathrm{v}_R = 0$ and voltage v_s appears as reverse bias across diode D as shown. At β, voltage v_D across diode jumps from zero to $V_m \sin\beta$ where $> \pi$. Here $\beta = \gamma$ is also the conduction angle of the diode.

$$V_{mean} = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\theta \, d\theta = \frac{V_m}{2\pi}(1 - \cos\beta) \quad \text{……………………………(10)}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{V_m}{2\pi R}(1 - \cos\beta) \quad \text{……………………………(11)}$$

A general expression for output current i_o for $0 < \theta < \beta$ can be obtained as under:

When diode is conducting, the circuit of fig.(3-a) gives

$$R\mathrm{i}_o + L\frac{d\mathrm{i}_o}{dt} = V_m \sin\theta$$

The load current i_o consists of two components, one steady state component i_s and the other transient component i_t. Here i_s is given by

$$\mathrm{i}_s = \frac{V_m}{\sqrt[2]{R^2 + X^2}}\sin(\theta - \emptyset)$$

Where $\emptyset = \tan^{-1}\frac{X}{R}$ and $X = \omega L$ .Here $\emptyset$ is the angle by which rms current $\mathrm{I_s}$ lage $\mathrm{V_s}$.

The transient component i_t can be obtained from equation

$$R\mathrm{i}_t + L\frac{d\mathrm{i}_t}{dt} = 0$$

Its solution gives $\mathrm{i}_t = Ae^{-\frac{R}{L}t}$

Total solution for current i_o is, therefore, gives by

$$\mathrm{i}_o = \mathrm{i}_s + \mathrm{i}_t = \frac{V_m}{Z}\sin(\theta - \emptyset) + A\,e^{-\frac{R}{L}t} \quad \text{………………………..(12)}$$

Where $Z = \sqrt[2]{R^2 + X^2}$

Constant A can be obtained from the boundary condition at $\omega t = 0$.

At $\omega t = 0$, or at $t = 0$, $i_o = 0$. Thus, from eq. (12)

$$0 = -\frac{V_m}{Z}\sin\emptyset + A$$

$$A = \frac{V_m}{Z}\sin\emptyset$$

Substitution of A in eq. (12) gives

$$i_o = \frac{V_m}{Z}\left[\sin(\theta - \emptyset) + \sin\emptyset e^{-\frac{R}{L}t}\right] \quad \text{……………………….(13)}$$

c- RL load with freewheeling diode

Performance of single phase half wave diode rectifier with RL load can be improved by connecting a freewheeling diode across the load as shown in fig.(4-a). Output voltage is $v_o = v_s$ for $0 \le \theta \le \pi$. At $\omega t = \pi$, source voltage v_s is zero, but output current i_o is not zero because of L in the load circuit. Just after $\omega t = \pi$, as v_s tends to reverse, negative polarity of v_s reaches cathode of FD through conducting diode D,whereas positive polarity of v_s reaches anode of FD direct. Freewheeling diode FD, therefore, gets forward biased. As a result, load current i_o is immediately transferred from D to FD as v_s tends to reverse. After $= \pi$, diode current $i_s = 0$ and it is subjected to reverse voltage with PIV equal to V_m.

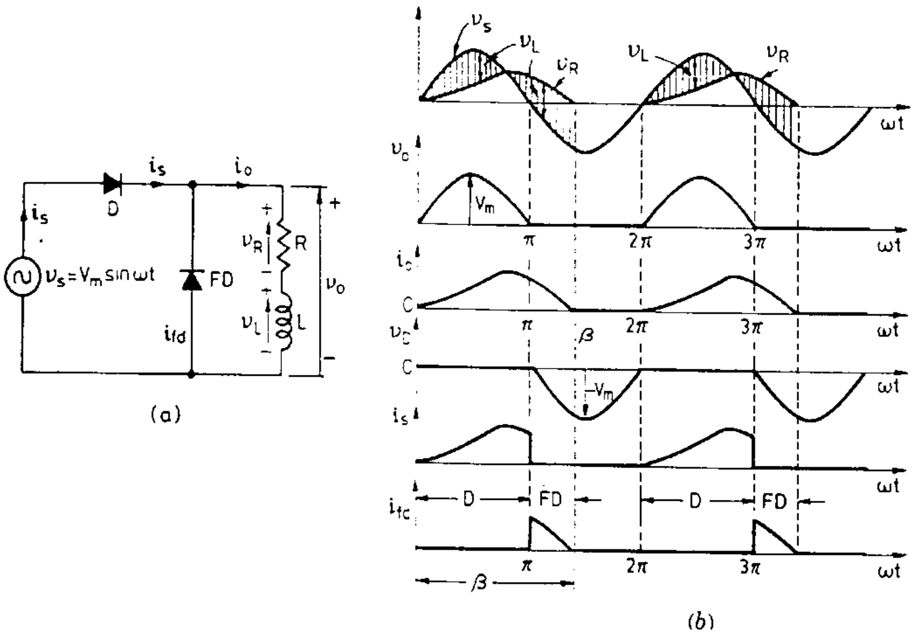


Fig.(4) Single phase half wave diode rectifier with RL load and freewheeling diode (a) circuit diagram and (b) waveforms

After $\omega t = \pi$, current freewheels through circuit RL and FD. The energy stored in L is now dissipated in R. When energy stored in L = energy dissipated in R, current fails to zero at $\omega t = \beta < 2\pi$.

The effects of using freewheeling diode are as under:

1- It prevents the output voltage from becoming negative.
2- As the energy stored in L is transferred to load R through FD, the system efficiency is improved.
3- The load current waveform is more smooth, the load performance is therefore improved.

The waveforms for the v_s , v_o , i_o , v_D , i_s and i_{fd} are drawn in fig.(4-b).

$$V_{mean} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta \, d\theta = \frac{V_m}{\pi} \qquad \text{……………………………(14)}$$

$$I_{mean} = \frac{V_m}{\pi R}$$

……………………………(15)

d- RE load

Single phase half wave rectifier with load resistance R and load E is shown in fig.(5-a).

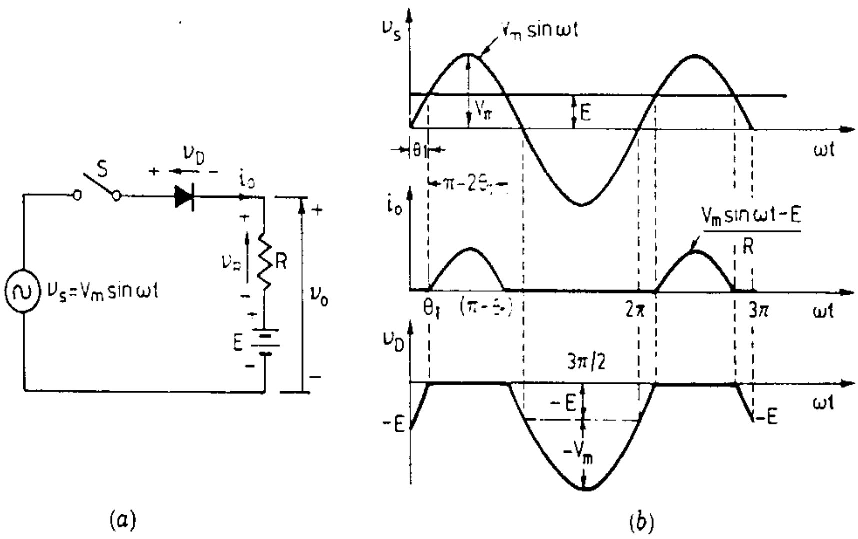


Fig.(5) Single phase half wave diode rectifier with RE load

The diode would not conduct at $\theta = 0$ because diode is reverse biased until source voltage v_s equals E . When $V_m \sin\theta_1 = E$, diode D starts conducting and turn-on angle θ_1 is given by

$v_s = E$

$V_m \sin\theta_1 = E$

$\theta_1 = \sin^{-1}\frac{E}{V_m}$ ……………………….(16)

The diode now conducts from $\theta = \theta_1$ to $\theta = (\pi - \theta_1)$, i.e. conduction angle for diode is $(\pi - 2\theta_1)$ as shown in fig.(5-b). During the conduction period of diode, the voltage equation for the circuit is

$V_m \sin\theta = E + i_o$

$i_o = \frac{V_m \sin\,\theta - E}{R}$ ……………………….(17)

$$I_{mean} = \frac{1}{2\pi R}\left[\int_{\theta_1}^{\pi-\theta_1}(V_m \sin\theta - E)\,d\theta\right]$$

$= \frac{1}{2\pi R}[2V_m \cos\theta_1 - E(\pi - 2\theta_1)]$ ……………………….(18)

$$I_{rms} = \left[\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1}\left(\frac{V_m \sin\theta - E}{R}\right)^2 d\theta\right]^{1/2}$$

$= \left[\frac{1}{2\pi R^2}\int_{\theta_1}^{\pi-\theta_1}(V_m^2\sin^2\theta + E^2 - 2V_m E\sin\theta)d\theta\right]^{1/2}$

$= \left[\frac{1}{2\pi R^2}\{(V_s^2 + E^2)(\pi - 2\theta_1) + V_s^2 \sin 2\theta_1 - 4V_m E\cos\theta_1\}\right]^{1/2}$…………..(19)

$P_o = EI_o + I_{rms}^2 R$ (watts) ………………………….(20)

From fig.(5-b) of v_D:

At $\theta = 0^o$ then $v_D = -E$, at $\theta = \theta_1$ then $v_D = 0$

During period diode conducts, v_D=0. When $\theta = 3\pi/2$, $v_s = -V_m$

and $v_D = -(V_m + E)$. Thus PIV for diode is $(V_m + E)$

1-2 **Single phase bridge uncontrolled rectifier**

a- R load

A single phase bridge rectifier employing diodes is shown in fig.(6-a).When point (a) is positive with respect to point (b) , diodes D_1 , D_2 conduct together so that output voltage is v_{ab} . Each of the diodes D_3 and D_4 is subjected to a reverse voltage of v_s as shown in fig.(6-b), When point (b) is positive with respect to point (a), diodes D_3 , D_4 conduct together and output voltage is v_{ba} . Each of the two diodes D_1 and D_2 experience a reverse voltage of v_s as shown.

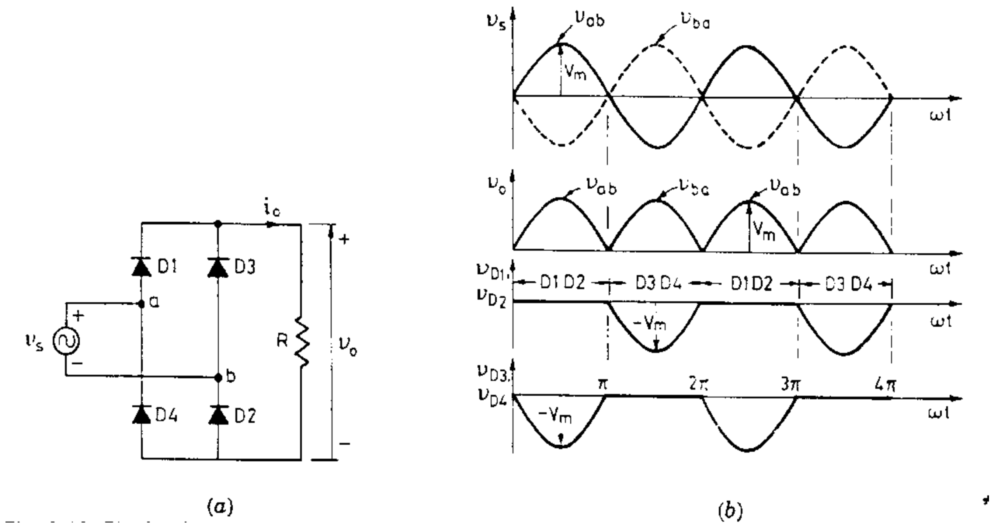


Fig.(6) Single phase bridge rectifier (a) circuit diagram and (b) waveforms

$$V_{mean} = V_o = \frac{2}{2\pi}\left[\int_0^{\pi} V_m \sin\theta d\theta\right] = \frac{2Vm}{\pi} \qquad \text{……………………(21)}$$

$$I_{mean} = I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \qquad \text{…………………….(22)}$$

$$V_{o\ rms} = \left[\frac{2}{2\pi}\int_0^{\pi}(V_m \ \sin\theta)^2 \ d\ \theta\right]^{1/2} = \frac{V_m}{\sqrt{2}} \qquad \text{………………………(23)}$$

$$I_{rms} = I_o = \frac{V_{o\ r}}{R} = \frac{V_m}{\sqrt{2}R} \qquad \text{……………………..(24)}$$

Ex:

The rectifier in fig.(6) has a purely resistive load of R. Determine a) the efficiency, b) the form factor , c) the ripple factor.

Solution:

$$V_{mean} = \frac{\boldsymbol{2V_m}}{\boldsymbol{\pi}} = 0.6366\ \mathrm{V}_{\boldsymbol{m}}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{0.6366V_m}{R}$$

$$\mathrm{P}_{mean} = \frac{(0.6366\ V_m)^2}{R}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\ V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707\ V_m}{R}$$

$$P_{rms} = \frac{(0.707\ V_m)^2}{R}$$

a) $\eta = \frac{(0.6366\ V_m)^2}{(0.707\ V_m)^2} = 81\ \%$

b) FF $= \frac{0.707\ V_m}{0.6366\ V_m} = 1.11 = 111\ \%$

c) RF $= \sqrt[2]{(1.11)^2 - 1} = 0.482 = 48.2\%$

The performance of a bridge rectifier is improved compared to that a half wave rectifier

b- **RL load**

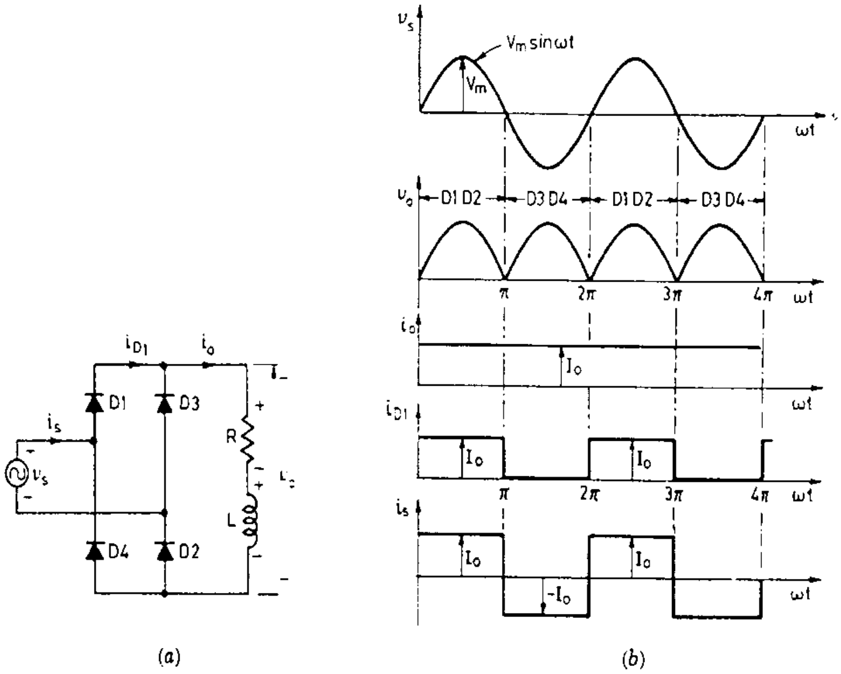


Fig.(7) Single phase bridge diode rectifier with RL load

(a) Circuit diagram and (b) waveforms

When L is very high the change of current and the ripple can be assumed to be zero. The load current can be assumed to be pure dc or the load current will be approximately pure dc.

$$I_{D_{1(rms)}} \& \, I_{D_{2(rms)}} = \left[\frac{1}{2\pi}\int_0^{\pi} I_{mean}^2 \, d\theta\right]^{1/2}$$

$$= \frac{I_{mean}}{\sqrt{2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

$$I_{s\,rms} = \left[\frac{2}{2\pi}\int_0^{\pi} I_{mean}^2 \, d\theta\right]^{1/2} = I_{mean} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

2- **Three phase half bridge uncontrolled rectifier**

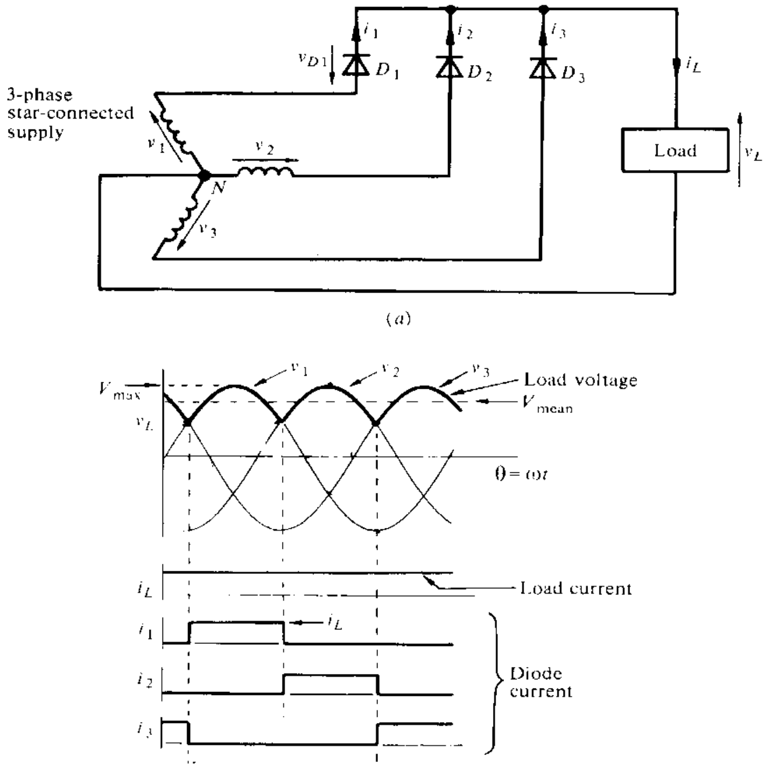


Fig. (8) Three phase half bridge rectifier (a) circuit diagram (b) waveform

In this case only one D will be at any instant this diode is the one connected to the branch having the highest voltage (e.g D on if V, it the highest voltage).When D_1 is on then when V_2 become higher than V_1, D_2 will be on and commutates D_1.

For usual case the load is inductive with L very high therefore it can be assumed to be continues level.

$$V_{mean} = \frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m \sin\theta \, d\theta = \frac{3\sqrt{3}}{2\pi} V_{m\,ph} \qquad \text{……………………………(27)}$$

$$I_{D_1\,rms} = \left[\frac{1}{2\pi} \int_{\pi/6}^{5\pi/6} I_{mean}^2 \, d\theta\right]^{1/2} = \frac{I_{mean}}{\sqrt{3}} \qquad \text{……………………………(28)}$$

3- Three phase bridge uncontrolled rectifier

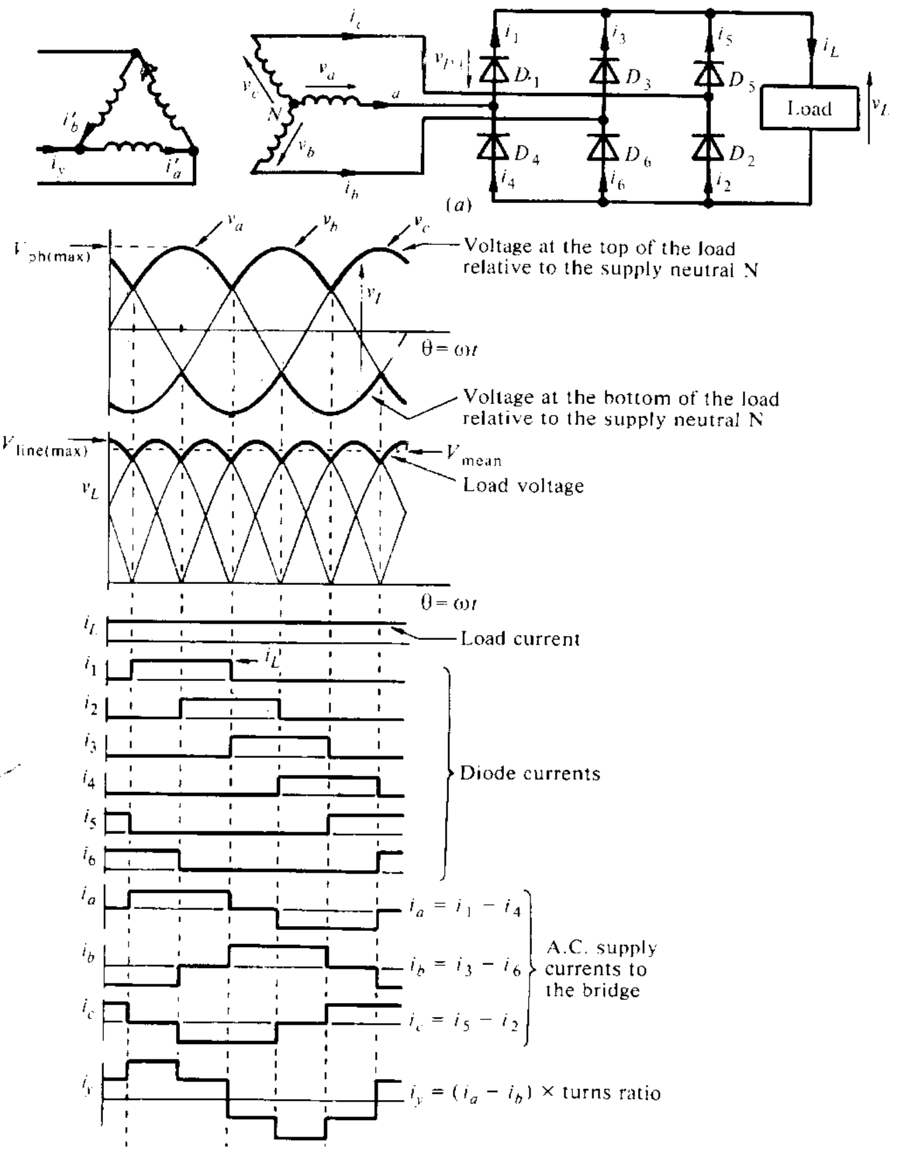


Fig. (9) Three phase bridge rectifier (a) circuit diagram (b)waveform

A three phase bridge rectifier is commonly used in high-power applications and it is shown in fig. This is a full wave rectifier .It can operate with or without a transformer and gives six-pulse ripples on the output voltage. The diodes are numbered in order of conduction sequences and each one conducts for (120°).

The conduction sequence for diodes is 12, 23, 34, 45, 56 and 61. The pair of diodes which are connected between that pair of supply lines having the highest amount of instantaneous line-to-line voltage will conduct. When v_a is the most positive phase diode D_1 conducts and during this period first v_b is the most negative

with diode D_6 conducting until v_c becomes more negative when the current in diode D_6 commutates to diode D_2.

The load voltage follows in turn six sinusoidal voltages during one cycle , these being $(v_a - v_b), (v_a - v_c), (v_b - v_c), (v_b - v_a), (v_c - v_a), (v_c - v_b)$,all having the maximum value of the line voltage that is $\sqrt{3}$ times the phase voltage. The line voltage can be found by V_{XN}, V_{YN}.

$$V_o = V_{XN} - V_{YN} = V_{XN} - (-V_{YN}) = V_{XN} + V_{YN}$$

$$V_{mean} = \frac{1}{\frac{2\pi}{6}} \int_{\pi/3}^{2\pi/3} V_{m\ line} \sin\theta \, d\,\theta = \frac{3V_{m\ line}}{\pi} \qquad \text{.................................(29)}$$

$$I_{a\ rms} = \left[\frac{2}{2\pi} \int_{\pi/6}^{5\pi/6} I_{mean}^2 \, d\theta\right]^{1/2} \ or \ \left[\frac{2}{2\pi} \int_{0}^{2\pi/3} I_{mean}^2 \, d\theta\right]^{1/2}$$

$$= \sqrt{\frac{2}{3}} I_{mean} \qquad \text{.................................(30)}$$

$$V_{rms} = \left[\frac{1}{\frac{2\pi}{6}} \int_{\pi/3}^{2\pi/3} (V_{m\ line})^2 \, (\sin\theta)^2 d\theta\right]^{1/2} = 1.6554\, V_m \qquad \text{......................(31)}$$

Ex:

A three phase bridge rectifier has a purely resistive load of R. Determine a) the efficiency, b) the form factor , c) the ripple factor.

Solution:

$$V_{mean} = \frac{\boldsymbol{3V_{m\,line}}}{\boldsymbol{\pi}} = \frac{3\sqrt{3}\mathrm{V_m}}{\pi} = 1.654\ \mathrm{V}_{\boldsymbol{m}}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{1.654V_m}{R}$$

$$\mathrm{P}_{mean} = \frac{(1.654\ V_m)^2}{R}$$

$$V_{rms} = 1.6554\, V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{1.6554\ V_m}{R}$$

$$P_{rms} = \frac{(1.6554\, V_m)^2}{R}$$

a) $\eta = \frac{(1.654\, V_m)^2}{(1.6554\, V_m)^2} = 99.83\ \%$

b) FF $= \frac{1.6554\, V_m}{1.654\, V_m} = 1.0008 = 100.08\ \%$

c) RF $= \sqrt[2]{(1.0008)^2 - 1} = 0.04 = 4\%$

	Single phase half wave rectifier	Single phase bridge rectifier	Three phase bridge rectifier
η	40.5%	81%	99.83%
FF	157%	111%	100.08%
RF	121%	48.2%	4%

3- **Single phase thyristor rectifiers (controlled rectifier)**

Controlled rectifiers are basically AC to DC converters. The power transferred to the load is controlled by controlling triggering angle of the devices. Fig.(10) shows this operation.

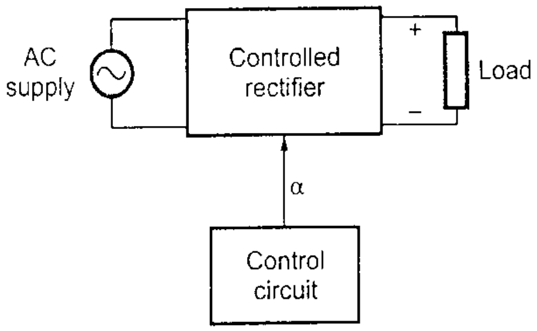


Fig. (10) Principle of operation of a controlled rectifier

The triggering angle (α) of the devices is controlled by control circuit. The input to the controlled rectifier is normally AC mains. The output of the controlled rectifier is adjustable DC voltage. Hence the power transferred across the load is regulated. Use thyristors (or one of its family traic, power transistor) as the main components.

3-**1 Single phase half wave controlled rectifier**

a- R load

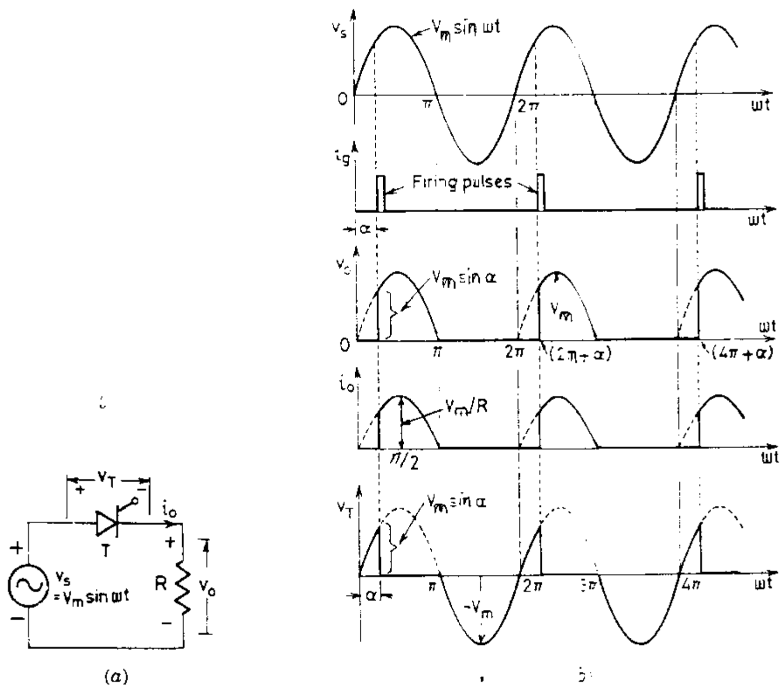


Fig.(11) Single phase half wave thyristor circuit with R load

An SCR can conduct only when anode voltage is positive and a gating signal is applied. The SCR will be off until a pulse is supplied to the gates which turn the SCR on. For resistive load, current i_o is in phase with v_o. Firing angle is therefore measured between the supply zero and the instant of pulse.

$$V_{mean} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\theta \, d\theta = \frac{V_m}{2\pi}(1+\cos\alpha) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

The maximum value of V_{mean} occurs at $\alpha = 0^{\circ}$

$$V_{mean} = \frac{V_m}{2\pi} \times 2 = \frac{V_m}{\pi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{V_m}{2\pi R}(1+\cos\alpha) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34)$$

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}(V_m \sin\theta)^2 d\theta\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(35)$$

$$I_{rms} = \frac{V_{rms}}{R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(36)$$

b- RL load

A single phase half wave thyristor circuit with RL load is shown in fig.(12-a). At $\omega t = \alpha$, thyristor is turned on by gating signal. The load voltage v_o at once becomes equal to source voltage v_s as shown. But the inductance L forces the load, or output, current i_o to rise gradually. After some time, i_o reaches maximum value and then begins to decrease. At $\omega t = \pi, v_o$ is zero but i_o is not zero because of the load inductance L. After $\omega t = \pi$, SCR is subjected to reverse anode voltage but it will not be turned off as load current i_o is not less than the holding current. At some angle $\beta > \pi, i_o$ reduces to zero and SCR is turned off as it is already reverse biased. After $\omega t = \beta, v_o = 0$ and $i_0 = 0$. At $\omega t = 2\pi + \alpha$, SCR is triggered again, v_o is applied to the load and load current develops as before.

The voltage equation for the circuit is

$$V_m \sin\theta = R i_o + L\frac{di_o}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(37)$$

The load current i_o consists of two components:

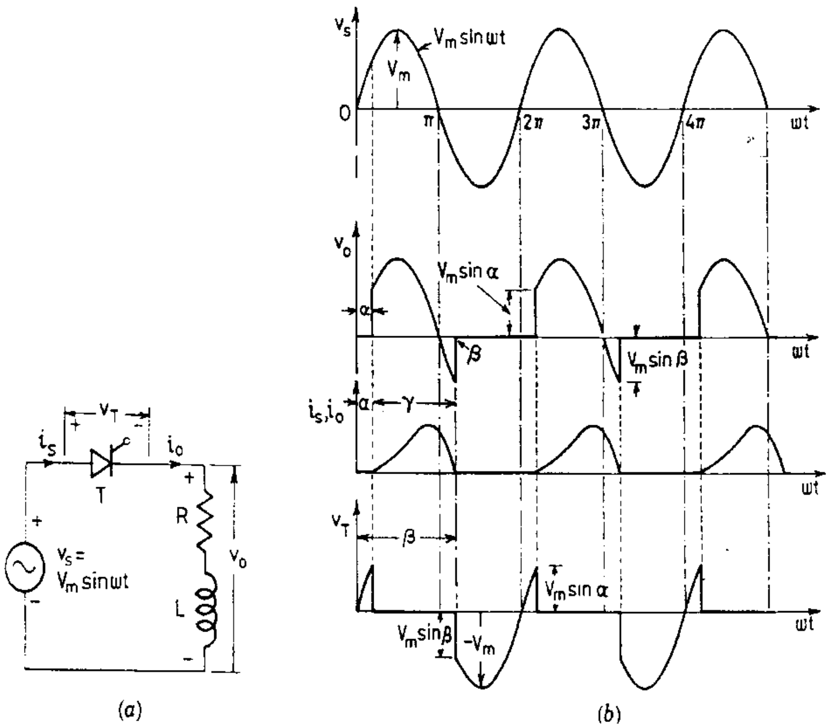


Fig. (12) Single phase half wave circuit with RL load

1- The steady sate component i_s

$$\mathrm{i}_s = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \emptyset)$$

Where $\emptyset = \tan^{-1}\frac{\omega L}{R}$. Here $\emptyset$ is the angle by which rms current I_s and V_s .

2- The transient component i_t

$$R\mathrm{i}_t + L\frac{d\mathrm{i}_t}{dt} = 0$$

It solution gives, $\mathrm{i}_t = Ae^{-(R/L)t}$

$$\mathrm{i}_o = \mathrm{i}_s + \mathrm{i}_t = \frac{V_m}{Z}\sin(\omega t - \emptyset) + Ae^{-(R/L)t} \qquad \text{……………………………….(38)}$$

Where $Z = \sqrt{R^2 + (\omega L)^2}$

Constant A can be obtained from the boundary condition at $\omega t = \alpha$.

At this time $t = \frac{\alpha}{\omega}$, $\mathrm{i}_o = 0$. Thus, from eq.(38),

$$0 = \frac{V_m}{Z}\sin(\alpha - \emptyset) + Ae^{-R\alpha/L\omega}$$

$$A = -\frac{V_m}{Z}\sin(\alpha - \emptyset)e^{-R\alpha/\omega L}$$

Substitution of A in eq.(38) gives

$$i_o = \frac{V_m}{Z}\sin(\omega t - \emptyset) - \frac{V_m}{Z}\sin(\alpha - \emptyset)\ exp.\left\{-\frac{R}{\omega L}(\omega t - \alpha)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(39)$$

For $\alpha < \theta < \beta$

$$V_{mean} = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin\theta d\theta = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) \quad \ldots\ldots\ldots\ldots\ldots\ldots..........(40)$$

$$I_{mean} = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(41)$$

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta}(V_m \sin\theta)^2\, d\theta\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

rms load current can be obtained from eq.(39) if required.

c- RL load with freewheeling diode

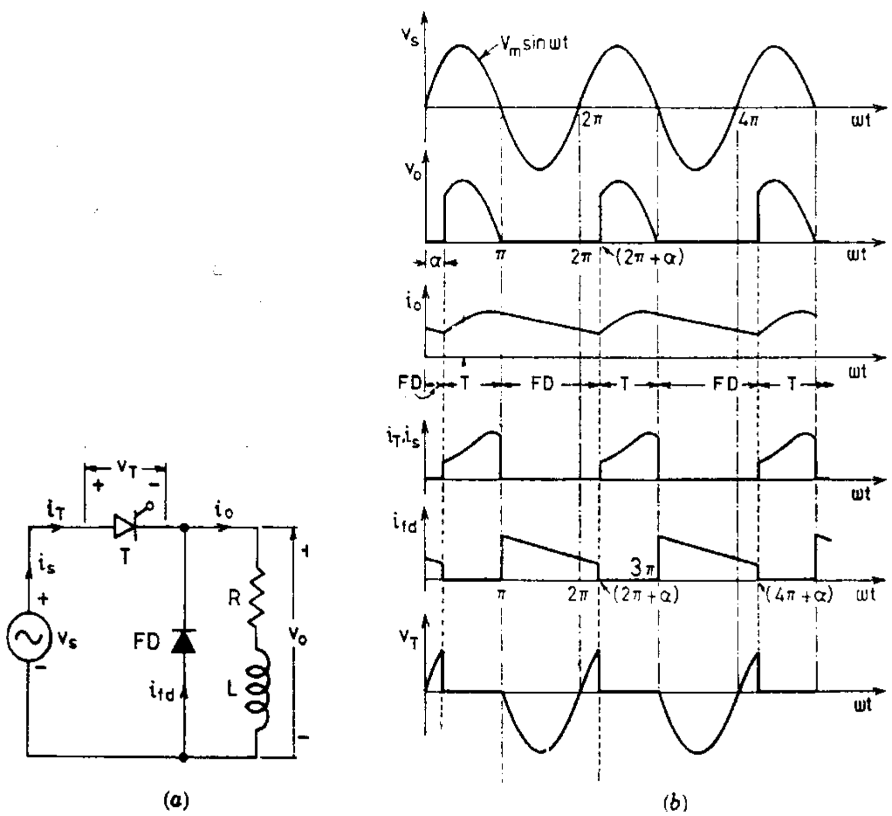


Fig.(13) Single phase half wave circuit with RL load and a freewheeling diode

$$V_{mean} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\theta \, d\theta = \frac{V_m}{2\pi}(1 + \cos\alpha) \qquad \text{.......................(43)}$$

$$I_{mean} = \frac{V_{mean}}{R} = \frac{V_m}{2\pi R}(1 + \cos\alpha) \qquad \text{........................(44)}$$

To remove the negative part form the load voltage wave form freewheeling diode is used as shown in the circuit diagram. Here, at zero crossing of v_s to negative half cycle , $L\frac{di}{dt}$ will cause f_d to forward biased. The thyristor will be reverse biased by the supply voltage and turned off. Load current will flow through the loop (L- R - f_d). As seen from the load current waveform that, during the positive half cycle of v_s , f_d is reversed biased and the load current equation is found from $V_o = Ri + L\frac{di}{dt}$. During the negative half cycle of v_s, f_d will be forward biased due to $L\frac{di}{dt}$, hence i_o is found from the equation $0 = Ri + L\frac{di}{dt}$. It can be seen from the load current waveform that increasing L to a very high value cause the load current waveform to approach a dc shape.

3-2 Single phase bridge controlled rectifier

a- Full controlled single phase bridge rectifier

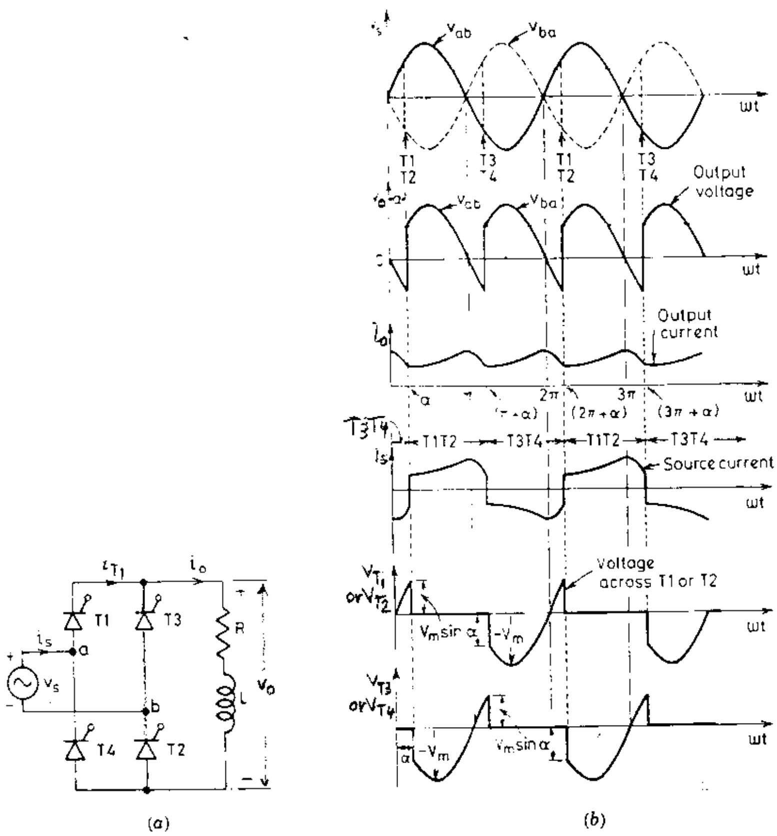


Fig.(14) Single phase full controlled bridge rectifier

$$V_{mean} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin\theta \, d\theta = \frac{2V_m}{\pi}\cos\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

The induced voltage $L\frac{di}{dt}$ in the inductance of the load keeps pairs of thyristors (T_1 & T_2 or T_3 & T_4) ON after the zero crossing point.

Therefore negative parts appears at the load voltage. However for example, when T_1 and T_2 are ON, then switching ON T_3 will cause T_1 to be reversed biased (can be seen from the circuit diagram and its voltages distributions) and hence switched off. The same can be said for T_4 and T_2.

The negative parts can be removed by adding freewheeling diode across the load. Also these negative parts can be removed by changing T_4 and T_2 to diodes.

Using f_d is preferred since it gives the required turn off time of the thyristors.

b- Half controlled single phase bridge rectifier

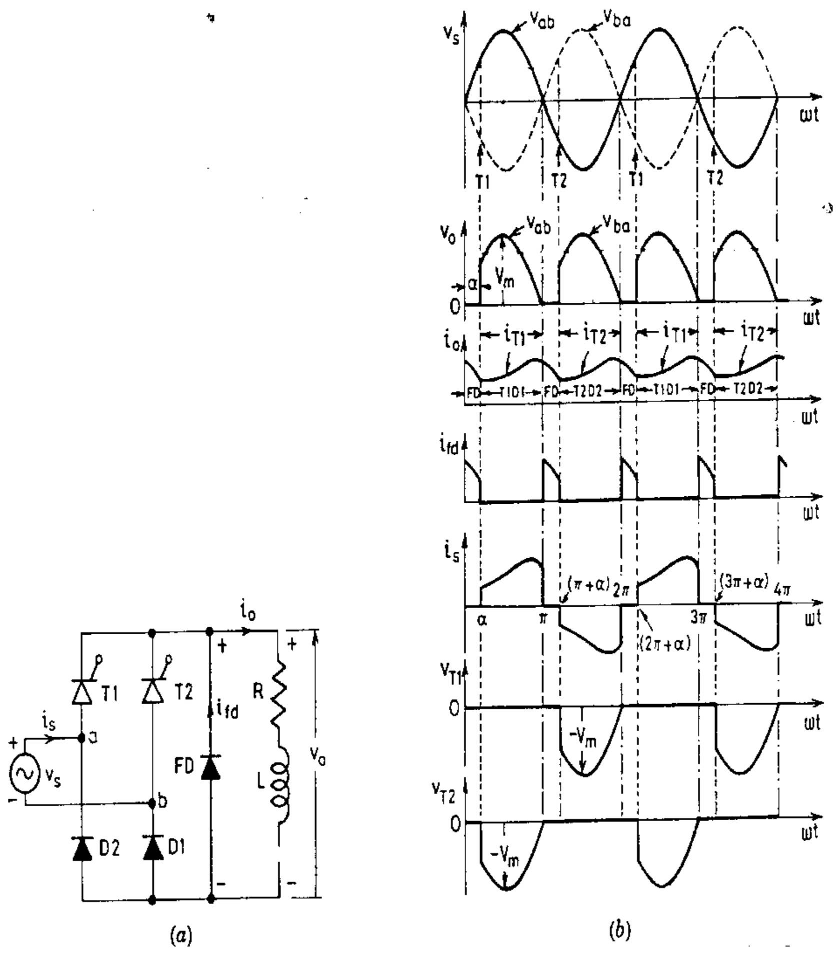


Fig. (15) Half controlled single phase bridge rectifier

$$V_{mean} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\theta \, d\theta = \frac{V_m}{\pi}(1+\cos\alpha) \qquad \text{.........................(46)}$$

$$I_{\text{fd } rms} = \left[\frac{2}{2\pi}\int_{0}^{\alpha}(I_{mean})^2 \, d\theta\right]^{1/2} = I_{mean}\sqrt{\frac{\alpha}{\pi}} \qquad \text{..........................(47)}$$

Compared to the full controlled circuit, the half controlled circuit is cheaper, but ac supply current is more distorted due to its zero periods.

4- Three phase half wave controlled rectifier

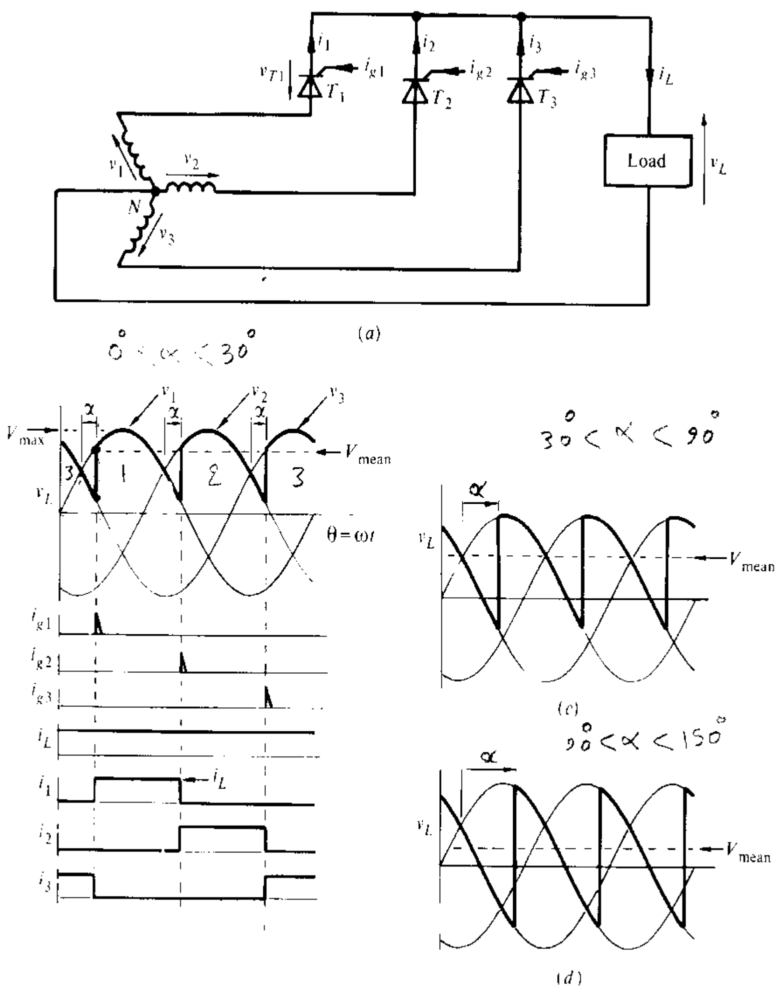


Fig. (16) Three phase half wave controlled circuit thyristors

a- From fig.(16-b) for $(0° < \alpha < 30°)$

$$V_{mean} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_{m\,ph} \sin\theta \, d\theta = \frac{3\sqrt{3}}{2\pi} V_{m\,ph} \cos\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(48)$$

b- From fig.(16-c) for $(30° < \alpha < 90°)$

$$V_{mean} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_{m\,ph} \sin\theta \, d\theta = \frac{3\sqrt{3}}{2\pi} V_{m\,ph} \cos\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(49)$$

c- Form fig.(16-d) for $(90° < \alpha < 150°)$

$$V_{mean} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_{m\,ph} \sin\theta \, d\theta = \frac{3V_{m\,ph}}{2\pi} \left[1 + \cos\left(\alpha + \frac{\pi}{6}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots(50)$$

5- **Three phase bridge controlled rectifier**

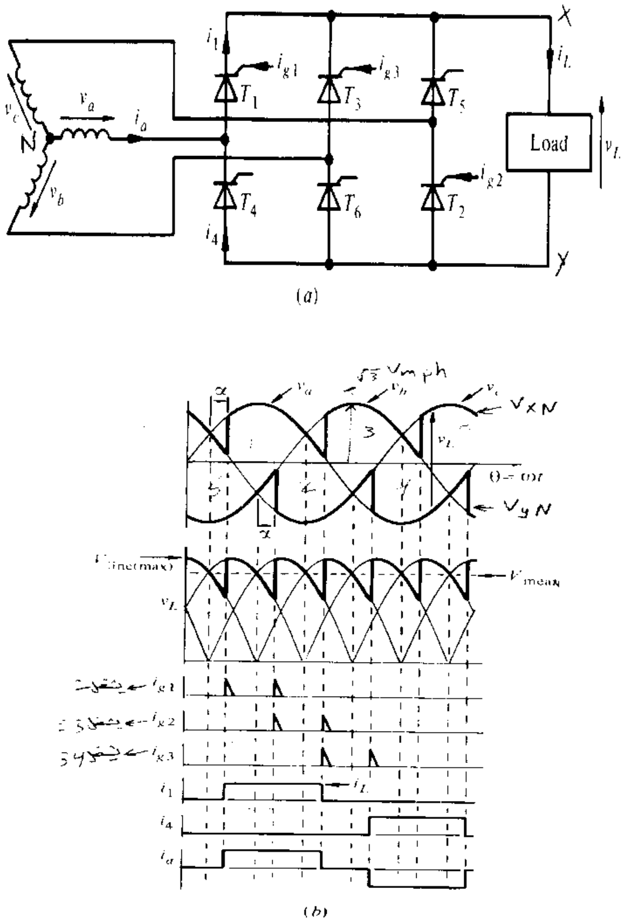


Fig.(17) Three phase bridge controlled circuit using thyristors

$$V_{mean} = \frac{1}{\frac{2\pi}{6}} \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_{m\ line} \sin\theta \, d\theta = \frac{3V_{m\ line}}{\pi} \cos\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(51)$$

6- Converter operation

6-1 Overlap

In previous, the assumption was made that the transfer or commutation of the current from one diode (or thyristor) to the next took place instantaneously.

In practice, inductance and resistance must be present in the supply source, and time is required for a current change to take place. The net result is that the current commutation is delayed, as it takes a finite time for the current to decay to zero in the outgoing diode (or thyristor) , while the current will rise at the same rate in the incoming diode.

The inductive reactance of the ac supply is normally much greater than its resistance and, as it is the inductance which delays the current change, it is reasonable to neglect the supply resistance. The ac supply may be represented by its Thevenin equivalent circuit , each phase being a voltage source in series with its inductance. The major contributor to the supply impedance is the transformer leakage reactance.

To explain the phenomenon associated with the current transfer, the three phase half wave rectifier connection will be used, as once the explanation with this circuit has been understood, it can be readily transferred to the other connections.

6-1-1 Three phase half bridge uncontrolled rectifier

Fig.(18-a) shows the three phase supply to be three voltages, each in series with an inductance (L). Reference to the waveforms in fig.(18-b) shows that at commutation there is an angular period (γ) during which both the outgoing diode and incoming diode are conducting.

This period is known as the overlap period, and (γ) is defined as the commutation angle or alternatively the angle of overlap.

The change of i_1 and i_2 during this period will be as shown in the current waveforms, (assuming a heavily inductive load), $i_1 + i_2 = i_o$ at any instant. The load voltage during this period equal to $\left(\frac{v_1+v_2}{2}\right)$ during (γ) .

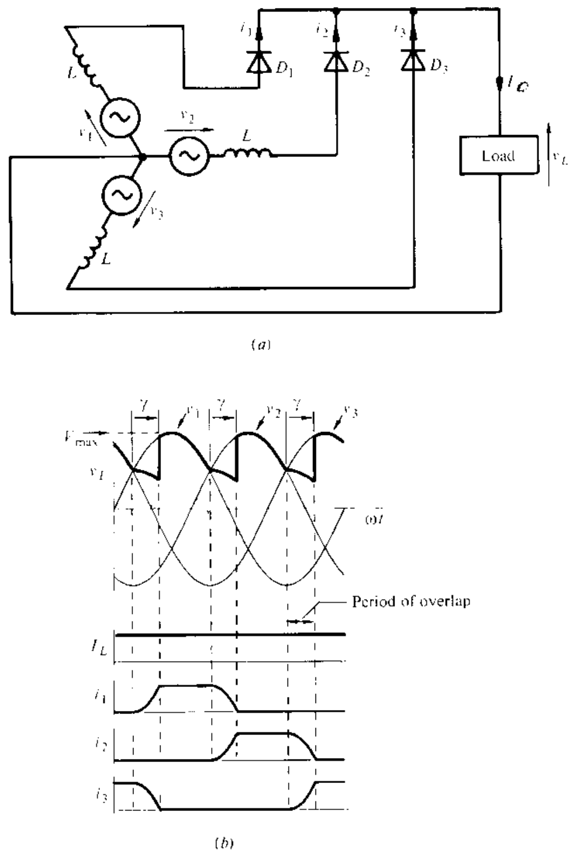


Fig.(18) Overlap in the three phase half wave uncontrolled rectifier

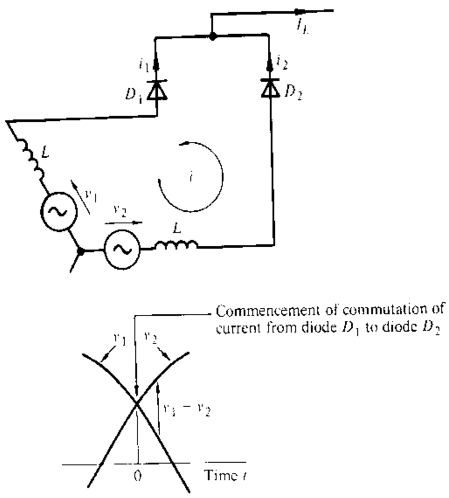


Fig. (19) Conditions during the overlap period

To determine the factors on which the overlap depends, and to derive an expression for the diode current, a circulating current i can be considered to flow in the closed path formed by the two conducting diodes D_1 and D_2 as shown in fig.(19).

$$v_2 - v_1 = L\frac{di}{dt} + L\frac{di}{dt} \qquad \text{……………………..(52)}$$

The voltage $v_2 - v_1$ is the difference between the two phases, having a zero value at t=0 , the time at which commutation commences. The voltage difference between two phases is the *line voltage* having a maximum value $\sqrt{3}V_m$ where V_m is of the phase voltage.

$$V_{line} = 2L\frac{di}{dt}$$

$$V_{m\ line}\ \sin\theta = 2L\frac{di}{dt}$$

$$di = \frac{V_{m\ line}}{2L}\ \sin\omega t\ dt$$

Integrating both sides,

$$i = \frac{\sqrt{3}V_{m\ ph}}{2L}\left(-\frac{\cos\omega t}{\omega}\right) + C$$

At $t = 0, i = 0, \therefore C = \frac{\sqrt{3}V_{m\ ph}}{2\omega L}$

$$\therefore i = \frac{\sqrt{3}V_{m\ ph}}{2\omega L}(1 - \cos\omega t) \qquad \text{……………………………..(53)}$$

The overlap is complete when $i = I_L = I_o = I_{mean}$, at which instant $\theta = \omega t = \gamma$, the overlap angle. Also $\omega L = X$, the supply source reactance. Hence,

$$I_{mean} = \frac{\sqrt{3}V_{m\ ph}}{2X}(1 - \cos\gamma) \qquad \text{…………………………..(54)}$$

$$\cos\gamma = 1 - \frac{2I_{mean}\ X}{\sqrt{3}V_{m\ ph}} \qquad \text{………………………………(55)}$$

From eq. (53), the current change in the diodes during overlap is consinusoidal, as illustrated in fig.(18-b).

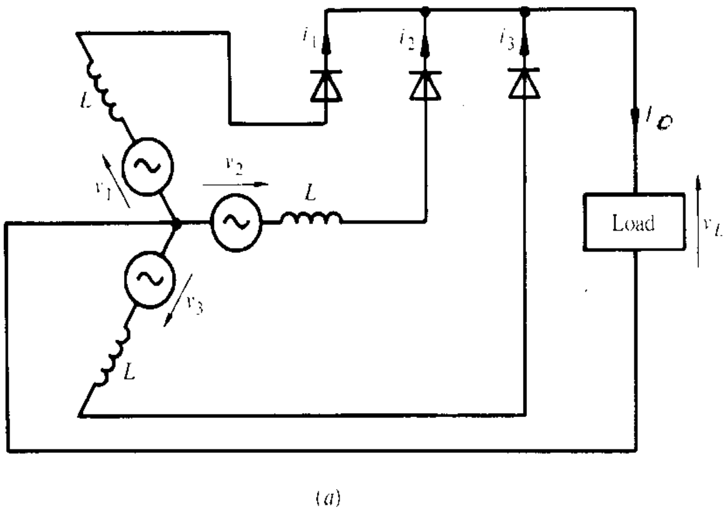
To determine the mean voltage of the waveform shown in fig.(18-a), one can use calculus to find the area under the two sections of the curve, one based on the sine wave shape after overlap is complete and the other during overlap.

During overlap, the load voltage is the mean between two sine waves, that is , the shape is sinusoidal, but if we consider the curve as a cosine wave, then the integration limits will be 0 to γ on a cosine wave of peak value $\left(V_{m\ \mathrm{ph}}\ \sin\frac{\pi}{6}\right)$, giving

$$V_{mean}\ =\frac{1}{\frac{2\pi}{3}}\left[\int_{\frac{\pi}{6}+\gamma}^{\frac{5\pi}{6}} V_{m\ ph}\sin\theta\ d\theta+\int_0^{\gamma} V_{m\ ph}\sin\frac{\pi}{6}\cos\emptyset\ d\emptyset\right]$$

$$=\frac{3\sqrt{3}V_{m\ ph}}{4\pi}(1+\cos\gamma) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(56)$$

6-1-2 Three phase half bridge controlled rectifier



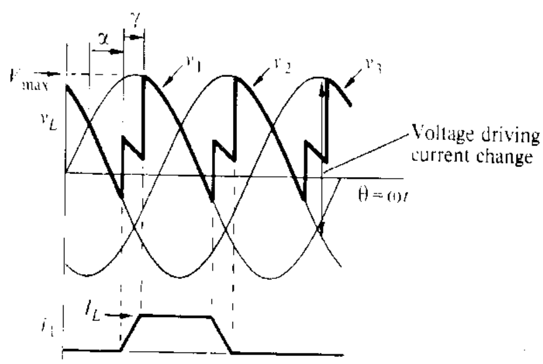


Fig.(20) Overlap in the three phase half wave controlled rectifier

In the controlled 3-pulse circuit, the overlap will lead to the waveform shown in fig.(20), where it can be seen that with a firing delay angle (α) a finite voltage is present form the start of commutation.

$$\mathrm{v}_2 - \mathrm{v}_1 = L\frac{d\mathrm{i}}{dt} + L\frac{d\mathrm{i}}{dt} \qquad \text{….. …………………….(57)}$$

$$V_{line} = 2L\frac{d\mathrm{i}}{dt}$$

$V_{m\ line}\ \sin(\omega\mathrm{t} + \alpha) = 2L\frac{d\mathrm{i}}{dt}$, where t is the time from the start of commutation, when i =0

$$d\mathrm{i} = \frac{V_{m\ line}}{2L}\ \sin(\omega t + \alpha)\ dt$$

Integrating both sides,

$$\mathrm{i} = \frac{\sqrt{3}V_{m\ ph}}{2L}\left(-\frac{\cos(\omega t + \alpha)}{\omega}\right) + C$$

At $t = 0, \mathrm{i} = 0, \therefore C = \frac{\sqrt{3}V_{m\ ph}}{2\omega L}\cos\alpha$

$$\therefore \mathrm{i} = \frac{\sqrt{3}V_{m\ ph}}{2\omega L}(\cos\alpha - \cos(\omega t + \alpha)) \qquad \text{……………………………..(58)}$$

The overlap is complete when $\mathrm{i} = I_L = I_o = I_{mean}$, at which instant $\theta = \omega t = \gamma$, the overlap angle. Also $\omega L = X$, the supply source reactance. Hence,

$$I_{mean} = \frac{\sqrt{3}V_{m\ ph}}{2X}[\cos\alpha - \cos(\gamma + \alpha)] \qquad \text{…………………………..(59)}$$

Compared to the uncontrolled case (α=0), the overlap angle γ will be shorter and the current change during commutation will be towards a linear variation. The mean load voltage is given by

$$V_{mean} = \frac{1}{\frac{2\pi}{3}}\left[\int_{\frac{\pi}{6}+\alpha+\gamma}^{\frac{5\pi}{6}+\alpha} V_{m\ ph}\sin\theta\ d\theta + \int_{\alpha}^{\alpha+\gamma} V_m \sin\frac{\pi}{6}\cos\emptyset\ d\emptyset\right]$$

$$= \frac{3\sqrt{3}V_{m\ ph}}{4\pi}[\cos\alpha + \cos(\alpha + \gamma)] \qquad \text{………………………………(60)}$$

6-2 Power factor

The power factor of a load fed from an ac supply is defined as:

$$power\ factor = \frac{mean\ \ power}{V_{rms}\ I_{rms}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(61)$$

In the usual ac system where the current is sinusoidal, the power factor is the cosine of the angle between current and voltage. The rectifier circuit, however, draws non-sinusoidal current from the ac system, hence the power factor cannot be defined simply as the cosine of the displacement angle.

The waveforms of the various controlled rectifiers in previous shows that firing delay has the effect of delaying the supply current relative to its phase voltage. The current dose contain harmonic components which result in its overall rms value being higher than the rms value of its fundamental component, therefore the power factor is less than that calculated from the cosine of its displacement angle.

Normally, the supply phase voltage can be taken as being sinusoidal, hence there will be no power associated with the harmonic current, which therefore result in

$$power = V_{1\ rms} I_{1\ rms} \cos \emptyset_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(62)$$

Where the suffix 1 relates to the fundamental component of the input to rectifier current, $\emptyset_1$ being the phase angle between the voltage and the fundamental component of the current.

For a sinusoidal voltage supply, substituting eq.(62) into eq.(61) yields

$$power\ factor = \frac{V_{1\ rms} I_{1\ rms} \cos \emptyset_1}{V_{rms} I_{rms}}$$

Where $\quad V_{1\ rms} = V_{rms}$

$$power\ factor = \frac{I_{1\ rms}}{I_{rms}} \cos \emptyset_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(63)$$

Where $\quad \frac{I_{1\ rms}}{I_{rms}} = input\ distortion\ factor \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(64)$

and $\quad \cos \emptyset_1 = input\ displacement\ factor \qquad \ldots\ldots\ldots\ldots...(65)$

$\emptyset_1$ will equal the firing delay angle α in the fully-controlled rectifier connection that have a continuous level load current.The power factor will always be less than unity when there are harmonic components in the supply current, even when the current is in phase with the voltage, as in the diode case.

University of Technology
Electromechanical Engineering Department
8 hrs / Four weeks
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Power Electronics and Electrical Drives
EME 401
Energy- Systems branch

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Contents

Definition: Inverters are static circuits that convert power from a DC to AC power at specified output voltage and frequency.

Typical Applications:Invertersare used in the following industrial applications:

1) Variable speed AC motor drivers.
2) Induction heating.
3) Aircraft power supply.
4) Uninterruptable power supplies for computers.
5) Traction.
6) High Voltage DC (HVDC).

Classification

Single phase (1**Φ**) and three phase (3**Φ**) inverters can be classified into:

1) Voltage source inverters (VSI).

The VSI convert the DC input voltage source into a square wave AC output voltage source as shown in Fig.1.The DC input voltage supply to the VSI can be a battery or the output of a controlled rectifier.A shunt capacitor in the input circuit of the VSI is used to provide the stiff voltage source. The VSI is commonly used for low and medium power application.

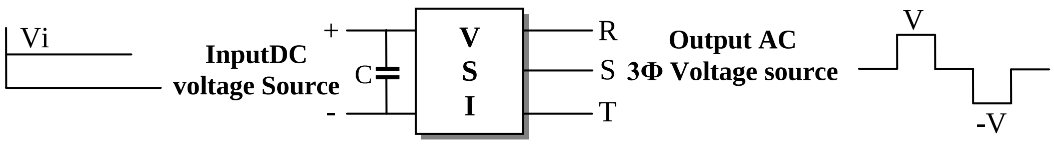


Fig.1 Voltage source inverter (VSI).

2) Current source inverters (CSI).

The CSI convert the DC input current source into a square wave AC output current source as shown in Fig.2.A series inductor in the input circuit of the CSI is used to provide the stiff current source.The CSI is usually used for high power application.

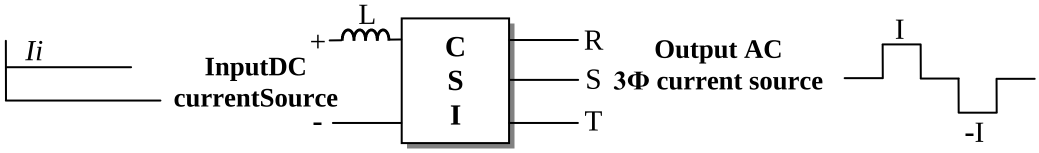


Fig.2 Current source inverter(CSI).

Voltage Source Inverter (VSI)

1) Single Phase Bridge Inverter (square wave)

(a) Resistive Load (R): The circuit of single phase bridge inverter fed resistive load is shown in Fig.3.

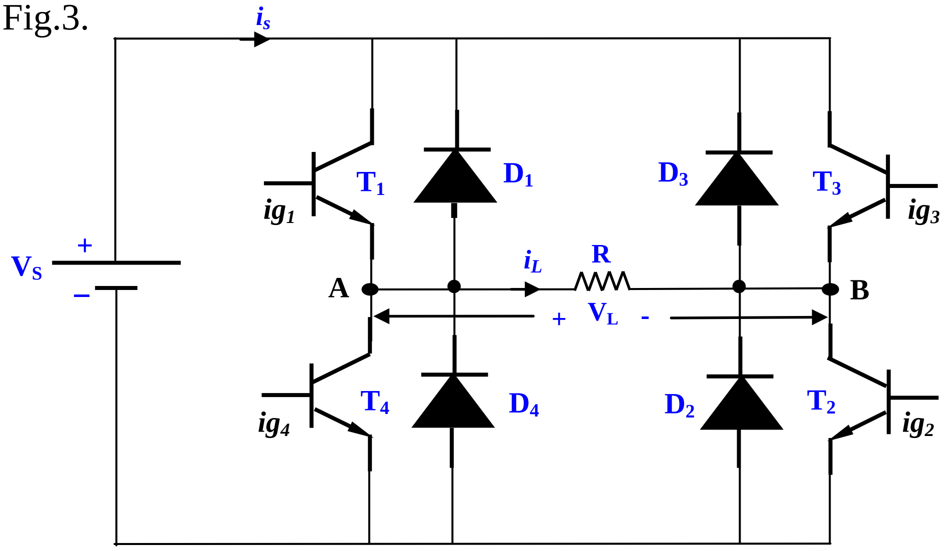
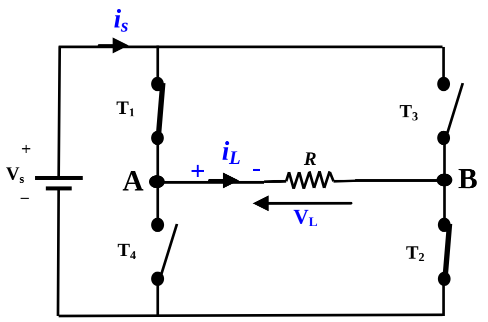
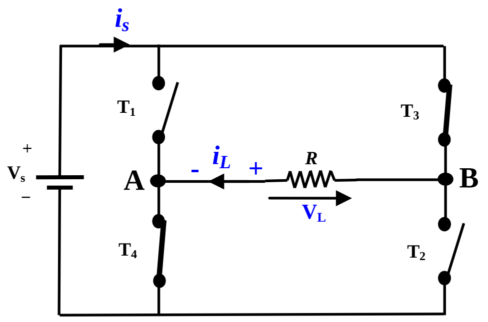


Fig.3 Single phase bridge inverter with R load.

Circuit Operation

- D_1,D_2,D_3,D_4 are eliminated from the circuit (always **OFF**) due to resistive load.
- From KVL T_1&T_4 and T_3&T_2 cannot be **ON** at the same time (short circuit).
- This circuit is operates in two modes as shown in table below.

Mode	Period	Conducting Devices	Equivalent Circuit	V_L	i_L
I	T/2	T_1&T_2		Vs	Vs/R
II	T	T_3&T_4		-Vs	-Vs/R

The inverter waveforms are shown below:

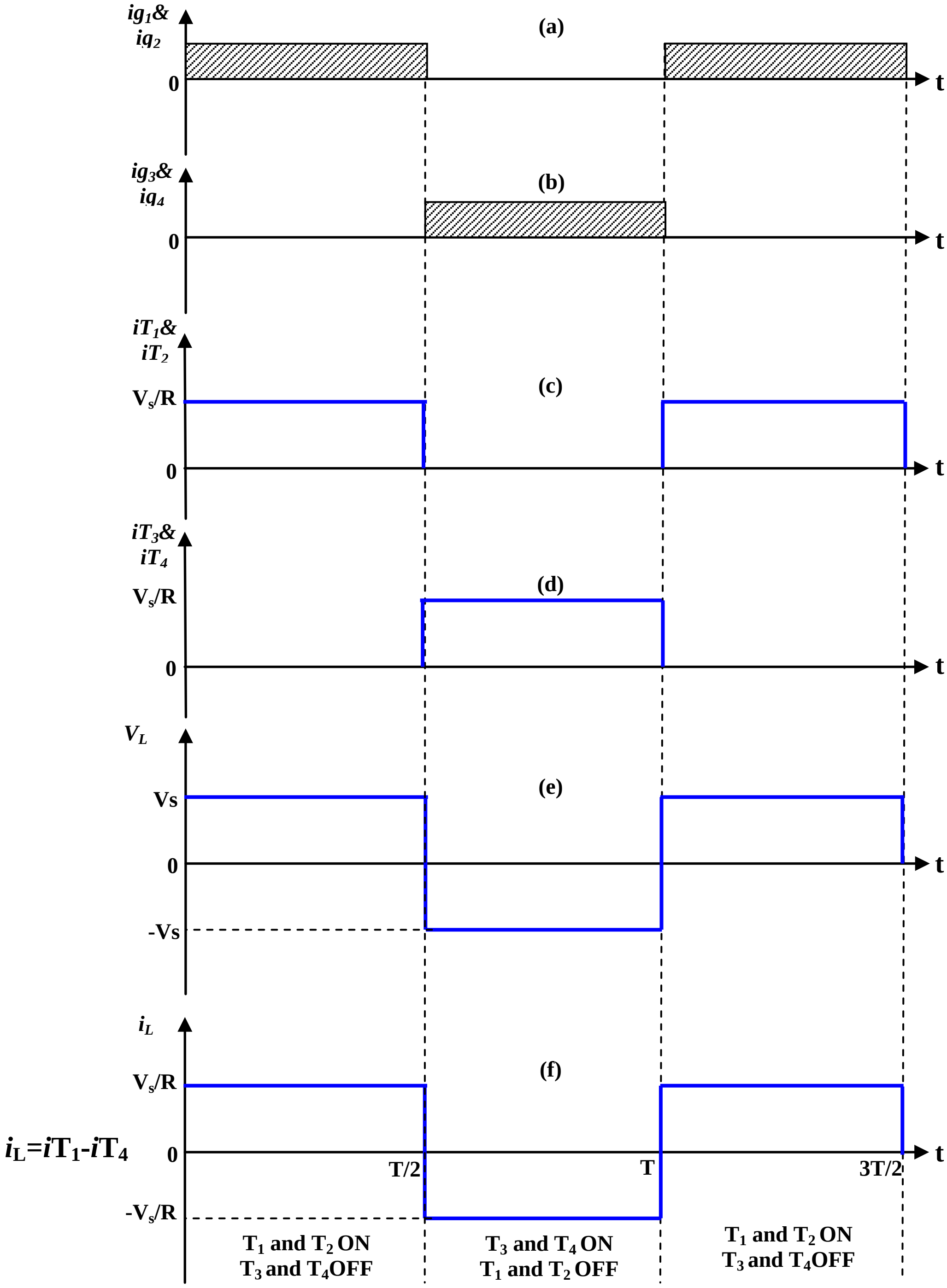


Fig.4 Waveforms (a) & (b) triggering, (c) &(d) transistors currents, (e) load voltage, and (f) load current.

Mathematical Analysis:

1) The rmsvalue of load voltage is given as,

$$V_{L(rms)} = \sqrt{\frac{1}{T/2}\int_0^{T/2} (V_s)^2 \, dt} = V_S$$

2) The instantaneous rmsvalue of load voltage in terms of Fourier series is:

$$v_{L(\omega t)} = V_{L(ave)} + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \varphi_n)$$

Where, $C_n = \sqrt{{a_n}^2 + {b_n}^2}$, $\varphi_n = \tan^{-1}\frac{a_n}{b_n}$ and T=period=2π

$$a_n = \frac{2}{T}\int_0^T v_{L(\omega t)} \cos(n\omega t) d\omega t$$

$a_n = \frac{2}{2\pi}\left[\int_0^{\pi}(V_s)cos(n\omega t)\, d\omega t \quad + \int_{\pi}^{2\pi}(-V_s)cos(n\omega t)\, d\omega t\right] = 0$ for all values of n

Now,the value of $\boldsymbol{b_n}$ can be calculated as:

$$b_n = \frac{2}{T}\int_0^T v_{L(\omega t)} \sin\,(n\omega t) d\omega t$$

$$b_n = \frac{2}{2\pi}\left[\int_0^{\pi}(V_s)\, sin\,(n\omega t)\, d\omega t \quad + \int_{\pi}^{2\pi}(-V_s)\, sin(n\omega t)\, d\omega t\right] = \frac{2\,V_s}{n\pi}(1 - cosn\pi)$$

$$b_n = \begin{Bmatrix} \frac{4V_s}{n\pi} & for\ odd\ values\ of\ n \\ 0 & for\ even\ values\ of\ n \end{Bmatrix}$$

$C_n = \sqrt{0 + {b_n}^2} = b_n$ and $\varphi_n = \tan^{-1} 0 = 0$

$$\therefore \quad C_n = \frac{4V_s}{n\pi} \qquad for\ odd\ values\ of\ n$$

Since the waveform (e) of Fig.4 has symmetric positive and negative cycles. Hence average value of such waveform is zero i.e.

$$V_{L(ave)} = 0$$

Therefore, the Fourier series for bridge inverter can be written as:

$$v_{L(\omega t)} = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

Where,**n** is the harmonic order

for**n= 2,4,6**,………$v_{L(\omega t)} = 0$, thus, output waveform contain only odd harmonics

3) Therms value of fundamental component of load voltage (n=1) :

$$v_{L1(\omega t)} = \frac{4V_s}{\pi} \sin \omega t$$

This has the same frequency(ωt) as that of square wave, the peak value of fundamental is,

$$C_1 = \frac{4V_s}{\pi}$$

The rms value of fundamental component is,

$$\boldsymbol{v_{L_1(rms)} = \frac{C_1}{\sqrt{2}} = \frac{4V_s}{\sqrt{2}\,\pi} = 0.9V_s}$$

The rms values of n^{th} harmonics can be obtained as,

$$v_{L_n(rms)} = \frac{4V_s}{\sqrt{2}\,n\pi} = \frac{0.9V_s}{n} \quad \text{for n} = 1,3,5, \dots \dots$$

4) The rms value of load current $I_{L(rms)} = \frac{V_{L(rms)}}{R}$

5) The value of output power $P_o = \frac{V_{L\ (rms)}^2}{R}$

(b) Heavily Inductive Load

1- Square wave output: The circuit of single phase bridge inverter fed heavily inductive load is shown in Fig.5.

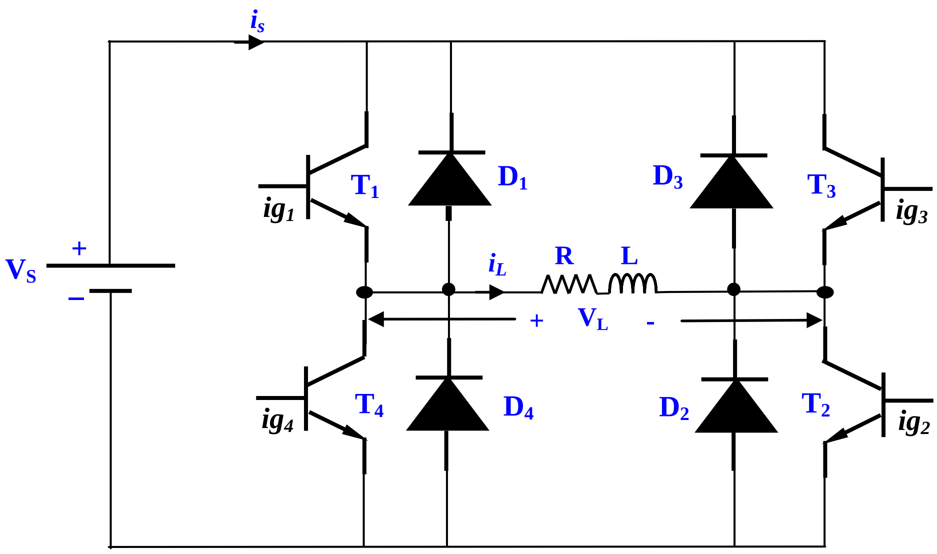
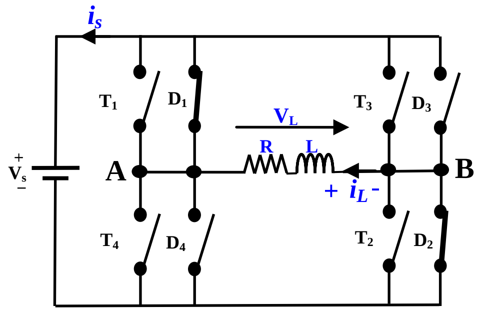
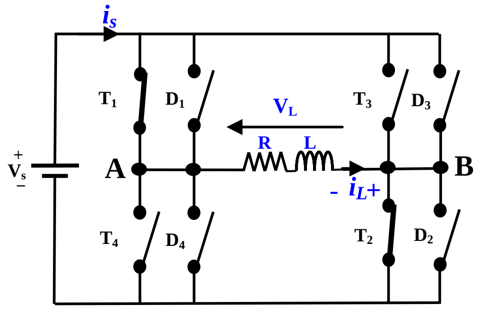
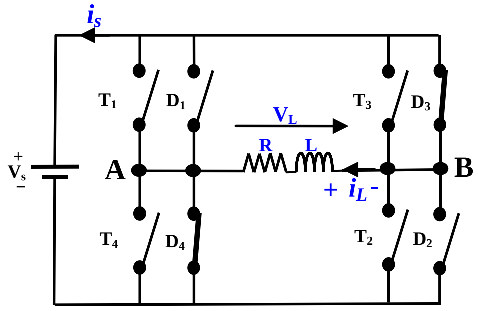
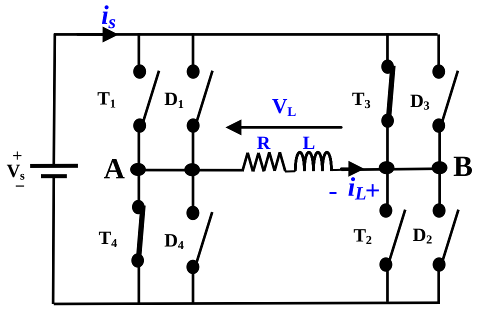


Fig.5 Single phase bridge inverter with RL load.

Circuit Operation

- D_1,D_2,D_3,D_4 are used to protect circuit against Ldi/dt .
- From KVL T_1&T_4 and T_3&T_2 cannot be **ON** at the same time (short circuit).
- This circuit is operates in four modes as shown in table below.

Mode	**Period**	**Conducting Devices**	**Equivalent Circuit**	V_L	i_L
I	$0\text{-}t_1$	D_1& D_2		Vs	-Imin
II	$t_1\text{-}T/2$	T_1&T_2		Vs	Imax
III	$T/2\text{-}t_2$	D_3& D_4		-Vs	Imax
IV	$t_2\text{-}T$	T_3&T_4		-Vs	-Imin

The inverter waveforms are shown in Fig.6.

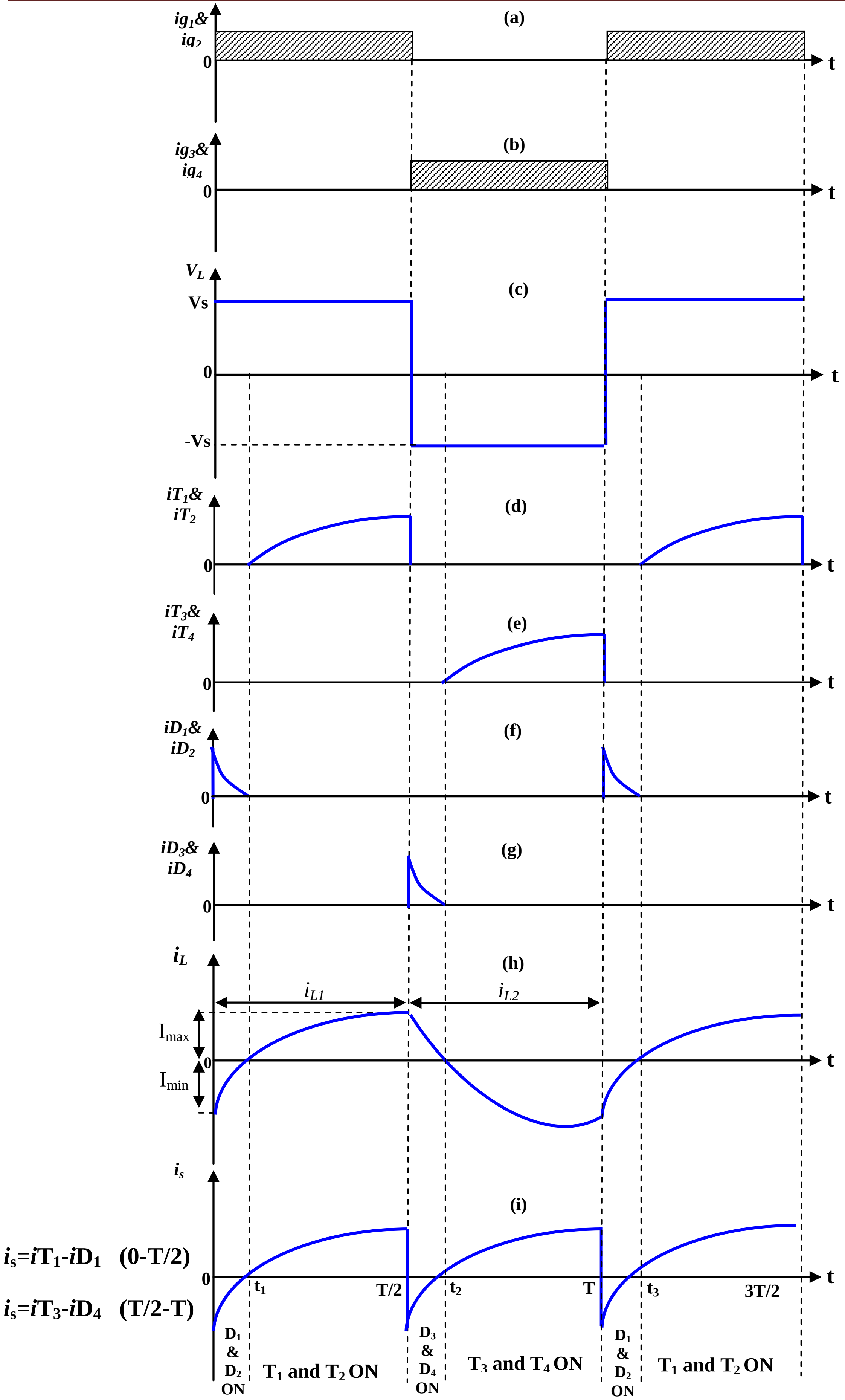


Fig.6 Waveforms (a) & (b) triggering, (c) load voltage,(d) & (e) transistors currents,(f) &(g) diodes currents ,(h) load current, and (i) supply current.

Mathematical Analysis:

1) Since the load voltagewaveform in inductive load shown in Fig.6(c) has similar shape to the load voltagewaveform in resistive load shown in Fig.4(e). Therefore, the load voltage equations for resistive load can be applied for inductive load.
2) The load current can be expressed as:

$$i_{L1}(t) = \frac{V_S}{R}\left(1 - e^{-(\frac{t}{\tau})}\right) - I_{min}\, e^{-(\frac{t}{\tau})} \qquad 0 \leq t \leq T/2$$

$$i_{L2}(t) = -\frac{V_S}{R}\left(1 - e^{-\left(t-\frac{T}{2}\right)/\tau}\right) + I_{max}\, e^{-\left(t-\frac{T}{2}\right)/\tau} \qquad T/2 \leq t \leq T$$

Where:-

τ:L/R Time constant

I_{min}: minimum value of load current

I_{max}:maximum value of load current

The maximum value of load current equal to minimum value but with negative sign as:

$$I_{max} = -I_{min}$$

$$I_{max} = \frac{V_S}{R}\left[\frac{1 - \boldsymbol{e}^{-(\frac{T}{2\tau})}}{1 + \boldsymbol{e}^{-(\frac{T}{2\tau})}}\right]$$

$$I_{min} = -\frac{V_S}{R}\left[\frac{1 - \boldsymbol{e}^{-(\frac{T}{2\tau})}}{1 + \boldsymbol{e}^{-(\frac{T}{2\tau})}}\right]$$

Example (1): A single phase bridge inverter shown in Fig.7used to supply inductive load with R=10Ω, L=25mH,the DC supplyvoltage V_S=100v, and the outputfrequency 50 Hz. Write an equations for instantaneous loadcurrent.

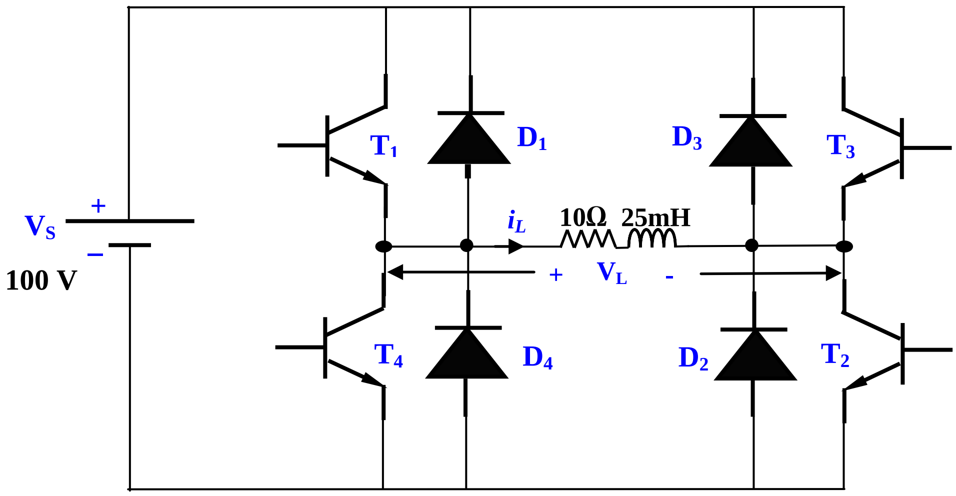


Fig.7 Single phase bridge inverter.

Solution: The period (T) of the load voltage and time constant ($\boldsymbol{\tau}$):

$$T = \frac{1}{f} = \frac{1}{50} = 0.02\ sec$$

$$\tau = \frac{L}{R} = \frac{0.025}{10} = 0.0025\ sec$$

Now, the minimum value and the maximum value of load currentsare:

$$I_{max} = -I_{min} = \frac{100}{10}\left[\frac{1 - e^{-(\frac{0.02}{2*0.0025})}}{1 + e^{-(\frac{0.02}{2*0.0025})}}\right] = 10\ \text{A}$$

The expressionsinstantaneous load currentsare:

$$i_{L1}(t) = \frac{100}{10}\left(1 - e^{-(\frac{t}{0.0025})}\right) - 10e^{-(\frac{t}{0.0025})}$$

$$= 10 - 20e^{-(\frac{t}{0.0025})} \quad 0 \le t \le 0.01$$

$$i_{L2}(t) = -\frac{100}{10}\left(1 - e^{-(t-0.01)/0.0025}\right) + 10e^{-(t-0.01)/0.0025}$$

$$= -10 + 20e^{-(t-0.01)/0.0025} \qquad 0.01 \le t \le 0.02$$

Example (2): Inthe single phase bridge inverter of Fig.8, the load current isi_L= 220 sin (ωt-45°) and the DC supply voltage V_S=100v. **(a)** Draw waveforms of V_L ,i_L, and i_s. Indicate on the waveforms of i_L, and i_s the device that are conducting during various intervals of time. **(b)**Determine the average value of the supply current i_s ,and the power from the supplyP_S.**(c)** Determine the power delivered to the load P_L.

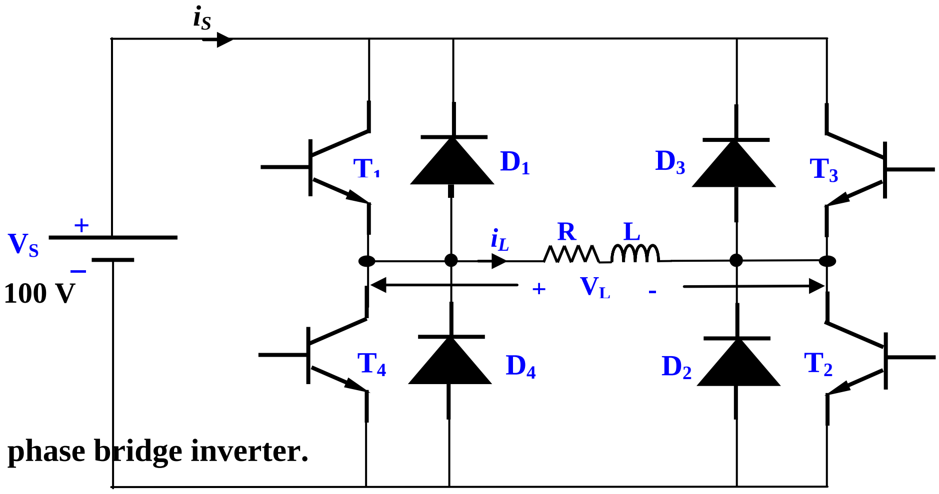


Fig.8 Single phase bridge inverter.

Solution:(a)the waveforms are shown in Fig.9.

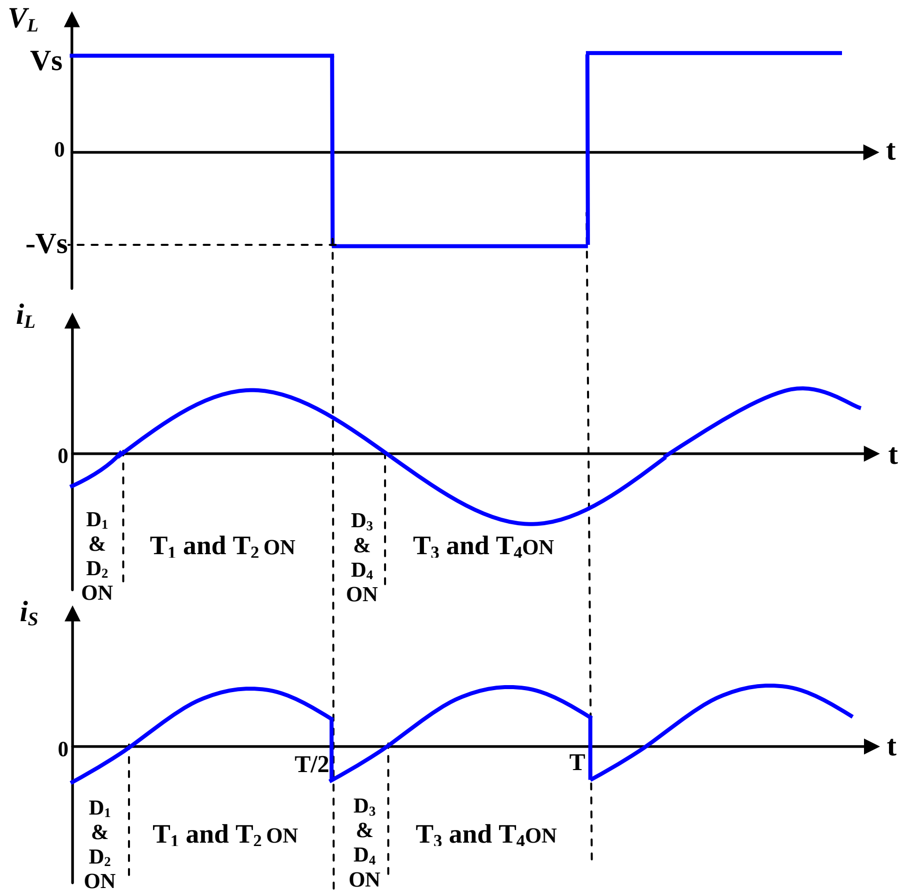


Fig.9waveforms.

(b)

$$i_{s\ ave} = i_s = \frac{1}{\pi}\int_0^{\pi} 220\sin(\omega t - 45^o)d\,(\omega t)$$

$$= \frac{220}{\pi}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = 99\ A$$

$$P_s = V_s\ x\ i_s = 100 * 99 = 9.9Kw$$

(c) From the Fourier analysis of V_L(square wave), the rms value of the fundamental load voltage is:

$$v_{L1(rms)} = \frac{4V}{\pi\sqrt{2}} = \frac{4 * 100}{\pi\sqrt{2}} = 90\ v$$

$$P_{out} = v_{L1(rms)} I_L\ \cos\theta$$

$$= 90 * \frac{220}{\sqrt{2}}\cos 45^o$$

$$= 99.89\ Kw$$

2- Quasi-square wave output (RL load) :the square waveformoutputfrom single phase bridge inverter can be improved by adding zero periods to obtain waveform called quasi-square. The circuit of quasi-square waveform is shown in Fig.5.

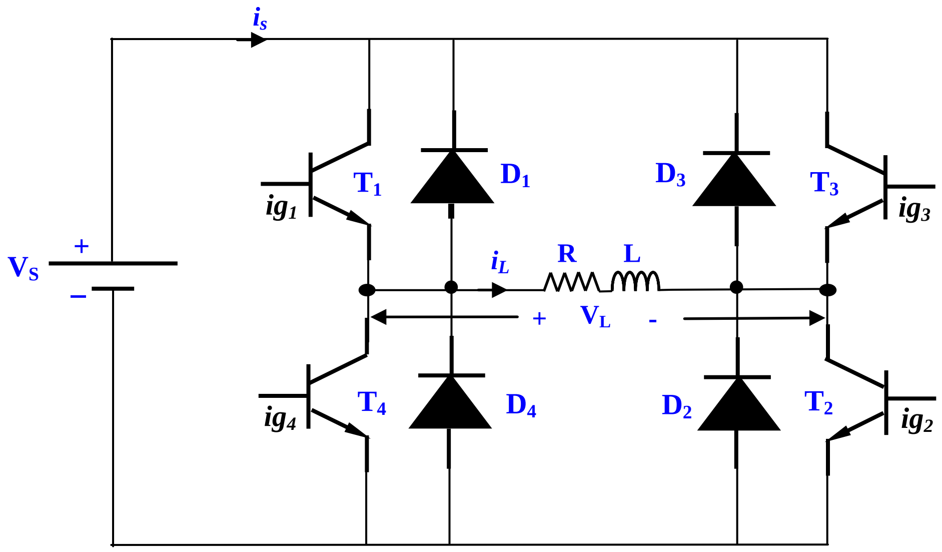


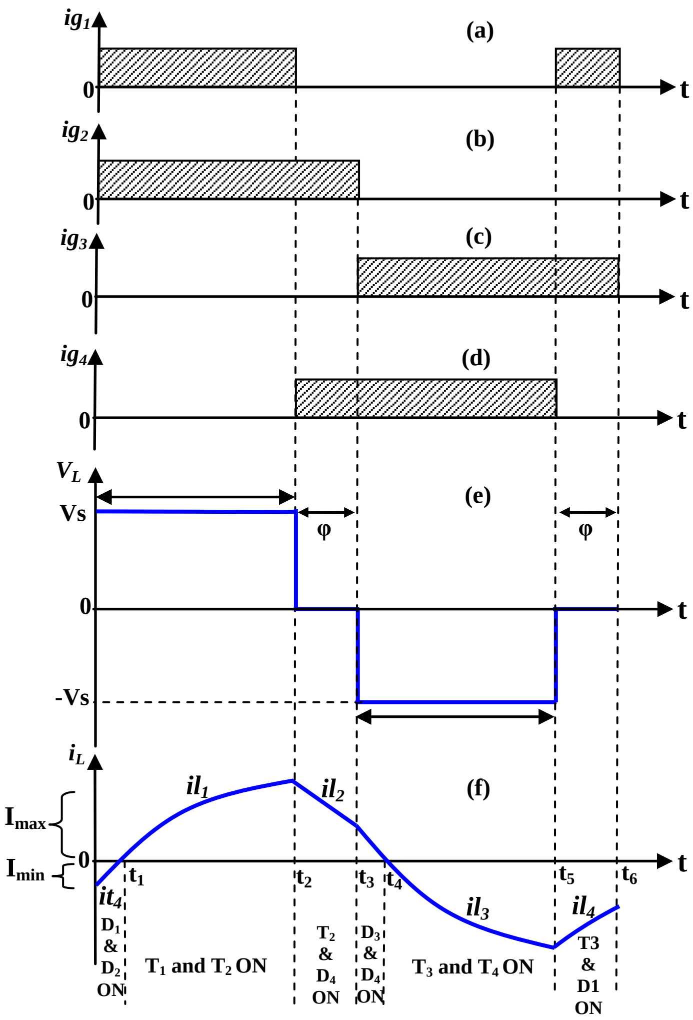
Fig.10 Single phase bridge inverter with RL load.

Circuit Operation

The operation of quasi square wave inverter can be summarized as shown in the following table

Mode	**Period**	**Conducting Devices**	V_L	i_L
I	$0\text{-}t_1$	D_1 D_2	V_s	il_4 (-)
II	$t_1\text{-}t_2$	T_1 T_2	V_s	il_1 (+)
III	$t_2\text{-}t_3$	T_2 D_4	0	il_2 (+)
IV	$t_3\text{-}t_4$	D_3 D_4	$-V_s$	il_3 (+)
V	$t_4\text{-}t_5$	T_3 T_4	$-V_s$	il_3 (-)
VI	$t_5\text{-}t_6$	T_3 D_1	0	il_4 (-)

The inverter waveforms are shown in Fig.6.



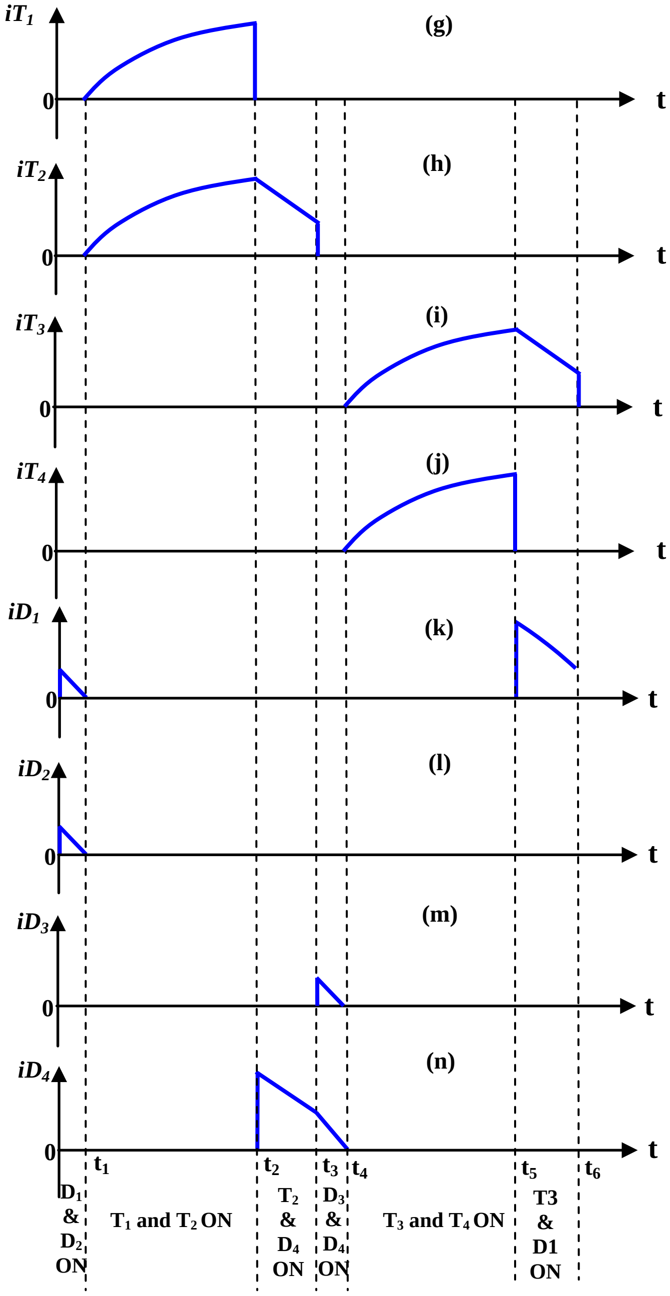


Fig.11 quasi square waveforms with inductive load: (a)-(d) triggering, (e) load voltage, (f) load current, (g)-(i)transistors currents, and (k)-(n) diodes currents.

Mathematical Analysis:

From Fig.11(f),the load current can be described as:

$$il_1(t) = \frac{\mathrm{V_s}}{\mathrm{R}}\left(1 - \mathrm{e}^{-\left(\frac{t}{\tau}\right)}\right) - I_{min}\,\mathrm{e}^{-\left(\frac{t}{\tau}\right)}$$

$$il_2(t) = I_{max}\,\mathrm{e}^{-\left(\frac{t}{\tau}\right)}$$

$$il_3(t) = -\left[\frac{\mathrm{V_s}}{\mathrm{R}}\left(1 - \mathrm{e}^{-\left(\frac{t}{\tau}\right)}\right) - I_1\mathrm{e}^{-\left(\frac{t}{\tau}\right)}\right]$$

$$il_4(\mathrm{t}) = I_2 \mathrm{e}^{-\left(\frac{\mathrm{t}}{\tau}\right)}$$

$$I_{min} = I_2 \mathrm{e}^{-\left(\frac{\mathrm{T}}{\tau}\right)}$$

$$I_{max} = \frac{\mathrm{V_s}}{\mathrm{R}}\left(1 - \mathrm{e}^{-\left(\frac{\mathrm{t2}}{\tau}\right)}\right) - I_{min}\, \mathrm{e}^{-\left(\frac{\mathrm{t2}}{\tau}\right)}$$

$$I_1 = I_{max}\, \mathrm{e}^{-\left(\frac{\mathrm{T}}{2\tau}\right)}$$

$$I_2 = -\left[\frac{\mathrm{V_s}}{\mathrm{R}}(1 - \mathrm{e}^{-\left(\frac{\mathrm{t5}}{\tau}\right)}\right] - I_1 \mathrm{e}^{-\left(\frac{\mathrm{t5}}{\tau}\right)}$$

Example (3): For a single phase quasi square wave inverter feeding heavily inductive load obtain an expression for rmsload voltage.

Solution:

From Fig.11 (e),the rms load voltage is

$$V_{L\,(rms)} = \sqrt{\frac{1}{T}\int_0^T (V_L)^2\, d\omega t}$$

$$V_{L\,(rms)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi-\varphi} (V_s)^2\, d\omega t + \int_{\pi}^{2\pi-\varphi} (-V_s)^2\, d\omega t\right]}$$

$$V_{L\,(rms)} = \sqrt{\frac{(V_s)^2}{2\pi}[(\pi-\varphi) + (2\pi-\varphi) - (\pi)]} = \sqrt{\frac{(V_s)^2}{2\pi}[(\pi-\varphi) + (\pi-\varphi)]}$$

$$V_{L\,(rms)} = V_s\sqrt{\frac{1}{\pi}(\pi-\varphi)}$$

For perfect quasi –square wave $(\boldsymbol{\varphi} - \boldsymbol{\pi}) = \frac{\mathbf{2\pi}}{\mathbf{3}}$ **i.e 120°** pulse width. Then therms load voltage will be,

$$V_{L\,(rms)} = V_s\sqrt{\frac{1}{\pi}\left(\frac{2\pi}{3}\right)}$$

$$V_{L\,(rms)} = V_s\sqrt{\frac{2}{3}} = 0.816\, V_s$$

Example (4): For a single phase quasi square wave inverter with RL load. Derive an expression for rms value of n[th] harmonic of load voltage.

Solution:

From Fig.11 (e), the general expression for Fourier series is given as

$$v_{L(\omega t)} = V_{L(ave)} + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \varphi_n)$$

Where, $C_n = \sqrt{{a_n}^2 + {b_n}^2}$ *and* $\varphi_n = \tan^{-1}\frac{a_n}{b_n}$

To obtain a_n

$$a_n = \frac{2}{T}\int_0^T v_{L(\omega t)} \cos(n\omega t) d\omega t$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi-\varphi} (V_s) cos(n\omega t)\, d\omega t \quad + \int_{\pi}^{2\pi-\varphi} (-V_s) cos(n\omega t)\, d\omega t\right]$$

$$= \frac{V_s}{\pi}\left[\left|\frac{sin(n\omega t)}{n}\right|_0^{\pi-\varphi} - \left|\frac{sin(n\omega t)}{n}\right|_{\pi}^{2\pi-\varphi}\right]$$

$$= \frac{V_s}{n\pi}[sin\, n(\pi-\varphi) - sin\, n(2\pi-\varphi) + sin\, n(\pi)]$$

Since $(\boldsymbol{\varphi} - \boldsymbol{\pi}) = \frac{\mathbf{2\pi}}{\mathbf{3}}$ $\therefore$ $\boldsymbol{\varphi} = \frac{\boldsymbol{\pi}}{\mathbf{3}}$Substitute the value ofzero periods($\boldsymbol{\varphi}$) in the above the equation will result

$$\therefore\ a_n = \frac{V_s}{n\pi}\left[sin\, n\left(\frac{2\pi}{3}\right) - sin\, n\left(\frac{5\pi}{3}\right) + sin\, n(\pi)\right] = 0 \ \text{ for all values of n}$$

To obtain b_n

$$b_n = \frac{2}{T}\int_0^T v_{L(\omega t)} sin(n\omega t) d\omega t$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi-\varphi} (V_s)\, sin\,(n\omega t)\, d\omega t \quad + \int_{\pi}^{2\pi-\varphi} (-V_s)\, sin(n\omega t)\, d\omega t\right]$$

$$= \frac{V_s}{\pi}\left[\left|\frac{-cos(n\omega t)}{n}\right|_0^{\pi-\varphi} + \left|\frac{cos(n\omega t)}{n}\right|_{\pi}^{2\pi-\varphi}\right]$$

$$= \frac{V_s}{n\,\pi}[-cos\ n(\pi-\varphi)+1+cos\ n(2\pi-\varphi)-cos\ n(\pi)]$$

$$b_n = \frac{V_s}{n\,\pi}\left[1-cos\ n\left(\frac{2\pi}{3}\right)+cos\ n\left(\frac{5\pi}{3}\right)-cos\ n(\pi)\right]$$

$b_n = 0$ for even values of n

and,

$$b_n = \frac{V_s}{n\,\pi}\left[1-cos\ n\left(\frac{2\pi}{3}\right)+cos\ n\left(\frac{5\pi}{3}\right)-cos\ n(\pi)\right]$$

for odd values of n except (3, 9,15 ……..) i.emultiples of 3

To obtain Fourier series

Since a_n=0 thus,

$$C_n = \sqrt{{a_n}^2+{b_n}^2} = b_n$$

$$\therefore\ C_n = \frac{V_s}{n\,\pi}\left[1-cos\ n\left(\frac{2\pi}{3}\right)+cos\ n\left(\frac{5\pi}{3}\right)-cos\ n(\pi)\right]$$

and

$$\varphi_n = \tan^{-1}\frac{a_n}{b_n} = 0$$

From the waveform of Fig.11 (e), it is clear that $V_{L(ave)} = 0$.Hence Fourier series will be

$$v_{L(\omega t)} = \sum_{n=1}^{\infty}\left\{\frac{V_s}{n\,\pi}\left[1-cos\ n\left(\frac{2\pi}{3}\right)+cos\ n\left(\frac{5\pi}{3}\right)-cos\ n(\pi)\right]\right\}\sin(n\omega t)$$

For odd values of n except multiples of 3

2) Three Phase Bridge Inverter (Six Step Inverter)

Three phase bridge inverter are used in high power applicationssuch as AC motors and three phase power system. The basic circuit of three phase bridge inverter is shown in Fig.9. The circuit is consists of six transistors or thyristors,and six diodes connected in parallel with each power semiconductor switch.

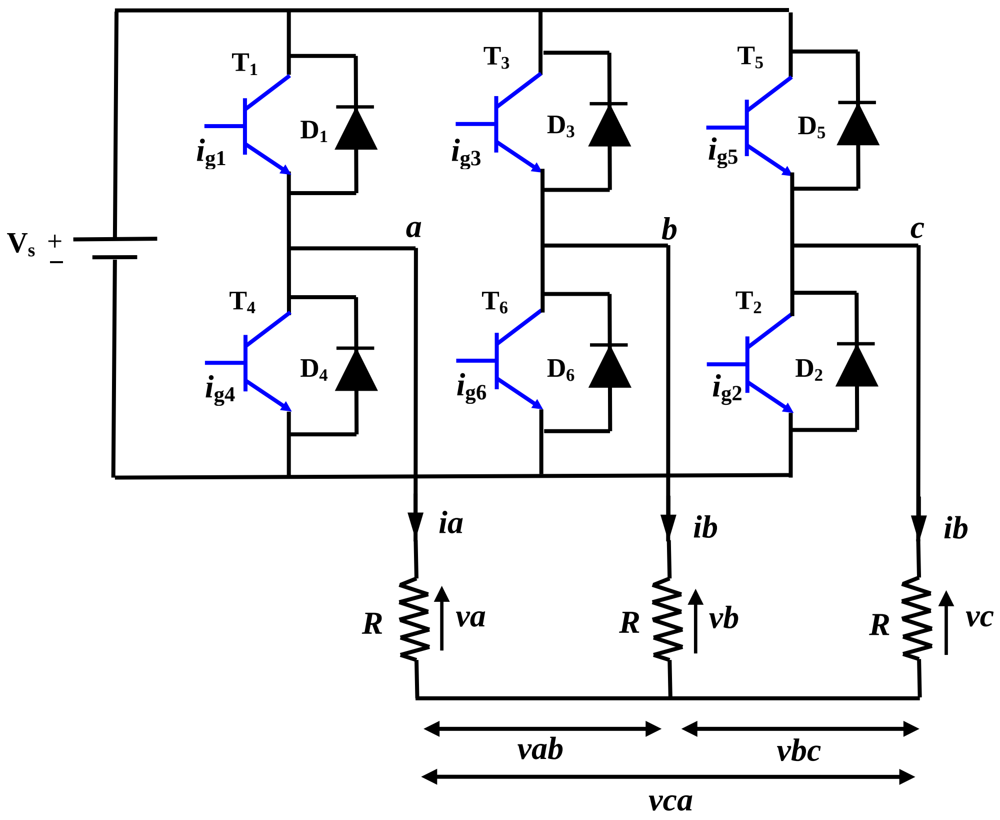


Fig. 9 basic three phase bridge inverter with resistive load.

Three phase bridge invertercan be controlled using the following conduction mode:

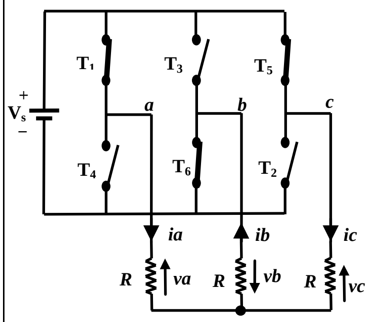
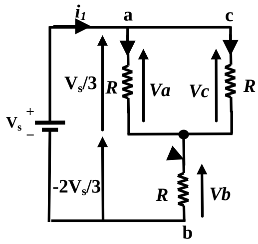
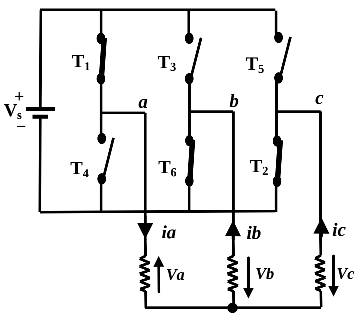
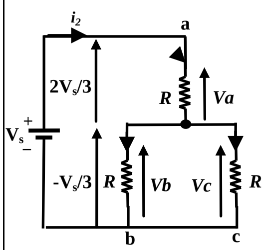
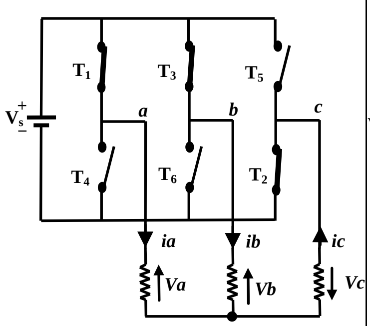
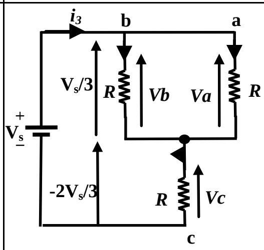
1) **180^o conduction**
2) **120^o conduction**

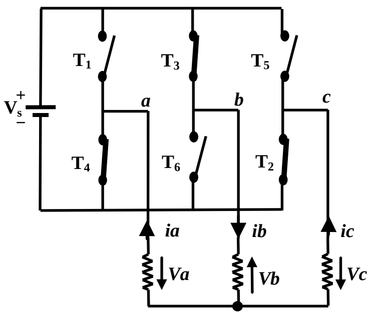
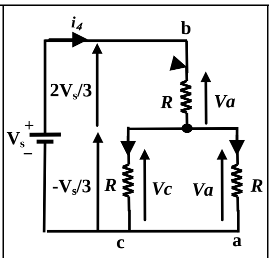
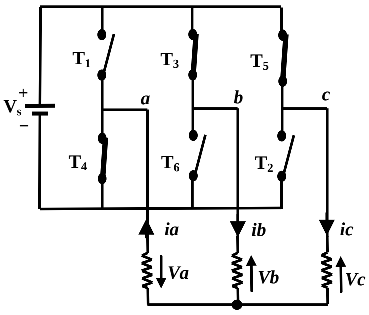
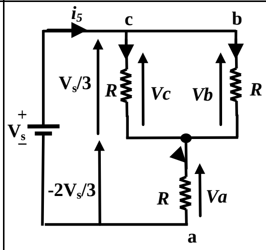
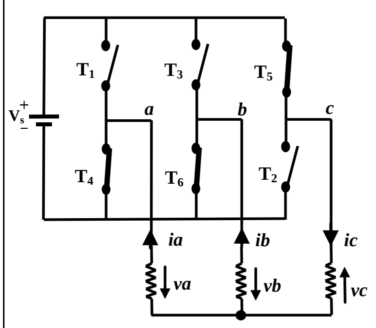
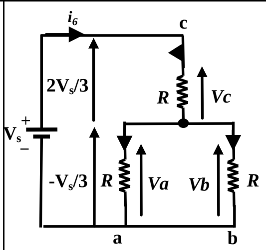
In both cases each power semiconductor switch will conductsevery 60^o interval.

1) 180^o conduction with resistive (R) load

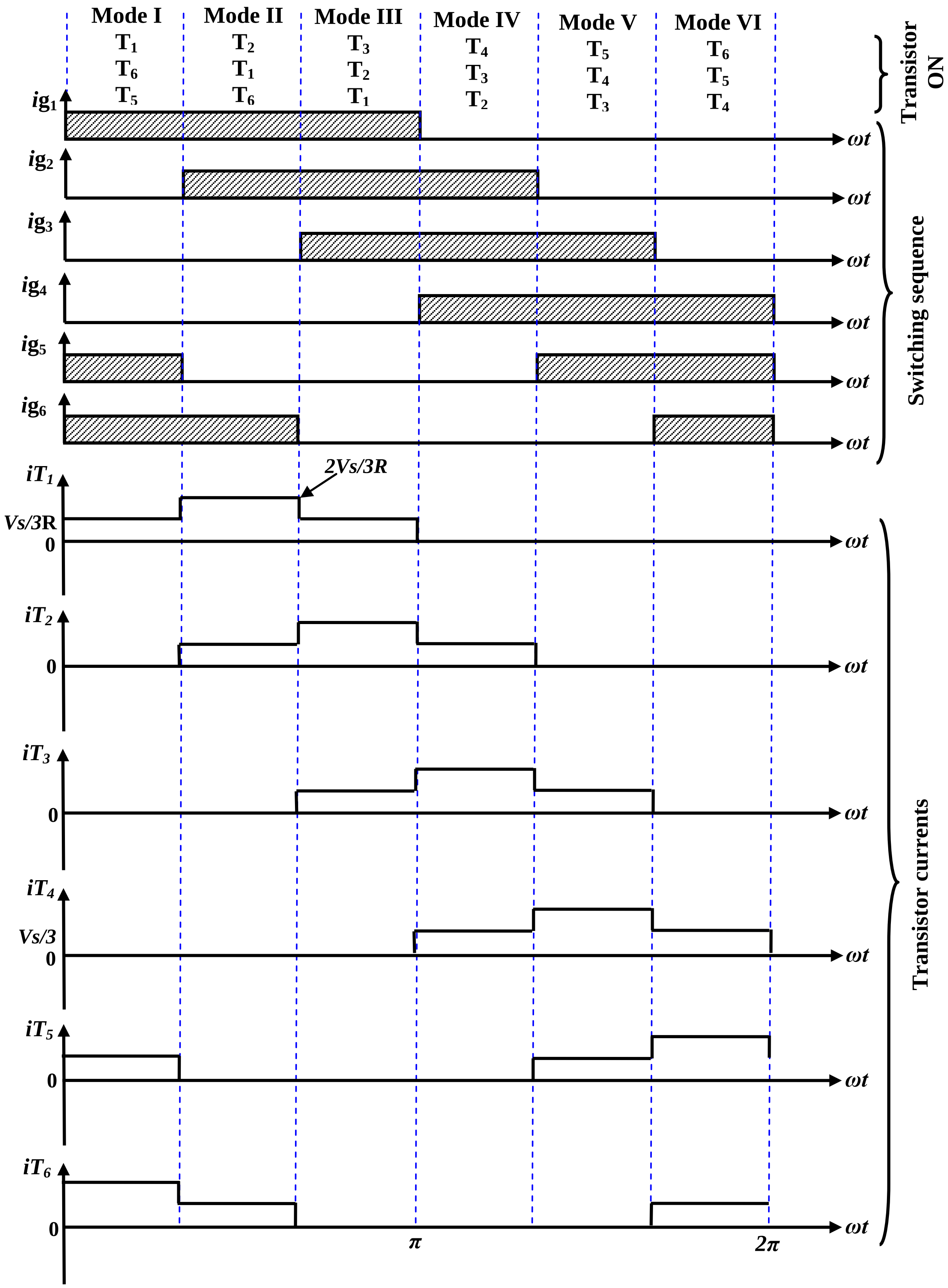
In this type of conduction each transistor in Fig.9 will conducts 180^o. The switching sequence:**561 (V_1)→ 612 (V_2) → 123 (V_3) → 234 (V_4) →345 (V_5)→ 456 (V_6)→ 561 (V_1)**

Next table summarize operation of three phase inverter with 180^o conduction angle and resistive load.

Mode	Conducting transistors	Actual cct.	Equivalent cct.	Phase voltage V_a	Phase voltage V_b	Phase voltage V_c	Line voltage V_{ab}	Line voltage V_{bc}	Line voltage V_{ca}
I (π/3)	T_1 T_6 T_5		 R_{eq}=(R//R)+R=3R/2 i_1=Vs/R_{eq}=2Vs/3R V_a=V_c= i_1(R//R)= i_1*(R/2)=Vs/3 V_b= - i_1*R=-2Vs/3 V_{ab}=V_a-V_b V_{bc}=V_b-V_c V_{ca}=V_c-V_a	$\frac{V_s}{3}$	$\frac{-2V_s}{3}$	$\frac{V_s}{3}$	V_s	- V_s	0
II (2π/3)	T_2 T_1 T_6		 R_{eq}= (R//R)+R=3R/2 i_2=v_s/R_{eq}=2V_s/3R V_a= i_2*R=2Vs/3 V_b = V_c= - i_2*(R//R) = - Vs/3 V_{ab}=V_a-V_b V_{bc}=V_b-V_c V_{ca}=V_c-V_a	$\frac{2V_s}{3}$	$\frac{-V_s}{3}$	$\frac{-V_s}{3}$	V_s	0	- V_s
III (π)	T_3 T_2 T_1		 R_{eq}= (R//R)+R=3R/2 i_3=Vs/R_{eq}=2Vs/3R V_a=V_b= i_3 (R//R)= i_3*(R/2)=Vs/3 V_c= - i_3*R=-2Vs/3 V_{ab}=V_a-V_b V_{bc}=V_b-V_c V_{ca}=V_c-V_a	$\frac{V_s}{3}$	$\frac{V_s}{3}$	$\frac{-2V_s}{3}$	0	V_s	- V_s

IV **($4\pi/3$)**	T_4 T_3 T_2		 $R_{eq}=(R//R)+R=3R/2$ $i_4=v_s/R_{eq}=2V_s/3R$ $V_b=i_4*R=2Vs/3$ $V_a=V_c=-i_4*(R//R)=-Vs/3$ $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	$\frac{-V_s}{3}$	$\frac{2V_s}{3}$	$\frac{-V_s}{3}$	$-V_s$	V_s	0
V **($5\pi/3$)**	T_5 T_4 T_3		 $R_{eq}=(R//R)+R=3R/2$ $i_5=Vs/R_{eq}=2Vs/3R$ $V_b=V_c=i_5(R//R)=$ $i_3*(R/2)=Vs/3$ $V_a=-i_5*R=-2Vs/3$ $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	$\frac{-2V_s}{3}$	$\frac{V_s}{3}$	$\frac{V_s}{3}$	$-V_s$	0	V_s
VI **(2π)**	T_6 T_5 T_4		 $R_{eq}=(R//R)+R=3R/2$ $i_6=v_s/R_{eq}=2V_s/3R$ $V_c=i_6*R=2Vs/3$ $V_a=V_b=-i_6*(R//R)=-Vs/3$ $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	$\frac{-V_s}{3}$	$\frac{-V_s}{3}$	$\frac{2V_s}{3}$	0	$-V_s$	V_s

The three phase bridge inverter waveforms with 180^o conduction angle and resistive load are shown in Fig.10.



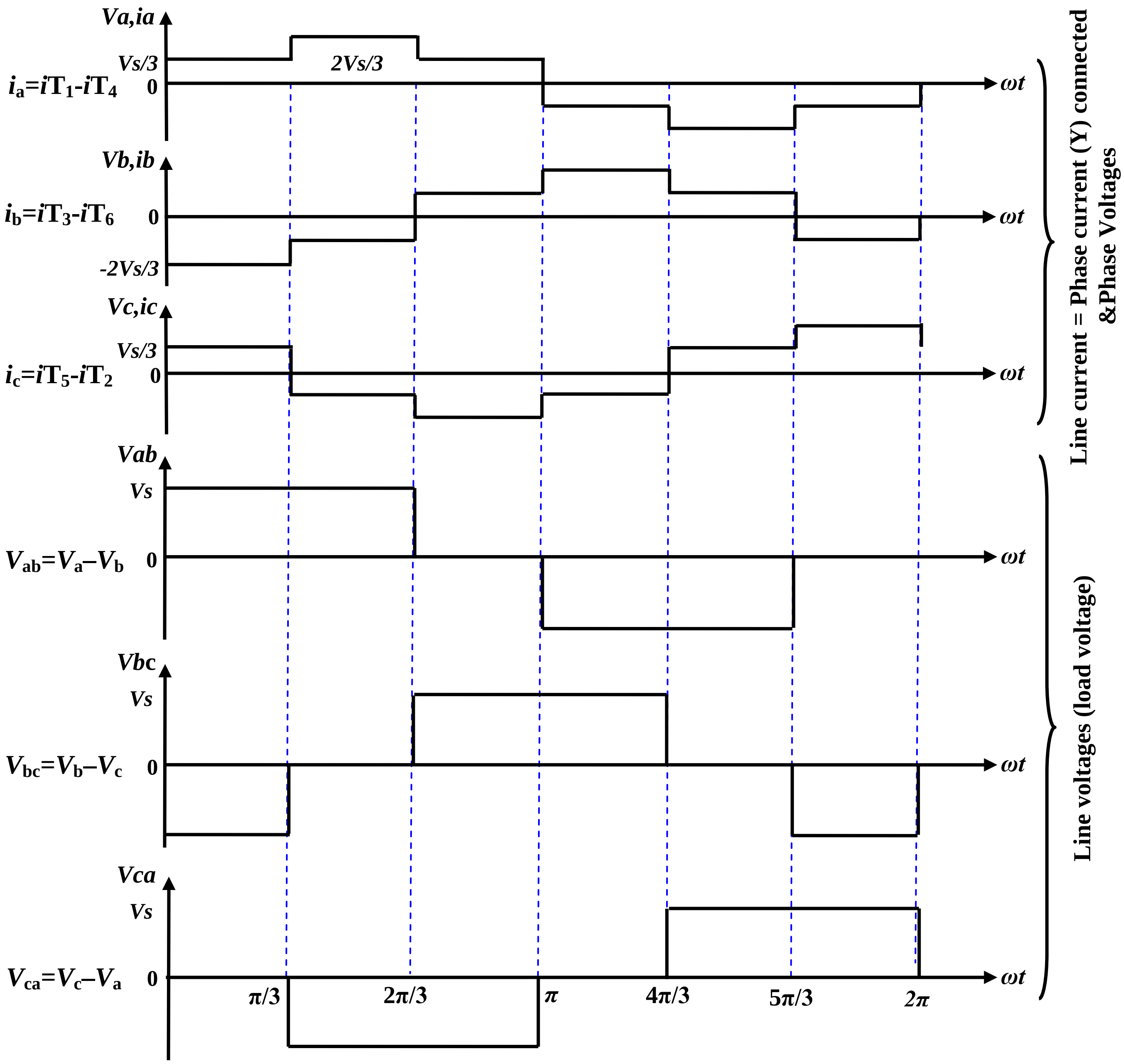


Fig.10 Three phase bridge inverter waveforms with 180° conduction and resistive load.

Mathematical Analysis:

The line voltage in terms of phase voltage in a 3- phase system with phase sequence (a,b,c) are:

$$V_{ab} = V_a - V_b$$

$$V_{bc} = V_b - V_c$$

$$V_{ca} = V_c - V_a$$

Where,

V_{ab}, V_{bc}, V_{ca} line voltage

V_a, V_b, V_c phase voltage

$$V_{ab} - V_{ca} = 2V_a - (V_b + V_c)$$

But in a balance three phase system, the sum of the three phase voltages is zero

$$\therefore \quad V_a + V_b + V_c = 0$$

$$\therefore \quad V_{ab} - V_{ca} = 3V_a$$

$$\therefore \quad V_a = \frac{(V_{ab} - V_{ca})}{3}$$

Similarly, the (b) and (c) phase voltages are:

$$\therefore \quad V_b = \frac{(V_{bc} - V_{ab})}{3}$$

$$\therefore \quad V_c = \frac{(V_{ca} - V_{bc})}{3} \quad ,$$

From Fig. 10,**the three phase output voltages** are six stepped and can be described by Fourier series.

$$V_a(t) = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi} \sin n(\omega t)$$

$$V_b(t) = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi} \sin n\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_c(t) = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t - \frac{4\pi}{3}\right)$$

Where k=0, 1, 2, 3,…..

For n=3,$\cos\frac{n\pi}{6} = 0$, thus, all multiples of 3rd harmonics are cancelled.

The three phase output voltage can be rewritten as:

$$V_a(t) = \frac{2V_s}{\pi}\left(\sin\omega t - \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t\right)$$

$$V_b(t) = \frac{2V_s}{\pi}\left(\sin(\omega t - \frac{2\pi}{3}) - \frac{1}{5}\sin 5(\omega t - \frac{2\pi}{3}) + \frac{1}{7}\sin 7(\omega t - \frac{2\pi}{3})\right)$$

$$V_c(t) = \frac{2V_s}{\pi}\left(\sin(\omega t - \frac{4\pi}{3}) - \frac{1}{5}\sin 5(\omega t - \frac{4\pi}{3}) + \frac{1}{7}\sin 7(\omega t - \frac{4\pi}{3})\right)$$

Also, **the three line output voltages** can be described by Fourier series as follows:

$$\therefore\ V_{ab}(t) = \sum_{1,3,5}^{\infty} \frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$\therefore\ V_{bc}(t) = \sum_{1,3,5}^{\infty} \frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$\therefore\ V_{ca}(t) = \sum_{1,3,5}^{\infty} \frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\sin n\left(\omega t + \frac{5\pi}{6}\right)$$

Also, all multiples of 3^{rd} harmonics are cancelled.Thus,

Note that $\cos\frac{\pi}{6} = 0.866 * 2 = \sqrt{3}$

$$V_{ab}(t) = \frac{2\sqrt{3}}{\pi}V_s\left(\sin\omega t - \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t\right)$$

$$V_{bc}(t) = \frac{2\sqrt{3}}{\pi}V_s\left(\sin\left(\omega t - \frac{2\pi}{3}\right) - \frac{1}{5}\sin 5\left(\omega t - \frac{2\pi}{3}\right) + \frac{1}{7}\sin 7\left(\omega t - \frac{2\pi}{3}\right)\right)$$

$$V_{ca}(t) = \frac{2\sqrt{3}}{\pi}V_s\left(\sin\left(\omega t - \frac{4\pi}{3}\right) - \frac{1}{5}\sin 5\left(\omega t - \frac{4\pi}{3}\right) + \frac{1}{7}\sin 7\left(\omega t - \frac{4\pi}{3}\right)\right)$$

The amplitude of the fundamental line voltage can be obtained as

$$V_{ab1} = \frac{2\sqrt{3}}{\pi} V_s$$

Where,

V_s:DC input voltage to the inverter

V_{ab1} : The amplitude of the fundamental line voltage

The RMS value of fundamental line voltage is

$$V_{ab1(rms)} = \frac{2\sqrt{3}}{\sqrt{2}\pi} V_s = \frac{\sqrt{6}}{\pi} = 0.7797 V_s$$

The phase voltages are shifted from the line voltages by 30^0

$$V_a = \left(\frac{2\sqrt{3}}{\pi} V_s\right) / \sqrt{3} = \frac{2}{\pi} V_s$$

The RMS value of fundamental phase voltage is

$$\therefore \quad V_{a1(rms)} = \left(\frac{2V_s}{\pi} \frac{1}{\sqrt{2}}\right) = \frac{V_{ab1(rms)}}{\sqrt{\mathbf{3}}} = 0.45\, V_s$$

It is seen from the line voltage waveform (V_{ab}) in Fig.10 that the line voltage is (V_s) from (0-120^o). Therefore, the rms value of line voltage (V_{ab}(rms)) is

$$\therefore \quad V_{ab(rms)} = \sqrt{\frac{1}{\pi} \int_0^{\frac{2\pi}{3}} V_s{}^2\, d\theta} = \sqrt{\frac{\mathbf{2}}{\mathbf{3}}} V_s = 0.8165 V_s$$

Also, the rms value of phase voltage (V_a(rms)) is

$$V_{a(rms)} = \sqrt{\frac{1}{\pi}\left[\int_0^{\frac{\pi}{3}} \left(\frac{V_s}{3}\right)^2 d\theta + \int_{\boldsymbol{\pi/3}}^{\frac{2\pi}{3}} \left(\frac{2V_s}{3}\right)^2 d\theta + \int_{\boldsymbol{2\pi/3}}^{\pi} \left(\frac{V_s}{3}\right)^2 d\theta\right]} = 0.4714 V_s$$

Orsince the load is star connected the phase voltage is

$$V_{a(rms)} = \frac{V_{ab(rms)}}{\sqrt{3}} = \frac{0.8165V_s}{\sqrt{3}} = 0.4714V_s$$

Total Harmonics Distortion (THD)

The THD of the line voltage is

$$THD_{line\ volatge} = \sqrt{\frac{(V_{ab})^2 - (V_{ab1})^2}{(V_{ab1})^2}} = \sqrt{\frac{(0.8165)^2 - (0.7797)^2}{(0.7797)^2}} = 31.08\ \%$$

The THD of the phase voltage is

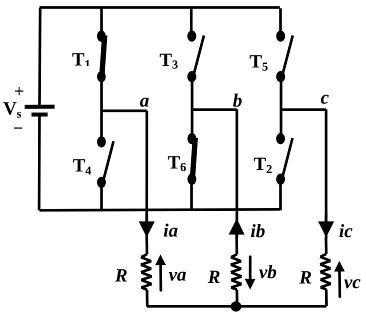
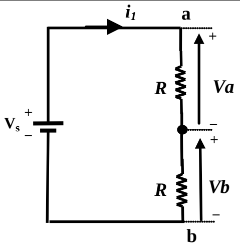
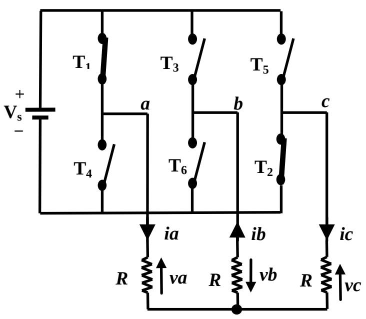
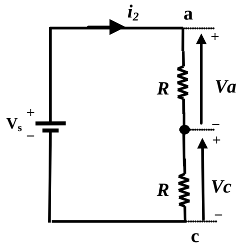
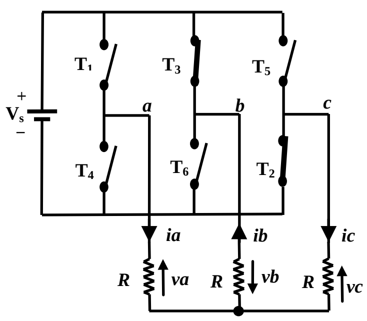
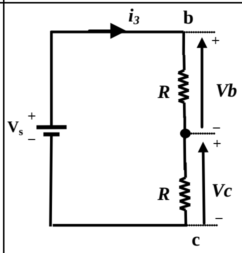
$$THD_{phase\ volatge} = \sqrt{\frac{(0.4714)^2 - (0.45)^2}{(0.45)^2}} = 31.1\ \%$$

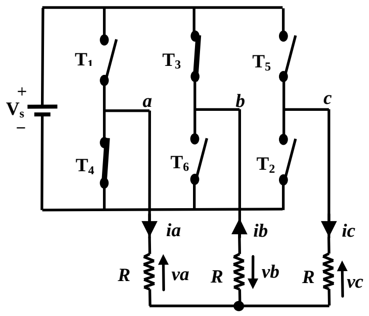
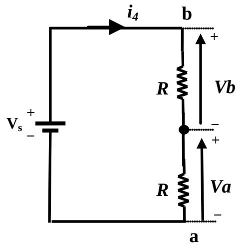
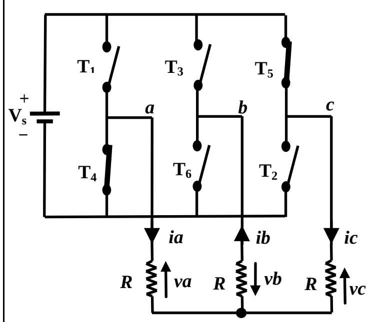
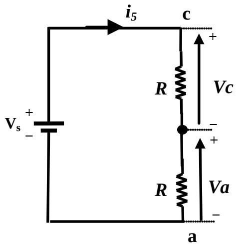
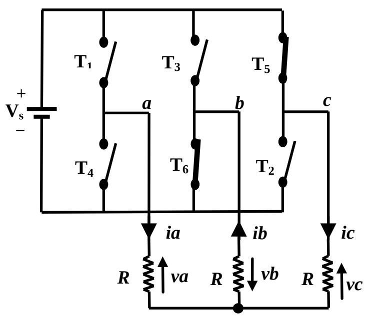
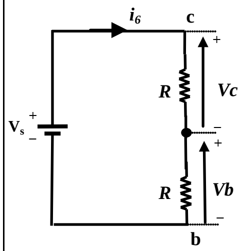
Note:Absence of the 3rd harmonics is the most important advantage of a three phase inverter.

2) 120^o conduction with resistive (R) load

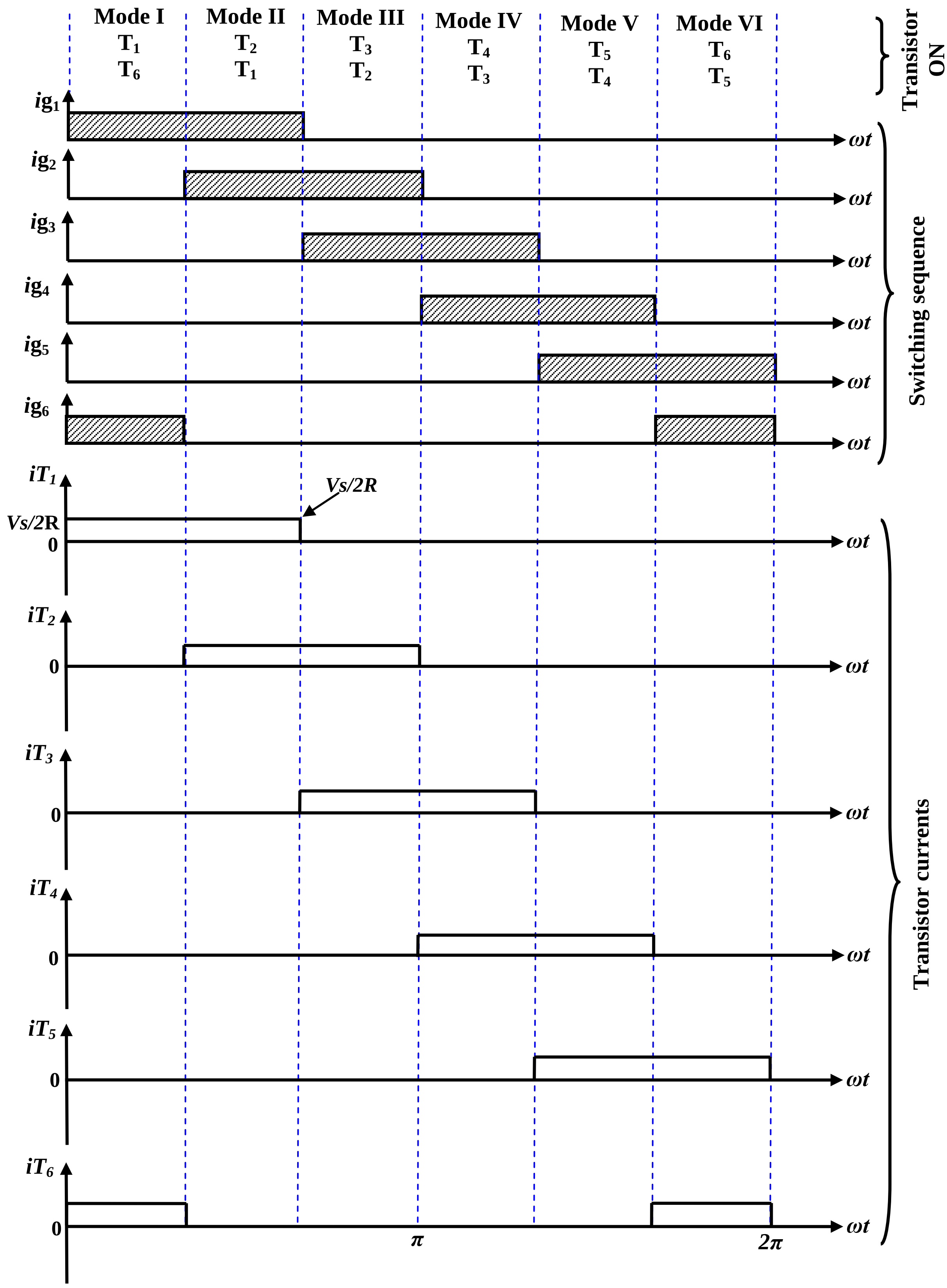
In this type of conduction each transistor in Fig.9 will conducts 120^o. The switching sequence:**61 (V_1)→ 12 (V_2) → 23 (V_3) → 34 (V_4) →45 (V_5)→ 56 (V_6)→ 61 (V_1)**

Next table summarize operation of three phase inverter with 120^o conduction angle and resistive load.

Mode	Conducting transistors	Actual cct.	Equivalent cct.	Phase voltage			Line voltage		
				V_a	V_b	V_c	V_{ab}	V_{bc}	V_{ca}
I ($\pi/3$)	T_1 T_6		R_{eq}= R+R=2R i_1=Vs/R_{eq}=Vs/2R V_a= i_1*R=Vs/2 V_b= - i_1*R=-Vs/2 V_c= 0 $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	$\frac{V_s}{2}$	$\frac{-V_s}{2}$	0	V_s	$\frac{-V_s}{2}$	$\frac{-V_s}{2}$
II ($2\pi/3$)	T_2 T_1		R_{eq}= R+R=2R i_2=Vs/R_{eq}=Vs/2R V_a= i_2 *R=Vs/2 V_c= - i_2*R=-Vs/2 V_b= 0 $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	$\frac{V_s}{2}$	0	$\frac{-V_s}{2}$	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$-V_s$
III (π)	T_3 T_2		R_{eq}= R+R=2R i_3=Vs/R_{eq}=Vs/2R V_b= i_3*R=Vs/2 V_c= - i_3*R=-Vs/2 V_a= 0 $V_{ab}=V_a-V_b$ $V_{bc}=V_b-V_c$ $V_{ca}=V_c-V_a$	0	$\frac{V_s}{2}$	$\frac{-V_s}{2}$	$\frac{-V_s}{2}$	V_s	$\frac{-V_s}{2}$

IV **($4\pi/3$)**	T_4 T_3		 $R_{eq} = R+R=2R$ $i_4 = Vs/R_{eq} = Vs/2R$ $V_b = i_4 * R = Vs/2$ $V_a = -\,i_4 * R = -Vs/2$ $V_c = 0$ $V_{ab} = V_a - V_b$ $V_{bc} = V_b - V_c$ $V_{ca} = V_c - V_a$	$\frac{-V_s}{2}$	$\frac{V_s}{2}$	**0**	$-\,V_s$	$\frac{V_s}{2}$	$\frac{V_s}{2}$
V **($5\pi/3$)**	T_5 T_4		 $R_{eq} = R+R=2R$ $i_5 = Vs/R_{eq} = Vs/2R$ $V_c = i_5 * R = Vs/2$ $V_a = -\,i_5 * R = -Vs/2$ $V_b = 0$ $V_{ab} = V_a - V_b$ $V_{bc} = V_b - V_c$ $V_{ca} = V_c - V_a$	$\frac{-V_s}{2}$	**0**	$\frac{V_s}{2}$	$\frac{-V_s}{2}$	$\frac{-V_s}{2}$	V_s
VI **(2π)**	T_6 T_5		 $R_{eq} = R+R=2R$ $i_6 = Vs/R_{eq} = Vs/2R$ $V_c = i_6 * R = Vs/2$ $V_b = -\,i_6 * R = -Vs/2$ $V_a = 0$ $V_{ab} = V_a - V_b$ $V_{bc} = V_b - V_c$ $V_{ca} = V_c - V_a$	**0**	$\frac{-V_s}{2}$	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$-V_s$	$\frac{V_s}{2}$

The three phase inverter waveforms with 120^o conduction angle and resistive load are shown in Fig.11.



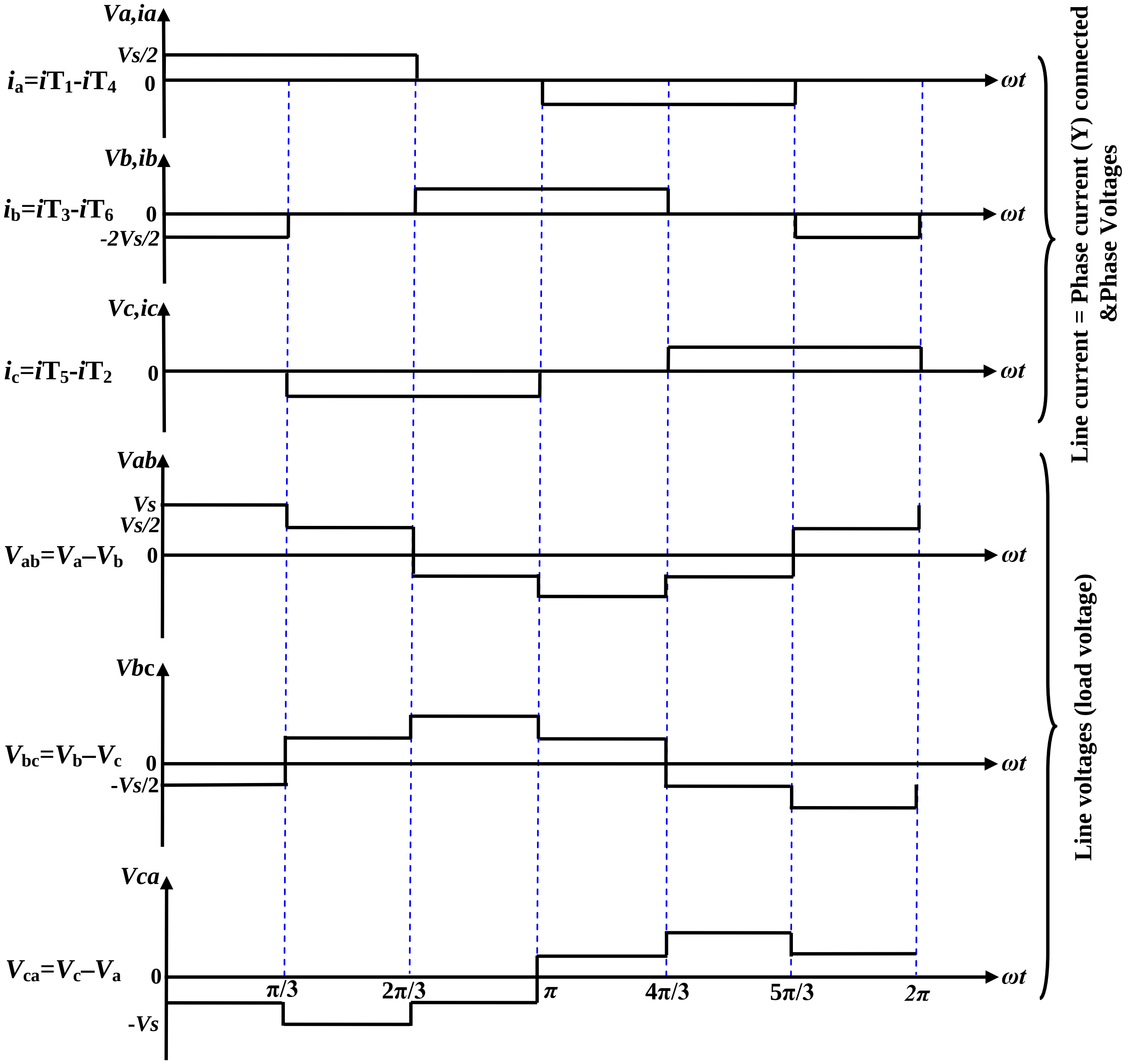


Fig.11Three phase bridge inverter waveforms with 120° conduction and resistive load.

Mathematical Analysis:

The line voltage in terms of phase voltage in a 3- phase system with phase sequence (a,b,c) are:

$$V_{ab} = V_a - V_b$$

$$V_{bc} = V_b - V_c$$

$$V_{ca} = V_c - V_a$$

Where,

V_{ab},V_{bc},V_{ca} line voltage

V_a,V_b,V_c phase voltage

$$V_{ab} - V_{ca} = 2V_a - (V_b + V_c)$$

But in a balance three phase system, the sum of the three phase voltages is zero

$$\therefore \quad V_a + V_b + V_c = 0$$

$$\therefore \quad V_{ab} - V_{ca} = 3V_a$$

$$\therefore \quad V_a = \frac{(V_{ab} - V_{ca})}{3}$$

Similarly, the (b) and (c) phase voltages are:

$$\therefore \quad V_b = \frac{(V_{bc} - V_{ab})}{3}$$

$$\therefore \quad V_c = \frac{(V_{ca} - V_{bc})}{3} \quad ,$$

From Fig. 11, **the three phase output voltages** can be described by Fourier series.

$$V_a\,(t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$V_b\,(t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$V_c\,(t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t + \frac{5\pi}{6}\right)$$

For n=3, $\cos\frac{n\pi}{6} = 0$, thus, all multiples of 3^{rd} harmonics are cancelled.

The three phase output voltage can be rewritten as:

Note that $\cos\frac{\pi}{6} = 0.866 * 2 = \sqrt{3}$

$$\mathrm{V_a}\ (\mathrm{t}) = \frac{\sqrt{3}V_s}{\pi}\left(sin\omega t - \frac{1}{5}sin5\omega t + \frac{1}{7}sin7\omega t\right)$$

$$\mathrm{V_b}(\mathrm{t}) = \frac{\sqrt{3}V_s}{\pi}\left(\sin(\omega t - \frac{2\pi}{3}) - \frac{1}{5}sin5(\omega t - \frac{2\pi}{3}) + \frac{1}{7}sin7(\omega t - \frac{2\pi}{3})\right)$$

$$\mathrm{V_c}(\mathrm{t}) = \frac{\sqrt{3}V_s}{\pi}\left(\sin(\omega t - \frac{4\pi}{3}) - \frac{1}{5}sin5(\omega t - \frac{4\pi}{3}) + \frac{1}{7}sin7(\omega t - \frac{4\pi}{3})\right)$$

Also, **the three line output voltages** can be described by Fourier series as follows:

$$\therefore\ \ \mathrm{V_{ab}}\ (\mathrm{t}) = \sum_{\mathrm{n=6k\pm1}}^{\infty} \frac{3\mathrm{V_s}}{\mathrm{n\pi}} \sin n\left(\omega\mathrm{t} + \frac{\pi}{3}\right)$$

$$\therefore\ \ \mathrm{V_{bc}}\ (\mathrm{t}) = \sum_{\mathrm{n=6k\pm1}}^{\infty} \frac{3\mathrm{V_s}}{\mathrm{n\pi}} \sin n\left(\omega\mathrm{t} + \frac{\pi}{3}\right)$$

$$\therefore\ \ \mathrm{V_{ca}}\ (\mathrm{t}) = \sum_{\mathrm{n=6k\pm1}}^{\infty} \frac{3\mathrm{V_s}}{\mathrm{n\pi}} \sin n\left(\omega\mathrm{t} + \frac{\pi}{3}\right)$$

Where k=0,1,2,3,…..

Also, all multiples of 3^{rd} harmonics are cancelled.Thus,

$$\mathrm{V_{ab}}\ (\mathrm{t}) = \frac{3}{\pi}V_s\left(sin\omega t - \frac{1}{5}sin5\omega t + \frac{1}{7}sin7\omega t\right)$$

$$\mathrm{V_{bc}}\ (t) = \frac{3}{\pi}V_s\left(sin\left(\omega t - \frac{2\pi}{3}\right) - \frac{1}{5}sin\ 5\left(\omega t - \frac{2\pi}{3}\right) + \frac{1}{7}sin\ 7\left(\omega t - \frac{2\pi}{3}\right)\right)$$

$$\mathrm{V_{ca}}\ (t) = \frac{3}{\pi}V_s\left(sin\left(\omega t - \frac{4\pi}{3}\right) - \frac{1}{5}sin5\left(\omega t - \frac{4\pi}{3}\right) + \frac{1}{7}sin7\left(\omega t - \frac{4\pi}{3}\right)\right)$$

The amplitude of the fundamental line voltage can be obtained as

$$\mathrm{V}_{\mathrm{ab}\,1} = \frac{3}{\pi} V_s$$

Where,

V_s:DC input voltage to the inverter

V_{ab1} : The amplitude of the fundamental line voltage

The RMS value of fundamental line voltage is

$$\mathrm{V}_{\mathrm{ab}\,1(\mathrm{rms})} = \frac{3}{\sqrt{2}\pi} V_s = \sqrt{3}\,\mathrm{V}_{\mathrm{a}\,1(\mathrm{rms})} = 0.6755\, V_s$$

The phase voltages are shifted from the line voltages by 30^0

$$\mathrm{V}_{\mathrm{a}} = \left(\frac{3}{\pi} V_s\right) / \sqrt{3} = 0.551 V_s$$

The RMS value of fundamental phase voltage is

$$\therefore \quad \mathrm{V}_{\mathrm{a}\,1(\mathrm{rms})} = \left(\frac{\sqrt{3} V_s}{\pi} \frac{1}{\sqrt{2}}\right) = \frac{\mathrm{V}_{\mathrm{ab}\,1(\mathrm{rms})}}{\sqrt{\mathbf{3}}} = 0.39\, V_s$$

It is seen from the phase voltage waveform (V_a) in Fig.11 that the phase voltage is ($V_s/2$) from ($0\text{-}120^o$). Therefore, the rms value of phase voltage (V_a(rms)) is

$$\therefore \qquad \mathrm{V}_{\mathrm{a}(\mathrm{rms})} = \sqrt{\frac{1}{\pi} \int_0^{\frac{2\pi}{3}} \left(\frac{V_s}{2}\right)^2 d\theta} = \sqrt{\frac{\mathbf{2}}{\mathbf{3}}} \frac{V_s}{2} = \frac{V_s}{\sqrt{6}} = 0.4082 V_s$$

Also, the rms value of line voltage (V_{ab}(rms)) is

$$\mathrm{V}_{\mathrm{ab}\,(\mathrm{rms})} = \sqrt{\mathbf{3}}\,\mathrm{V}_{\mathrm{a}(\mathrm{rms})} = \sqrt{\mathbf{3}} * 0.4082 V_s = 0.707 V_s$$

Total Harmonics Distortion (THD)

The THD of the line voltage is

$$THD_{line\ volatge} = \sqrt{\frac{(V_{ab})^2 - (V_{ab1})^2}{(V_{ab1})^2}} = \sqrt{\frac{(0.707)^2 - (0.6755)^2}{(0.6755)^2}} = 30.89\ \%$$

The THD of the phase voltage is

$$THD_{phase\ volatge} = \sqrt{\frac{(0.4082)^2 - (0.39)^2}{(0.39)^2}} = 30.89\ \%$$

The square waveform output from the inverter supplied to load is not good enough. But, the sinusoidal load voltage is usually the most desirable. But how do we approximate a sinusoidal output with only three states (+Vdc, –Vdc, 0) ?

The answer: Unipolar PWM modulation

3) Single Phase Bridge Pulse Width Inverter (PWM)

The PWM is very commonlyusedmethod to control the output voltage and frequency fromthe inverter through controlling the **switching instants**of the inverter transistors (ON and OFF).The PWM technique produces lower order harmonic contents in comparison to other technique. However, there are two types of single phase bridge PWM inverter.

a) Bipolar PWM

In this method,a triangular carrier waveform of high frequency (V_c) is compared with a low frequency sinusoidal reference waveform (V_r) and the crossover points are

used to determine **switching instants**as shown in Fig.12. The dotted curve is the desired output; also the fundamental frequency.

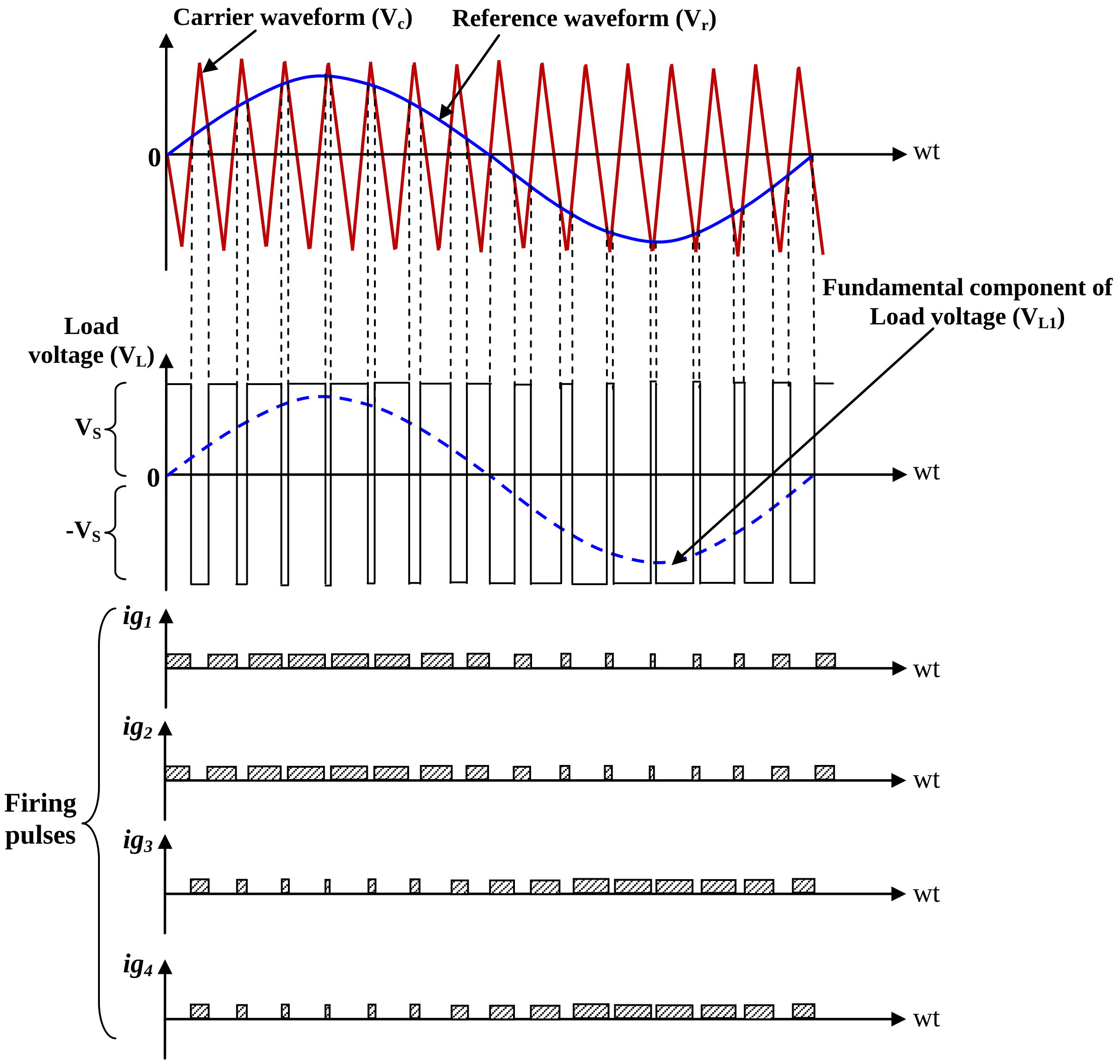


Fig.12Bipolar switching scheme.

$$V_r > V_c \therefore \ V_L = V_s$$

$$V_r < V_c \therefore \ V_L = -V_s$$

$$v_{control} < v_{tri} \qquad T_{A-} \ on \ and \qquad V_{AN} = 0$$

$$-v_{control} > v_{tri} \qquad T_{B+} \ on \ and \qquad V_{BN} = V_d$$

b) Unipolar PWM

In this method,triangular carrier waveform of high frequency (V_c) is compared with two low frequency sinusoidal reference waveforms (V_{ra} and V_{rb}),and the crossover points are used to determine **switching instants** as shown in Fig.13.The phase shift between these two waveforms is (180^o). Unipolar switching schemeproduces better harmonics. But it is moredifficult to implement.

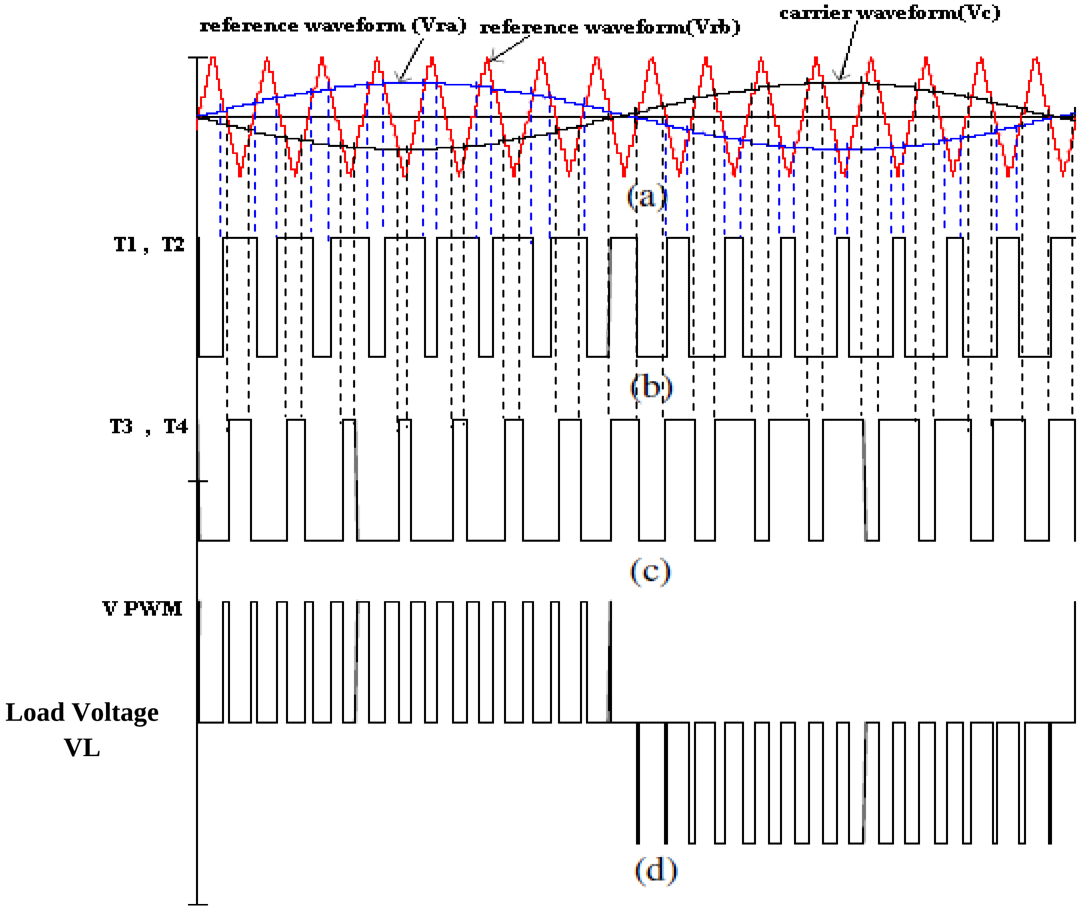


Fig.13Unipolar switching scheme.

Modulation Index(*MI*):defined as the ration of the sinusoidal reference waveforms amplitude (V_r) to the carrier waveform amplitude (V_c).

$$\boldsymbol{MI} = \frac{\text{Amplitude of the sinusoidal reference waveform}}{\text{Amplitude of the carrier waveform}}$$

MI is related to the fundamental (sinusoidalwaveform)output voltagemagnitude. If M is high, thenthe sinusoidalwaveform output is high and vice versa.

If $0 \leq MI \geq 1$

$$V_1 = MI\, V_{in}$$

Where, V_1 and V_{in} are fundamental of the outputvoltage and input (DC) voltage,respectively.

Fig.14a showsa maximum output, a reduction to half this value being made by simply reducing the reference sinusoidal waveform to half value as shown in Fig.14b. Fig.14c shows how a reduction in frequency of the reference sinusoidal waveform increases the number of pulses within each half cycle.

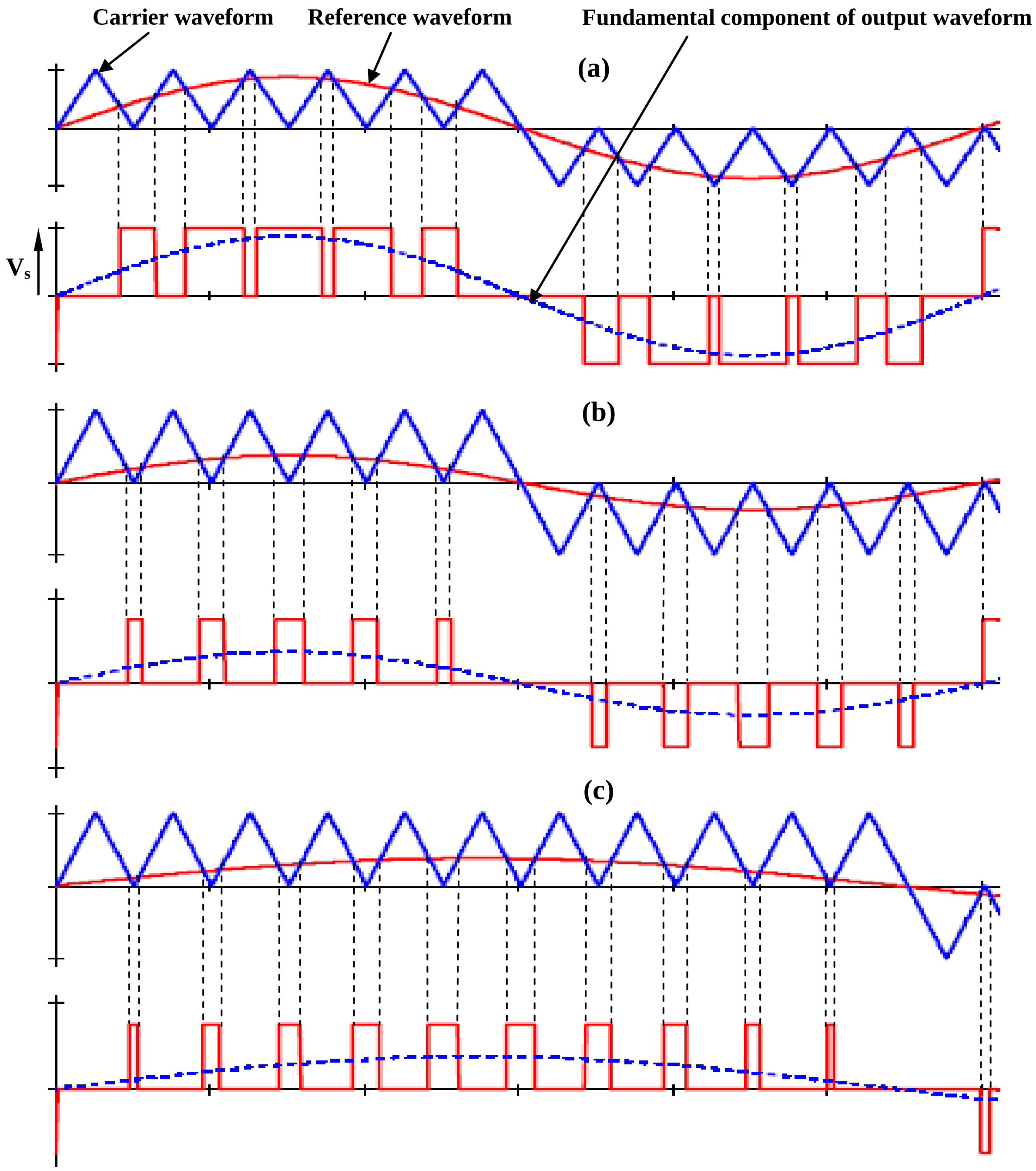


Fig.14 Switching instants forPWM waveform (a) At maximum output voltage (b) At half maximum output voltage and (c) At half voltage and half frequency.

4) Three Phase Bridge Inverter (PWM Inverter)

The circuit of three phase bridge PWM inverter is shown in Fig.15. In this type, the PWM waveform generation is identical to the six step inverter, but the switching sequence is more complex.

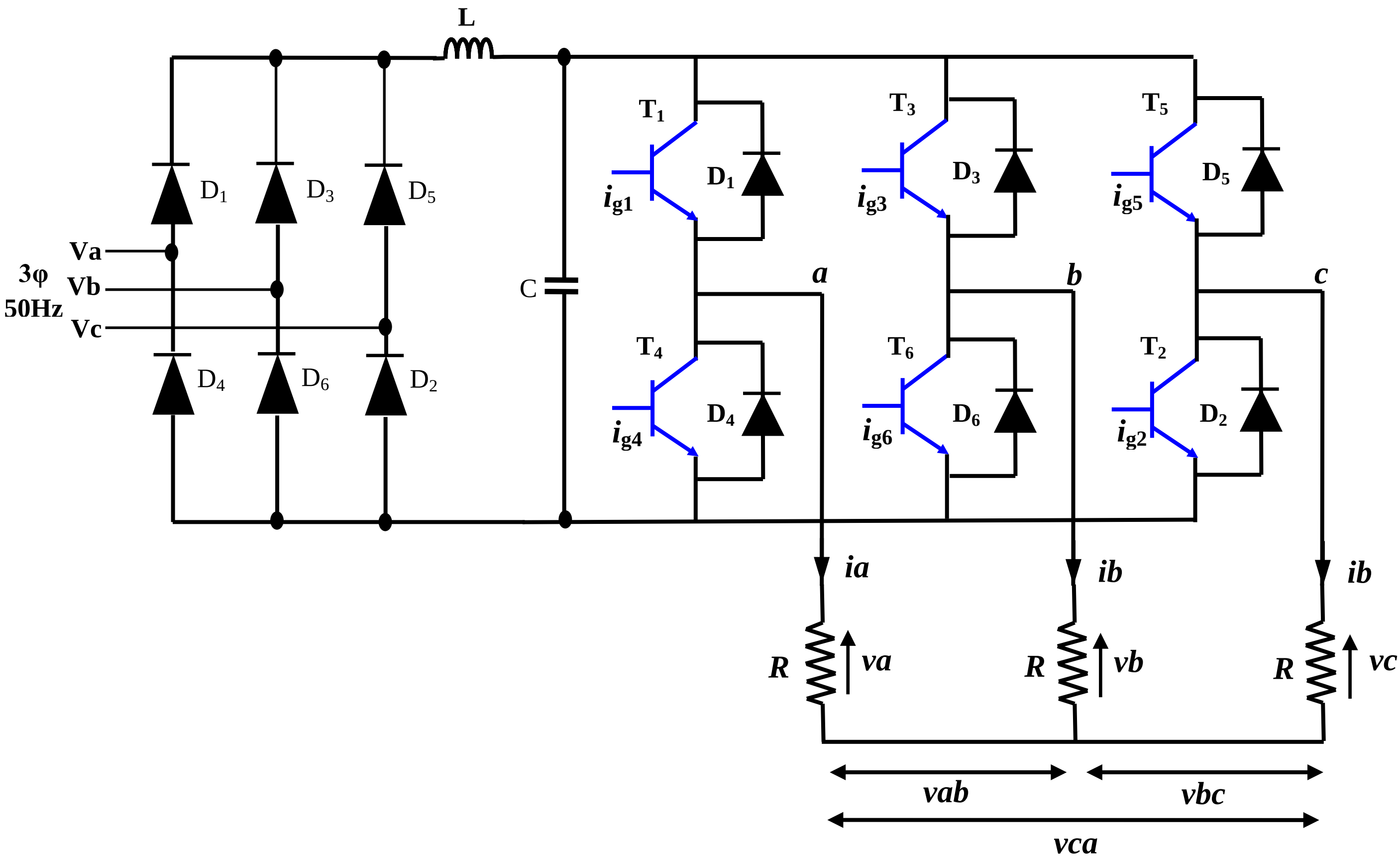


Fig.15 Three phase bridge PWM inverter.

The operation of PWM inverter is depending on the technique that may be used to generate the firing signal waveforms. Conventionally, these waveforms have been produced by comparing a triangular carrier waveformof high frequency (V_c) with a low frequency sinusoidal reference waveform(V_r).

3-1 Classification of PWM

The PWM can be broadly classified into the following types:

1) Natural (sinusoidal) sampling

- Problems with analogue circuitry, e.g. Drift,sensitivity etc.

2) Regular sampling

- Simplified digital version of natural sampling appropriate for digital hardware or microprocessor implementation.
- Widely used in industry.

3) Optimum PWM

- PWM waveform is constructed based oncertain performance criteria, e.g. THD.

4) Harmonic elimination/minimisation PWM

- PWM waveforms are constructed to eliminatesome undesirable harmonics from the outputwaveform spectra.
- Highly mathematical in nature.

5) Space-vector modulation (SVM)

- Advanced computationally intensive technique that offers superior performance in variablespeed drives.
- Used to minimize harmonic content of the three-phase isolated neutral load.

1) Three Phase sinusoidal PWM

In this method, a triangular carrier waveform of high frequency (V_c) is compared with a three low frequency sinusoidal reference waveforms (V_{ra} ,V_{rb} ,V_{rc}) to determine the switching instants of each transistor. Principle of sinusoidal PWM for three phase bridge inverteris shown Fig.16.

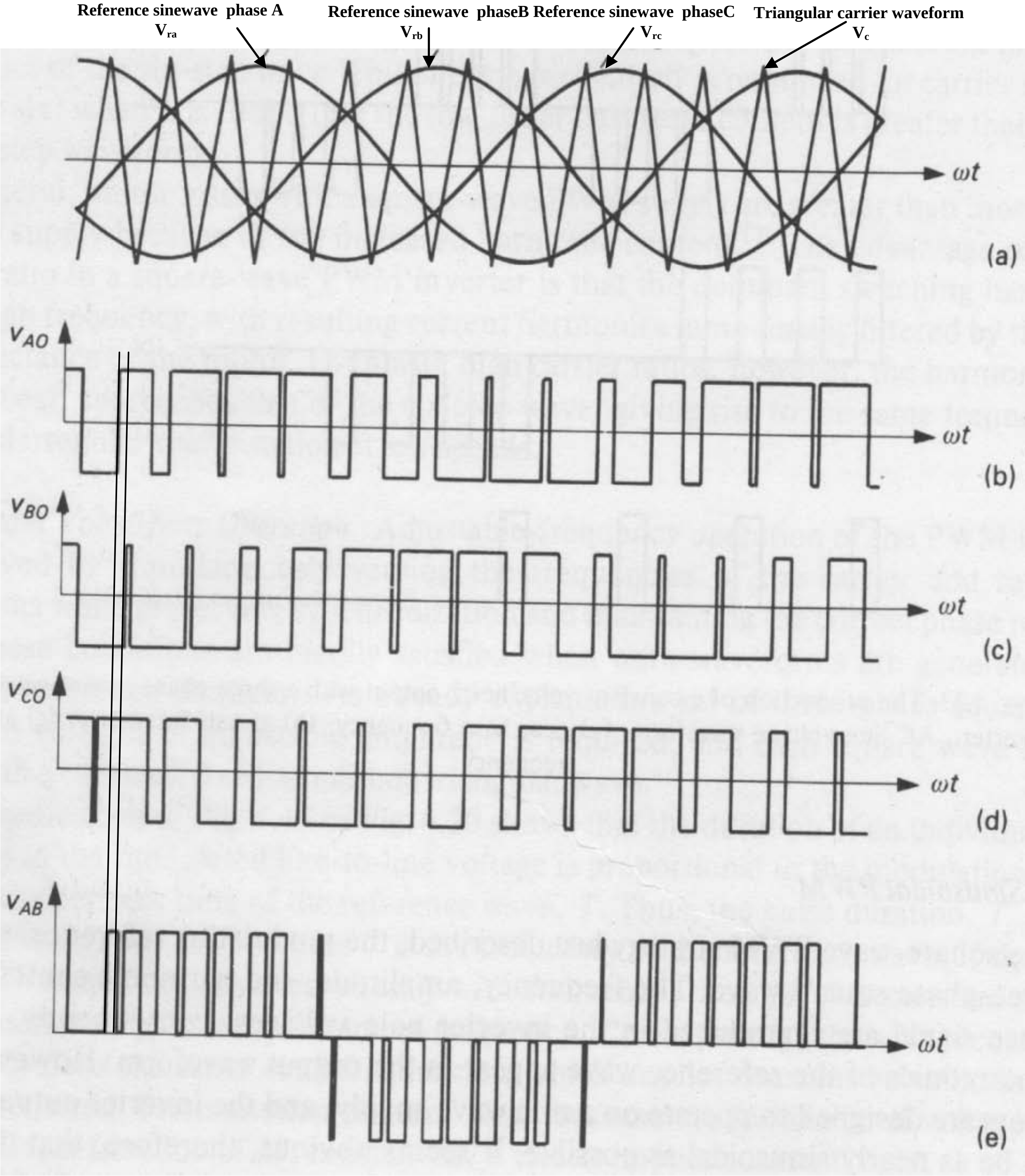


Fig.16Principle of sinusoidal PWM for three phase bridge inverter.

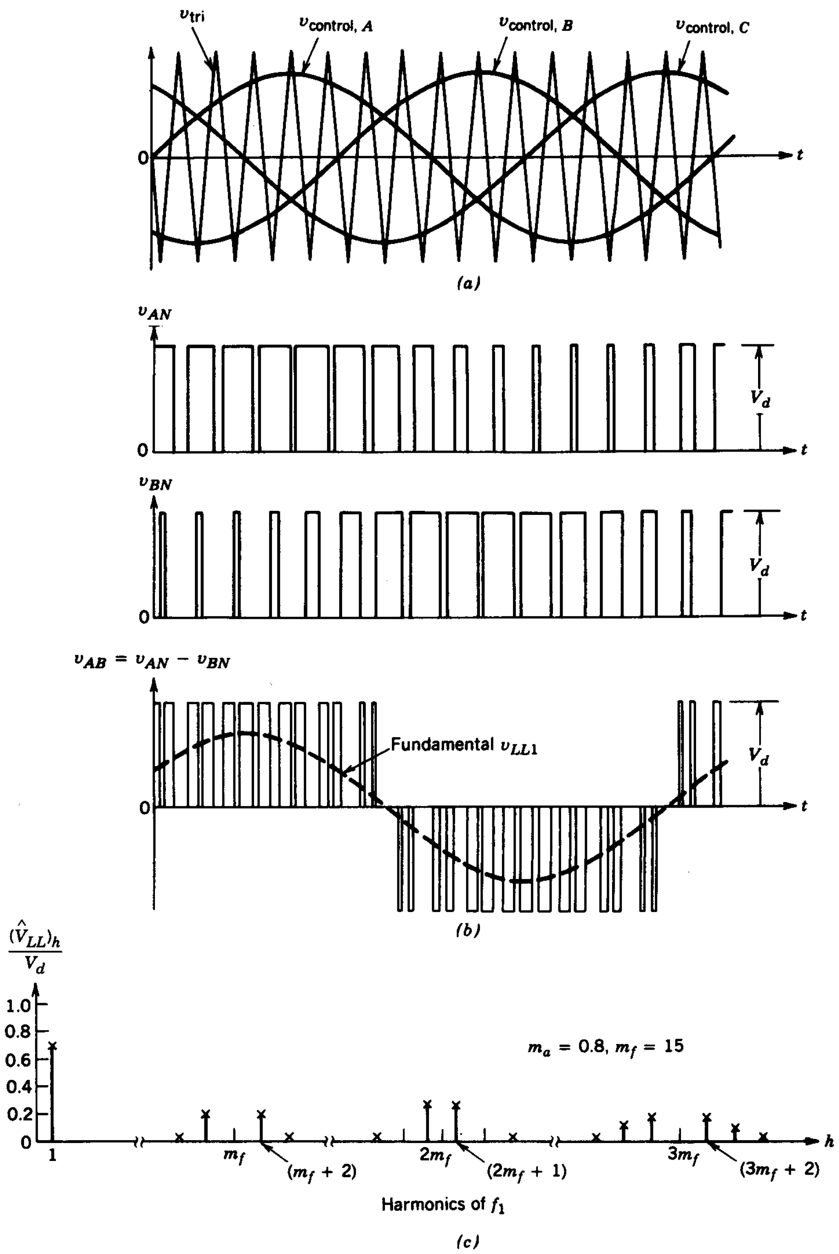


Figure 8-22 Three-phase PWM waveforms and harmonic spectrum.

DC/DC Converter (DC Chopper)

Many industrial applications require power from dc voltage sources. Several of these applications. However, perform better in case these are fed from variable dc voltage sources. Examples of such dc systems are subway cars, trolley buses, battery-operated vehicles, and battery-charging etc.

From ac supply systems, variable dc output voltage can be obtained through the use of phase-controlled converters or motor-generator sets . The conversion of fixed dc voltage to an adjustable dc output voltage, through the use of semiconductor devices, can be carried out by the use of two types of dc to dc converters given below.

AC Link Chopper. In the ac link chopper, dc is first converted to ac by an inveter (dc to ac converter). AC is then stepped-up or stepped down by transformer which is then converted back to dc by a diode rectifier fig. 1.a. As the conversion is in two stages, dc to ac and then ac to dc , ac link chopper is costly , bulky and less efficient.

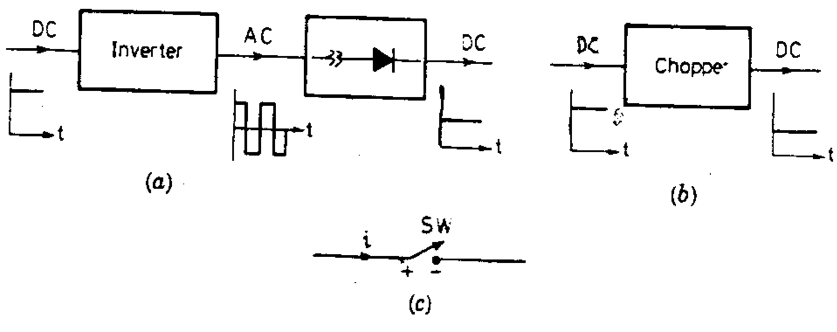


Fig.1 (a) AC link chopper (b) dc chopper (c) reproduction of a power semiconductor device.

DC Chopper. A chopper is a static devices that converts fixed dc input voltage to a variable dc output voltage directly fig.1 .b. A chopper may be thought of as dc equivalent of an ac transformer since they behave in an identical manner. As choppers involve one stage conversion, these are more efficient.

The power semiconductor device used for a chopper circuit can be power BJT, power MOSFET, GTO. These device, in general , can be represented by a switch SW with an arrow as shown in fig.1.c. When the switch is off, no current can flow. When the switch is on, current flows in the direction of arrow only. The power semiconductor device have on-state voltage drop of 0.5V to 2.5V across them. For the sake of simplicity, this voltage drop across these devices is neglected.

As stated above , a chopper is dc equivalent to an ac transformer having continuously variable turns ratio. Like a transformer, a choppers can be used to step down or step up the fixed dc input voltage. As step-down dc chopper are more common, a dc chopper in this chapter would mean a step-down dc chopper unless stated otherwise.

1- Principle of chopper operation

A chopper is a high speed on/off semiconductor switch. It connects source to load and disconnects the load from source at a fast speed. In this manner, a chopper load voltage as shown in fig.2.b is obtained from a constant dc supply of magnitude v_s. In fig.2.a, chopper is represented by a switch SW inside a dotted rectangle, which may be turned-on or turned-off as desired. For the sake of highlighting the principle of chopper operation, the circuitry used for controlled the on,off periods of this switch is not shown.

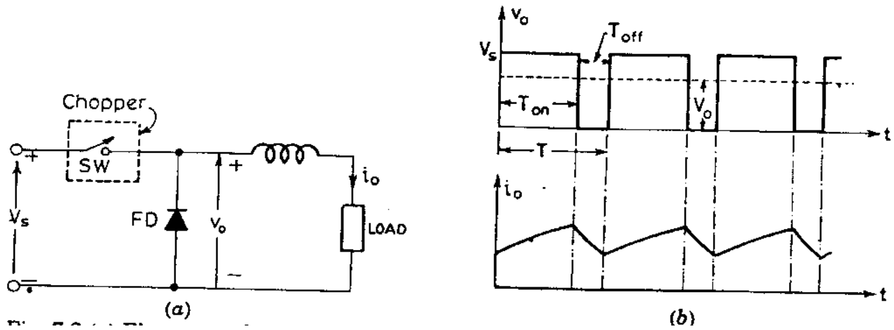


Fig.2(a) elementry chopper circuit (b) output voltage and current waveforms

During the period T_{on}, chopper is on and load voltage is equal to source voltage v_s .During the interval T_{off} , chopper is off , load current flows through the freewheeling diode FD. As a result, load terminals are short circuited by FD and load voltage is therefore zero during T_{off} . In this manner , a chopped dc voltage is produced at the load terminals. The load current as shown in fig.2.b is continuous. Form fig.2 b , average load voltage v_o is given by

$$v_o = \frac{T_{on}}{T_{on} + T_{off}} v_s = \frac{T_{on}}{T} v_s = Dv_s \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (1)$$

Where T_{on}=on-time; T_{off} =off-time

$T=T_{on} +T_{off}$ =chopping period

$$D = \frac{T_{on}}{T} = \text{duty } cycle$$

Thus load voltage can be controlled by varying duty cycle D.Eq.(1) shows that load voltage is independent of load current.Eq.(1) can also be written as

$$v_o= f.T_{on}.v_s$$

where f=(1/T)=chopping frequency

2-Control straegies

It is seen from eq.(1) that average value of output voltage v_o can be controlled through D by opening and closing the semiconductor switch periodically. The various control strategies for varying duty cycle D are as follows:

2-1 Constant frequency system

In this scheme, the on-time T_{on} is varied but chopping frequency f (or chopping period T) is kept constant. Variation of T_{on} means adjustment of pulse width, as such this scheme is also called *pulse-width-modulation* scheme. This scheme has also been referred to as *time –ratio control* (TRC) by some authors.

Fig.3 illustrates the principle of pulse-width-modulation. Here chopping period T is constant. In fig.3.a ,T_{on}=(1/4)T so that(D =0.25) or (D =25%) . In fig.3.b , T_{on} = (3/4)T so that(D = 0.75) or(75%).Ideally D can be varied from zero to infinty . Therefor output voltage v_o can be varied between zero and source voltage v_s .

2-2 Variable frequency system

In this scheme , the chopping frequency f (or chopping period T) is varied and either(1) on-time T_{on} is kept constant or (2) off-time T_{off} is kept constant . This method of controlling D is also called frequency- modulation scheme.

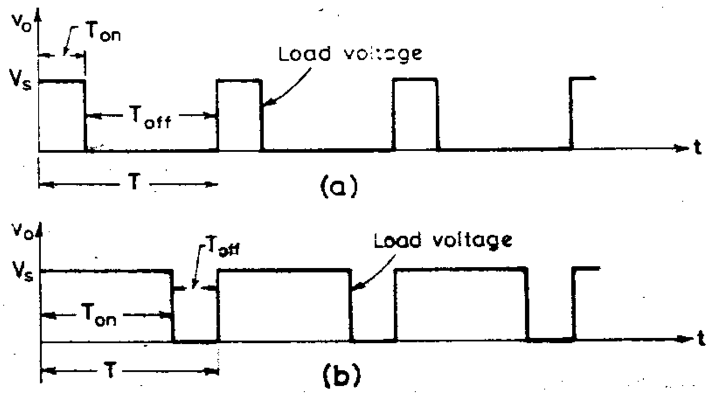


Fig.3. principle of pulse-width modulation (constant T)

Fig.4. illustrates the principle of frequency modulation. In fig.4.a , T_{on} is kept constant but T is varied. In the upper diagram of fig.4.a , $T_{on}=(1/4)T$ so that (D = 0.25). In the lower diagram of fig.4.a. ,T_{on} =(3/4)T so that (D = 0.75) . In fig.4. b ,T_{off} is kept constant and T is varied. In the upper diagram of this figure, $T_{on}=(1/4)T$ so that(D = 0.25) and in the lower diagram T_{on} =(3/4)T so that (D = 0.75) .

Frequency modulation scheme has some disadvantages as compared to pulse-width modulation scheme. These are as under:

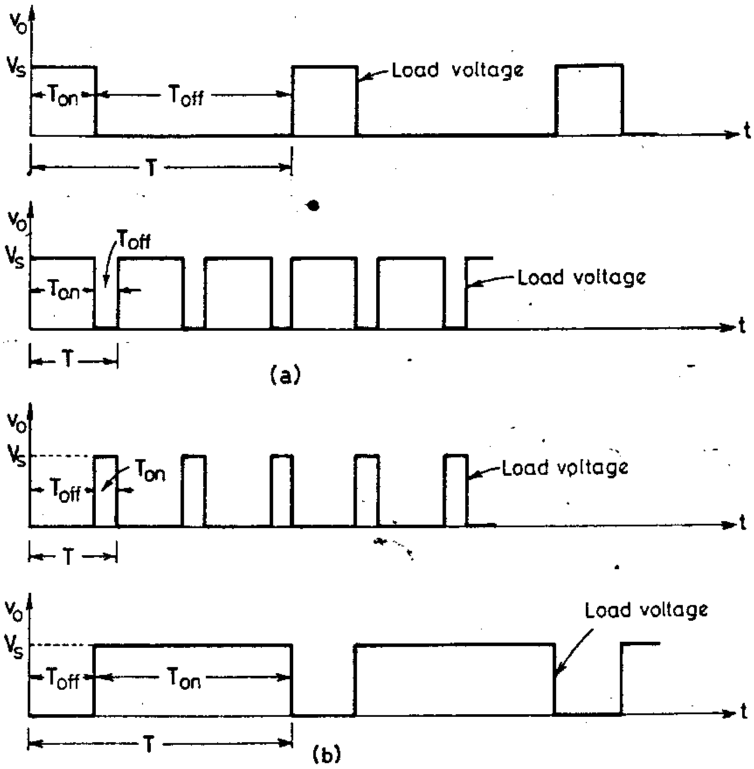


Fig.4. principle of frequency modulation

(a) On-time T_{on} constant (b) off-time T_{off} constant.

(1) The chopping frequency has to be varied over a wide range for the control of output voltage in frequency modulation. Filter design for such wide frequency variation is, therefore, quite difficult.
(2) For the control of D , frequency variation would be wide. As such, there is a possibility of interference with signalling and telephone lines in frequency modulation scheme.
(3) The large off-time frequency modulation scheme may make the load current discontinuous which is undesirable.

It is seen from above that constant frequency (PWM) scheme is better than variable frequency scheme.(PWM) technique has, however, a limitation .In this technique, T_{on} cannot be reduced to near zero for most of the commutation circuits used in choppers. As such, low range of D control is not possible in (PWM). This can , however, be achieved by increasing the chopping period (or decreasing the chopping frequency) of the chopper.

3- Step –up choppers

For the chopper configuration of fig.2.a , average output voltage v_o is less than the input voltage v_s , i.e.($v_o < v_s$); this configuration is therefore called step-down chopper. Avarge output voltage v_o greater than input voltage v_s can, however, be obtained by a chopper called step-up chopper. Fig.5.a. illustrates an elementary form of a step-up chopper.In this article, working principle of a step-up chopper is presented.

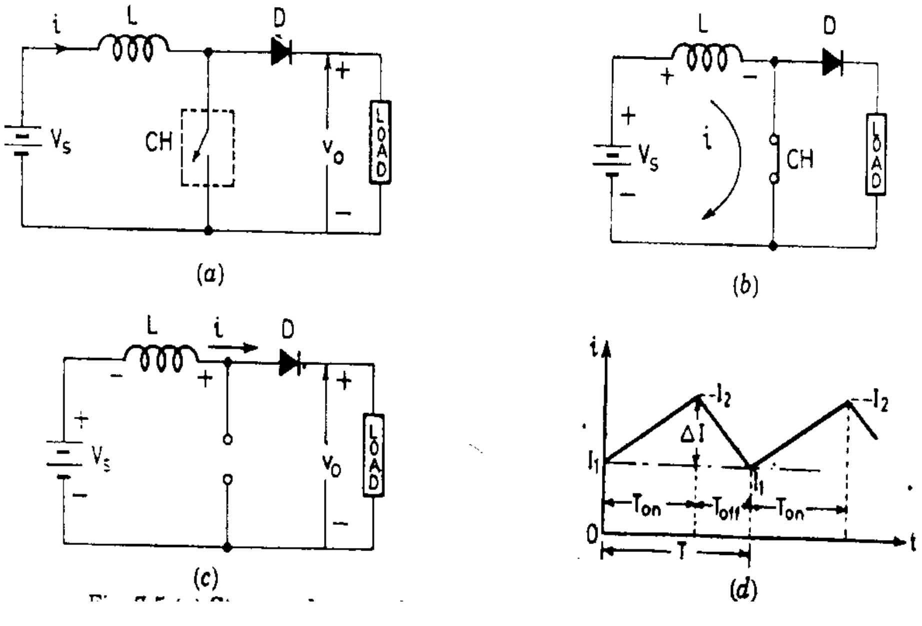


Fig.5.(a) step-up chopper (b) L store eneragy
(c) L.di /dt is added to v_s (d) current waveform

In this chopper, a large inductor L in series with source voltage v_s is essential as shown in fig.5.a. When the chopper CH is on , the closed current path is as shown in fig.5.b and inductor stores energy during T_{on} period. When the chopper CH is off, as the inductor current cannot die down instantaneously, this current is forced to flow through the diode and load for a time T_{off} , fig.5.c. As the current tends to decrease, polarity of the emf induced in L is reversed as shown in fig.5.c. As a result, voltage across the load, given by [$v_o = v_s + L(di/dt)$], exceed the source voltage v_s . In this manner, the circuit of fig.5.a. acts as a step-up chopper and the energy stored in L is released to the load.

When CH is on , current through the load would increase from I_1 to I_2 as shown in fig.5.d. When CH is off , current would fall from I_2 to I_1. With CH on, source voltage is applied to L i.e. ($v_L = v_s$).

When CH is off, for fig.5.c gives [$v_L - v_o + v_s = 0$], or [$v_L = v_o - v_s$].

Here v_L = voltage across L. **Assuming linear variation of output current, the energy input to inductor from the source**, during the period T_{on} , is

$$W_{in} = \text{(voltage across L)(average current through L)}\ T_{on}$$

$$= v_s \,.\left(\frac{I_1+I_2}{2}\right) .\, T_{on} \qquad \dots\dots\dots\dots\dots\dots\dots (3)$$

During the time T_{off} , when chopper is off , the energy released by inductor to the load is

$$W_{off} = \text{(voltage across L)(average current through L)}\ T_{off}$$

$$=(v_o - v_s)\,.\left(\frac{I_1+I_2}{2}\right) .\, T_{off} \qquad \dots\dots\dots\dots\dots\dots\dots (4)$$

Considering the system to be lossless, these two energies given by eq.(3) and (4) will be equal.

$$v_s \,.\left(\frac{I_1+I_2}{2}\right) .\, T_{on} = (v_o - v_s)\,.\left(\frac{I_1+I_2}{2}\right) .\, T_{off}$$

$$v_o = v_s \frac{T}{T-T_{on}} = v_s \frac{1}{1-D} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

It is seen from eq.(5) that average voltage across the load can be stepped up by varying the duty cycle . If chopper of fig.5.a. is always off , (D = 0) and ($v_o = v_s$). If this chopper is always on , (D = 1) and
($v_o = \infty$ infinity).

In practice , chopper is turned on and off so that D is variable and the required step-up average output voltage , more than source voltage , is obtained.

The principle of step-up chopper can be employed for regenerative braking of dc motors. In fig.5.a, if v_s represents the motor armature voltage and v_o the dc source voltage , the power can be fed back to the dc source in case [v_s / (1-D)] is more than v_o . In this manner , regenerative braking of dc motor occurs . Even at decreasing motor speeds , regenerative braking can be made to take place provided duty cycle D is so adjusted that [v_s / (1-D)] exceeds the fixed source voltage v_o .

4 – For the basic dc to dc converter of fig.2.a , express the following variables as function of v_s , R ,and duty cycle D in case load is resistive :

The load voltage variation is shown in fig.2.b. For a resistive load, output or load current waveform is similar to load voltage waveform.

(a) Average output voltage and current

$$\mathrm{v_{av}} = \mathrm{v_o} = \frac{\mathrm{T_{on}}}{\mathrm{T}}\,\mathrm{v_s} = \mathrm{D\,v_s} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\mathrm{I_{av}} = \mathrm{I_o} = \frac{\mathrm{v_o}}{\mathrm{R}} = \frac{\mathrm{T_{on}}}{\mathrm{T}} \cdot \frac{\mathrm{v_s}}{\mathrm{R}} = \mathrm{D}\,\frac{v_s}{R} \ldots\ldots\ldots\ldots (7)$$

(b) The output current is commutated by the thyristor (chopper) at the instant ($t = T_{on}$). Therefore, output current at the instant of commutation is (v_s / R).

(c) For a resistive load , freewheeling diode FD does not come into play. Therefore, average and rms values of freewheeling diode currents are zero.

(d) Rms value of the output voltage

$$\mathrm{v_{rms}} = \left[\frac{\mathrm{T_{on}}}{\mathrm{T}} \cdot \mathrm{v}_s^2\right]^{1/2} = \sqrt{\mathrm{D}} \cdot \mathrm{v_s} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

(e) average and rms thyristor(chopper) current

$$\mathrm{I_{T\,av}} = \frac{\mathrm{T_{on}}}{\mathrm{T}} \cdot \frac{\mathrm{v_s}}{\mathrm{R}} = \mathrm{D}\,\frac{\mathrm{v_s}}{\mathrm{R}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\mathrm{I_{T\,rms}} = \left[\frac{\mathrm{T_{on}}}{\mathrm{T}} \cdot \left(\frac{\mathrm{v_s}}{\mathrm{R}}\right)^2\right]^{1/2} = \sqrt{\mathrm{D}} \cdot \frac{\mathrm{v_s}}{\mathrm{R}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

(f) average source current = average thyristor current = $\mathrm{D} \cdot \frac{\mathrm{v_s}}{\mathrm{R}} \quad \ldots\ldots\ldots (11)$

Effective input resistance of the chopper = $\frac{\text{dc source voltage}}{\text{average source current}}$

$$= \frac{\mathrm{v_s}\,.\mathrm{R}}{\mathrm{D}\,.\mathrm{v_s}} = \frac{\mathrm{R}}{\mathrm{D}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

(g) to obtain chopper efficiency

The load power : $P_o = \frac{D(v_s - v_T)}{R}$ ... ... (13)

If the chopper is lossless , then v_T =0 and output power will be ,

$$P_o = \frac{D\ .v_s^2}{R} \text{ ... ... } (14\ )$$

The supply power (input power)

$$P_i = D\ .\ v_s\ .\frac{v_s - v_T}{R} \text{ ... ... } (15\ \ )$$

If the chopper is lossless , then v_T = 0 and input power will be

$$P_i = \frac{D\ .v_s^2}{R} \text{ ... ... } (\ 16\ \ )$$

$$\eta = \frac{P_0}{P_i} = 1 \text{ ... ... } (\ \ 17)$$

the efficiency is 100% since chopper lossless.

5 – types of chopper circuits

Power semiconductor devices used in chopper circuits are unidirectional devices; polarities of output voltage v_o and the direction of output current I_o are , therefore, restricted.

A chopper can , however , operate in any of the four quadrants by an appropriate arrangement of semiconductor devices . This characteristic of their operation in any of the four quadrants forms the basis of their classification as type –A chopper , type – B chopper etc. Some authors describe this chopper classification as class A , class B ,… in place of type – A , type – B ….respectively.

In the chopper –circuit configirations drawn henceforth, the current directions and voltage polarities marked in the power circuit would be treated as positive. In case current directions and voltage polarities turn out to be opposite to those shown in the circuit, these currents and voltages must be treated as negative.

In this section , the classification of various chopper configurations is discussed.

5-1 First – quadrant , or type – A , chopper

This type of chopper is shown in fig.6.a. It is observed that chopper circuit of fig.2.a. is also type – A chopper. In fig.6.a , when chopper CH1 is on ,($v_o = v_s$)and current i_o flows in the arrow direction shown. When CH1 is off , 9v_o = 0) but i_o in the load continues flowing in the same direction through freewheeling diode FD , fig.2.b. It is thus seen that average values of both load voltage and current , i.e. v_o and i_o are always positive :this fact is shown by the hatched area in the first quadrant of (v_o –I_o) plane in fig.6.b.

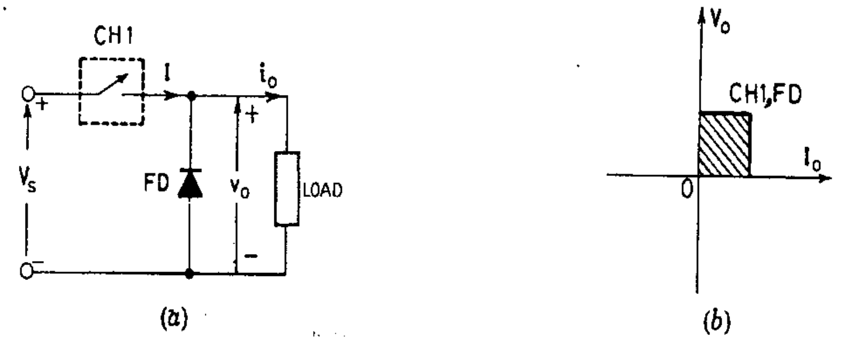


Fig.6. first –quadrant , or type – A chopper

The power flow in type – A chopper is always from source to load. This chopper is also called ***step-down chopper*** as average output voltage v_o is always less than the input dc voltage v_s .

5-2 second – quadrant , or type – B , chopper

Power circuit for this type of chopper is shown in fig.7.a. Note that load must contain a dc source E , like a battery (or a dc motor) in this chopper.

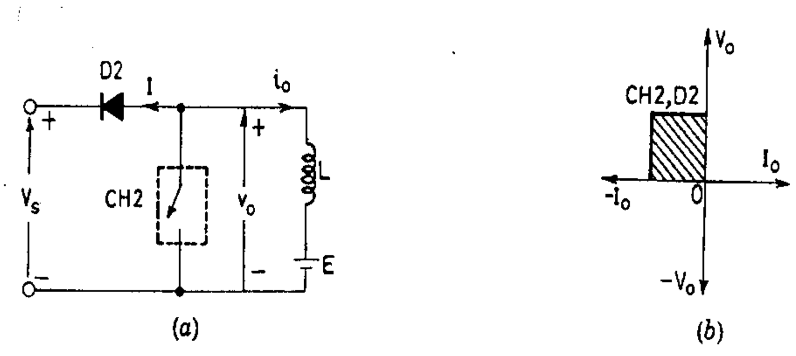


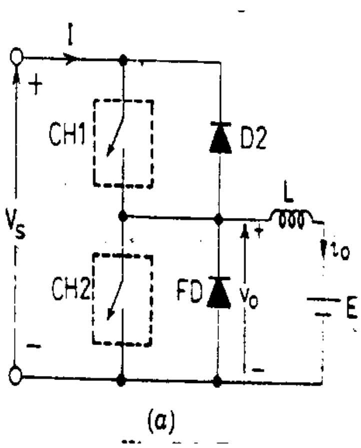
Fig.7.second – quadrant , or type – B ,chopper

When CH2 is on , v_o = 0 but load voltage E drives current through L and CH2 . Inductance L stores energy during T_{on}(= on period) of CH2 . when CH2 is off ,

[v_o=E+L(di /dt)] exceeds source voltage v_s . As a result , diode D_2 is forward biased and begins conduction , thus allowing power to flow to the source . Chopper CH2 may be on or off , current I_o flows out of the load ,current i_o is therefore treated as negative . Since v_o is always positive and I_o is negative ,power flow is always from load to source . As load vltage [v_o=E+L(di /dt)] is more than source voltage v_s , type –B chopper is also called ***step – up chopper*** .

5 – 3 two – quadrant type – A chopper , or type – C chopper

This type of chopper is obtained by connecting type – A and type-B choppers in parallel as shown in fig.8.a. The output voltage v_o is always positive because of the presence of freewheeling diode FD across the load . When chopper CH2 is on , or freewheeling diode FD conducts , output voltage v_o = 0 and in case chopper CH1 is on or diode D2 conducts, output voltage v_o = v_s . The load current i_o can , however, reverse its direction. Current i_o flows in the arrow direction marked in fig.8.a ,i.e. load current is positive when CH1 is on or FD conducts. Load current is negative if CH2 is on or D2 conducts. In other words, CH1 and FD operate together as type-A chopper in first quadrant . Likewise, CH2 and D2 operate together as type-B chopper in second quadrant.



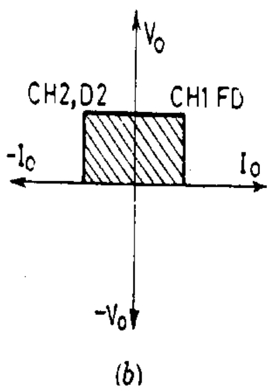


Fig.8.Two – quadrant type – A chopper, or type – C chopper

Average load voltage is always positive but average load current may be positive or negative as explained above. Therefore , power flow may be from source to load (first-quadrant operation) or from load to source (second – quadrant operation). Choppers CH1 and CH2 should not be on simultaneously as this would lead to a direct short circuit on the supply lines . This type of chopper configuration is used for

motoring and regenerative braking of dc motors. The operating region of this type of chopper is shown in fig.8.b. by hatched area in first and second quadrants.

6 – steady state time – domain analysis of type –A chopper

Normally the choppers are used to drive the dc motors .These motors are considered as RL (inductive) load (fig.6.a), hence back emf 'E' is also shown in the circuit diagram as a part of load .Such loads are also called as (RLE) load.

Fig.9.a shows the waveforms of this circuit. The CH1 is on for 0 to T_{on} (=DT). Here D is the duty cycle of the chopper. The output voltage is equal to supply voltage ($v_o = v_s$) when the CH1 is on. The equivalent circuit in fig.9.b shows the current flow when the CH1is on. At T_{on} , the output current reaches to maximum value I_{max} .

For this mode of operation, the differential equation governing its performance is

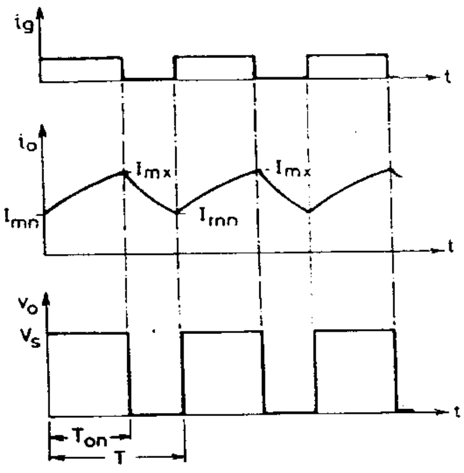
$$v_s = Ri + L\frac{di}{dt} + E \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (18)$$

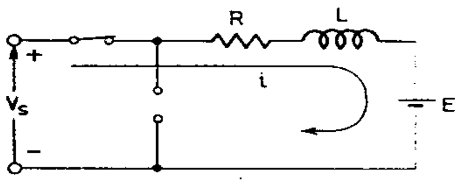
For $\quad 0 \leq t \leq T_{on}$

From T_{on} to T the CH1 is off . At T_{on} , the output current is at I_{max} . When the CH1 is off , the load inductance tries to maintain the output current in the same direction. This current flows through the freewheeling diode FD . The equivalent circuit in fig.9.c. shows this situation. The freewheeling diode is forward biased due to load inductance voltage L (di_o /dt). Output voltage is zero when freewheeling diode conducts, hence i_s and swith i_T are same. For this circuit , the differential equation is

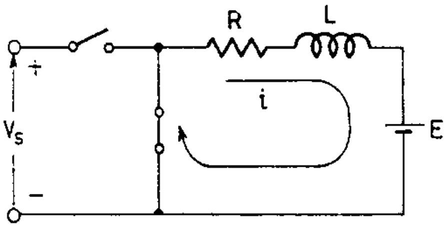
$$0 = Ri + L\frac{di}{dt} + E \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (19)$$

For $\quad T_{on} \leq t \leq T$





(b)



(c)

Fig.9 (a) waveforms for type – A chopper with load RLE (b) equivalent circuit for type –A chopper with CH1 on and (c) CH1 off

The maximum I_{max} and minimum I_{min} values of load current can be obtained from equations (20) and (21) respectively for given v_s , R , D, T_a ,E .

$$I_{max} = \frac{v_s}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}}\right] - \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Where $T_a = \frac{L}{R}$

and

$$I_{min} = \frac{v_s}{R}\left[\frac{e^{T_{on}/T_a} - 1}{e^{T/T_a} - 1}\right] - \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(21)$$

The values of I_{max} and I_{min} depends upon the load inductance.

In case CH1 conducts continuously, then T_{on} = T and from eqs. (20) and (21).

$$I_{max} = I_{min} = \frac{v_s - E}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

6 -1 steady state ripple

The ripple current ($I_{max} - I_{min}$) can be obtained from eqs. (20) and (21) as follows:

Ripple current = $\Delta I_{o(p-p)} = I_{max} - I_{min}$

$$= \frac{v_s}{R}\left[\frac{1-e^{-T_{on}/T_a}}{1 - e^{-T/T_a}} - \frac{e^{T_{on}/T_a} - 1}{e^{T/T_a} - 1}\right]$$

$$= \frac{v_s}{R}\left[\frac{\left(1 - e^{-T_{on}/T_a}\right)\left(1 - e^{-(T-T_{on})/T_a}\right)}{\left(1 - e^{-T/T_a}\right)}\right] \quad \ldots\ldots\ldots(23)$$

The ripple current gives be eq.(23) is seen to be independent of the load counter emf E.With T_{on} = DT and T- T_{on} =(1-D) T , eq. (23), can be written as :

Ripple current = $\Delta I_{o(p-p)} = I_{max} - I_{min}$

$$= \frac{v_s}{R}\left[\frac{\left(1 - e^{-DT/T_a}\right)\left(1 - e^{-(1-D)T/T_a}\right)}{\left(1 - e^{-T/T_a}\right)}\right] \ldots\ldots\ldots\ldots\ldots\ldots(24)$$

Per unit ripple current = p.u. $\Delta I_o = \frac{I_{max} - I_{min}}{v_s/R}$

$$= \left[\frac{\left(1 - e^{-DT/T_a}\right)\left(1 - e^{-(1-D)T/T_a}\right)}{\left(1 - e^{-T/T_a}\right)}\right] \ldots\ldots\ldots..(25)$$

For (D = 0.5) and (T / T_a = 5), p.u. ripple current = 0.848 . For (D = 0.5) and (T / T_a = 25), p.u. ripple current = 1 .

In this manner , the variation of p.u.ripple current as a function of duty cycle D and ratio (T / T_a) can be plotted as shown in fig.10.

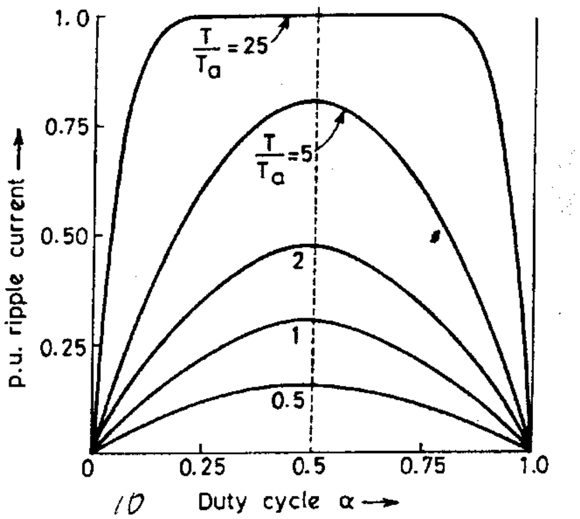


Fig.10 per unit ripple current as a function of D and T / T_a

Its value is maximum when (D = 0.5). As L increases , T_a = (L / R) increases and (T / T_a) reduces and p.u. ripple current decreases, fig.10.

The **peak to peak ripple current has maximum** value (ΔI_{max}) when duty cycle (D=0.5) in eq.(25).

Putting (T / T_a = x) for convenience, (ΔI_{max}) from eq.(25) is

$$\Delta I_{max} = \frac{v_s}{R}\left[\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{(1-e^{-x})}\right]$$

$$= \frac{v_s}{R}\left[\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{(1-e^{-0.5x})(1-e^{-0.5x})}\right]$$

$$= \frac{v_s}{R}\left[\frac{(1-e^{-0.5x})}{(1+e^{-0.5x})}\right] = \frac{v_s}{R}\tanh\frac{1}{4}\,x = \frac{v_s}{R}\tanh\frac{T}{4T_a} \dots\dots\dots\dots (26)$$

But $T = \frac{1}{f}$ and $T_a = \frac{L}{R}$

$$\therefore \Delta I_{max} = \frac{v_s}{R}\tanh\frac{R}{4f\,L} \dots\dots\dots\dots\dots\dots\dots\dots\dots..(27)$$

7 – Summary

7-1 – Step down chopper with resistive load

Average output voltage , $v_{o(av)} = D \cdot v_s$

Rms output voltage , $v_{o(rms)} = \sqrt{D} \cdot v_s$

Load power , $P_o = \frac{D \cdot v_s^2}{R}$ (neglecting drop in chopper)

Supply power , $P_i = \frac{D \cdot v_s^2}{R}$ (neglecting drop in chopper)

output current , $I_{o(av)} = \frac{v_{o(av)}}{R}$

7-2- step down chopper with RLE load

$$I_{min} = \frac{v_s}{R}\left[\frac{e^{T_{on}/T_a} - 1}{e^{T/T_a} - 1}\right] - \frac{E}{R}$$

$$I_{max} = \frac{v_s}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}}\right] - \frac{E}{R}$$

$\Delta I_{o(p-p)} = I_{max} - I_{min}$

$v_{o(av)} = D \cdot v_s$, $I_{o(av)} = \frac{v_{o(av)}}{R}$

$$I_{o(av)} = \frac{I_{max} + I_{min}}{2}$$

7-3- step up chopper

Output average voltage , $v_{o(av)} = \frac{v_s}{1-D}$

University of Technology
Electromechanical Engineering Department
8 hrs / Four week
Fall 2013-2014

Power Electronics and Electrical Drives
EME 401
Energy- Systems branch

Lecturers: Dr.Ali Hussein Numan & Dr.Shatha K. Baqir

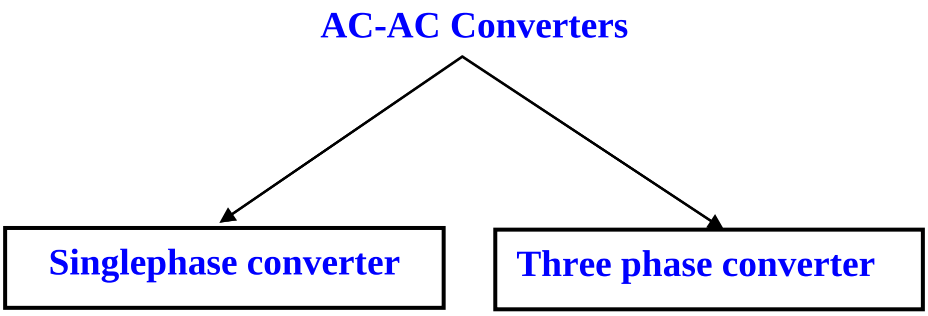
AC-AC Converters

Contents

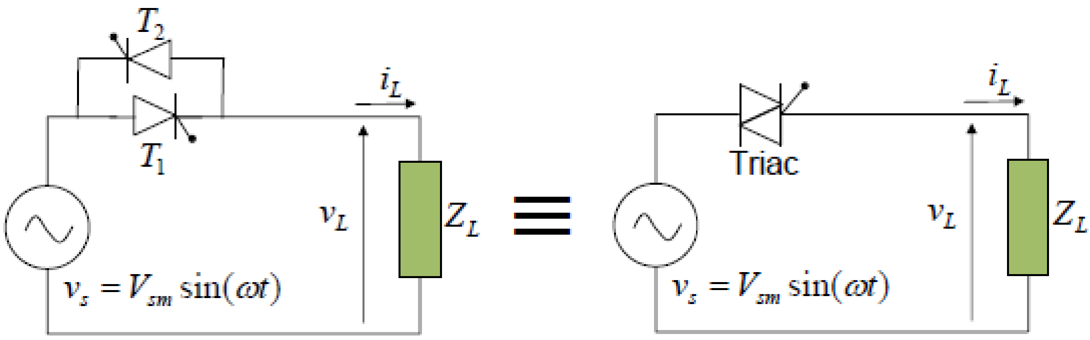
Definition: The AC-AC converters or so called AC voltage controllers are used to control the output RMS voltage using thyristor orTriactype switch.

Classifications

The AC-ACconverters can be broadly classified into the following types:



1-Phase AC-AC Converter



(a) **(b)**

Fig.4.1 Single phase AC-AC converter using (a) :thyristor and (b) Triac

Control methods of AC-AC converter

a) ON-OFF Control.

b) Phase Control.