Basics of Measurements

1.1 Introduction

Measurement is the process by which one can convert physical parameters to meaningful number. The measurement of any quantity plays very important role not only in science but in all branches of engineering, medicine, and in almost all the human day by day activities. A measurement generally involves using instruments as a physical means of determining a quantity or variable.

The ***measurement*** of a given parameter or quantity is the act of result of quantitative comparison between a predefined standard and an unknown quantity to be measured.

The ***measuring instrument*** is a device for determining the value or magnitude of a quantity or variable.

1.2 Functional Elements of an Instrument

Any instrument or a measuring system can be described in general with the help of a block diagram. While describing the general form of a measuring system, it is not necessary to go into details of the physical aspects of a specific instrument. The block diagram indicates the necessary elements and their functions in a general measuring system. The entire operation of an instrument can be studied in terms of these functional elements. Fig (1) shows the block diagram showing functional elements of an instrument.

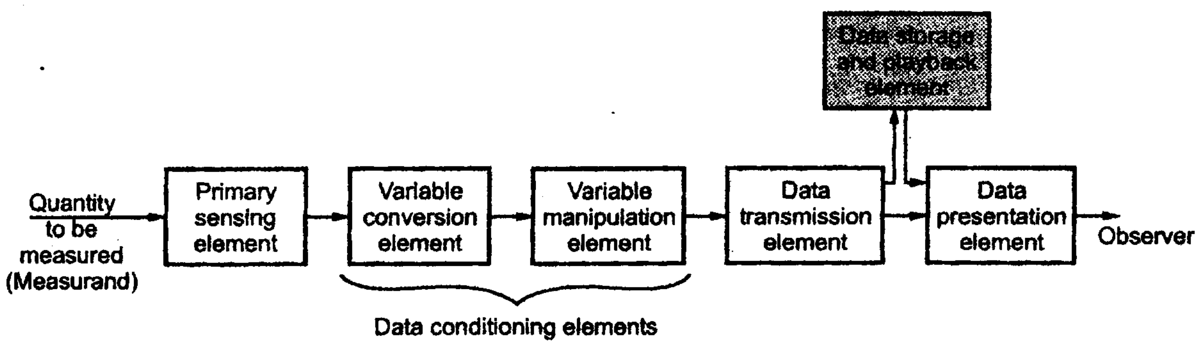


Fig (1) Functional elements of an instrument

The various elements can be grouped as:-

1- Primary sensing element.
2- Data conditioning elements.
3- Data presentation element.

Each element is made up of number of distinct components which perform a particular function in the measurement procedure. In the block diagram, the function of each element is important rather than the construction of the element.

1.2.1 Primary Sensing Element

An element of an instrument which makes first, the contact with the quantity to be measured is called primary sensing element. In ammeter, coil carrying current to be measured is a primary sensing element. In most of the cases, a transducer follows primary sensing element which converts the measurand into a corresponding electrical signal.

1.2.2 Variable Conversion Element

The output of the primary sensing element is in electrical form such as voltage, frequency or any other electrical parameter. Such an output may not be suitable for the actual measurement system. For example if the measurement system is digital, then the analog obtained from the primary sensing element is not suitable for the digital system. Thus analog to digital converter is required, which is nothing but variable conversion element.

1.2.3 Variable Manipulation Element

The level of the output from the previous stage may not be enough to drive the next stage. Thus variable manipulation element manipulates the signal, preserving the original nature of the signal. So amplifiers and attenuators are used as the variable manipulation elements.

Note: - sometimes the output of the transducer may get affected due to unwanted signals like noise. Thus such signals are required to be processed with some process like modulation, clipping, clamping etc.., to obtain the signal in pure and acceptable form from highly distorted form. Such a process is called signal conditioning. Thus in addition to variable conversion and variable manipulation, the signal conditioning is also done in the second stage. Hence second stage called data conditioning or signal conditioning elements.

1.2.4 Data Transmission Element

When the elements of the system are physically separated, it is necessary to transmit the data from one stage to other. This achieved by data transmission element. The signal conditioning and data transmission together is called intermediate stage of an instrument.

1.2.5 Data Presentation Element

The transmitted data may be used by the system, finally for monitoring, controlling or analyzing purposes. Thus the person handling the instrument must get the information in the proper form, according to the purpose for which it is intended. The data presentation stage may be called terminating stage of an instrument.

Example: - consider a simple analog meter used to measure current or voltage as shown in fig (2) the moving coil is the primary sensing element. The magnets and coil together act as data conditioning stage to convert current in a coil to a force. This force is transmitted to the pointer through mechanical linkage which acts as data transmission element. The pointer and scale act as data presentation element.

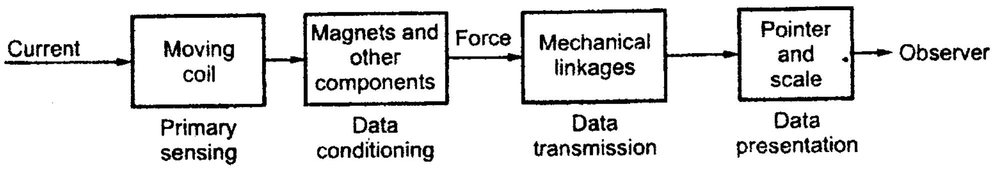


Fig (2) Block schematic of an ammeter

1.3 Performance Characteristics

The performance characteristics of an instrument are mainly divided in two categories:-

1- ***Static characteristics***

Accuracy
Resolution
Sensitivity
Threshold
Reproducibility
Zero drift
Stability
Linearity
Precision

2- ***Dynamic characteristics***

Speed of response
Fidelity
Lag
Dynamic Error

1.4 Static Characteristics

The static characteristics are defined for the instruments which measure the quantities which do not vary with time.

1.4.1 Accuracy
It is the degree of closeness with which the instrument reading approaches the true value of the quantity to be measured.

1.4.2 Precision
It is the degree of agreement within a group of measurements or instruments.

1.4.3 Range or Span
It is the minimum and maximum values of a quantity for which instrument are designed to measure.

1.4.4 Threshold
If the input quantity is slowly varied from zero on words, the output does not change until some minimum value of the input is exceeded. This minimum value of the input is called **threshold.** In other words the threshold is defined as the smallest measurable input.

1.4.5 Resolution
It is the smallest increment of quantity being measured, which can be detected with certainty by an instrument. In other words, the minimum change in the input which causes a change in the output is called **resolution.**

1.4.6 Stability
It is defined as the ability of an instrument to retain its performance throughout its specified operating life and the storage life.

1.4.7 Sensitivity
Sensitivity denotes the smallest change in the measured variable to which the instrument responds. It's also defined as the ratio of the change in the value of the quantity to be measured.

$$\text{sensitivity} = \frac{\text{infinitesimal change in output}}{\text{infinitesimal change in input}} = \frac{\Delta q_o}{\Delta q_i}$$

$$\text{Deflection factor} = \frac{\Delta q_i}{\Delta q_o}$$

Thus, if the calibration curve is not linear, as shown in fig (3b), then the sensitivity varies with the input. But if the calibration curve is linear as shown in fig (3a), the sensitivity of the instrument is the slope of the calibration curve.

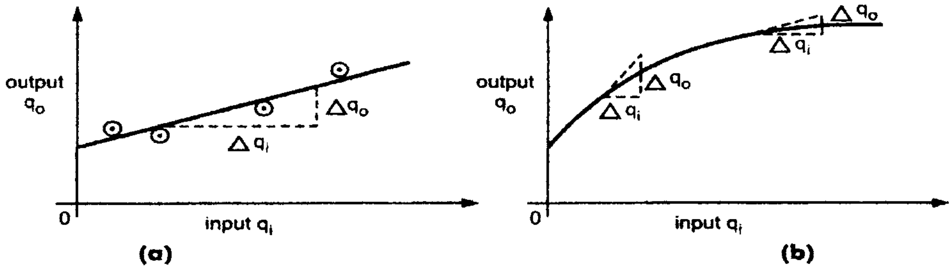


Fig (3) Sensitivity

<u>Example:</u> - A particular ammeter requires a change of 2A in its coil to produce a change in deflection of the pointer by 5mm. Determine its sensitivity and deflection factor.

$$\text{sensitivity} = \frac{\Delta q_o}{\Delta q_i} = \frac{5\text{mm}}{2\text{A}} = 2.5\ \text{mm/A}$$

$$\text{deflection factor} = \frac{\Delta q_i}{\Delta q_o} = \frac{1}{\text{sensitivity}} = \frac{1}{2.5} = 0.4\ \text{A/mm}$$

1.4.8 <u>Linearity</u>

The linearity is defined as maximum deviation of the actual calibration curve [the graph of output against the input] from the idealized straight line, expressed as a percentage of full scale reading or a percentage of the actual reading.

$$\%\ \text{Linearity} = \frac{\text{max.deviation of output from idealized straight line}}{\text{full scale deflection}} \times 100$$

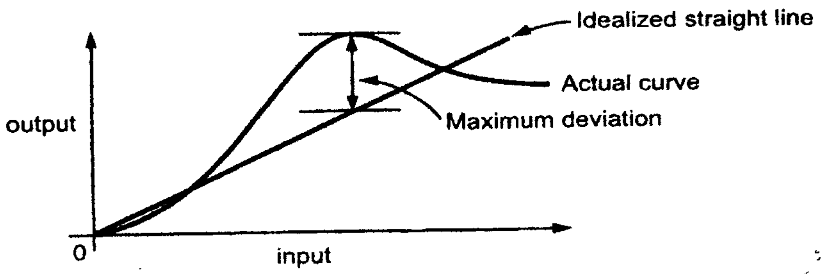


Fig (4) Linearity

1.5 Calibration

Calibration is the process of making an adjustment or marking a scale so that the readings of an instrument agree with the accepted and the certified standard. In other words, it is the procedure for determining the correct values of measurand by comparison with measured or standard ones. The particular instrument is compared with a primary standard, secondary standard with high accuracy or an instrument with known accuracy.

Measuring Units, Dimensions and Standards

When a particular instrument indicates a reading, to specify the reading and use it in the further calculations, it is necessary to specify type and magnitude for that reading. The magnitude is nothing but the reading obtained on the instrument. The type of reading is nothing but the unit of the physical quantity which is measured by the instrument. Without unit, only magnitude has no physical meaning.

In the past the system of units most commonly used where the English and metric .The metric is sub divided into two interrelated standards .The MKS and CGS .fundamental quantities of these systems are compared in the table (1) below along with their abbreviations .The MKS system uses ,meters ,kilograms and second ,while the CGS system uses centimeters ,Grams and second .

Table 1

Comparison of the English and Metric systems of units

English	Metric		SI
	MKS	CGS	
Length Yard(yd)=(0.914m)	Meter(m) (39.37 in) (100 cm)	Centimeter(cm) 2.54 cm=1 in	Meter(m)
Mass Slug(14.6 Kg)	Kilogram(kg) (1000g)	Gram(gram)	Kilogram(kg)
Force Pound(lb)=4.45 N	Newton(N)=100.000 dyne	Dyne	Newton
Temperature Fahrenheit(F)= 9/5 c°+32	Celsius or Centigrade(c°)= 5/9(F-32)	Centigrade(c°)	Kelvin(k) K=273.13+c°
Energy Foot-Pound(ft-lb)= (1.356 joules)	Newton-meter(N-m) Or joule(J)=0.7378 ft-lb	Dyne-centimeter or Erg (1 joule=10^7 erg)	Joule(J)
Time Second(s)	Second(s)	Second(s)	Second(s)

The use of more than one system of unit the world would introduce unnecessary complications to the basic understanding of any technical data. The need for a standard set of units to be adopted by all nations has become increasingly obvious .In 1960 a general conference adopted the International system of unit (SI).

The Three Classes of SI Units and the SI Prefixes

SI units are currently divided into three classes:

- Base units (fundamental)
- Derived units
- Supplementary units

SI Base units

The following table gives the seven base quantities, assumed to be mutually independent, on which the SI is founded, and the names and symbols of their respective units, called "SI base units." or fundamental units.

Table (2)
Basic SI quantities, units and symbols

SI Base Units		
Base Quantity	**Name**	**Symbol**
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Primary fund

Auxiliary.

The derived units are expressed in terms of these seven basic units by defining equation.
Table (3) lists together with fundamentals quantities, the supplementary and derived units in the SI system.

Table (3)

FUNDAMENTAL, SUPPLEMENTARY, AND DERIVED UNITS

Quantity	Symbol	Dimension	Unit	Unit Symbol
Fundamental				
Length	l	L	meter	m
Mass	m	M	kilogram	kg
Time	t	T	second	s
Electric current	I	I	ampere	A
Thermodynamic temperature	T	Θ	degree Kelvin	°K
Luminous intensity			candela	cd
Supplementary*				
Plane angle	α, β, γ	$[L]^0$	radian	rad
Solid angle	Ω	$[L^2]^0$	steradian	sr
Derived				
Area	A	L^2	square meter	m²
Volume	V	L^3	cubic meter	m³
Frequency†	f	T^{-1}	hertz	Hz (1/s)
Density	ρ	$L^{-3}M$	kilogram per cubic meter	kg/m³
Velocity	v	LT^{-1}	meter per second	m/s
Angular velocity	ω	$[L]^0T$	radian per second	rad/s
Acceleration	a	LT^{-2}	meter per second squared	m/s²
Angular acceleration	α	$[L]^0T^{-2}$	radian per second squared	rad/s²
Force	F	LMT^{-2}	newton	N (kg m/s²)
Pressure, stress	p	$L^{-1}MT^{-2}$	newton per square meter	N/m²
Work, energy	W	L^2MT^{-2}	joule	J (N m)
Power	P	L^2MT^{-3}	watt	W (J/s)
Quantity of electricity	Q	TI	coulomb	C (A s)
Potential difference, electromotive force	V	$L^2MT^{-3}I^{-1}$	volt	V (W/A)
Electric fieldstrength	E, ϵ	$LMT^{-3}I^{-1}$	volt per meter	V/m
Electric resistance	R	$L^2MT^{-3}I^{2}$	ohm	Ω (V/A)
Electric capacitance	C	$L^{-2}M^{-1}T^4I^2$	farad	F (A s/V)
Magnetic flux	Φ	$L^2MT^{-2}I^{-1}$	weber	Wb (v s)
Magnetic fieldstrength	H	$L^{-1}I$	ampere per meter	A/m
Magnetic flux density†	B	$MT^{-2}I^{-1}$	tesla	T (Wb/m²)
Inductance	L	$L^2MT^{-2}I^{2}$	henry	H (V s/A)
Magnetomotive force	U	I	ampere	A
Luminous flux			lumen	lm (cd sr)
Luminance			candela per square meter	cd/m²
Illumination			lux	lx (lm/m²)

The first column in table 3 shows the quantities (fundamental, supplementary and derived).The second column gives the equation symbol for each quantity .The third column lists the dimension of each derived unit in terms of the six fundamental

dimensions. The fourth column gives the name of each unit, the fifth, and the unit symbol.

The multiples and sub multiples of the above units are described by the use of prefixes listed in table (4).

Table (4)

SI Prefixes					
Factor	**Prefix**	**Symbol**	**Factor**	**Prefix**	**Symbol**
$10^{24} = (10^3)^8$	yotta	Y	10^{-1}	deci	d
$10^{21} = (10^3)^7$	zetta	Z	10^{-2}	centi	c
$10^{18} = (10^3)^6$	exa	E	$10^{-3} = (10^3)^{-1}$	milli	m
$10^{15} = (10^3)^5$	peta	P	$10^{-6} = (10^3)^{-2}$	micro	μ
$10^{12} = (10^3)^4$	tera	T	$10^{-9} = (10^3)^{-3}$	nano	n
$10^9 = (10^3)^3$	giga	G	$10^{-12} = (10^3)^{-4}$	pico	p
$10^6 = (10^3)^2$	mega	M	$10^{-15} = (10^3)^{-5}$	femto	f
$10^3 = (10^3)^1$	kilo	k	$10^{-18} = (10^3)^{-6}$	atto	a
10^2	hecto	h	$10^{-21} = (10^3)^{-7}$	zepto	z
10^1	deka	da	$10^{-24} = (10^3)^{-8}$	yocto	y

ELECTRICAL MEASUREMENTS

Errors in Measurement and Their Analysis

Definitions:-

True value (A_t): it is the value of the unknown quantity obtained on making measurements with primary standard instruments

Absolute error (e): it is the difference between the measured value (A_m) and the true value (A_t) of the unknown quantity.

$$\boxed{e = A_t - A_m}$$

Relative error(e_r): it is the ratio of the absolute error of measurement to the true value of the unknown quantity expressed a fraction.

$$\boxed{e_r = e / A_t \qquad \text{or} \qquad (A_t - A_m) / A_t}$$

Also could be expressed as a percentage:

$$\boxed{e_r\% = e/A_t \times 100\%}$$

Accuracy (A_{CC}): it is the degree of closeness with which the instrument reading an approaches the true value of the quantity to be measured.

$$\boxed{\%\,Acc = 100 - e_r\%}$$

Precision (p): it is the degree of agreement within a group of measurement or instrument. The precision can be expressed mathematically as

$$p = 1 - \left|\frac{x_n - \overline{x_n}}{\overline{x_n}}\right|$$

Where x_n=value of n^{th} measurement

$\overline{x}_n$=average of set of n measured values.

Type of error: error may come from different sources and are usually classified under three main heading:

1-**Gross Error:** the gross error mainly occurs due to carelessness or lack of experience of a human being. These cover human mistakes in readings, recordings and calculating results and these errors cannot be treated mathematically and also called personal errors.

2-systematic error : there are three types of systematic errors as :

A-Instrumental errors: the errors can be mainly due to three reasons:

1-Shortcoming of instruments.

2-Misuse of instrument.

3- Loading effects.

B-Environmental error: this error due to condition external to device such as Temperature, humidity, barometric pressure.
C-Observational errors: there are many sources of observational error such as parallax while reading a meter, wrong scale selection .to eliminates these error, one should use the instruments with mirrors, knife edge pointers.
3-Random errors :these error are due to unknown causes and occur even when all systematic errors have been accounted for .They are of a variable magnitude and sign that do not obey any known law.

STATISTICAL ANALYSIS

1- Arithmetic Mean $(\bar{x}) = \dfrac{x1 = x2 + x3 + x4 ...... xn.}{n} = \dfrac{\sum x}{n}$

Where: x_n=n^{th} reading taken
n=total number of readings

2-Deviation (d): it is the departure of a given reading from the arithmetic mean of group of readings .

$d_n = x_n - \bar{x}$

n=1, 2, 3, 4.....................number of readings

3-Average Deviation (D):

$$D = \frac{|d_1| + |d_2| + |d_3| + ...|d_n|}{n}$$

4-Standerd Deviation (σ):

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + ...d_n^2}{n-1}} \quad \text{or} \quad \sigma = \sqrt{\frac{\sum dt^2}{n-1}} \quad ..... \text{For } n < 20$$

5-percentage error = $\dfrac{D}{\bar{X}} \times 100$

6-probable error $= 0.6745 \times \sigma$

7-Variance (V) $= \sigma^2$

Probability of Errors

Normal Distribution of Errors

A practical point to note is that, whether the calculation is done on the whole "population" of data or on a sample drawn from it, the population itself should at least approximately fall into a so called "normal (or Gaussian)" distribution.

For example, 50 readings of voltage were taken at small time intervals and recorded to the nearest 0.1 V. The nominal value of the measured graphically in the form of a block diagram or histogram in which the number of observations is plotted against each observed voltage reading. The histogram and the table data are given in Figure 3.7. The figure shows that the largest number of readings (19) occurs at the central value of 100.0 V while the other readings are placed more or less symmetrically on either side of the central value. If more readings were taken at smaller increments, say 200 readings at 0.05-V intervals, the distribution of observations would remain approximately symmetrical about the central value and the shape of the histogram would be about the same as before. With more and more data taken at smaller and smaller increments, the contour of the histogram would finally become a smooth curve as indicated by the dashed line in the figure. This bell shaped curve is known as a Gaussian curve. The sharper and narrower the curve, the more definitely an observer may state that the most probable value of the true reading is the central value

Tabulation of Voltage Readings

Voltage reading (volts)	# of reading
99.7	1
99.8	4
99.9	12
100.0	19
100.1	10
100.2	3
100.3	1

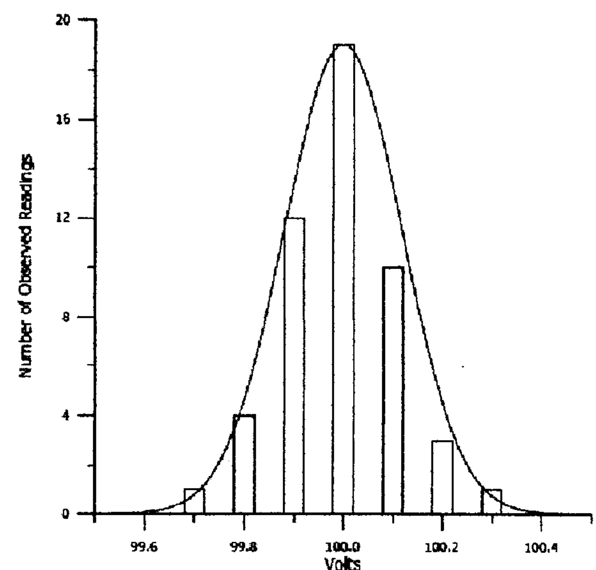


Figure 3.7 Distribution of 50 voltage readings

or mean reading.

For unbiased experiments all observations include small disturbing effects, called random errors. Random errors undergo a Normal (Gaussian) law of distribution shown in Figure 3.8. They can be positive or negative and there is equal probability of positive and negative random errors. The error distribution curve indicates that:

1- Small errors are more probable than large errors.
2- Large errors are very improbable.
3- There is an equal probability of plus and minus errors so that the probability of a given error, Will be symmetrical about the zero value.

$$Probability\,of\,error = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{x^2}{2\sigma}\right)$$

Area Under the Probability Curve

Deviation $\pm\sigma$	Fraction of total area
0.6745	0.5000
1.0	0.6828
2.0	0.9546
3.0	0.9972

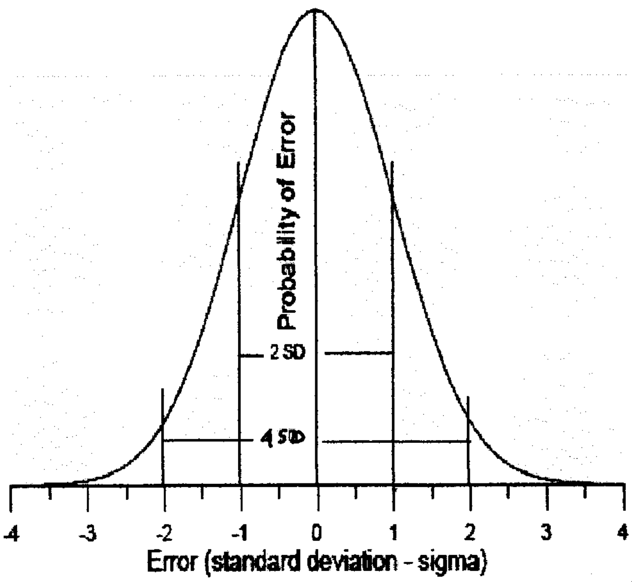


Figure 3.8 The error distribution curve for a normal (Gaussian) distribution

Table 3.2 Deviations in readings

Reading, x	Deviation d	Deviation d^2
101.	-0.1	0.01
101.7	0.4	0.16
101.3	0.0	0.00
101.0	-0.3	0.09
101.5	0.2	0.04
101.3	0.0	0.00
101.2	-0.1	0.01
101.4	0.1	0.01
101.3	0.0	0.00
101.1	-0.2	0.04
Σx=1013.0	$\Sigma\|d\|$=1.4	Σd^2=0.36

The error distribution curve in Figure 3.8 is based on the Normal (Gaussian) law and shows a symmetrical distribution of errors. This normal curve may be regarded as the limiting form of the histogram in which the most probable value of the true voltage is the mean value of 100.0V. Table 3.2 lists the readings, deviations and deviation squares of readings from the mean value. The reason why the standard deviation is such a useful measure of the scatter of the observations is illustrated in the figure. If the observations follow a "normal" distribution, a range covered by one standard deviation above the mean and one

standard deviation below it (i.e. $x \pm 1$ SD) includes about 68% of the observations. A range of 2 standard deviations above and below ($x \pm 2$ SD) covers about 95% of the observations. A range of 3 standard deviations above and below ($x \pm 3$ SD) covers about 99.72% of the observations.

Range of a Variable

If we know the mean and standard deviation of a set of observations, we can obtain some useful information by simple arithmetic. By putting 1, 2, or 3 standard deviations above and below the mean we can estimate the ranges that would be expected to include about 68%, 95% and 99.7% of observations. Ranges for ± SD and ± 2 SD are indicated by vertical lines. The table in the inset (next to the figure) indicates the fraction of the total area included within a given standard deviation range.

Acceptable range of possible values is called the confidence interval. Suppose we measure the resistance of a resistor as (2.65 ± 0.04) kΩ. The value indicated by the color code is 2.7 kΩ. Do the two values agree? Rule of thumb: if the measurements are within 2 SD, they agree with each other. Hence, ± 2 SD around the mean value is called the range of the variable.

Probable Error

The table also shows that half of the cases are included in the deviation limits of $\pm 0.6745\sigma$. The quantity *r* is called the *probable error* and is defined as

$$probable\ error\ r = \pm 0.6745\sigma$$

This value is *probable* in the sense that there is an even chance that any one observation will have a random error no greater than ±r. Probable error has been used in experimental work to some extent in the past, but standard deviation is more convenient in statistical work and is given preference.

Example 1. : *The set of independent measurement of voltages are recorded as 101.2, 101.4, 101.7, 101.3, 101.3, 101.2, 101.0, 101.3, 101.5 and 101.1 Calculate : i) Arithmetic mean ii) Deviation from mean iii) Standard deviation and iv) Probable error.*

Solution : The result is tabulated as shown where di is the deviation from mean.

No. (n)	x	$d_i = x - \bar{x}$	d_i^2
1	101.2	–0.1	0.01
2	101.4	0.1	0.01
3	101.7	0.4	0.16
4	101.3	0	0
5	101.3	0	0
6	101.2	–0.1	0.01
7	101.0	–0.3	0.09
8	101.3	0	0
9	101.5	0.2	0.04
10	101.1	–0.2	0.04
n = 10	$\sum x = 1013$	$\sum \lvert d_i \rvert = 1.4$	$\sum d_i^2 = 0.36$

i) Arithmetic mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{1013}{10} = 101.3$ **V**

ii) Deviation from mean = Average deviation

$$= \frac{\Sigma |d_i|}{n} = \frac{1.4}{10} = 0.14 \text{ V}$$

iii) Standard deviation, $\sigma = \sqrt{\frac{\Sigma d_i^2}{n-1}} = \sqrt{\frac{0.36}{9}} = 0.2 \text{ V}$

iv) Probable error of the readings = 0.6745 σ

$$= 0.6745 \times 0.2 = 0.1349 \text{ V}$$

Methods of measurements

The methods of measurement are classified as:

1-Direct method :in the direct method, the quantity to be measured is used to produce certain effect which directly gives the indication on the meter .For example ,measurement of current by an ammeter , the current to be measured is passed through the coil which deflects due to current effect . This directly produces deflection of the pointer which is attached to the coil. This gives the required measurement; some of the examples are the voltmeters, wattmeter's…etc.

2-Indirect method of Measurement:

in the indirect method of measurement, the quantity to be measured is not directly measured but other parameters related to the quantity are measured .for example, if power consumed by a resistance is to obtained then instead of measuring it directly, the voltage across the resistance (V) and current through the resistance (I) are measured and then the power is calculated by the product of (V) and (I) i.e. $p=V\,I$ .Thus by measuring related quantities, the actual quantity can be measured.

Limiting error:

the manufacturers specify the accuracy of the instruments within a certain percentage of full scale reading .the components like the resistor, inductor ,capacitor are guarantee to be within a certain percentage of rated value .this percentage indicates the deviations from the nominal or specified value of the particular quantity .these deviations from the specified value are called limiting error .these are also called guarantee errors .for example , the manufacturer of a certain instrument may specify that the instrument is accurate within ±1%of full scale deflection . This means that the full scale reading is guaranteed to be within ±1 of perfectly accurate reading .but for a reading less than full scale, the limiting error increases. Another example is say a resistor is specified by the manufacturer as 4.7 k ohm with tolerance of ±5%. Then the actual value of the resistance is guaranteed to be within the limits:

$R=4.7k \pm (5\% of\ 4.7\ k) = 4.7\ k \pm 0.235k\Omega$

$=4.935k$ and $4.465\ K\Omega$

Thus, the actual value with the limiting error can be expressed Mathematically as:

$$A_a = A_s \pm \delta A$$

Where:

A_a=Actual value

A_s= Specified or rated value

δA=limiting error

error as a percentage of full scale reading

$= \frac{A_s - A_a}{f.s.d} \times 100$

Relative Limiting Error

This is also called **fractional error.** It is the ratio of the error to the specified magnitude of a quantity.

Thus $$\boxed{e = \frac{\delta A}{A_s}}$$

where e = relative timing error

From the above equation, we can write,

$$\delta A = e \cdot A_s$$

and $$A_a = A_s \pm \delta A$$

$$= A_s \pm e\, A_s$$

$$\therefore \quad \boxed{A_a = A_s [\, 1 \pm e \,]}$$

The percentage relative limiting error is expressed as

$$\boxed{\% \, e = e \times 100}$$

The relative limiting error can be also be expressed as,

$$\boxed{e = \frac{\text{Actual value } (A_a) - \text{Specified value } (A_s)}{\text{Specified value } (A_s)}}$$

Example 2 : *A 0-50 V voltmeter is specified to be accurate within ± 1 % of full scale. Calculate the limiting error when the instrument reading is 15 V.*

Solution : The limiting error at full scale is,

$$\delta A = \pm 1 \text{ \% of } 50$$

$$= \pm \frac{1}{100} \times 50$$

$$= \pm 0.5 \text{ V}$$

For a reading of 15 V, it is

$$\% \, e = \frac{0.5}{15} \times 100 = \mathbf{3.33 \ \%}$$

Thus as reading is less than full scale, the limiting error is more.

Combination of Quantities with Limiting Errors

When the two quantities are combined, each having limiting error, then it is necessary to calculate the overall limiting error. Let us consider the various combinations of two quantities and methods to obtain the corresponding limiting error.

1. **Sum of the two quantities :** Let a_1 and a_2 be the two quantities which are to be added to obtain the result as A_T.

Consider the relative increment of the function which the ratio of change in function to the value of the function.

$$\therefore \quad \frac{dA_T}{A_T} = \frac{d(a_1 + a_2)}{A_T} \qquad \text{as } A_T = a_1 + a_2$$

$$\therefore \quad \frac{dA_T}{A_T} = \frac{da_1}{A_T} + \frac{da_2}{A_T}$$

$$\therefore \quad \frac{dA_T}{A_T} = \frac{a_1}{A_T} \cdot \frac{da_1}{a_1} + \frac{a_2}{A_T} \cdot \frac{da_2}{a_2} \qquad \text{... (1)}$$

Let δa_1 be the limiting error of a_1

and δa_2 be the limiting error of a_2

Hence, the corresponding relative limiting errors are

$$e_1 = \frac{\delta a_1}{a_1}$$

and $$e_2 = \frac{\delta a_2}{a_2} \qquad \text{as } e = \frac{\delta A}{A_s}$$

Thus, the relative error in the result A_T can be expressed as :-

$$e_T = \frac{\delta A_T}{A_T}$$

$$e_T = \frac{a_1}{A_T} \cdot \frac{\delta a_1}{a_1} + \frac{a_2}{A_T} \cdot \frac{\delta a_2}{a_2} \qquad \text{... from (1)}$$

$$\boxed{e_T = \pm\left[\frac{a_1}{A_T} \cdot e_1 + \frac{a_2}{A_T} \cdot e_2\right]}$$

Key Point : *Thus, the resultant (total) limiting error is sum of the products obtained by multiplying the individual limiting error by the ratio of each term to the resultant function.*

2. **Difference of the two quantities**

let $\quad A_T = a_1 - a_2$

$$\frac{dA_T}{A_T} = \frac{d(a_1 - a_2)}{A_T} = \frac{da_1}{A_T} - \frac{da_2}{A_T}$$

$$\frac{dA_T}{A_T} = \frac{a_1}{A_T} \cdot \frac{da_1}{a_1} - \frac{a_2}{A_T} \cdot \frac{da_2}{a_2}$$

Now, if $\pm \delta a_1$ and $\pm \delta a_2$ are the errors in a_1 and a_2 respectively, then maximum possible error will result when the signs of δa_1 and δa_2 are opposite to each other. For same signs of δa_1 and δa_2 the error will be very small. Hence, considering worst possible discrepancy i.e. when δa_1 is positive, δa_2 is negative and vice-versa.

$$\therefore \quad \frac{\delta A_T}{A_T} = \frac{a_1}{A_T} \cdot \frac{\delta a_1}{a_1} - \frac{a_2}{A_T} \cdot \frac{\delta a_2}{a_2}$$

But, as δa_1 and δa_2 are of opposite sign, the resultant sign can be taken common to express the result as,

$$\boxed{e_T = \pm \left[\frac{a_1}{A_T} \cdot e_1 + \frac{a_2}{A_T} \cdot e_2 \right]}$$

If a_1 and a_2 are almost same, then as A_T is $a_1 - a_2$, $A_T << a_1$ and $A_T << a_2$ also, then the relative error e_T in A_T would be very large.

If there is difference of more than two quantities, then

$$A_T = \pm a_1 \pm a_2 \pm a_3 \pm \ldots$$

$$\therefore \quad \boxed{e_T = \pm \left[\frac{a_1}{A_T} e_1 + \frac{a_2}{A_T} e_2 + \frac{a_3}{A_T} e_3 + \ldots \right]}$$

where $\quad e_1 = \frac{\delta a_1}{a_1}, \quad e_2 = \frac{\delta a_2}{a_2}, \quad e_3 = \frac{\delta a_3}{a_3} \ldots$

3. Product of the two quantities

Now, let $\quad A_T = a_1 a_2$

$$\therefore \quad \log A_T = \log (a_1 a_2) = \log a_1 + \log a_2$$

Differentiating with respect to A_T,

$$\therefore \quad \frac{1}{A_T} = \frac{1}{a_1} \cdot \frac{d a_1}{d A_T} + \frac{1}{a_2} \cdot \frac{d a_2}{d A_T}$$

Multiplying both sides by dA_T,

$$\therefore \quad \frac{d A_T}{A_T} = \frac{d a_1}{a_1} + \frac{d a_2}{a_2}$$

Thus, if δa_1 and δa_2 are the limiting errors of a_1 and a_2, then,

$$\therefore \quad \frac{\delta A_T}{A_T} = \frac{\delta a_1}{a_1} + \frac{\delta a_2}{a_2}$$

$$\therefore \quad \boxed{e_T = \pm (e_1 + e_2)}$$

Key Point : *Thus, the relative limiting error of the product is equal to the sum of the relative limiting errors.*

4. Division of the two quantities

Let $A_T = \frac{a_1}{a_2}$

$\therefore \quad \log A_T = \log\left[\frac{a_1}{a_2}\right] = \log a_1 - \log a_2$

Differentiating with respect to A_T.

$$\therefore \quad \frac{1}{A_T} = \frac{1}{a_1}\cdot\frac{d a_1}{d A_T} - \frac{1}{a_2}\cdot\frac{d a_2}{d A_T}$$

$$\therefore \quad \frac{d A_T}{A_T} = \frac{d a_1}{a_1} - \frac{d a_2}{a_2}$$

$$\therefore \quad \frac{\delta A_T}{A_T} = \frac{\delta a_1}{a_1} - \frac{\delta a_2}{a_2}$$

But, again consider the worst case i.e. δa_1 and δa_2 are of opposite signs. Hence, taking the common sign outside, the result can be expressed as

$$\boxed{e_T = \pm (e_1 + e_2)}$$

The result is same as that obtained for the product.

If there is product or division of more than two quantities, then,

$$A_T = a_1\, a_2\, a_3 \ldots$$

or $$A_T = \frac{a_1}{a_2\, a_3 \ldots}$$

or $$A_T = \frac{1}{a_1\, a_2\, a_3 \ldots}$$

then $$\boxed{e_T = \pm [\, e_1 + e_2 + e_3 + \ldots\,]}$$

5. Power of a factor

Let $A_T = (a_1)^n$

$$\log A_T = \log [a_1]^n = n \log a_1$$

$$\frac{1}{A_T} = n \cdot \frac{1}{a_1} \cdot \frac{d a_1}{d A_T}$$

$$\frac{d A_T}{A_T} = \frac{n}{a_1} \cdot d\, a_1$$

$$\frac{\delta A_T}{A_T} = n \cdot \frac{\delta a_1}{a_1}$$

$$\boxed{e_T = \pm n\, e_1}$$

where e_T is relative limiting error in result A_T while e_1 is relative limiting error in a_1.

If the result is the product of different powers of two quantities i.e.

$$A_T = (a_1)^n \cdot (a_2)^m$$

Then as it is a product of two quantities, each having some power, the resultant limiting error can be expressed as,

$$\boxed{e_T = \pm [\, n\, e_1 + m\, e_2 \,]}$$

Example 3 : *The r.m.s. current passing through a resistor of 120 ± 0.5 ohms is 2 ± 0.02 A. Calculate the limiting error in the value of power dissipation.*

Solution : $P = I^2 R$

δa_1 = limiting error in current = 0.02

δa_2 = limiting error in resistor = 0.5

$$\therefore \quad e_1 = \frac{\delta a_1}{A_1} = \frac{0.02}{2} = 0.01$$

and $$e_2 = \frac{\delta a_2}{A_2} = \frac{0.5}{120} = 4.167 \times 10^{-3}$$

The current term in power appears as I^2 so it is second power of I.

Hence, the contribution by I^2 to the resultant error is ne_1

where n = power = 2

e_1 = limiting error

while the limiting error due to resistance is e_2.

As power is the product of I^2 and R, the resultant error is the sum of the contributions by I^2 and R.

$$\therefore \quad e_T = n\, e_1 + e_2 \qquad \text{where } n = 2$$

Hence, the limiting error in the power calculation is

$$e_T = \pm (2 \times e_1 + e_2)$$

$$= \pm [\, 2 \times 0.01 + 4.167 \times 10^{-3} \,]$$

$$= \pm\, \mathbf{0.02417 \text{ i.e. } \pm 2.417\ \%}$$

Analog Meters.

Electromechanical Indicating Instruments

The various electrical instrument may in a very broad sense be divided into (i) absolute instruments and (ii) secondary instruments .Absolute instruments are those which give the value of the quantity to be measured in term of the constant of the instrument and their deflection only .no previous calibration or comparison in necessary in their case. For example tangent galvanometer. Secondary instruments are those in which the value of electrical quantity to be measured can be determined from the deflection of the instruments, only when they have been pre-calibrated by comparison with an absolute instrument .without calibration, the deflection of such instruments is meaningless. All electrical measuring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are:

- Magnet effect — Ammeters, Voltmeters, Wattmeters
- Electrodynamics effect. — Dynamometer.
- induction ~~Electromagnetic~~ effect. — Ammeters, Voltmeters, Wattmeter, Energy meters,
- Thermal effect. — Ammeters, Voltmeters.
- Chemical effect. — DC Ampere hour meters
- Electrostatic effect. — Voltmeters.
- Hall Effect — Flux meter, Clamp meter.

Another way to classify secondary instrument is to divide them in into:

1-**Indicating instruments:** are those which indicate the instantaneous value of electrical quantity being measured at the time at which it is being measured their indications are given by pointers moving over calibrated dials.

a- Electromechanical Indicating Inst.
b- Electronic Instruments.

2-**Recording instruments:** are those ,which ,instead of indicating by means of a pointer and a scale, the instantaneous value of an electrical quantity, give a continuous record or the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked pen which rests lightly on a chart or graph.

3-**Integrating Instruments:** are those, which measure and register by a set of dials and pointers either the total quantity of electricity (in ampere-hours) or the

total amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time).

Essentials of indicating instruments

For satisfactory operation of ~~ay~~ any analog instrument the following systems must be present in the instrument:

(moving system)

1. **Deflecting System**: this system provides the deflecting or operating torque proportional to the quantity to be measured and moves the pointer from its zero position.

   convert electrical current or potential into mechanical force (deflecting force)

2. * **Controlling System:** The controlling force is equal and opposite to the deflecting torque in order to make the deflection of the pointer proportional to the magnitude of the quantity to be measured .the controlling force also brings the pointer back to zero position when the force which causes the movement of the pointer is removed.

   1- Gravity control 2- Spring control

3. **Damping System:** Before coming to the rest, the pointer oscillates about the equilibrium position. The damping system provides the damping torque so that the pointer quickly comes to the final steady state position without any swing or oscillations. If the system oscillates with decreasing amplitude before coming to it final position, it is called under damped system. If the system takes some time and slowly comes to reset without oscillation, it is called over damped system .if the system reaches to its final position rapidly but without oscillations, it is called critically damped system.

The controlling torque is provided by either spring or gravity while the damping torque is provided by air friction, fluid friction or eddy current in the analog instruments. The main types of analog instruments used as ammeter, voltmeter and power meter.

* In the absence of a controlling system, the pointer will shoot (swing) beyond the final steady position for any magnitude of current.

Moving Coil Instruments

There are two types of moving coil instruments namely, ***permanent magnet moving coil*** type which can only be used for ***direct*** current, voltage measurements and the ***dynamometer*** type which can be used on either ***direct or alternating*** current, voltage measurements.

Permanent Magnet Moving Coil Mechanism (PMMC)

In PMMC meter or (D'Arsonval) meter or galvanometer all are the same instrument, a coil of fine wire is suspended in a magnetic field produced by permanent magnet. According to the fundamental law of electromagnetic force, the coil will rotate in the magnetic field when it carries an electric current by electromagnetic (EM) torque effect. A pointer which attached the movable coil will deflect according to the amount of current to be measured which applied to the coil. The (EM) torque is counterbalance by the mechanical torque of control springs attached to the movable coil also. When the torques are balanced the moving coil will stopped and its angular deflection represent the amount of electrical current to be measured against a fixed reference, called a scale. If the permanent magnet field is uniform and the spring linear, then the pointer deflection is also linear.

Mathematical Representation of PMMC Mechanism

Assume there are (N) turns of wire and the coil is (L) in long by (W) in wide. The force (F) acting perpendicular to both the direction of the current flow and the direction of magnetic field is given by:

$$F = N \cdot B \cdot I \cdot L$$

where N: turns of wire on the coil I: current in the movable coil
B: flux density in the air gap L: vertical length of the coil

Electromagnetic torque is equal to the multiplication of force with distance to the point of suspension

$T_{I1} = NBIL\frac{W}{2}$ in one side of cylinder $T_{I2} = NBIL\frac{W}{2}$ in the other side of cylinder

The total torque for the two cylinder sides

$$T_I = 2\left(NBIL\frac{W}{2}\right) = NBILW = NBIA$$

where A: effective coil area

This torque will cause the coil to rotate until an equilibrium position is reached at an angle θ with its original orientation. At this position

Electromagnetic torque = control spring torque

$$T_I = T_s$$

Since $T_s = K\theta$

K = Spring constant Nm/rad or Nm/deg.

So $\theta = \frac{NBA}{K} I$ where $C = \frac{NBA}{K}$ Thus $\theta = CI$

The angular deflection proportional linearly with applied current

... 5 or 1mA

... to 200µA

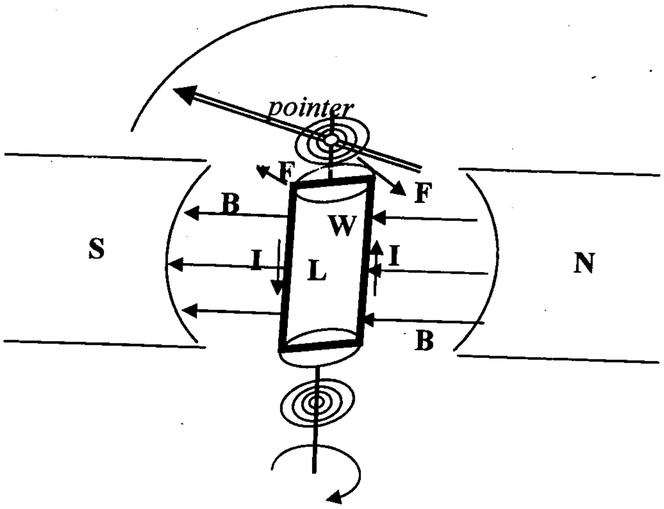
Flemings left hand rule

Accuracy 2 to 5% of full scale reading

Lenz's LAW

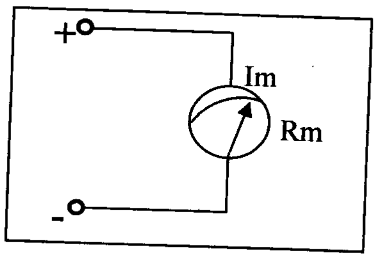
Faraday's law

4.



1- D.c Ammeter:

An Ammeter is always connected in series with a circuit branch and measures the current flowing in it. Most d.c ammeters employ a d'Arsonval movement, an ideal ammeter would be capable of performing the measurement without changing or distributing the current in the branch but real ammeters would possess some internal resistance.



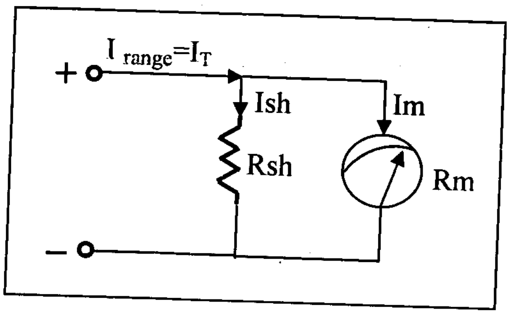
Power consumed $= (I_{range})^2 \cdot R_{amm}$

$R_{sh} // R_m$

Extension of Ammeter Range:

Since the coil winding in PMMC meter is *small and light,* they can carry only small currents (μA-1mA). Measurement of large current requires ***a shunt external resistor*** to connect with the meter movement, so only a fraction of the total current will passes through the meter.

$$Vm = Vsh$$
$$Im\,Rm = IshRsh$$
$$Ish = I_T - Im$$
$$\boxed{Rsh = \frac{Im\,Rm}{I_T - Im}} \quad \div \; I_m$$



$$R_{sh} = \frac{R_m}{\frac{I_T}{I_m} - 1}$$

$$\frac{I_T}{I_m} - 1 = \frac{R_m}{R_{sh}} \quad \text{or} \quad \frac{I_T}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$$\frac{I_T}{I_m} = m = \text{Multiplying factor}$$

$$\therefore m = 1 + \frac{R_m}{R_{sh}}$$

$$R_{sh} = \frac{R_m}{m - 1}$$

5.

Example:

If PMMC meter have internal resistance of 10Ω and full scale range of 1mA. Assume we wish to increase the meter range to 1A.

Sol.

So we must connect shunt resistance with the PMMC meter of

$$\boxed{Rsh = \frac{\text{Im}\,Rm}{I_T - \text{Im}}} \qquad Rsh = \frac{1\times10^{-3}\cdot 10}{1-1\times10^{-3}} = 0.01001\Omega$$

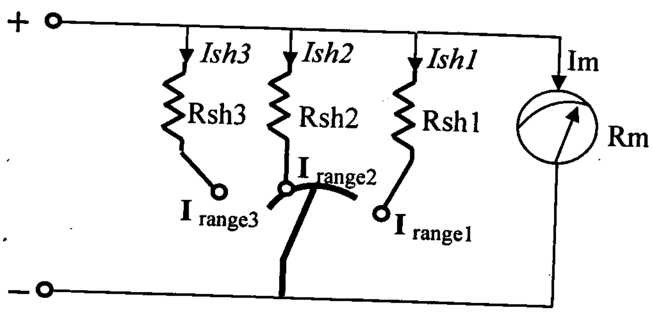
a) Direct D.c Ammeter Method ~~(Ayrton Shunt)~~:

The current range of d.c ammeter can be further extended by a number of shunts selected by a range switch; such ammeter is called a multirange ammeter.

$$\boxed{Rsh_* = \frac{\text{Im}\,Rm}{Ir_* - \text{Im}}}$$

Ir : Range Current

Rsh : Range Shunt.

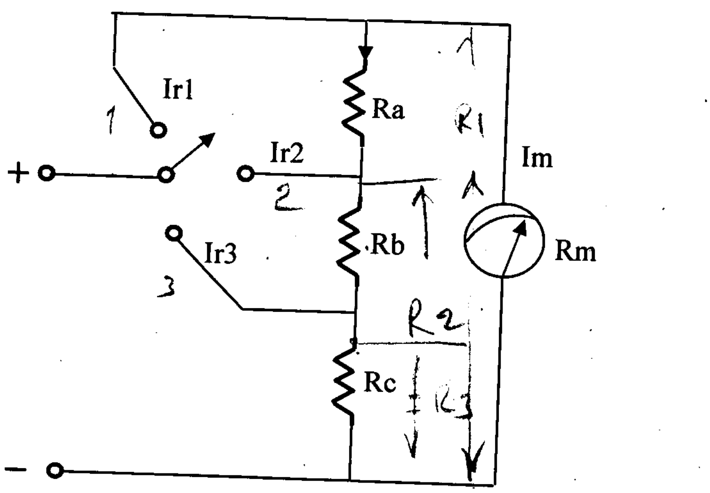


b) Indirect D.C Ammeter Method: (Ayrton Shunt) (Universal Shunt)

$$\boxed{\frac{Ir_*}{\text{Im}} = \frac{Rm + R}{r_*}}$$

Where R=Ra+ Rb+ Rc

And r = parallel resistors branch with the meter



Ra+Rb+Rc = R1

$I_m R_m = (I_1 - I_m) R_1$

$m_1 = \frac{I_1}{I_m} = 1 + \frac{R_m}{R_1}$

$R_1 = \frac{R_m}{m_1 - 1}$

at position 2 $R_2 = \frac{R_1 + R_m}{m_2}$

at posit 3 $R_3 = \frac{R_1 + R_m}{m_3}$

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Example (1):

Design a multirange ammeter by using ***direct method*** to give the following ranges 10mA, 100mA, 1A, 10A, and 100A. If d'Arsonval meter have internal resistance of 10Ω and full scale current of 1mA.

Sol:

Rm=10Ω Im=1mA

$$\boxed{Rsh_* = \frac{\text{Im}\,Rm}{Ir_* - \text{Im}}}$$

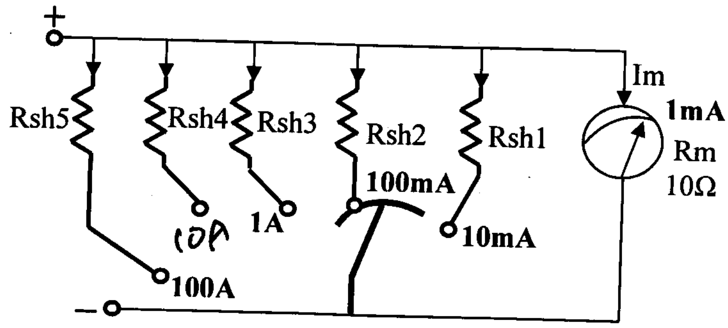
$$Rsh1 = \frac{1\times10^{-3}\cdot10}{(10-1)\times10^{-3}} = 1.11\Omega$$

$$Rsh2 = \frac{1\times10^{-3}\cdot10}{(100-10)\times10^{-3}} = 0.101\Omega$$

$$Rsh3 = \frac{1\times10^{-3}\cdot10}{1-10\times10^{-3}} = 0.0101\Omega$$

$$Rsh4 = \frac{1\times10^{-3}\cdot10}{10-1\times10^{-3}} = 0.0011\Omega$$

$$Rsh5 = \frac{1\times10^{-3}\cdot10}{100-1\times10^{-3}} = 0.00011\Omega$$



Example (2):

Design an Ayrton shunt by ***indirect method*** to provide an ammeter with current ranges 1A, 5A, and 10A, if PMMC meter have internal resistance of 50Ω and full scale current of 1mA.

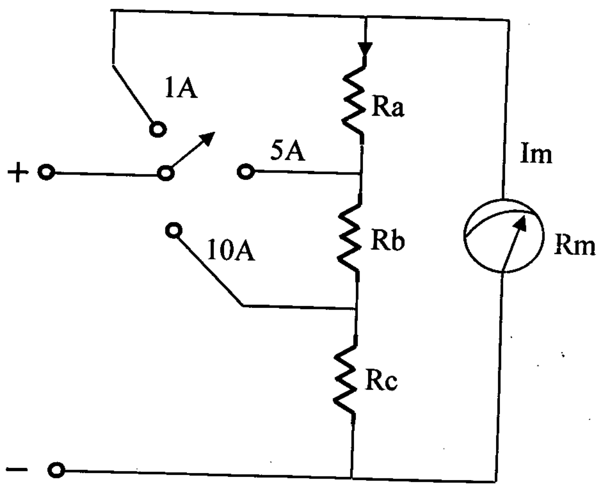
Sol.:

Rm=50Ω I_{FSD}=Im=1mA

$$\boxed{\frac{Ir_*}{\text{Im}} = \frac{Rm+R}{r_*}}$$

Where R=Ra+ Rb+ Rc

And r = parallel resistors branch with the meter



1- ***For 1A Range:***

$$\frac{I1}{\text{Im}} = \frac{Rm+R}{R}$$

$$\frac{1A}{1mA}=\frac{50+R}{R}$$ R=0.05005Ω

2- For 5A Range:

$$\frac{I2}{\text{Im}}=\frac{Rm+R}{Rb+Rc}$$ r =Rb+Rc

$$\frac{5A}{1mA}=\frac{50+0.05005}{Rb+Rc}$$ Rb+Rc= 0.01001Ω

Ra=R-(Rb+Rc) **Ra**=0.05-0.01001=0.04004 Ω

3- For 10A Range:

$$\frac{I3}{\text{Im}}=\frac{Rm+R}{Rc}$$ r =Rc

$$\frac{10A}{1mA}=\frac{50+0.05005}{Rc}$$ **Rc**=5.005×10^{-3} Ω

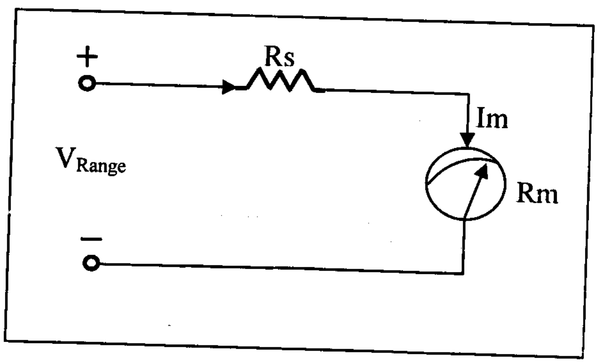
Rb=0.01001-5.005×10^{-3}= 5.005×10^{-3} Ω

2- D.C Voltmeter:

A voltmeter is always connect in parallel with the element being measured, and measures the voltage between the points across which its' connected. Most d.c voltmeter employ PMMC meter with series resistor as shown. The series resistance should be much larger than the impedance of the circuit being measured, and they are usually much larger than Rm.

power consumed $= (V_{range})^2 / R_{total}$

$$\boxed{\begin{aligned} Rs &= R_T - Rm \\ Rs &= \frac{V_{range}}{\text{Im}} - Rm \end{aligned}}$$



Im=I_{FSD}

The ohm/volt sensitivity of a voltmeter Is given by:

$$\boxed{S_v=\frac{Rm}{V_{FSD}}=\frac{1}{I_{FSD}}=\frac{\Omega}{V}\,rating}$$

$$S_{Range}=\frac{Rm+Rs}{V_{Range}}=\frac{1}{I_{Range}}=\frac{\Omega}{V}$$

So the internal resistance of voltmeter or the input resistance of voltmeter is

$$\boxed{Rv = V_{FSD}\ \text{x sensitivity}}$$

Example:

We have a micro ammeter and we wish to adapted it so as to measure 1volt full scale, the meter has internal resistance of 100Ω and I_{FSD} of 100μA.

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Sol.:

$$Rs = \frac{V}{Im} - Rm \qquad Rs = \frac{1}{0.0001} - 100 = 9900\Omega = 9.9K\Omega$$

So we connect with PMMC meter a series resistance of 9.9KΩ to convert it to voltmeter

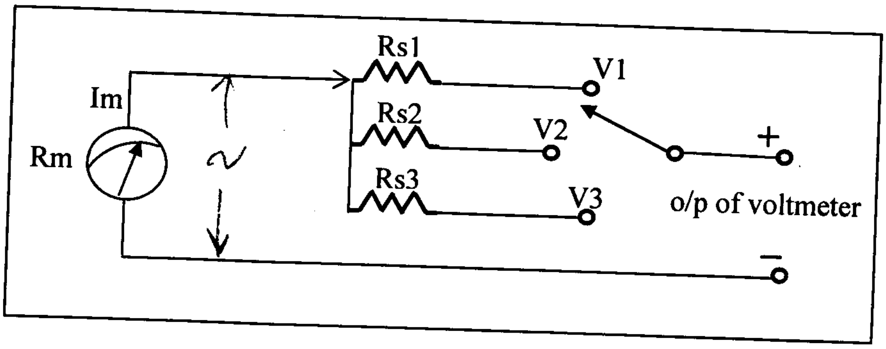
Extension of Voltmeter Range:

Voltage range of d.c voltmeter can be further extended by a number of series resistance selected by a range switch; such a voltmeter is called multirange voltmeter.

a) Direct D.c Voltmeter Method:

In this method each series resistance of multirange voltmeter is connected in direct with PMMC meter to give the desired range.

$$\boxed{Rs_* = \frac{V_*}{Im} - Rm}$$



$Rs_1 = (m_1 - 1) Rm$

$Rs_2 = (m_2 - 1) Rm$

$Rs_3 = (m_3 - 1) Rm$

$m_1 = \frac{V_1}{v}$, $m_2 = \frac{V_2}{v}$

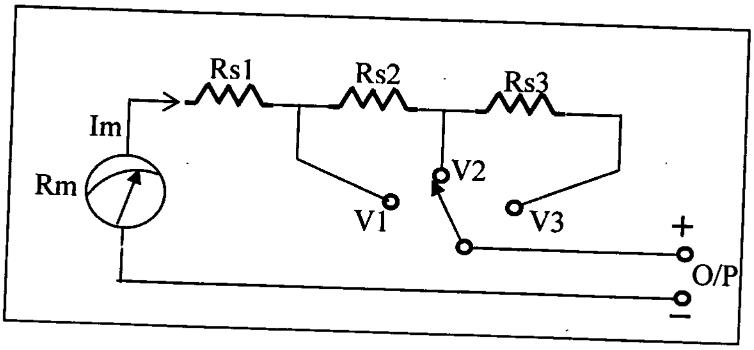
b) Indirect D.c Voltmeter Method:

In this method one or more series resistances of multirange voltmeter is connected with PMMC meter to give the desired range.

$$Rs1 = \frac{V1}{Im} - Rm$$

$$Rs2 = \frac{V2 - V1}{Im}$$

$$Rs3 = \frac{V3 - V2}{Im}$$



Example (1):

A basic d'Arsonval movement with internal resistance of 100Ω and half scale current deflection of 0.5 mA is to be converted by indirect method into a multirange d.c voltmeter with voltages ranges of 10V, 50V, 250V, and 500V.

Sol:

$I_{FSD} = I_{HSD} \times 2$

$I_{FSD} = 0.5mA \times 2 = 1mA$

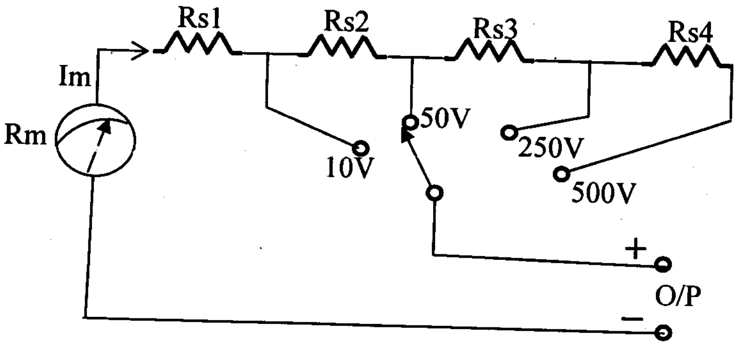
$$Rs1 = \frac{V1}{Im} - Rm \qquad Rs1 = \frac{10}{1mA} - 100 = 9.9K\Omega$$

$$Rs2 = \frac{V2 - V1}{\text{Im}}$$

$$Rs2 = \frac{50 - 10}{1 \times 10^{-3}} = 40K\Omega$$

$$Rs3 = \frac{250 - 50}{1 \times 10^{-3}} = 200K\Omega$$

$$Rs4 = \frac{500 - 250}{1 \times 10^{-3}} = 250K\Omega$$



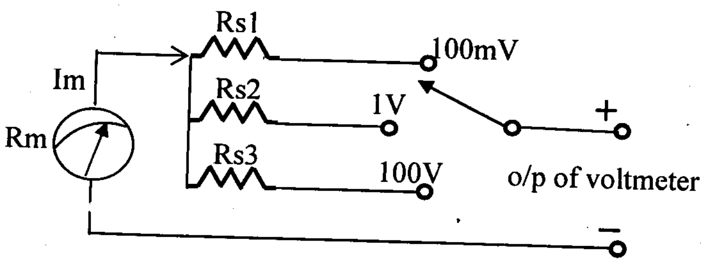
Example (2):

Design d.c voltmeter by using direct method with d'Arsonval meter of 100Ω and full scale deflection of 100μA to give the following ranges: 10mV, 1V, and 100V.

Sol:

$$\boxed{Rs_* = \frac{V_*}{\text{Im}} - Rm}$$

$$Rs1 = \frac{V1}{\text{Im}} - Rm$$



$$Rs1 = \frac{10mV}{100\mu A} - 100 = 0\Omega$$

$$Rs2 = \frac{1}{100 \times 10^{-6}} - 100 = 9.9K\Omega$$

$$Rs3 = \frac{100}{100 \times 10^{-6}} - 100 = 99.9K\Omega$$

3- Ohmmeter and Resistance measurement:

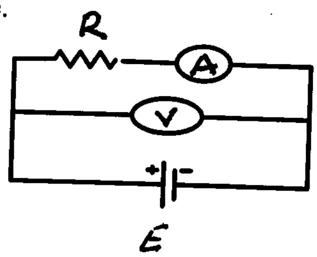
When a current of 1A flows through a circuit which has an impressed voltage of 1volt, the circuit has a resistance of 1Ω.

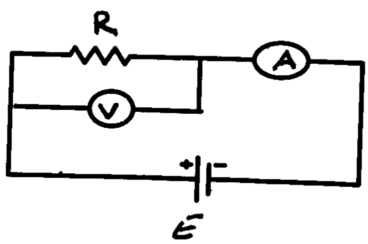
$$R = \frac{V}{I}$$

There are several methods used to measure unknown resistance:

a) Indirect method by ammeter and voltmeter.

This method is inaccurate unless the ammeter has a small resistance and voltmeter have a high resistance.





10✓

D'Arsonval Meter as a DC Ohmmeter

The d'Arsonval meter can also be transformed as an ohmmeter for any circuit's resistance measurement. The basic ohmmeter circuit employing d'Arsonval meter is shown in Figure

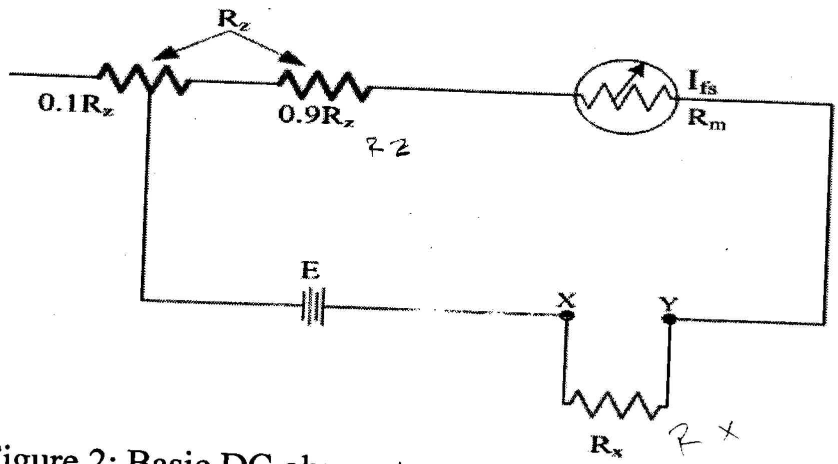


Figure 2: Basic DC ohmmeter

Before Rx is measured, the ohmmeter must first be calibrated. The "zero" calibration is performed by shorting the terminal x-y and adjusting Rz so that full-scale deflection on the meter movement is obtained. Without Rx, the current equation for full-scale deflection becomes:

$$I_{fs} = \frac{E}{R_z + R_m}$$

With Rx, the current equation will now becomes:

$$I = \frac{E}{R_x + R_z + R_m}$$

Notice that, by introducing Rx, the current I will always less than the full-scale current I < Ifs

The relationship between the full-scale deflection with the value of Rx can be given as

$$\frac{I}{I_{fs}} = \frac{R_z + R_m}{R_z + R_m + R_x}$$

$I = \frac{R_z + R_m}{R_z + R_m + R_x} \cdot I_{fs}$ (non linear)

Practically, this equation is being used to develop the marking scale on the meter face to indicate the value of the measured resistor.

A.c Measuring Instrument

Review on Alternating Signal:

The instantaneous values of electrical signals can be graphed as they vary with time. Such graphs are known as the ***waveforms*** of the signal. If the value of waveform remains constant with time, the signal is referred to as ***direct (d.c)*** signal; such as the voltage of a battery. If a signal is time varying and has positive and negative instantaneous values, the waveform is known as ***alternating (a.c)*** waveform. If the variation of a.c signal is continuously repeated then the signal is known as ***periodic*** waveform.

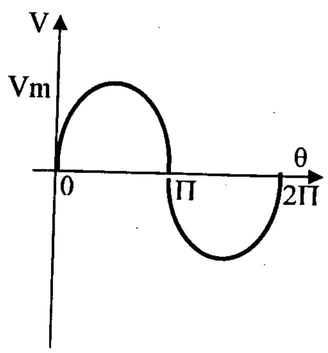
The ***frequency of a.c signal*** is defined as the *number of cycles traversed in one second.* Thus the time duration of *one cycle per second* for a.c signal is known as the ***period (T)***. Where the complete variation of a.c signal before repeated itself is represent one ***cycle***.

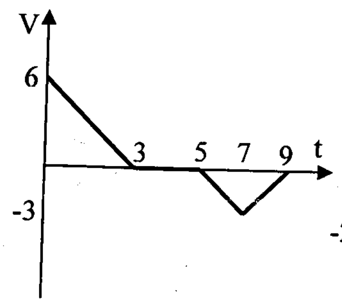
Average Values:

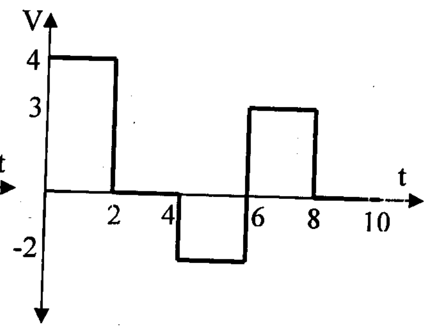
It is found by dividing the area under the curve of the waveform in one period (T) by the time of the period.

Average value= $\dfrac{\text{Algebraic sum of the areas under the curve}}{\text{Length of the curve}}$

$$Av = \frac{\sum areas}{T} \quad \text{.................(1)} \qquad \text{or} \qquad Av = \frac{1}{T}\int_0^T f(t)dt \quad \text{.................(2)}$$







$$Av = \frac{1}{2\pi}\int_0^{2\pi} VmSin\theta d\theta \qquad Av = \frac{\frac{1}{2}\times 3\times 6+\frac{1}{2}\times 4\times(-3)}{9} \qquad Av = \frac{4\times 2+(-2)\times 2+3\times 2}{10}$$

$$Av = -\frac{Vm}{2\pi}\left(Cos\theta \uparrow_0^{2\pi}\right)$$

$$Av = -\frac{Vm}{2\pi}(1-1) = 0$$

The average value for the figure below by using equation (2) is:

$Av = \dfrac{1}{T}\displaystyle\int_0^T f(t)dt$ we use the tangent equation for (xo,yo)=(0,0), and (x_1,y_1)=(3,6) to find the function of f(t)

$\int \cos x$

$\int \sin x = -\cos x$

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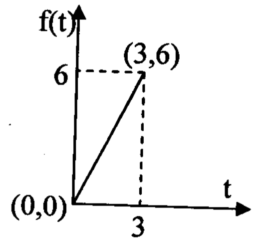
$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \quad \rightarrow \quad \frac{y-0}{x-0}=\frac{6-0}{3-0}\Rightarrow\frac{y}{x}=\frac{6}{3}=2\Rightarrow\Rightarrow y=2x \qquad \boxed{f(t)=2t}$$

$$Av=\frac{1}{3}\int_0^3 (2t)dt$$



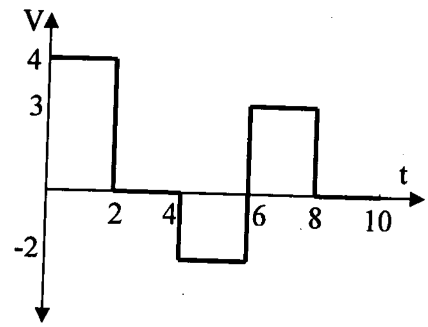
$$Av=\frac{2}{3}\left(\frac{t^2}{2}\uparrow_0^3\right) \qquad Av=\frac{1}{3}\left((3)^2-(0)^2\right)=\frac{9}{3}=3$$

Root Mean Square Value(effective value of a.c signal):

The r.m.s value of a waveform refers to its power capability. It is refer to the effective value of a.c signal because the r.m.s value equal to the value of a d.c signal which would deliver the same power if it replaced with a.c signal.

$$r.m.s=\sqrt{\frac{\sum area(V)^2}{T}} \quad \textit{(for square waveform only)}$$

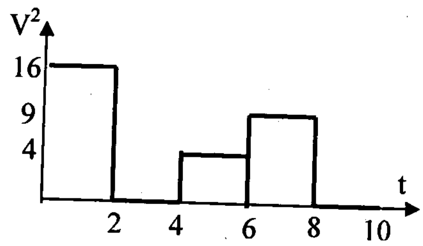
1- $r.m.s=\sqrt{\dfrac{16\times2+4\times2+9\times2}{10}}$



In general form the r.m.s value has the following aqua.

r.m.s= √Average $f(t)^2$

$$\boxed{r.m.s=\sqrt{\frac{1}{T}\int_0^T f(t)^2dt}}$$

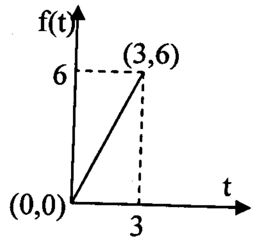


2- If f(t) = 2t then its r.m.s value is:

$$r.m.s=\sqrt{\frac{1}{3}\int_0^3 (2t)^2\,dt}$$

√12

$$r.m.s=\sqrt{\frac{4}{3}\left(\frac{t^3}{3}\uparrow_0^3\right)}=\sqrt{\frac{4}{9}\left((3)^3-(0)^3\right)}=\sqrt{\frac{4\times27}{9}}=3.46$$



3- If f(t) = Vm Sinθdθ

$$r.m.s=\sqrt{\frac{1}{2\pi}\int_0^{2\pi} Vm^2 Sin^2\theta d\theta} \qquad r.m.s=\sqrt{\frac{Vm^2}{2\pi}\int_0^{2\pi}\frac{1-Cos2\theta}{2}d\theta}$$

$$r.m.s=\left\{\frac{Vm^2}{4\pi}\left[\int_0^{2\pi} d\theta-\int_0^{2\pi}Cos2\theta d\theta\right]\right\}^{\frac{1}{2}} \qquad r.m.s=\sqrt{\frac{Vm^2}{4\pi}\left[\theta\Big|_0^{2\pi}-\frac{1}{2}Sin2\theta\Big|_0^{2\pi}\right]}$$

rms (H.W) = $\sqrt{\frac{1}{\pi}[\theta]_0^{\pi}-0}$

$= \sqrt{\frac{Vm^2}{4}} = \frac{Vm}{2}$

$E_{rms}(F.W) = \frac{\sqrt{2}}{2} E_m$ | $E_{average} = \frac{2}{\pi} E_m$

H.W $\frac{\sqrt{2}}{2} E_m$ | $E_{H.W} = \frac{E_m}{\pi}$

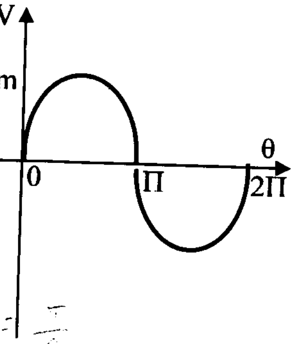
13.

$$r.m.s = \sqrt{\frac{Vm^2}{4\pi}[2\pi - 0]} = \sqrt{\frac{Vm^2}{2}} = \frac{Vm}{\sqrt{2}}$$

$$\boxed{FormFactor = \frac{r.m.s}{average}}$$ for Sine wave F.F=1.11 (F.W.R)

F.F=1.57 (H.W.R)

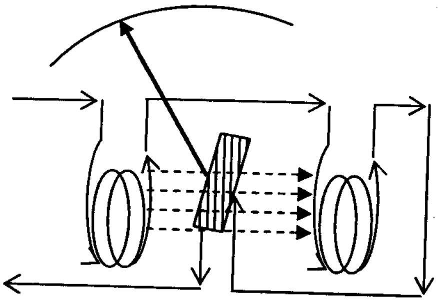
$$\boxed{CrestFactor = \frac{PeakValue}{r.m.s}}$$

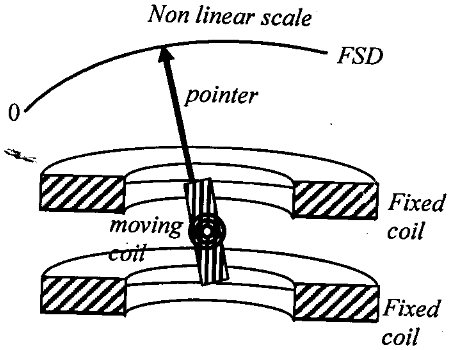


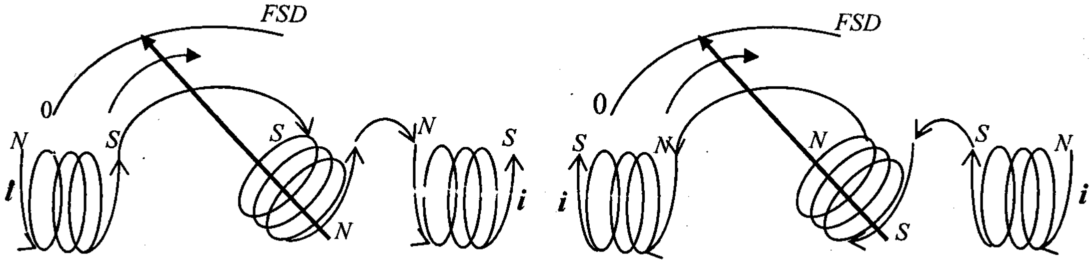
Dynamometer:

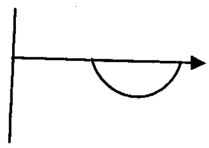
This instrument is suitable for the measurement of direct and alternating current, voltage and power. The deflecting torque in dynamometer is relies by the interaction of magnetic field produced by a pair of fixed air cored coils and a third air cored coil capable of angular movement and suspended within the fixed coil.

$S = 10 - 30\ \Omega/V$









Transfer equip.

Calibrated with DC and used on AC w.o. any modification

$T_i = N\vec{B}i_m A$, $\vec{B} \alpha\, i_f$ thus $T_i \alpha\, i_m i_f A \Rightarrow\Rightarrow$ so $T_i \alpha\, i^2$

θ α average i^2 , since $r.m.s = \sqrt{average f(t)^2}$

$T_c = k\theta$

$T_d \alpha$ average i^2

$T_d = T_c$

$k\theta$ = average i^2

∴ θ α average i^2

Damping ...

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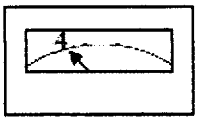
The *output scale is calibrated to give the r.m.s value of a.c signal* by taking the square roots of the inside measured value.

O/P scale $= r.m.s = \sqrt{average(i)^2}$, for example if (average i^2) = 16 inside the measuring device, the output scale of the device will indicate (4)



d.c → *dynamometer* → d.c

a.c → *dynamometer* → *r.m.s*

$$r.m.s = \sqrt{\frac{1}{T}\int_0^T f(t)^2 dt}$$

Meters

- d.c meters — measure d.c or Av values
  - PMMC

    $T \alpha i$
    B constant
    linear scale
    $\theta \alpha i$

  - dynamometer

    $T \alpha i^2$
    B varied
    non linear scale
    $\theta \alpha$ average i^2

- a.c meters — measure r.m.s values
  - Average Responding a.c meter ⇓ Rectifier + PMMC
  - True Responding a.c meter ⇓ dynamometer

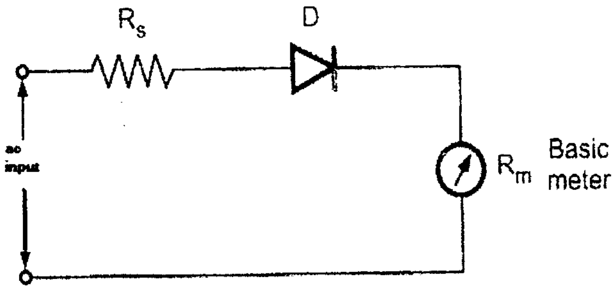
1- Average Responding a.c Meter:

Ac measurement are made with ac to dc converters which produce a dc current proportional to the ac input being measured and use this current for, either meter deflection, or application to the dc circuitry of digital or analogue multi-meter. Average responding ac meters uses half wave or full wave rectification with the meter scale calibrated in terms of the rms value of a waveform instead of the average value.

I- AC Voltmeter Using Half Wave Rectifier:

The ac voltmeter using half wave rectifier is achieved by introducing a diode in a basic dc voltmeter. This is shown in the fig.

$$V_{rms} = \frac{Vmax}{\sqrt{2}}$$



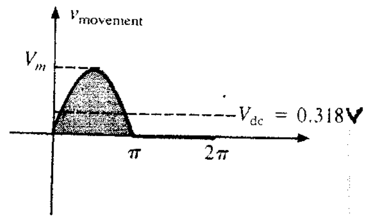
- $V_{av} = V_{dc} = \frac{V_{max}}{\pi} = 0.318 V_{max}$

$\quad = \frac{\sqrt{2}}{\pi} V_{rms} = 0.45 V_{rms}$

The value of series resistance R_s can be obtained
As:

$R_s = \frac{V_{av}}{I_{dc}} - (R_m + R_D)$

$R_s = \frac{0.45\, E_{rms}}{I_{dc}} - (R_m + R_D)$

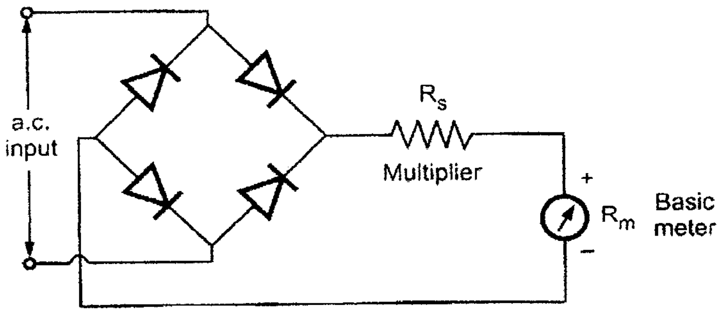


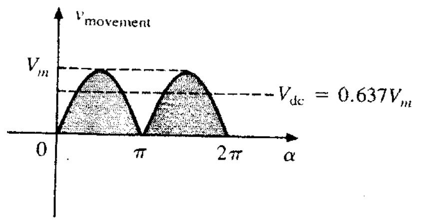
Where, I_{dc} is the full scale deflection current. And R_D is the forward resistance of the diode.

Sensitivity (ac) = 0.45 x sensitivity (dc).....for half wave

II- AC Voltmeter Using Full Wave Rectifier:

The ac voltmeter using full wave rectifier is achieved by using bridge rectifier consisting of four diodes, as shown in fig.





$V_{av} = V_{dc} = \frac{2V_{max}}{\pi} = 0.636 V_{max}$

$= \frac{2\sqrt{2} V_{rms}}{\pi} = 0.9 V_{rms}$

The multiplier resistance can be obtained as,

$R_s = \frac{V_{dc}}{I_{dc}} - (R_m + 2R_D)$

$R_s = \frac{0.9 V_{rms}}{I_{dc}} - (R_m + 2R_D)$

Sensitivity (ac) =0.9 x sensitivity (dc) .... For full wave

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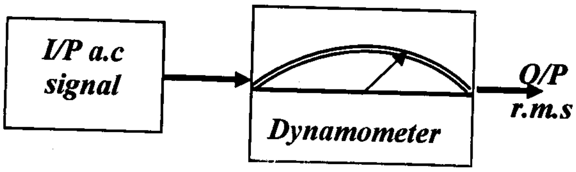
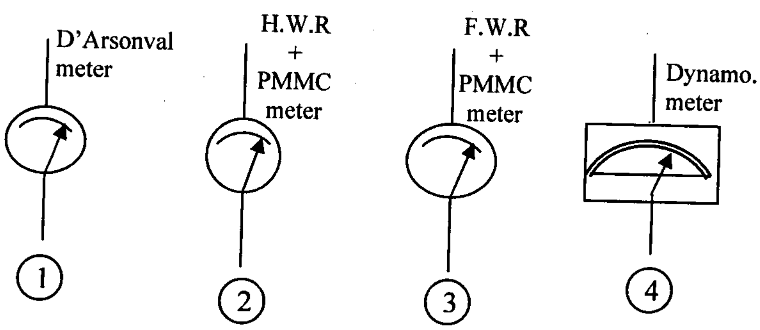
2-*True Responding a.c Meter (Dynamometer):*

O/P (r.m.s) = Av x $F.F_{true}$ $F.F_{true}$ = The form factor of any input signal
(true) = (measured)



Example:

What will be the out put of the following meters, if an average responding a.c meter of half-wave rectifier read (4.71v), and true form factor of input waveform is (1.414).



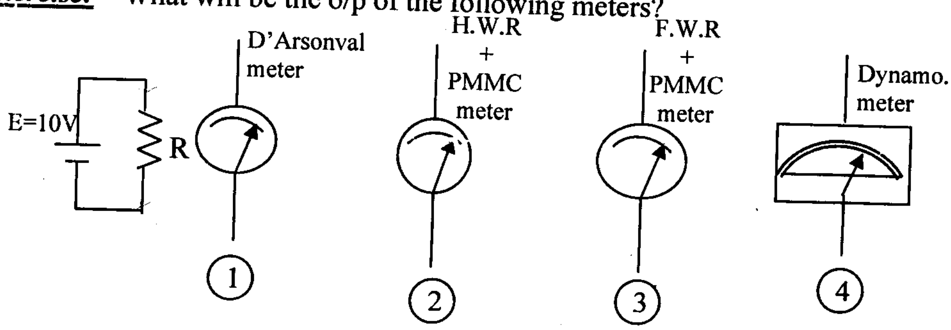
Sol:

$r.m.s_{measured} = 1.57 \times Av$ for average responding a.c meter of half wave rectifier

$4.71 = 1.57 \times Av$ $\Rightarrow Av = \dfrac{4.71}{1.57} = 3V$

1. D'Arsonval meter read Av = 3V
2. HWR+PMMC (***Average responding of halve wave rectifier***) meter = 4.71V
3. FWR+PMMC (***Average responding of full wave rectifier)*** meter = 1.11 x 3 = 3.33V
4. Dynamometer =$F.F_{(true)}$ x Av
   $r.m.s_{(true)}$= 1.414 x 3 = 4.242V

Exercise: What will be the o/p of the following meters?



10V 10V

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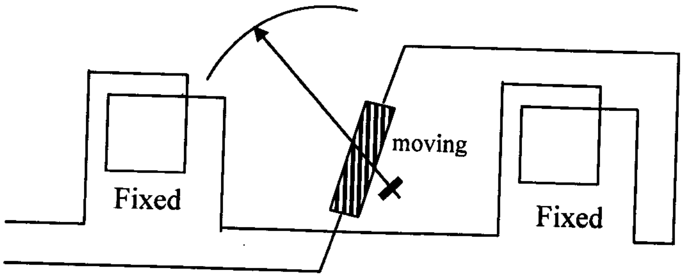
Dynamometer As Ammeter And Voltmeter:

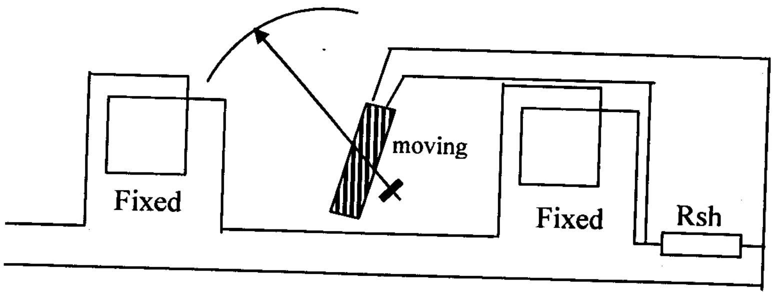
For small current measurement (5mA to 100mA), fixed and moving coils are connect in series. While larger current measurement (up to 20A) , the moving coil is shunted by a small resistance.



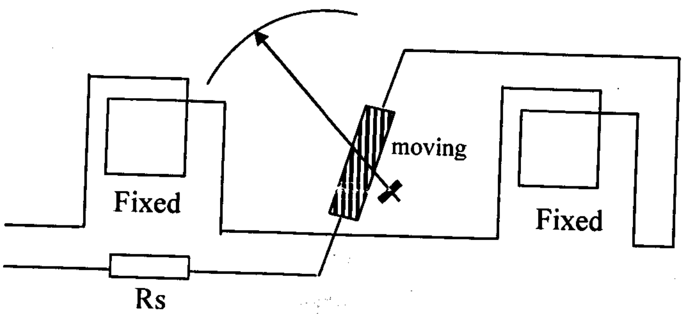
$T_d = I_1 I_2 \frac{dM}{d\theta}$

I_1, I_2 either d.c or A.C (rms)

$\therefore T_d = I^2 \frac{dM}{d\theta}$



To convert such an instrument to a voltmeter only a rather big series resistance is connected with the moving coil.



$T_d = I^2 \frac{dM}{d\theta}$

$\frac{dM}{d\theta}$ = constant over usual working range.

$\therefore T_d \propto I^2$

$I^2 = \frac{V^2}{Z^2}$

Clamp on Meters (Average Responding A.C meter):

One application of average responding a.c meters is the ***clamp on meter*** which is used to measured a.c current, voltage in a wire ***with out having to break*** the circuit being measured. The meter having use the transformer principle to detect the current. That is, the clamp on device of the meter serves as the core of a transformer. The current carrying wire is the primary winding of the transformer, while the secondary winding is in the meter. The alternating current in the primary is coupled to the secondary winding by the core, and after being rectified the current is sensed by a d'Arsonval meter.

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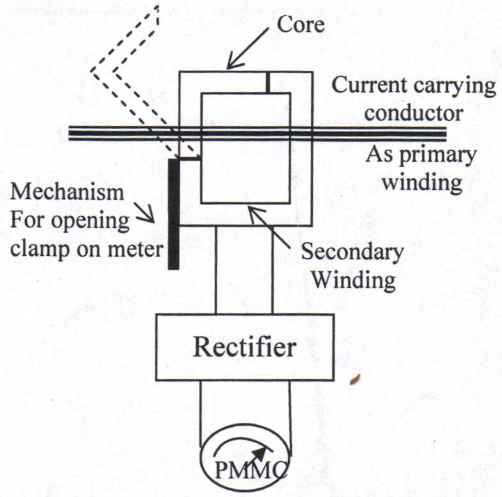
$V_2 = N \cdot 4.44 \cdot \phi_m \cdot f$



$\frac{N_1}{N_2} = \frac{V_1}{V_2}$

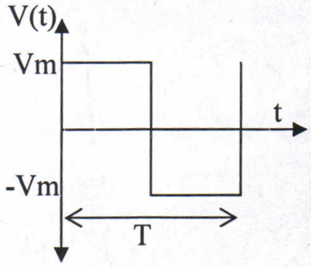
$N_1 = 1$
$N_2 = N$
$V_1 = 4.44 \cdot \phi_m \cdot f$
V_2 ?

Example:

The symmetrical square wave voltage is applied to an average responding a.c voltmeter with a scale calibrated in term of the r.m.s value of a sine wave. Calculate:

1. The form factor of square wave voltage.
2. The error in the meter indication.

Sol:



$$Vrms_{(True)} = \sqrt{\frac{1}{T}\int_o^T V(t)^2\,dt} = Vm$$

$$Vaverage_{(True)} = \frac{2}{T}\int_o^{\frac{T}{2}} V(t)dt = Vm$$

$$F.F_{(True)} = \frac{Vrms}{Vav.} = \frac{Vm}{Vm} = 1$$

$$Vrms_{(measured)} = 1.11 \times Av. = 1.11 \times Vm = 1.11Vm$$

meas true

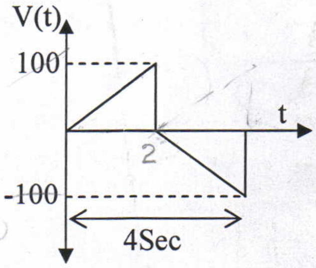
$$Error = \frac{Vrms_{(True)} - Vrms_{(measured)}}{Vrms_{(True)}} \times 100\%$$

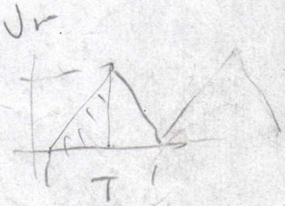
$$\%\,Error = \frac{1 - CF}{CF} \times 100$$

$$Error = \frac{Vm - 1.11Vm}{Vm} \times 100\%$$

Exer.:

Repeat the above example for saw tooth waveform shown





Sol:

V(t)=25t 50t

Vav.=50V

$Vrms_{(True)}$=57.75V

$Vrms_{(Measured)}$=55.5V

$F.F_{(True)}$=1.154

Error=0.0389%

3.89%

3.89%