Chapter 10

Heat Exchangers:

A heat exchanger is a component that allows the transfer of heat from one fluid (liquid or gas) to another fluid. Reasons for heat transfer include the following:

1. To heat a cooler fluid by means of a hotter fluid
2. To reduce the temperature of a hot fluid by means of a cooler fluid
3. To boil a liquid by means of a hotter fluid
4. To condense a gaseous fluid by means of a cooler fluid
5. To boil a liquid while condensing a hotter gaseous fluid

In order to transfer heat the fluids involved must be at different temperatures and they must come into thermal contact. Heat can flow only from the hotter to the cooler fluid.

In a heat exchanger there is no direct contact between the two fluids. The heat is transferred from the hot fluid to the metal isolating the two fluids and then to the cooler fluid.

TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment.. The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 10-1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow**, both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow,** on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

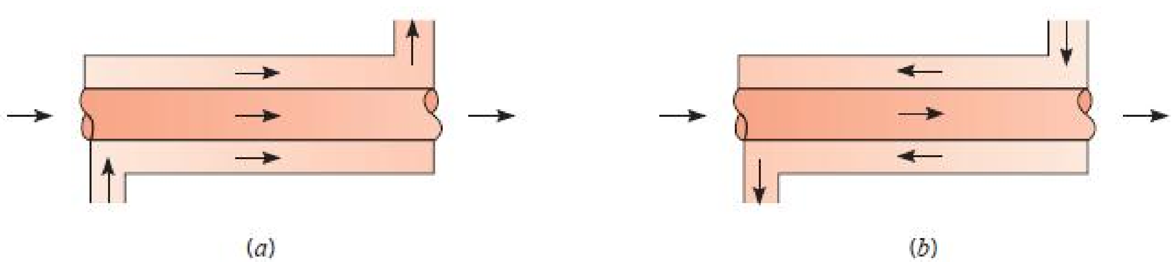


FIGURE 10.1 Concentric tube heat exchangers. (*a*) Parallel flow. (*b*) Counterflow

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger.

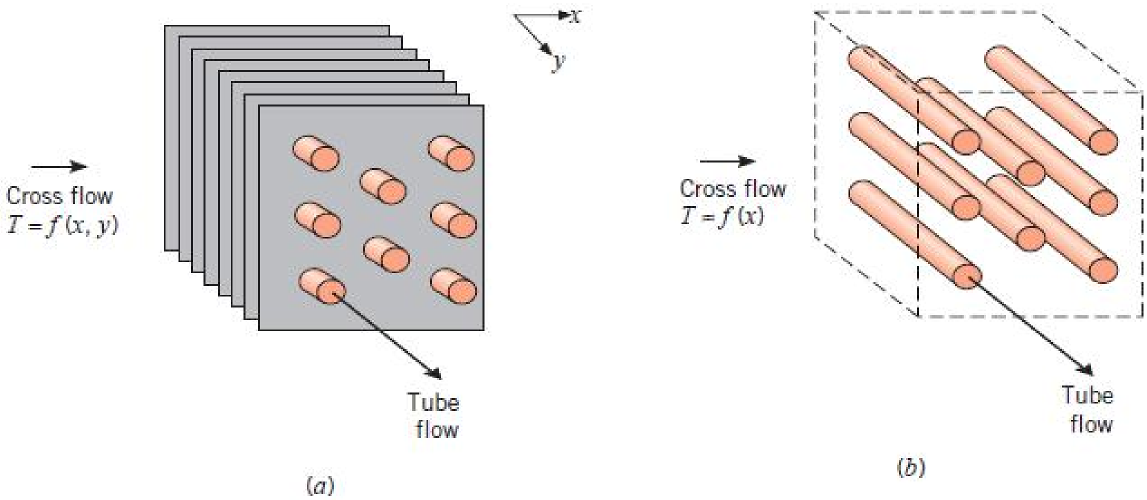


FIGURE 10.2 Cross-flow heat exchangers. (*a*) Finned with both fluids unmixed. (*b*) Unfinned with one fluid mixed and the other unmixed.

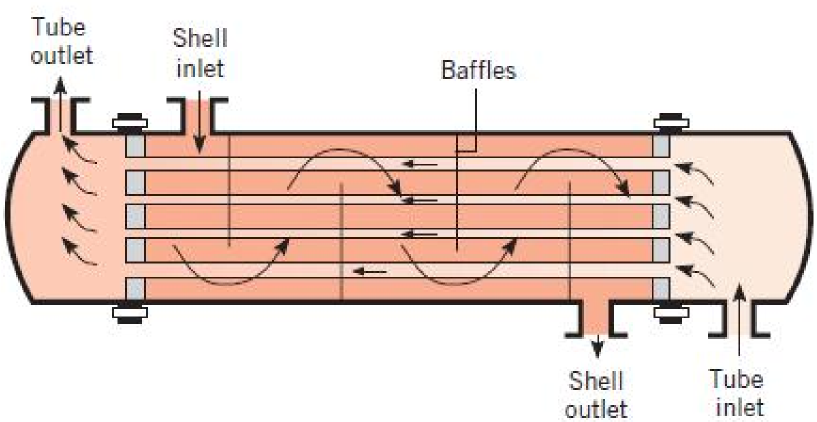


FIGURE 10.3 Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of Operation).

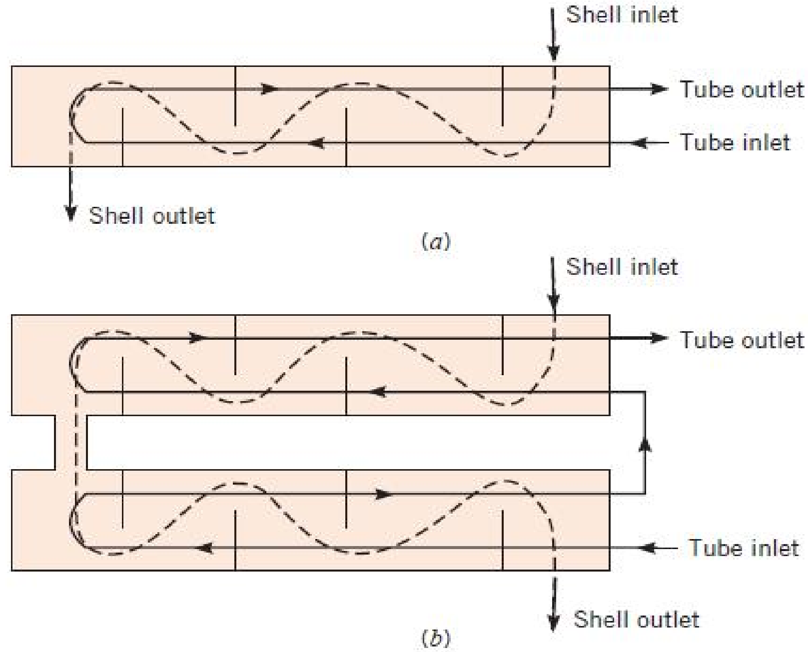


FIGURE 10.4 Shell-and-tube heat exchangers. (*a*) One shell pass and two tube passes. (*b*) Two shell passes and four tube passes.

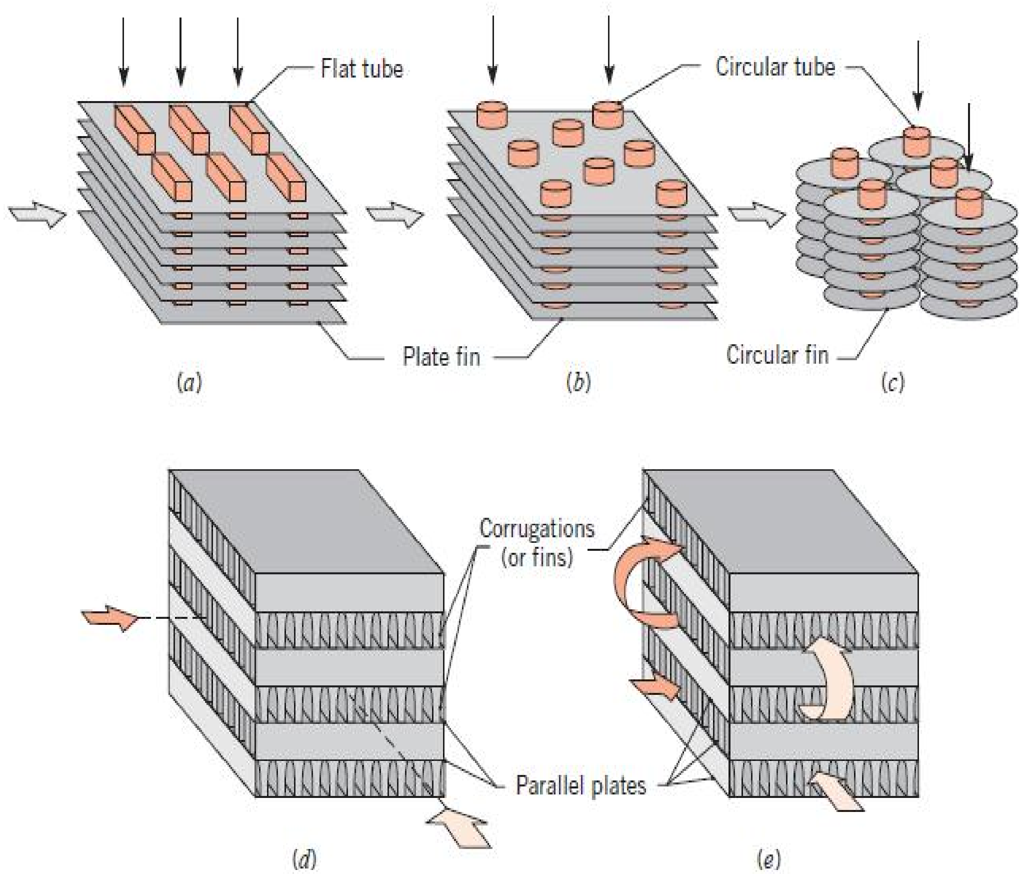


FIGURE 10.5 Compact heat exchanger cores. (*a*) Fin–tube (flat tubes, continuous plate fins).
(*b*) Fin–tube (circular tubes, continuous plate fins). (*c*) Fin–tube (circular tubes, circular fins).
(*d*) Plate–fin (single pass). (*e*) Plate–fin (multipass).

10–2 THE OVERALL HEAT TRANSFER COEFFICIENT

We have already discussed the overall heat-transfer coefficient in Section 2-4 with the heat transfer through the plane wall of Figure 10-6 expressed as

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} \qquad \textbf{[10-1]}$$

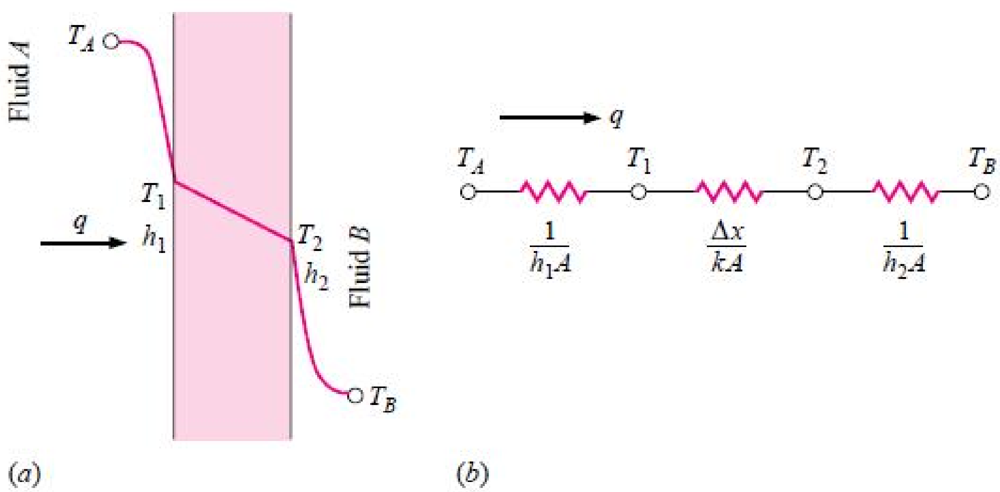


Figure 10-6 Overall heat transfer through a plane wall

The overall heat-transfer coefficient *U* is defined by the relation

$$q = U A \Delta T_{\text{overall}} \qquad \textbf{[10-2]}$$

The overall heat transfer is obtained from the thermal network of Figure 10-7*b* as

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}} \qquad \textbf{[10-3]}$$

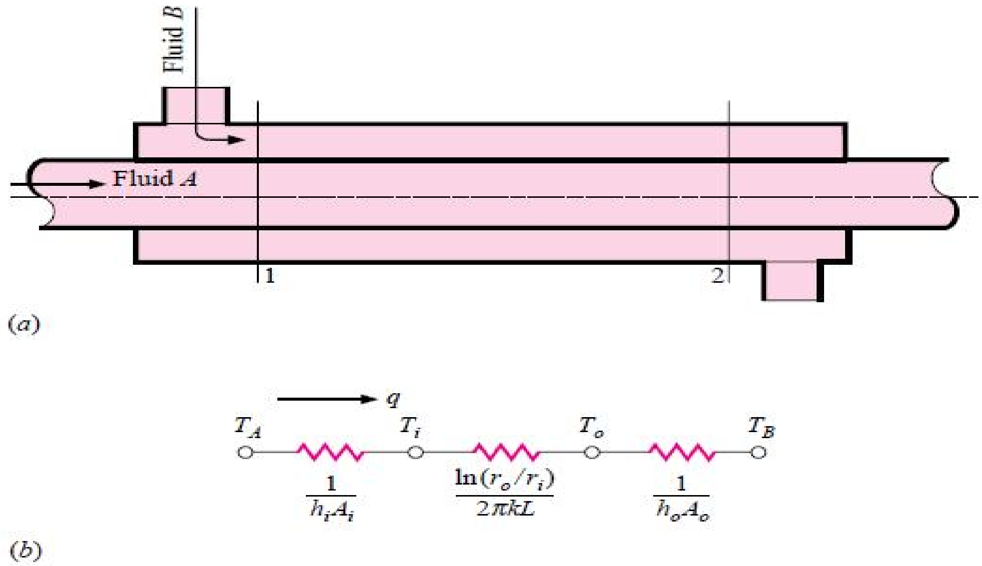


Figure 10-7 Double-pipe heat exchange: (*a*) schematic; (*b*) thermal-resistance network for overall heat transfer

where the subscripts *i* and *o* pertain to the inside and outside of the smaller inner tube. The overall heat-transfer coefficient may be based on either the inside or outside area of the tube at the discretion of the designer. Accordingly

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi k L} + \frac{A_i}{A_o}\frac{1}{h_o}} \qquad [10\text{-}4a]$$

$$U_o = \frac{1}{\frac{A_o}{A_i}\frac{1}{h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o}} \qquad [10\text{-}4b]$$

Overall Heat-Transfer Coefficient for Pipe in Air

Hot water at 98◦C flows through a 2-in schedule 40 horizontal steel pipe [k =54 W/m· ◦C] and is exposed to atmospheric air at 20◦C. The water

velocity is 25 cm/s. Calculate the overall heat transfer coefficient for this situation, based on the outer area of pipe.

Solution

From Appendix A the dimensions of 2-in schedule 40 pipe are

$$\text{ID} = 2.067 \text{ in} = 0.0525 \text{ m}$$
$$\text{OD} = 2.375 \text{ in} = 0.06033 \text{ m}$$

The heat-transfer coefficient for the water flow on the inside of the pipe is determined from the flow conditions with properties evaluated at the bulk temperature. The free-convection heat-transfer coefficient on the outside of the pipe depends on the temperature difference between the surface and ambient air. This temperature difference depends on the overall energy balance. First, we evaluate h_i and then formulate an iterative procedure to determine h_o.

The properties of water at 98°C are

$$\rho = 960 \text{ kg/m}^3 \qquad \mu = 2.82 \times 10^{-4} \text{ kg/m}\cdot\text{s}$$
$$k = 0.68 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 1.76$$

The Reynolds number is

$$\text{Re} = \frac{\rho u d}{\mu} = \frac{(960)(0.25)(0.0525)}{2.82 \times 10^{-4}} = 44{,}680 \qquad [a]$$

and since turbulent flow is encountered, we may use Equation (6-4):

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$$
$$= (0.023)(44{,}680)^{0.8}(1.76)^{0.4} = 151.4$$
$$h_i = \text{Nu}\frac{k}{d} = \frac{(151.4)(0.68)}{0.0525} = 1961 \text{ W/m}^2\cdot{}^\circ\text{C} \quad [345 \text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}] \qquad [b]$$

For unit length of the pipe the thermal resistance of the steel is

$$R_s = \frac{\ln(r_o/r_i)}{2\pi k} = \frac{\ln(0.06033/0.0525)}{2\pi(54)} = 4.097 \times 10^{-4} \qquad [c]$$

Again, on a unit-length basis the thermal resistance on the inside is

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i 2\pi r_i} = \frac{1}{(1961)\pi(0.0525)} = 3.092 \times 10^{-3} \qquad [d]$$

The thermal resistance for the outer surface is as yet unknown but is written, for unit lengths,

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_o} \qquad [e]$$

From Table 7-2, for laminar flow, the simplified relation for h_o is

$$h_o = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} = 1.32\left(\frac{T_o - T_\infty}{d}\right)^{1/4} \qquad [f]$$

where T_o is the unknown outside pipe surface temperature. We designate the inner pipe surface as T_i and the water temperature as T_w; then the energy balance requires

$$\frac{T_w - T_i}{R_i} = \frac{T_i - T_o}{R_s} = \frac{T_o - T_\infty}{R_o} \qquad [g]$$

Combining Equations (e) and (f) gives

$$\frac{T_o - T_\infty}{R_o} = 2\pi r_o \frac{1.32}{d^{1/4}}(T_o - T_\infty)^{5/4} \qquad [h]$$

This relation may be introduced into Equation (g) to yield two equations with the two unknowns T_i and T_o:

$$\frac{98 - T_i}{3.092 \times 10^{-3}} = \frac{T_i - T_o}{4.097 \times 10^{-4}}$$

$$\frac{T_i - T_o}{4.097 \times 10^{-4}} = \frac{(\pi)(0.06033)(1.32)(T_o - 20)^{5/4}}{(0.06033)^{1/4}}$$

This is a nonlinear set that may be solved by iteration to give

$$T_o = 97.6°\text{C} \qquad T_i = 97.65°\text{C}$$

As a result, the outside heat-transfer coefficient and thermal resistance are

$$h_o = \frac{(1.32)(97.6 - 20)^{1/4}}{(0.06033)^{1/4}} = 7.91 \text{ W/m}^2 \cdot °\text{C} \quad [1.39 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

$$R_o = \frac{1}{(0.06033)(7.91)\pi} = 0.667$$

The calculation clearly illustrates the fact that the free convection controls the overall heat-transfer because R_o is much larger than R_i or R_s. The overall heat-transfer coefficient based on the outer area is written in terms of these resistances as

$$U_o = \frac{1}{A_o(R_i + R_s + R_o)} \qquad [i]$$

With numerical values inserted,

$$U_o = \frac{1}{\pi(0.06033)(3.092 \times 10^{-3} + 4.097 \times 10^{-4} + 0.667)}$$
$$= 7.87 \text{ W/Area} \cdot °\text{C}$$

In this calculation we used the outside area for a 1.0-m length as

$$A_o = \pi(0.06033) = 0.1895 \text{ m}^2\text{/m}$$
$$U_o = 7.87 \text{ W/m}^2 \cdot °\text{C}$$

Thus, we find that the overall heat-transfer coefficient is almost completely controlled by the value of h_o. We might have expected this result strictly on the basis of our experience with the relative magnitude of convection coefficients; free-convection values for air are very low compared with forced convection with liquids.

Overall Heat-Transfer Coefficient for Pipe Exposed to Steam

EXAMPLE 10-2

The pipe and hot-water system of Example 10-1 is exposed to steam at 1 atm and 100°C. Calculate the overall heat-transfer coefficient for this situation based on the outer area of pipe.

■ Solution

We have already determined the inside convection heat-transfer coefficient in Example 10-1 as

$$h_i = 1961 \text{ W/m}^2 \cdot °\text{C}$$

The convection coefficient for condensation on the outside of the pipe is obtained by using Equation (9-12),

$$h_o = 0.725\left[\frac{\rho(\rho-\rho_v)gh_{fg}k_f^3}{\mu_f d(T_g - T_o)}\right]^{1/4} \quad [a]$$

where T_o is the outside pipe-surface temperature. The water film properties are

$$\rho = 960\ \text{kg/m}^3 \qquad \mu_f = 2.82\times10^{-4}\ \text{kg/m}\cdot\text{s}$$
$$k_f = 0.68\ \text{W/m}\cdot{}^\circ\text{C} \qquad h_{fg} = 2255\ \text{kJ/kg}$$

so Equation (a) becomes

$$h_o = 0.725\left[\frac{(960)^2(9.8)(2.255\times10^6)(0.68)^3}{(2.82\times10^{-4})(0.06033)(100-T_o)}\right]^{1/4}$$
$$= 17{,}960(100-T_o)^{-1/4} \quad [b]$$

and the outside thermal resistance per unit length is

$$R_o = \frac{1}{h_o A_o} = \frac{(100-T_o)^{1/4}}{(17{,}960)\pi(0.06033)} = \frac{(100-T_o)^{1/4}}{3403} \quad [c]$$

The energy balance requires

$$\frac{100-T_o}{R_o} = \frac{T_o - T_i}{R_s} = \frac{T_i - T_w}{R_i} \quad [d]$$

From Example 10-1

From Example 10-1

$$R_i = 3.092\times10^{-3} \qquad R_s = 4.097\times10^{-4} \qquad T_w = 98^\circ\text{C}$$

and Equations (c) and (d) may be combined to give

$$3403(100-T_o)^{3/4} = \frac{(T_o - T_i)}{4.097\times10^{-4}}$$

$$\frac{T_o - T_i}{4.097\times10^{-4}} = \frac{T_i - 98}{3.092\times10^{-3}}$$

This is a nonlinear set of equations that may be solved to give

$$T_o = 99.91^\circ\text{C} \qquad T_i = 99.69^\circ\text{C}$$

The exterior heat-transfer coefficient and thermal resistance then become

$$h_o = 17{,}960(100-99.91)^{-1/4} = 32{,}790\ \text{W/m}^2\cdot{}^\circ\text{C} \quad [e]$$

$$R_o = \frac{(100-99.91)^{1/4}}{3403} = 1.610\times10^{-4} \quad [f]$$

Based on unit length of pipe, the overall heat-transfer coefficient is

$$U_o = \frac{1}{A_o(R_i + R_s + R_o)}$$
$$= \frac{1}{\pi(0.06033)(3.092\times10^{-3} + 4.097\times10^{-4} + 1.610\times10^{-4})} \quad [g]$$
$$= 1441\ \text{W/}^\circ\text{C}\cdot\text{Area}$$

Since A_o and the R's were both per unit length,

$$U_o = 1441\ \text{W/m}^2 \cdot {}^\circ\text{C} \qquad [254\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

In this problem the water-side convection coefficient is the main controlling factor because h_o is so large for a condensation process. In fact, the outside thermal resistance is smaller than the conduction resistance of the steel. The approximate *relative* magnitudes of the resistances are

$$R_o \sim 1 \qquad R_s \sim 2.5 \qquad R_i \sim 19$$

Fouling Factor

Fouling factors must be obtained experimentally by determining the values of *U* for both clean and dirty conditions in the heat exchanger. The fouling factor is thus defined as

$$R_f = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}$$

The overall heat transfer coefficient is modified as follows:

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$= \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

Table 10-2 | Table of selected fouling factors, according to Reference 2.

Type of fluid	Fouling factor, $\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}$	$\text{m}^2 \cdot {}^\circ\text{C/W}$
Seawater, below 125°F	0.0005	0.00009
Above 125°F	0.001	0.002
Treated boiler feedwater above 125°F	0.001	0.0002
Fuel oil	0.005	0.0009
Quenching oil	0.004	0.0007
Alcohol vapors	0.0005	0.00009
Steam, non-oil-bearing	0.0005	0.00009
Industrial air	0.002	0.0004
Refrigerating liquid	0.001	0.0002

Influence of Fouling Factor

Suppose the water in Example 10-2 is seawater above 125◦F and a fouling factor of

0.0002 $m^2 \cdot {}^\circ C/W$ is experienced. What is the percent reduction in the convection heat-transfer coefficient?

The fouling factor influences the heat-transfer coefficient on the inside of the pipe. We have

$Rf = 0.0002 = 1/h_{dirty} - 1/h_{clean}$

Using $h_{clean} = 1961$ $W/m^2 \cdot {}^\circ C$ we obtain

$h_{dirty} = 1409$ $W/m^2 \cdot {}^\circ C$

This is a 28 percent reduction because of the fouling factor.

10–3 ANALYSIS OF HEAT EXCHANGERS , THE LOG MEAN TEMPERATURE DIFFERENCE

The *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one.

That is,

$$q_h = \dot{m}_h c_{p,h} (T_{h,I} - T_{h,o}) \qquad \text{(a)}$$

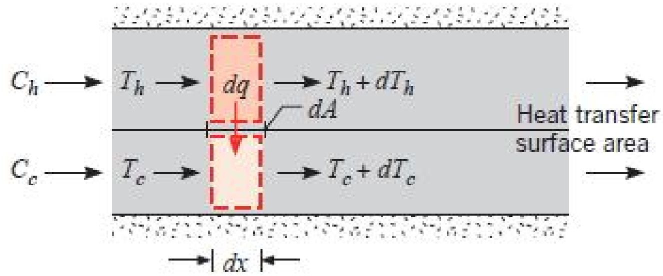
and

$$q_c = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) \qquad \text{(b)}$$

Another useful expression may be obtained by relating the total heat transfer rate q to
the temperature difference ΔT between the hot and cold fluids, Such an expression would be an extension of Newton's law of cooling, with the overall
heat transfer coefficient U used in place of the single convection coefficient h. However,

$$\Delta T = T_h - T_c$$

$$q = U A \Delta Tm \qquad \text{(c)}$$



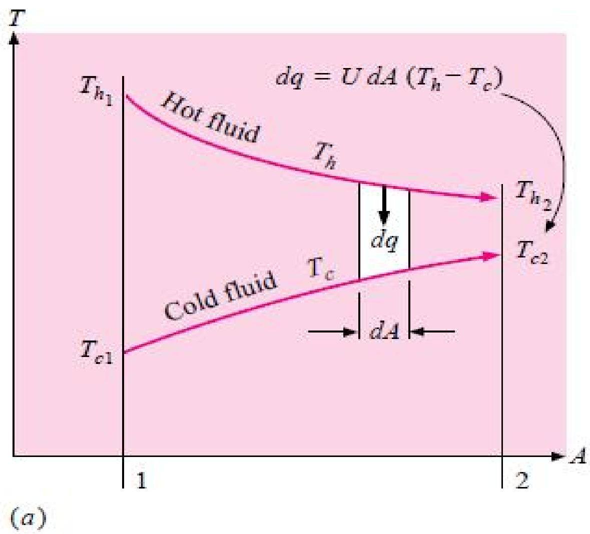


FIGURE 10.8 Temperature distributions for a parallel-flow heat exchanger

For the parallel-flow heat exchanger shown in Figure 10-8, the heat transferred through an element of area *dA* may be written

$$dq = -\dot{m}_h c_{p,h}\, dT_h = -\, C_h\, dT_h$$

$$dT_h = -\, dq / C_h \qquad \text{(d)}$$

$$dq = \dot{m}_c c_{p,c}\, dT_c = C_c\, dT_c$$

$$dT_c = dq / C_c \qquad \text{(e)}$$

where C_h and C_c are the hot and cold fluid *heat capacity rates*, respectively.

The heat transfer across the surface area dA may also be expressed as

$$dq = U\, \Delta T\, dA \qquad \text{(f)}$$

where $\Delta T = T_h - T_c$ is the *local* temperature difference between the hot and cold fluids. so

$$d\ (\Delta T) = dT_h - dT_c \qquad (g)$$

we begin by substituting Equations(d)and (e)into the differential form of equation (g)

to obtain

$$d(\Delta T) = -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \qquad (h)$$

Substituting for dq from Equation (f) and integrating across the heat exchanger, we
obtain

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

or

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Substituting for C_h and C_c from Equations (a) and (b), respectively, it follows that

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right)$$
$$= -\frac{UA}{q}[(T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})]$$

Recognizing that, for the parallel-flow heat exchanger, $\Delta T_1 = (T_{h,i} - T_{c,i})$
and $\Delta T_2 = (T_{h,o} - T_{c,o})$, we then obtain

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

we conclude that the appropriate average temperature difference is a *log mean temperature difference,* ΔT_{lm}. Accordingly, we may write

$$q = UA\ \Delta T_{lm}$$

where

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

Remember that, for the *parallel-flowexchanger* ,

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o} \end{bmatrix}$$

And for the Counter-Flow Heat Exchangers the endpoint temperature differences must now be defined as

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i} \end{bmatrix}$$

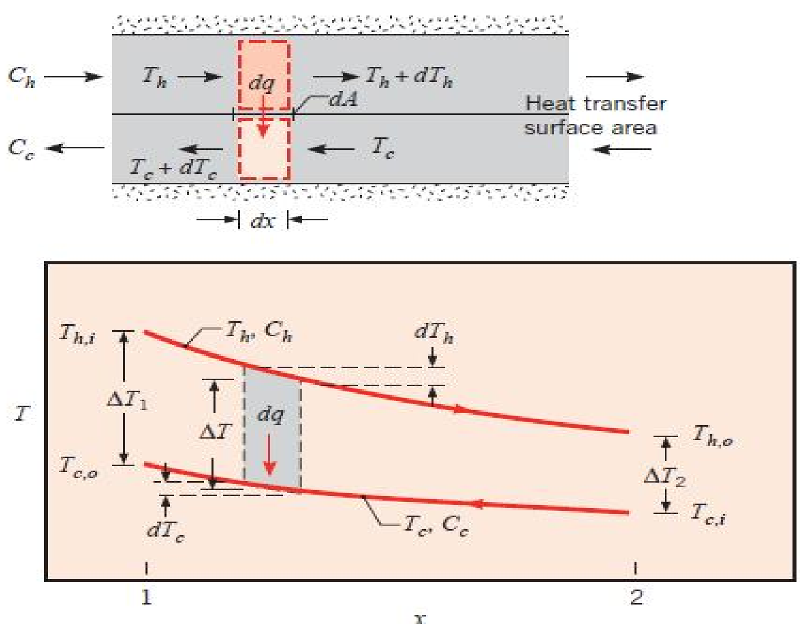


FIGURE 10.9 Temperature distributions for a counterflow heat exchanger

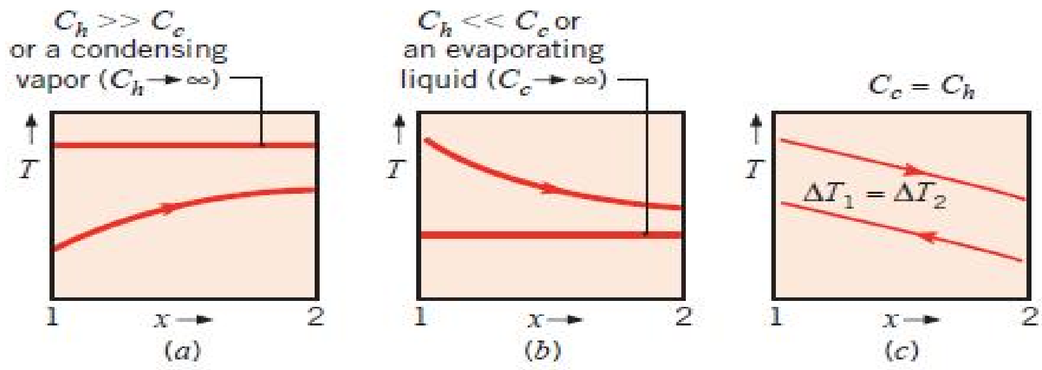


FIGURE 11.10 Special heat exchanger conditions. (*a*) $C_h \gg C_c$ or a condensing vapor. (*b*) An evaporating liquid or $C_h \ll C_c$. (*c*) A counterflow heat exchanger with equivalent fluid heat capacities ($C_h = C_c$).

If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTD ***for a counterflow double-pipe arrangement with the same hot and cold fluid temperatures.*** The heat-transfer equation then takes the form

$$q = U\,A\,F\,\Delta T_m \qquad \textbf{[10-13]}$$

Values of the correction factor *F* according to Reference 4 are plotted in Figures 10-8 to 10-11 for several different types of heat exchangers

Figure 10-9 | Correction-factor plot for exchanger with two shell passes and four, eight, or any multiple of tube passes.

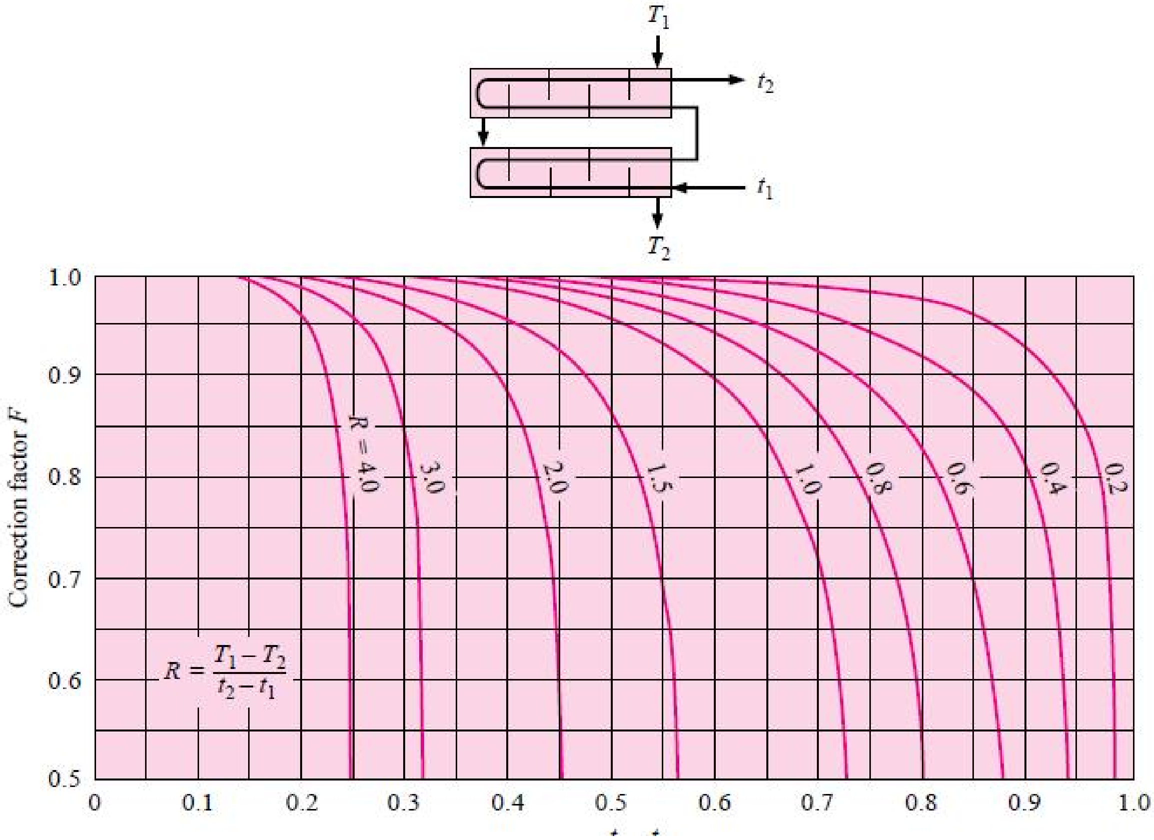


Figure 10-8 | Correction-factor plot for exchanger with one shell pass and two, four, or any multiple of tube passes.

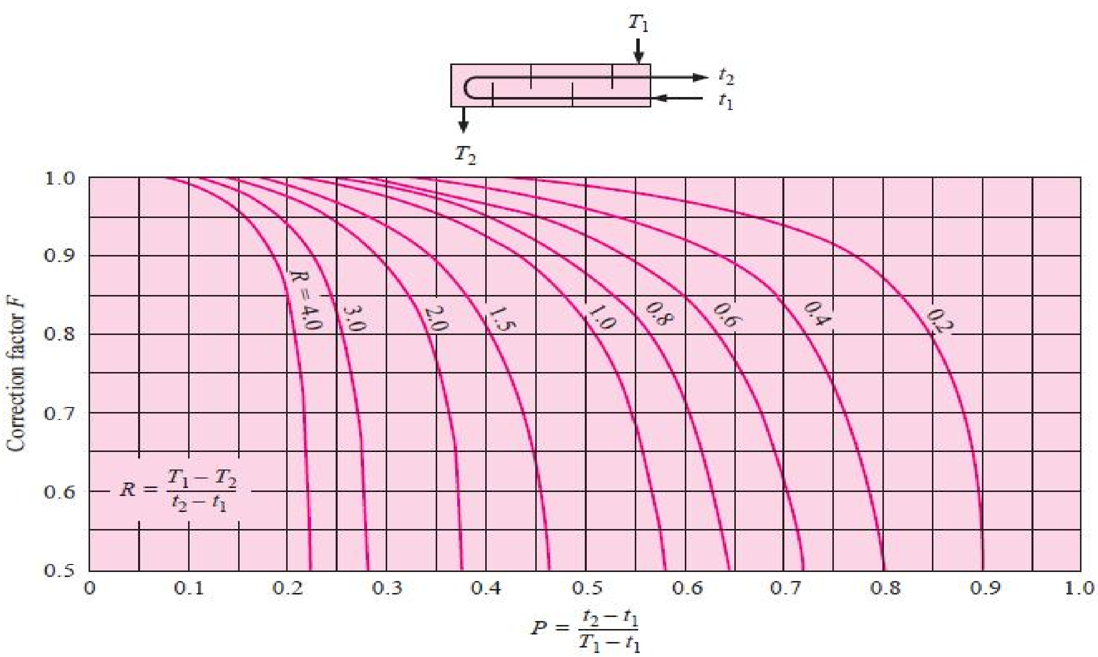


Figure 10-10 | Correction-factor plot for single-pass cross-flow exchanger, both fluids unmixed.

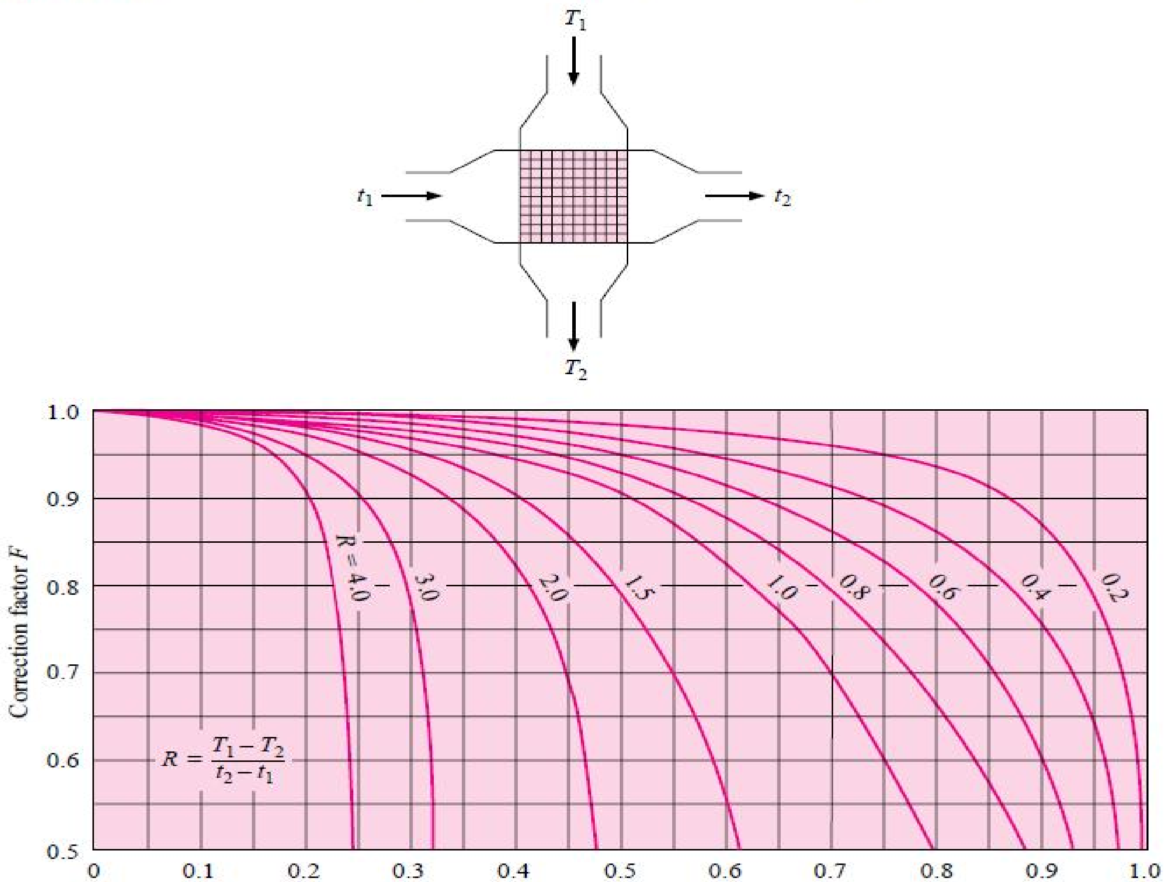
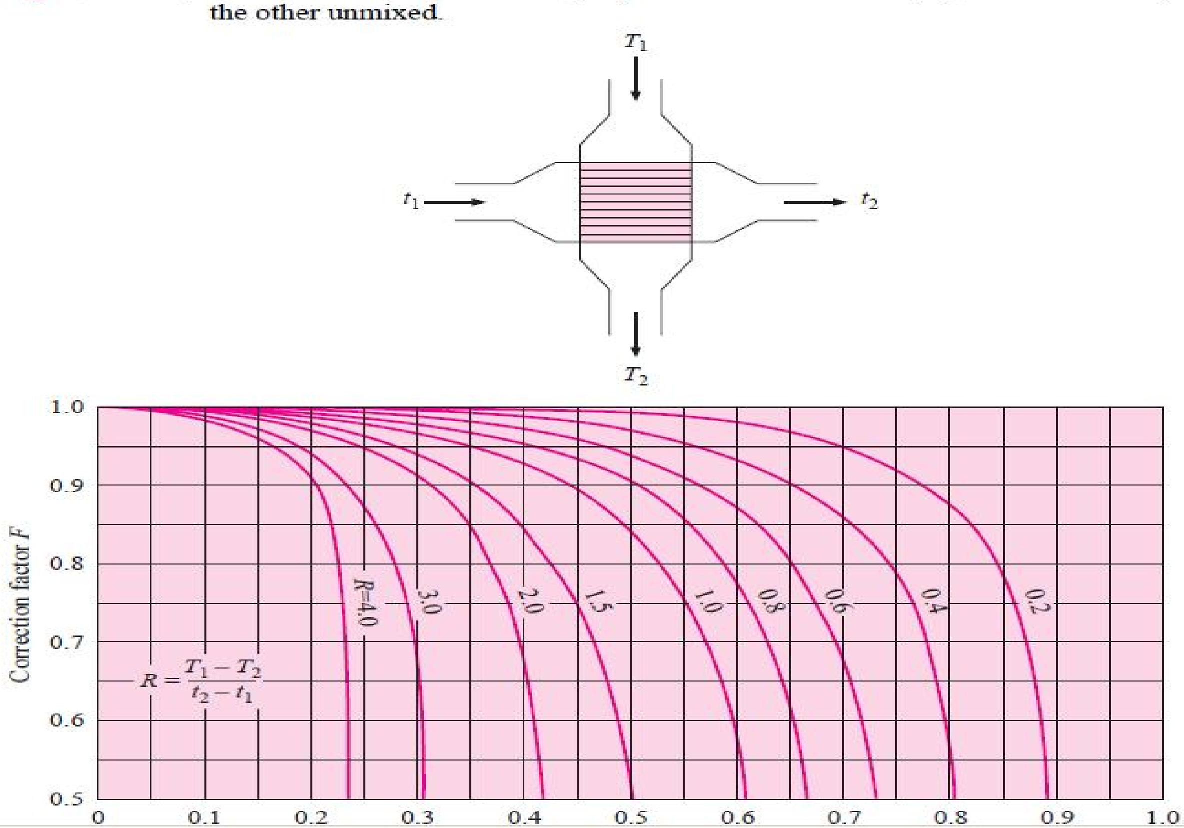


Figure 10-11 | Correction-factor plot for single-pass cross-flow exchanger, one fluid mixed, the other unmixed.



EXAMPLE 10-4 Calculation of Heat-Exchanger Size from Known Temperatures

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg · °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m^2 · °C. Calculate the heat-exchanger area.

Solution

The total heat transfer is determined from the energy absorbed by the water:

$$q = \dot{m}_w c_w \Delta T_w = (68)(4180)(75 - 35) = 11.37 \text{ MJ/min} \qquad [a]$$

$$= 189.5 \text{ kW} \qquad [6.47 \times 10^5 \text{ Btu/h}]$$

Since all the fluid temperatures are known, the LMTD can be calculated by using the temperature scheme in Figure 10-7*b*:

$$\Delta T_m = \frac{(110 - 75) - (75 - 35)}{\ln[(110 - 75)/(75 - 35)]} = 37.44°\text{C} \qquad [b]$$

Then, since $q = UA\,\Delta T_m$,

$$A = \frac{1.895 \times 10^5}{(320)(37.44)} = 15.82 \text{ m}^2 \qquad [170 \text{ ft}^2]$$

EXAMPLE 10-5 Shell-and-Tube Heat Exchanger

Instead of the double-pipe heat exchanger of Example 10-4, it is desired to use a shell-and-tube exchanger with the water making one shell pass and the oil making two tube passes. Calculate the area required for this exchanger, assuming that the overall heat-transfer coefficient remains at 320 W/m^2 · °C.

Solution

To solve this problem, we determine a correction factor from Figure 10-8 to be used with the LMTD calculated on the basis of a counterflow exchanger. The parameters according to the nomenclature of Figure 10-8 are

$$T_1 = 35°\text{C} \qquad T_2 = 75°\text{C} \qquad t_1 = 110°\text{C} \qquad t_2 = 75°\text{C}$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{75 - 110}{35 - 110} = 0.467$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{35 - 75}{75 - 110} = 1.143$$

so the correction factor is

$$F = 0.81$$

and the heat transfer is

$$q = UAF\,\Delta T_m$$

so that

$$A = \frac{1.895 \times 10^5}{(320)(0.81)(37.44)} = 19.53 \text{ m}^2 \qquad [210 \text{ ft}^2]$$

Design of Shell-and-Tube Heat Exchanger

EXAMPLE 10-6

Water at the rate of 30,000 lb_m/h [3.783 kg/s] is heated from 100 to 130°F [37.78 to 54.44°C] in a shell-and-tube heat exchanger. On the shell side one pass is used with water as the heating fluid, 15,000 lb_m/h [1.892 kg/s], entering the exchanger at 200°F [93.33°C]. The overall heat-transfer coefficient is 250 Btu/h · ft^2 · °F [1419 W/m^2 · °C], and the average water velocity in the $\frac{3}{4}$-in [1.905-cm] diameter tubes is 1.2 ft/s [0.366 m/s]. Because of space limitations, the tube length must not be longer than 8 ft [2.438 m]. Calculate the number of tube passes, the number of tubes per pass, and the length of the tubes, consistent with this restriction.

Solution

We first assume one tube pass and check to see if it satisfies the conditions of this problem. The exit temperature of the hot water is calculated from

$$q = \dot{m}_c c_c \, \Delta T_c = \dot{m}_h c_h \, \Delta T_h$$

$$\Delta T_h = \frac{(30{,}000)(1)(130-100)}{(15{,}000)(1)} = 60^\circ\mathrm{F} = 33.33^\circ\mathrm{C} \qquad [a]$$

so

$$T_{h,\mathrm{exit}} = 93.33 - 33.33 = 60^\circ\mathrm{C}$$

The total required heat transfer is obtained from Equation (a) for the cold fluid:

$$q = (3.783)(4182)(54.44 - 37.78) = 263.6\ \mathrm{kW} \quad [8.08 \times 10^5\ \mathrm{Btu/h}]$$

For a counterflow exchanger, with the required temperature

$$\mathrm{LMTD} = \Delta T_m = \frac{(93.33 - 54.44) - (60 - 37.78)}{\ln[(93.33 - 54.44)/(60 - 37.78)]} = 29.78^\circ\mathrm{C}$$

$$q = UA\,\Delta T_m$$

$$A = \frac{2.636 \times 10^5}{(1419)(29.78)} = 6.238\ \mathrm{m}^2 \quad [67.1\ \mathrm{ft}^2] \qquad [b]$$

Using the average water velocity in the tubes and the flow rate, we calculate the total flow area with

$$\dot{m}_c = \rho A u$$

$$A = \frac{3.783}{(1000)(0.366)} = 0.01034\ \mathrm{m}^2 \qquad [c]$$

This area is the product of the number of tubes and the flow area per tube:

$$0.01034 = n\frac{\pi d^2}{4}$$

$$n = \frac{(0.01034)(4)}{\pi(0.01905)^2} = 36.3$$

or $n = 36$ tubes. The surface area per tube per meter of length is

$$\pi d = \pi(0.01905) = 0.0598 \text{ m}^2/\text{tube} \cdot \text{m}$$

We recall that the total surface area required for a one-tube-pass exchanger was calculated in Equation (*b*) as 6.238 m^2. We may thus compute the length of tube for this type of exchanger from

$$n\pi d\, L = 6.238$$
$$L = \frac{6.238}{(36)(0.0598)} = 2.898 \text{ m}$$

This length is greater than the allowable 2.438 m, so we must use more than one tube pass. When we increase the number of passes, we correspondingly increase the total surface area required because of the reduction in LMTD caused by the correction factor F. We next try two tube passes. From Figure 10-8, $F = 0.88$, and thus

$$A_{\text{total}} = \frac{q}{UF\Delta T_m} = \frac{2.636 \times 10^5}{(1419)(0.88)(29.78)} = 7.089 \text{ m}^2$$

The number of tubes per pass is still 36 because of the velocity requirement. For the two-tube-pass exchanger the total surface area is now related to the length by

$$A_{\text{total}} = 2n\pi d\, L$$

so that

$$L = \frac{7.089}{(2)(36)(0.0598)} = 1.646 \text{ m} \quad [5.4 \text{ ft}]$$

This length is within the 2.438-m requirement, so the final design choice is

Number of tubes per pass = 36

Number of passes = 2

Length of tube per pass = 1.646 m   [5.4 ft]

Cross-Flow Exchanger with One Fluid Mixed

EXAMPLE 10-7

A heat exchanger like that shown in Figure 10-4 is used to heat an oil in the tubes ($c = 1.9$ kJ/kg · °C) from 15°C to 85°C. Blowing across the outside of the tubes is steam that enters at 130°C and leaves at 110°C with a mass flow of 5.2 kg/sec. The overall heat-transfer coefficient is 275 W/m^2 · °C and c for steam is 1.86 kJ/kg · °C. Calculate the surface area of the heat exchanger.

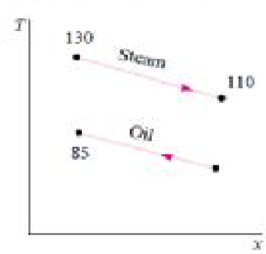


Figure Example 10-7

Solution

The total heat transfer may be obtained from an energy balance on the steam

$$q = \dot{m}_s c_s \Delta T_s = (5.2)(1.86)(130 - 110) = 193 \text{ kW}$$

We can solve for the area from Equation (10-13). The value of ΔT_m is calculated *as if the exchanger were counterflow double pipe* (i.e., as shown in Figure Example 10-7). Thus,

$$\Delta T_m = \frac{(130 - 85) - (110 - 15)}{\ln\left(\dfrac{130 - 85}{110 - 15}\right)} = 66.9°\text{C}$$

Now, from Figure 10-11, t_1 and t_2 will represent the unmixed fluid (the oil) and T_1 and T_2 will represent the mixed fluid (the steam) so that

$$T_1 = 130 \qquad T_2 = 110 \qquad t_1 = 15 \qquad t_2 = 85°\text{C}$$

and we calculate

$$R = \frac{130 - 110}{85 - 15} = 0.286 \qquad P = \frac{85 - 15}{130 - 15} = 0.609$$

Consulting Figure 10-11 we find

$$F = 0.97$$

so the area is calculated from

$$A = \frac{q}{UF\,\Delta T_m} = \frac{193{,}000}{(275)(0.97)(66.9)} = 10.82\ \text{m}^2$$

Effects of Off-Design Flow Rates for Exchanger in Example 10-7

EXAMPLE 10-8

Investigate the heat-transfer performance of the exchanger in Example 10-7 if the oil flow rate is reduced in half while the steam flow remains the same. Assume U remains constant at 275 W/m$^2\cdot{}^\circ$C.

Solution

We did not calculate the oil flow in Example 10-7 but can do so now from

$$q = \dot{m}_o c_o\,\Delta T_o$$

$$\dot{m}_o = \frac{193}{(1.9)(85-15)} = 1.45\ \text{kg/s}$$

The new flow rate will be half this value or 0.725 kg/s. We are assuming the inlet temperatures remain the same at 130°C for the steam and 15°C for the oil. The new relation for the heat transfer is

$$q = \dot{m}_o c_o (T_{e,o} - 15) = \dot{m}_s c_{p_s}(130 - T_{e,s}) \qquad [a]$$

but the exit temperatures, $T_{e,o}$ and $T_{e,s}$ are unknown. Furthermore, ΔT_m is unknown without these temperatures, as are the values of R and P from Figure 10-11. This means we must use an iterative procedure to solve for the exit temperatures using Equation (a) and

$$q = UAF\Delta T_m \qquad [b]$$

The general procedure is to assume values of the exit temperatures until the q's agree between Equations (a) and (b).

The objective of this example is to show that an iterative procedure is required when the inlet and outlet temperatures are not known or easily calculated. There is no need to go through this iteration because it can be avoided by using the techniques described in Section 10-6.

10-6 EFFECTIVENESS-NTU METHOD

The **effectiveness–NTU method** greatly simplified heat exchanger analysis.

This method is based on a dimensionless parameter called the **heat transfer** effectiveness _, defined as

$$\text{Effectiveness} = \epsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

The actual heat transfer may be computed, for the parallel-flow exchanger See fig(10-8)

$$q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m}_c c_c (T_{c_2} - T_{c_1})$$

and for the counterflow exchanger

$$q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m}_c c_c (T_{c_1} - T_{c_2})$$

See fig(10-9)

Maximum possible heat transfer is expressed as

$$q_{max} = (\dot{m}c)_{min}(T_{h\,inlet} - T_{c\,inlet})$$

In a general way the effectiveness is expressed as

$$\epsilon = \frac{\Delta T(\text{minimum fluid})}{\text{Maximum temperature difference in heat exchanger}}$$

The grouping of terms UA/C_{min} is called the *number of transfer units* (NTU)

Kays and London [3] have presented effectiveness ratios for various heat-exchanger arrangements, and some of the results of their analyses are available in chart form in
Figures 10-12 to 10-17. Examples 10-9 to 10-14 illustrate the use of the effectiveness-NTU method in heat-exchanger analysis.

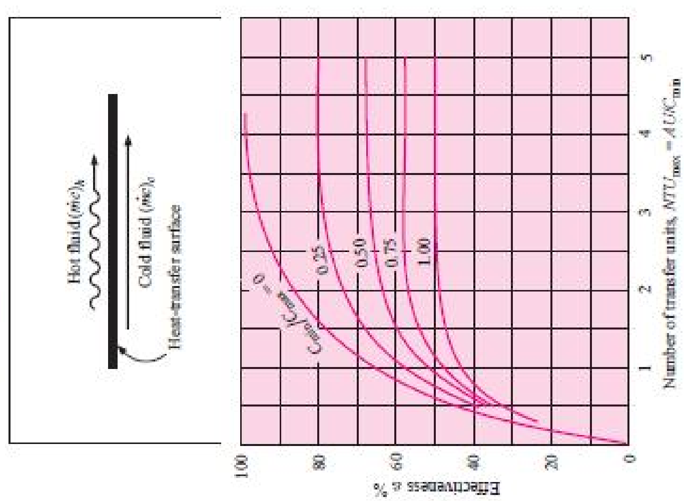


Figure 10-12 | Effectiveness for parallel-flow exchanger performance.

Figure 10-14 | Effectiveness for cross-flow exchanger with one fluid mixed.

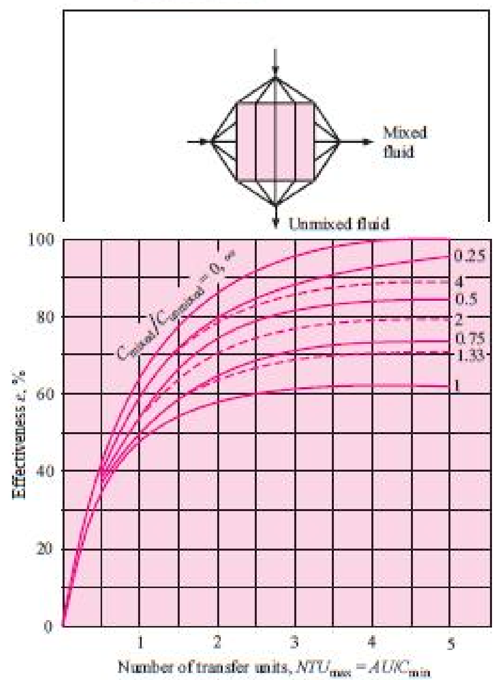


Figure 10-15 | Effectiveness for cross-flow exchanger with fluids unmixed.

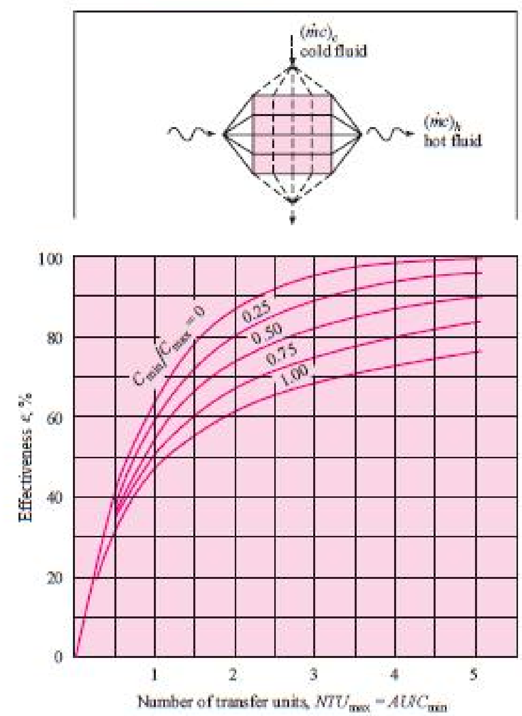
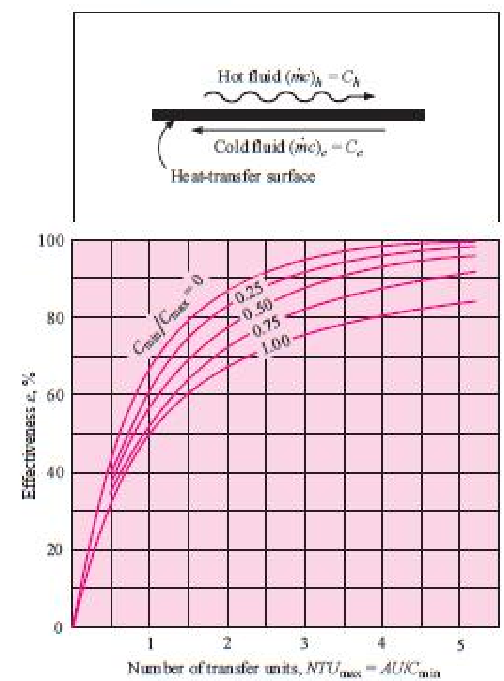


Figure 10-13 | Effectiveness for counterflow exchanger performance.



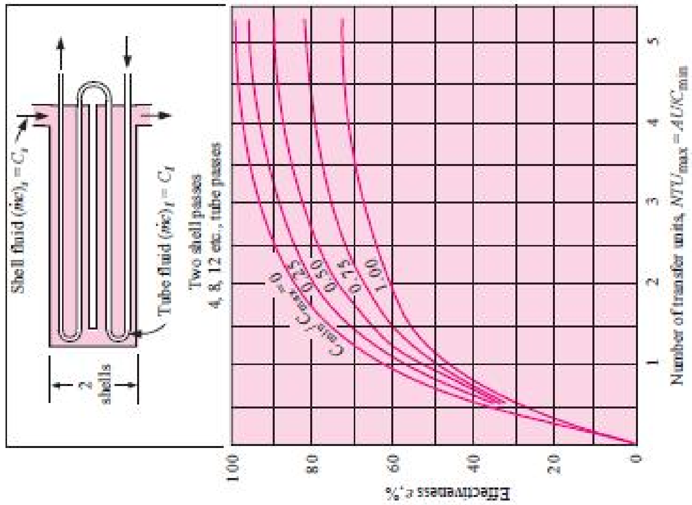


Figure 10-17 | Effectiveness for 2-4 multipass counterflow exchanger performance.

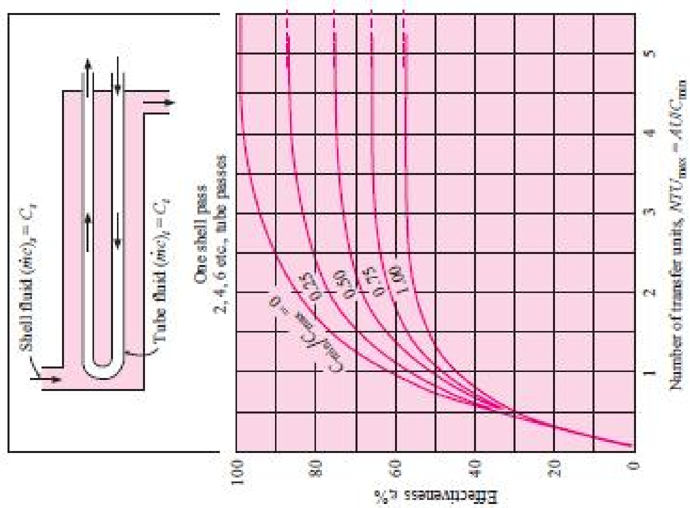


Figure 10-16 | Effectiveness for 1-2 parallel counterflow exchanger performance.

Off-Design Calculation of Exchanger in Example 10-4

EXAMPLE 10-10

The heat exchanger of Example 10-4 is used for heating water as described in the example. Using the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg/min of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under these new conditions.

■ **Solution**

The flow rate of oil is calculated from the energy balance for the original problem:

$$\dot{m}_h c_h \, \Delta T_h = \dot{m}_c c_c \, \Delta T_c \qquad [a]$$

$$\dot{m}_h = \frac{(68)(4180)(75-35)}{(1900)(110-75)} = 170.97 \text{ kg/min}$$

The capacity rates for the new conditions are now calculated as

$$\dot{m}_h c_h = \frac{170.97}{60}(1900) = 5414 \text{ W/°C}$$

$$\dot{m}_c c_c = \frac{40}{60}(4180) = 2787 \text{ W/°C}$$

so that the water (cold fluid) is the minimum fluid, and

$$\frac{C_{\text{min}}}{C_{\text{max}}} = \frac{2787}{5414} = 0.515$$

$$\text{NTU}_{\text{max}} = \frac{UA}{C_{\text{min}}} = \frac{(320)(15.82)}{2787} = 1.816 \qquad [b]$$

where the area of 15.82 m^2 is taken from Example 10-4. From Figure 10-13 or Table 10-3 the effectiveness is

$$\epsilon = 0.744$$

and because the cold fluid is the minimum, we can write

$$\epsilon = \frac{\Delta T_{\text{cold}}}{\Delta T_{\text{max}}} = \frac{\Delta T_{\text{cold}}}{110-35} = 0.744 \qquad [c]$$

$$\Delta T_{\text{cold}} = 55.8\text{°C}$$

and the exit water temperature is

$$T_{w,\text{exit}} = 35 + 55.8 = 90.8\text{°C}$$

The total heat transfer under the new flow conditions is calculated as

$$q = \dot{m}_c c_c \Delta T_c = \frac{40}{60}(4180)(55.8) = 155.5 \text{ kW} \quad [5.29 \times 10^5 \text{ Btu/h}] \qquad [d]$$

Notice that although the flow rate has been reduced by 41 percent (68 to 40 kg/min), the heat transfer is reduced by only 18 percent (189.5 to 155.5 kW) because the exchanger is more effective at the lower flow rate.

$$T_{w,\text{exit}} = 35 + 55.8 = 90.8\text{°C}$$

The total heat transfer under the new flow conditions is calculated as

$$q = \dot{m}_c c_c \Delta T_c = \frac{40}{60}(4180)(55.8) = 155.5 \text{ kW} \quad [5.29 \times 10^5 \text{ Btu/h}] \qquad [d]$$

Notice that although the flow rate has been reduced by 41 percent (68 to 40 kg/min), the heat transfer is reduced by only 18 percent (189.5 to 155.5 kW) because the exchanger is more effective at the lower flow rate.

EXAMPLE 10-13 Shell-and-Tube Exchanger as Air Heater

Hot oil at 100°C is used to heat air in a shell-and-tube heat exchanger. The oil makes six tube passes and the air makes one shell pass; 2.0 kg/s of air are to be heated from 20 to 80°C. The specific heat of the oil is 2100 J/kg · °C, and its flow rate is 3.0 kg/s. Calculate the area required for the heat exchanger for $U = 200\ \mathrm{W/m^2 \cdot {}^\circ C}$.

Solution

The basic energy balance is

$$\dot{m}_o c_o \,\Delta T_o = \dot{m}_a c_{pa}\,\Delta T_a$$

or

$$(3.0)(2100)(100 - T_{oe}) = (2.0)(1009)(80 - 20)$$
$$T_{oe} = 80.78^\circ\mathrm{C}$$

We have

$$\dot{m}_h c_h = (3.0)(2100) = 6300\ \mathrm{W/^\circ C}$$
$$\dot{m}_c c_c = (2.0)(1009) = 2018\ \mathrm{W/^\circ C}$$

so the air is the minimum fluid and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{2018}{6300} = 0.3203$$

The effectiveness is

$$\epsilon = \frac{\Delta T_c}{\Delta T_{\max}} = \frac{80 - 20}{100 - 20} = 0.75$$

Now, we may use either Figure 10-16 or the analytical relation from Table 10-4 to obtain NTU. For this problem we choose to use the table.

$$N = -(1 + 0.3203^2)^{-1/2} \ln\left[\frac{2/0.75 - 1 - 0.3203 - (1 + 0.3203^2)^{1/2}}{2/0.75 - 1 - 0.3203 + (1 + 0.3203^2)^{1/2}}\right]$$
$$= 1.99$$

Now, with $U = 200$ we calculate the area as

$$A = \mathrm{NTU}\frac{C_{\min}}{U} = \frac{(1.99)(2018)}{200} = 20.09\ \mathrm{m^2}$$

Example 10-14

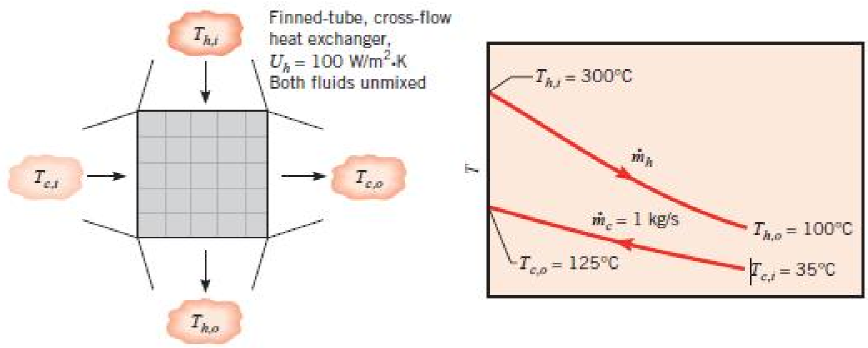
Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at 300°C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125°C. The overall heat transfer coefficient based on the gas-side surface area is $U_h = 100\ \mathrm{W/m^2 \cdot K}$. Determine the required gas-side surface area A_h using the NTU method.

Solution

Known: Inlet and outlet temperatures of hot gases and water used in a finned-tube, cross-flow heat exchanger. Water flow rate and gas-side overall heat transfer coefficient.

Find: Required gas-side surface area.

Schematic:



Assumptions:

1. Negligible heat loss to the surroundings and kinetic and potential energy changes.
2. Constant properties.

Properties: Table A.6, water ($\bar{T}_c = 80°\mathrm{C}$): $c_{p,c} = 4197\ \mathrm{J/kg \cdot K}$.

Analysis: The required surface area may be obtained from knowledge of the number of transfer units, which, in turn, may be obtained from knowledge of the ratio of heat capacity rates and the effectiveness. To determine the minimum heat capacity rate, we begin by computing

$$C_c = \dot{m}_c c_{p,c} = 1\ \mathrm{kg/s} \times 4197\ \mathrm{J/kg \cdot K} = 4197\ \mathrm{W/K}$$

Since $\dot{m}_h$ is not specified, C_h is obtained by combining the overall energy balances, Equations 11.6b and 11.7b:

$$C_h = \dot{m}_h c_{p,h} = C_c \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}} = 4197 \frac{125 - 35}{300 - 100} = 1889\ \mathrm{W/K} = C_{\min}$$

From Equation 11.18

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) = 1889\ \mathrm{W/K}\ (300 - 35)°\mathrm{C} = 5.00 \times 10^5\ \mathrm{W}$$

From Equation 11.7b the actual heat transfer rate is

$$q = C_c(T_{c,o} - T_{c,i}) = 4197 \text{ W/K } (125 - 35)^\circ\text{C}$$

$$q = 3.78 \times 10^5 \text{ W}$$

Hence from Equation 11.19 the effectiveness is

$$\varepsilon = \frac{q}{q_{\max}} = \frac{3.78 \times 10^5 \text{ W}}{5.00 \times 10^5 \text{ W}} = 0.755$$

With

$$\frac{C_{\min}}{C_{\max}} = \frac{1889}{4197} = 0.45$$

it follows from Figure 11.14 that

$$\text{NTU} = \frac{U_h A_h}{C_{\min}} \approx 2.0$$

or

$$A_h = \frac{2.0(1889 \text{ W/K})}{100 \text{ W/m}^2 \cdot \text{K}} = 37.8 \text{ m}^2$$

Example 10 -15

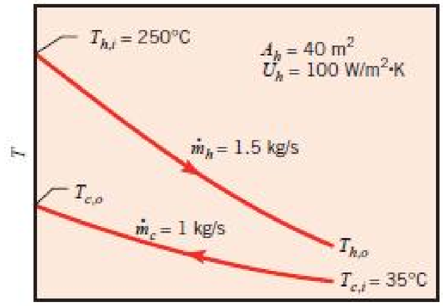
Consider the heat exchanger design of Example 10.14, that is, a finned-tube, cross-flow heat exchanger with a gas-side overall heat transfer coefficient and area of 100 W/m^2 . K and 40 m^2, respectively. The water flow rate and inlet temperature remain at 1 kg/s and 35˚C. However, a change in operating conditions for the hot gas generator causes the gases to now enter the heat exchanger with a flow rate of 1.5 kg/s and a temperature of 250˚C. What is the rate of heat transfer by the exchanger, and what are the gas and water outlet temperatures?

SOLUTION

Known: Hot and cold fluid inlet conditions for a finned-tube, cross-flow heat exchanger
of known surface area and overall heat transfer coefficient.

Find: Heat transfer rate and fluid outlet temperatures

Schematic:



Assumptions:

1. Negligible heat loss to surroundings and kinetic and potential energy changes.
2. Constant properties (unchanged from Example 11.3).

Analysis: The problem may be classified as one requiring a heat exchanger *performance calculation*. The heat capacity rates are

$$C_c = \dot{m}_c c_{p,c} = 1 \text{ kg/s} \times 4197 \text{ J/kg}\cdot\text{K} = 4197 \text{ W/K}$$

$$C_h = \dot{m}_h c_{p,h} = 1.5 \text{ kg/s} \times 1000 \text{ J/kg}\cdot\text{K} = 1500 \text{ W/K} = C_{\min}$$

in which case

$$\frac{C_{\min}}{C_{\max}} = \frac{1500}{4197} = 0.357$$

The number of transfer units is

$$\text{NTU} = \frac{U_h A_h}{C_{\min}} = \frac{100 \text{ W/m}^2\cdot\text{K} \times 40 \text{ m}^2}{1500 \text{ W/K}} = 2.67$$

From Figure 11.14 the heat exchanger effectiveness is then $\varepsilon \approx 0.82$, and from Equation 11.18 the maximum possible heat transfer rate is

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) = 1500 \text{ W/K } (250 - 35)°\text{C} = 3.23 \times 10^5 \text{ W}$$

Accordingly, from the definition of ε, Equation 11.19, the actual heat transfer rate is

$$q = \varepsilon q_{\max} = 0.82 \times 3.23 \times 10^5 \text{ W} = 2.65 \times 10^5 \text{ W} \qquad \triangleleft$$

It is now a simple matter to determine the outlet temperatures from the overall energy balances. From Equation 11.6b

$$T_{h,o} = T_{h,i} - \frac{q}{\dot{m}_h c_{p,h}} = 250°\text{C} - \frac{2.65 \times 10^5 \text{ W}}{1500 \text{ W/K}} = 73.3°\text{C} \qquad \triangleleft$$

and from Equation 11.7b

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 35°\text{C} + \frac{2.65 \times 10^5 \text{ W}}{4197 \text{ W/K}} = 98.1°\text{C} \qquad \triangleleft$$