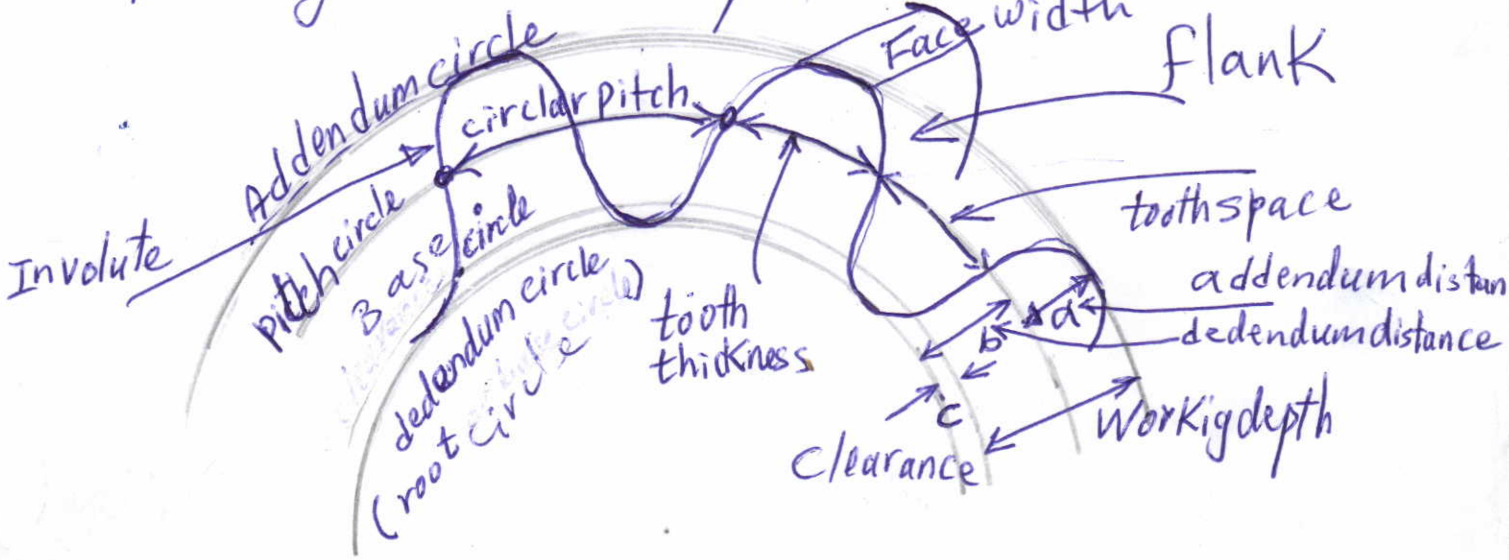
Gear Theory

The following are some important terms



· pitch circle: The circle on a gear (wheel) that corresponds to contact surface of the gears in contact rolls on each other.

Addendum circle: The circle drawn through the top of gear tooth surface.

Addendum distance (a): The radial distance from pitch circle to addendum circle.

Dedendum circle: The circle drawn through the bottom of the gear tooth (root circle).

Dedendum distance (b): the radial distance from pitch circle to dedendum circle.

clearance circle: The smallest circle centred at the gear centro, which is not penetrated by the teeth of the mating gear.

clearance distance (c): The radial distance from clearance circle to root circle, or the differance between the dedendum of one gear and the addendum of the mating gear

$$\boxed{c = b - a}$$

Working depth (h): The radial distance between the addendum circle and clearance circle, or the sum of addend (a) of two mating gears

Circular pitch (P_c): The circular (circumference distance) distance measured along the pitch circle from a point on one tooth to correspond point on the adjacent tooth of the gear

$$\boxed{P_c = \frac{\pi D}{T} \text{ or } \frac{2\pi r}{T}}$$

(d) ≡ D ≡ pitch diameter, T ≡ number of teeth
(r) ≡ R ≡ raduis of pitch circle.

(The circumferance of circle divided by number of teeth)

Diametral pitch (P_d): The diametral pitch is equal to the number of teeth (T) divided by the diameter of the pitch circle

$$\boxed{P_d = \frac{T}{d}}$$

The circular and diameteral pitch are extremely important in gear analysis. The pitch is an indication of the spacing and sizes of the gear teeth, additionaly in order for two gears to mesh perfectly so P_c of the two must be equal.

Relation between P_c & P_d

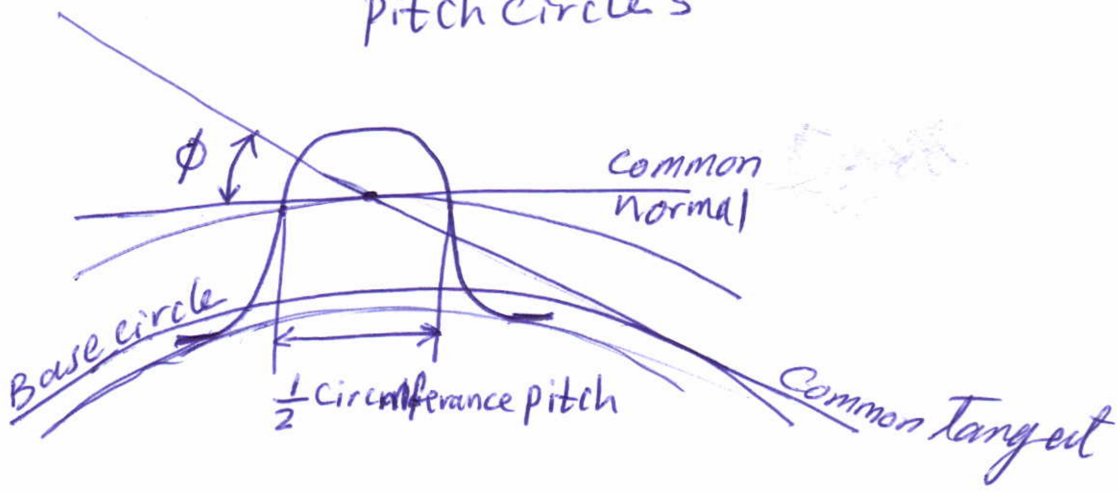
$$\boxed{P_c = \frac{\pi}{P_d}}$$

i.e product always equal to constant value. (π)
($P_c * P_d = \pi$).

Module (m): The diameter of pitch circle divided by number of teeth. It is used to determine the size of gear.

$\boxed{m = \frac{D}{T}}$; $\boxed{P_c = \pi m}$; $\boxed{m = \frac{1}{P_d}}$

Pressure angle (ϕ): The angle between the common normal to two teeth in contact and the common tangent to the pitch circles
(obliquity angle)



Standard proportions: The proportions recommended by British standards institution in B.S 436–1940 are

$$\text{addendum } (a) = \frac{1}{P_d} = m$$

$$\text{dedendum } (b) = \frac{1.25}{P_d} = 1.25m$$

$$\text{Working depth } (h) = \frac{2}{P_d} = 2m$$

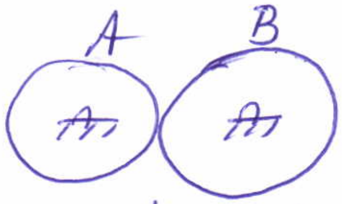
$$\text{pressure angle } (\phi) = 20^\circ$$

Gear Trains

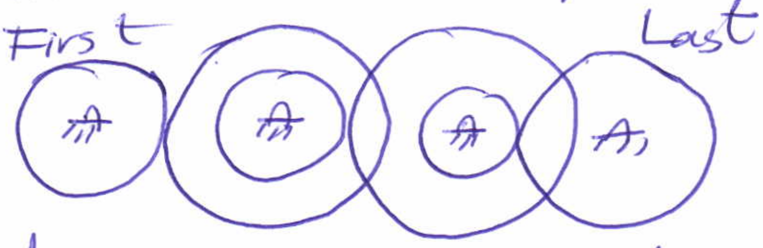
Any compination of gear wheels by means of which motion is transmitted from one shaft to another shaft is called a gear train

Types of Train:

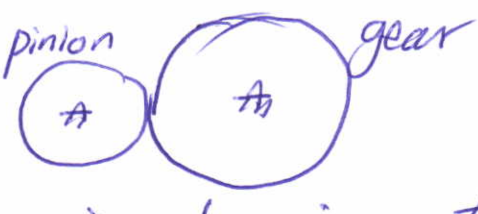
(1) Simple Train: A train which each shaft carries one gear only.



(2) Compound Train: A train in which each shaft except the first and last, have two gears fastened (meshed) together on the same shaft to operate as an integral part



Pinion and gear: with respect to two mating gears, the Smaller is called pinion, the larger is called gear

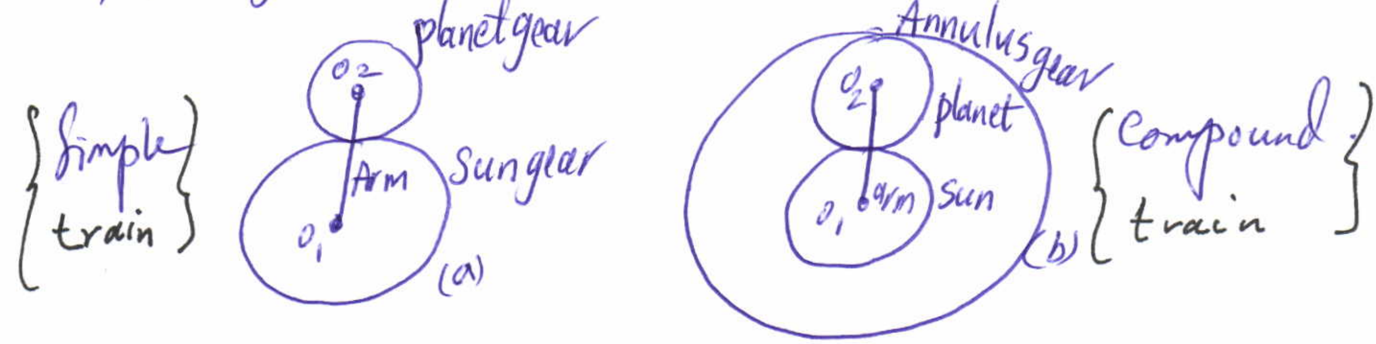


Idlear gear: A gear in a train, other than the first and last gear, that is mounted on a shaft by itself. It act both as driver and follower. It does not affect the speed ratio (train value) of the train, it serves only to fill up space and reverse direction of motion

(3) Epicyclic gear Train:

(a) simple (b) compound (c) Complex

It may be described as a train in which one gear is fixed (the sun gear) and other gear (planet gear) carried by revolving arm (carrier), rotate not only about their own centre but their centers rotate about fixed gear.



Transmission or Gear Box: A gear train that has several speed (velocity) ratios that can be selected by moving one or more of the gears by means of a shifting lever.

The important purposes of gear trains:

1- transmit motion from one point to another point
2- modifying motion by changing its speed or direction
3. obtaining required speed ratio in a limited space.

<u>Velocity (speed) ratio (VR)</u>: it is the ratio of speed of the output to that of input (simple train, compound train or complex train)

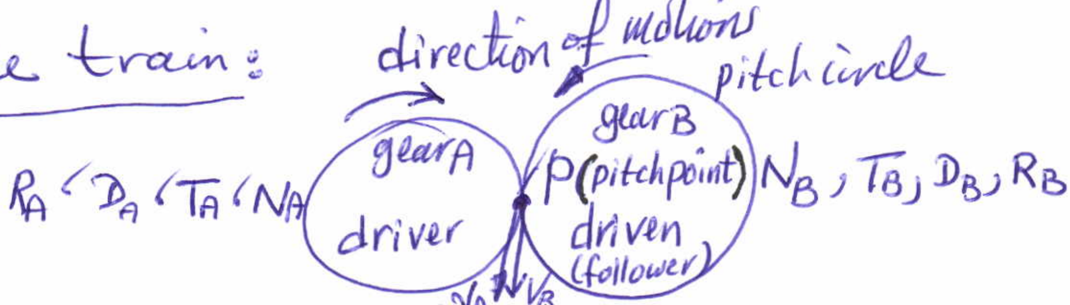
$$V.R = \frac{N_{out}}{N_{in}} \quad or \quad = \frac{\omega_{out}}{\omega_{in}}$$

$N \equiv$ speed in RPM
$\omega \equiv$ " " rad/sec
$(\omega = \frac{2\pi}{60} * N)$

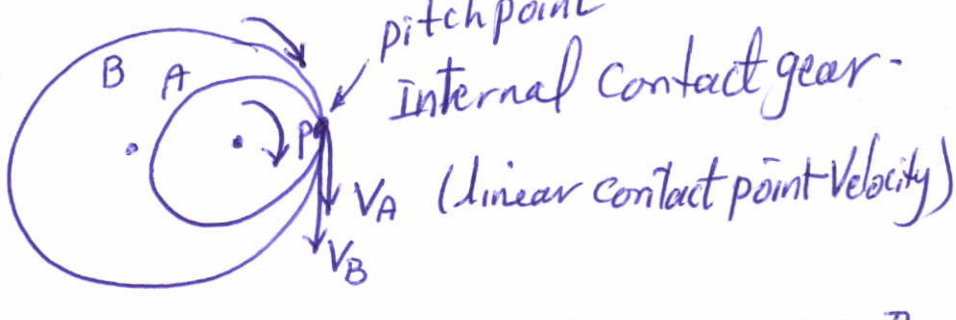
If outputspeed (follower speed) is rotating in same direction as the inputspeed (driverspeed) their sign (+ve), other wise is (−ve)

V.R can be expressed in term of pitch circle diameter (or radius) and Number of teeth on the two gears

<u>Simple train:</u>



External contact gears



Since $V = R\omega$ (OR) $V = \frac{2\pi}{60} * N * R$ (OR) $V = \frac{2\pi}{60} N * \frac{D}{2}$

pitch point is common contact point on pitch circle of the two mating gears, which have same velocity, also in order to have perfect mesh circular pitch or module must be equal

$$V_A = V_B$$
$$R_A \omega_A = R_B \omega_B$$
$$\frac{\omega_B}{\omega_A} = \frac{R_A}{R_B}$$

$$V_A = V_B$$
$$\frac{2\pi}{60} N_A * \frac{D_A}{2} = \frac{2\pi}{60} N_B * \frac{D_B}{2}$$
$$\frac{N_B}{N_A} = \frac{D_A}{D_B}$$

Since $P_{cA} = P_{cB}$ (OR) $m_A = m_B$

So $\frac{\pi D_A}{T_A} = \frac{\pi D_B}{T_B}$ (OR) $\frac{D_A}{T_A} = \frac{D_B}{T_B}$

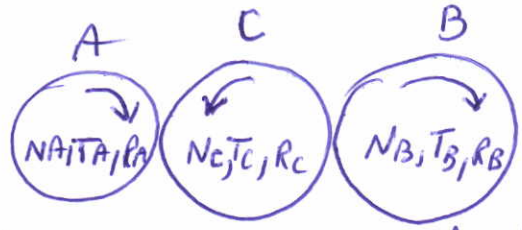
$\therefore \frac{D_A}{D_B} = \frac{T_A}{T_B}$

from which the speed ratio can be expressed between any two meshing gears in term of diameter, number of teeth

$$\frac{N_B}{N_A} = \pm \frac{D_A}{D_B} = \pm \frac{T_A}{T_B}$$

(+ ve same direction
– ve opposite ")

Compound train speed ratio:



$$\frac{N_B}{N_A} = \pm \frac{N_C}{N_A} * \frac{N_B}{N_C}$$

+ve if in-out same direction
–ve " " , opposite direction

$$\frac{N_B}{N_A} = \pm \frac{T_A}{T_C} * \frac{T_C}{T_B}$$

$$\frac{N_B}{N_A} = \pm \frac{D_A}{D_C} * \frac{D_C}{D_B}$$

In general the relation of speed ratio can be:

$$VR = \pm \frac{\text{product of speeds of follower}}{\text{product of speeds of driver}}$$

$$= \pm \frac{\text{product of number of teeth of driver}}{\text{product of number of teeth of follower}}$$

$$= \pm \frac{\text{product of diameter (radius) of driver}}{\text{product of diameter (radius) of follower}}$$

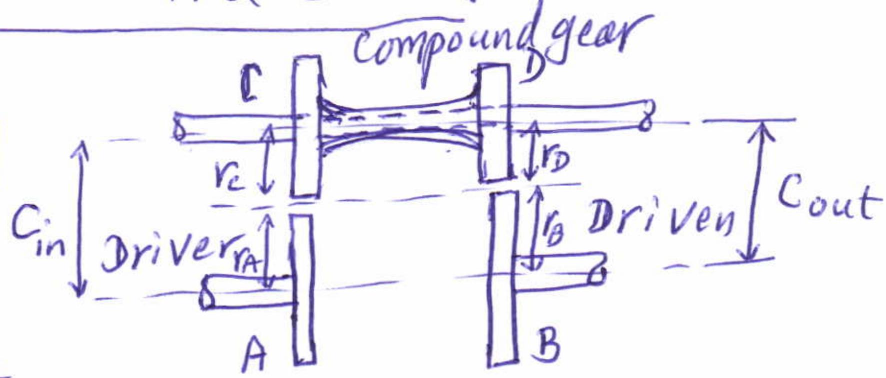
In a reverted or co-axial train:

$C_{in} = C_{out}$

$\therefore \boxed{r_C + r_A = r_D + r_B \quad \text{— (1)}}$

$P_{CA} = P_{CC}$, $P_{CD} = P_{CB}$

$m_A = m_C$, $m_D = m_B$



also equation (1) can be

$$\frac{m_A T_A}{2} + \frac{m_C T_C}{2} = \frac{m_D T_D}{2} + \frac{m_B T_B}{2}$$

OR $\boxed{T_A + T_C = T_D + T_B}$

* In compound gear (C−D) the speed of gear C&D must be equal common shaft, but not necessary number of gear teeth equal.

speed Ratio in reverted train: $\frac{V_B}{V_A}$ or $\frac{N_B}{N_A}$

$$\frac{N_C}{N_A} = \frac{-T_A}{T_C} = \frac{-D_A}{D_C} \quad ; \quad N_D = N_C \text{ compound gear}$$

$$\frac{N_B}{N_D} = -\frac{T_D}{T_B} = \frac{-D_D}{D_B} = \frac{N_B}{N_C}$$

$$\therefore \frac{N_B}{N_A} = \frac{T_A * T_D}{T_C * T_B} \quad ; \quad \frac{N_B}{N_A} = \frac{-N_C}{N_A} * \frac{-N_B}{N_D}$$

Epicyclic gear Train: To determine speed of gears and speed ratio the tabulation method is used.

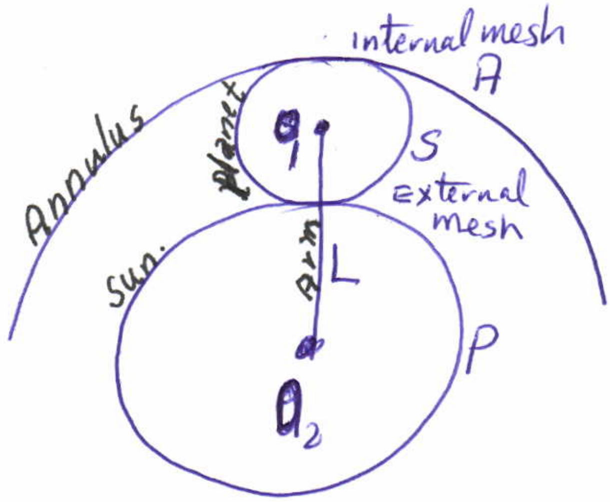
This method and procedure can be applied if any gear in train fixed or all gears rotates (revolve)

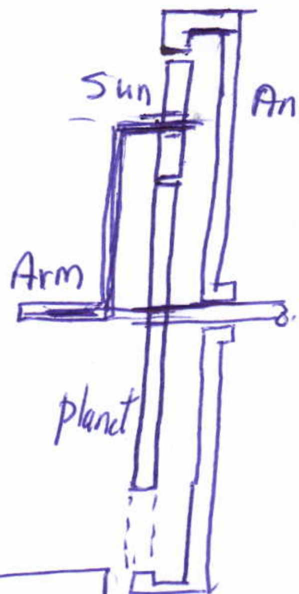
step(1) Fix (lock) all gears and arm (carrier) and give (+a) number of revolution in c.c.w direction

step(2) Fix arm (i.e give zero revolution) and give any gear in train (+b) revolution in c.c.w direction ((must be as step(1))). and determine speed ratio of each gear

step(3) sum motion, (i.e sum step(1) + step(2))

Exple

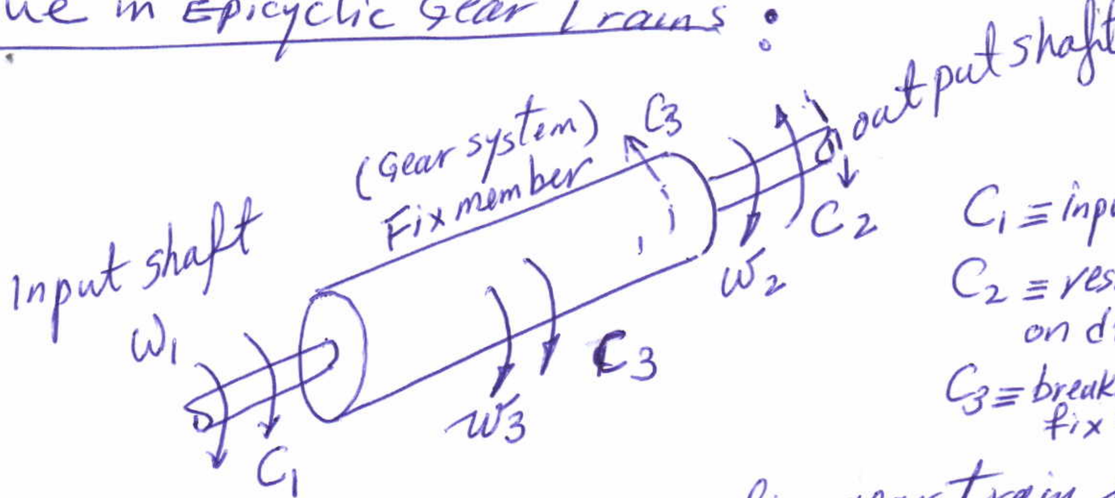




step	Conditions	speed of gear (wheels) N: Arm(L)	gear A	gear S	gear P
1	Fix all give +a c.c.w rev	a	a	a	a
2	Fix arm, give gear S +b rev c.c.w	0	$+b\frac{T_A}{T_S}$	b	$-b\frac{T_P}{T_S}$
3	Sum (1+2) Total motion	a	$a+b\frac{T_A}{T_S}$	$a+b$	$a-b\frac{T_P}{T_S}$

Now any condition given can be applied to determine any speed or ratios between gears.

Torque in Epicyclic Gear Trains :



$C_1 \equiv$ input torque
$C_2 \equiv$ resisting torque on driven shaft
$C_3 \equiv$ breaking torque on fix member.

When rotating parts of an epicyclic gear train have no angular acceleration, the gear is kept in equilibrium by the three externally applied torque, so The net torque applied to train must be zero

r = radius
$F \equiv$ force

$$C_1 + C_2 + C_3 = 0 \quad \text{———} \quad (1)$$

$$\therefore \quad F_1 * r_1 + F_2 * r_2 + F_3 * r_3 = 0 \quad \text{———} \quad (2)$$

Further if ω_1, ω_2 and ω_3 are angular speeds, and friction in train be neglected, so the Kinetic energy dissipated by the train must be zero, so

$$C_1 \omega_1 + C_2 \omega_2 + C_3 \omega_3 = 0 \quad \text{———} \quad (3)$$

But for fix member $\omega_3 = 0$,

$$\therefore \quad C_1 \omega_1 + C_2 \omega_2 = 0 \quad \text{———} \quad (4)$$

$\therefore$ If allowing for friction, the efficiency of the unit is η then equation (4) become

$$\eta = \frac{\text{output power}}{\text{input power}}$$

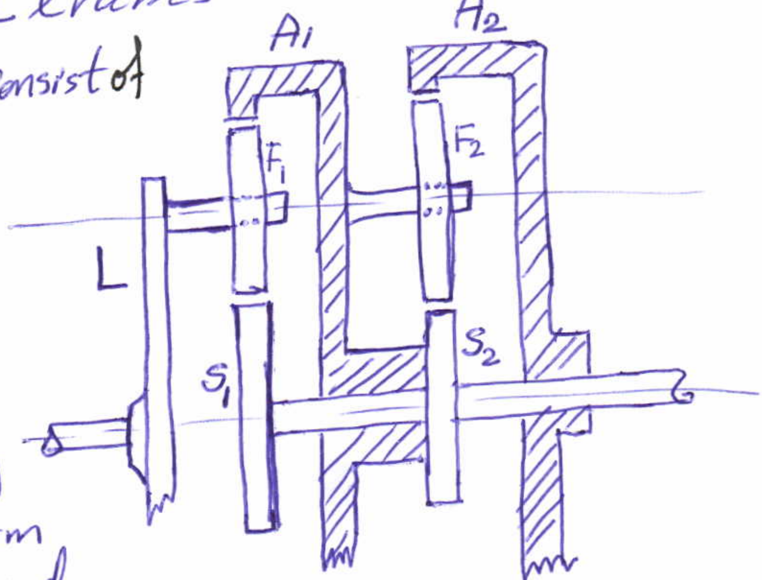
so $\quad \eta C_1 \omega_1 + \omega_2 C_2 = 0 \quad \text{———} \quad (5)$

** Direction of rotation and applied torque must be noticed, to take the correct sign.

Compound epicyclic gear trains :

A compound train consist of two or more co-axial simple epicyclic trains with members forming part of two consecutive trains -

Compound train consist of two epicyclic $L S_1 A_1$ and $A_1 S_2 A_2$ where L is the arm in first and A_1 (Annulus) of epicyclic (1) form the arm of second.



Let A_2 (Annulus) be fixed

So will first obtain the ratio of arm (L) to S_1 when A_1 fixed.

conditions	Speed of wheels (gears)		
	Arm(L)	A_1	Sun S_1
1 Fix all give +1 rev c.c.w	+1	+1	+1
2 Fix arm, give A_1 (-1) rev ccw	0	-1	$+1 * \frac{T_{A_1}}{T_{S_1}}$
3 total motion	+1	0	$1 + \frac{T_{A_1}}{T_{S_1}}$

$\frac{N_{S_1}}{N_{A_1}} = \frac{-T_{A_1} * \cancel{T_{F_1}}}{\cancel{T_{F_1}} * T_{S_1}}$

since $N_{A_1} = -1$

$\therefore N_{S_1} = 1 * \frac{T_{A_1}}{T_{S_1}}$

$\therefore \frac{N_L}{N_{S_1}} = \frac{1}{1 + T_{A_1}/T_{S_1}}$

when A_1 is fixed

Then now consider the whole train with A_1 as arm

Conditions	Speed of gears			
	Arm(L)	A_1	$S_1 S_2$	A_2
1 give all +1	+1	+1	+1	+1
2 Fixed A_1 give -1 to A_2	$\frac{T_{A_2}/T_{S_2}}{1 + T_{A_1}/T_{S_1}}$	0	$1 * \frac{T_{A_2}}{T_{S_2}}$	-1
3 Sum motion	$1 + \frac{T_{A_2}/T_{S_2}}{1 + T_{A_1}/T_{S_1}}$	1	$1 + \frac{T_{A_2}}{T_{S_2}}$	0

$\frac{N_{S_1 S_2}}{N_{A_2}} = \frac{-T_{A_2} * T_{F_2}}{T_{S_2} * T_{F_2}}$

$N_{A_2} = -1$

$\therefore N_{S_1 S_2} = \frac{1 * T_{A_2}}{T_{S_2}}$

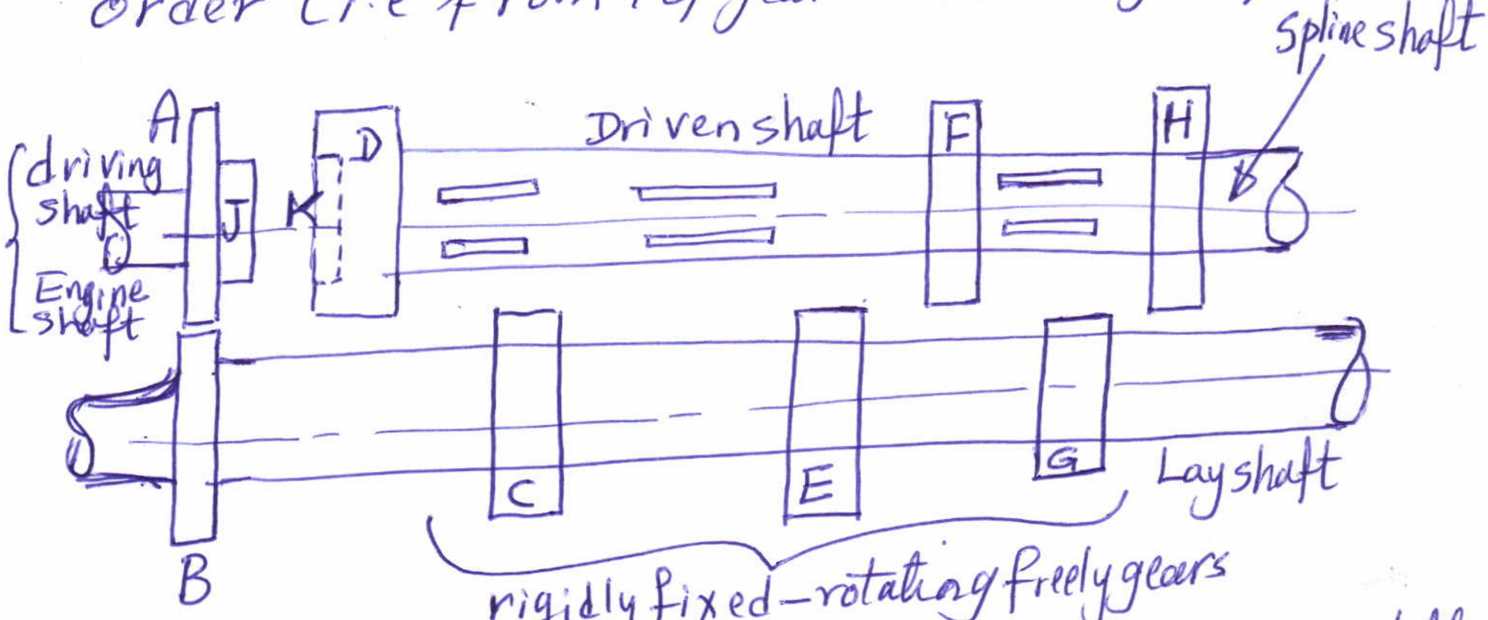
$\because N_{S_1} = N_{S_2} = N_{S_1 S_2}$

So $\frac{N_L}{N_{S_1 S_2}} = \frac{N_L}{N_{S_1}} = \frac{N_L}{N_{S_2}}$

Thus $\frac{N_L}{N_{S_1}} = \left(1 + \frac{(T_{A_2}/T_{S_1})}{1 + T_{A_1}/T_{S_1}}\right) / \left(1 + T_{A_2}/T_{S_2}\right)$

Sliding Gear Box for Automobile :

Four speed sliding gearbox shown, it is in desending order (i.e from Top gear ⟶ First gear)



gear (A) is pinion and in constant mesh with gear (B) on lay shaft (J and K) for as coupling element, gears (D, F, H) can slide on spline shaft.

Top (high) speed gear : J mate with K by sliding (D) on driven shaft where now speed of Driven shaft equal engine speed

$\boxed{N_D = N_A}$ - so Auto in top gear.

Third gear : Slide gear D to right to mesh with gear C so motion from A to B then from C to D, (E, G) Free

$$\boxed{V.R = \frac{T_A}{T_B} \times \frac{T_C}{T_D}}$$

second gear : Slide F to left mesh with E, (C and G) Free

so motion

$$\boxed{V.R = \frac{T_A}{T_B} \times \frac{T_E}{T_F}}$$

First gear : Slide H to left to mesh with G, (F, D) Free

so motion

$$\boxed{V.R = \frac{T_A}{T_B} \times \frac{T_G}{T_H}}$$